First Report of The Project

Problem: Dam Control

Problem Description: *The Clear Lake Dam controls the water level in Clear Lake, a well-known resort in Dreamland. The Dam Commission is trying to decide how much water to release in each of the next four months. The lake is currently 150 mm below flood stage. The dam is capable of lowering the water level 200 mm each month, but additional precipitation and evaporation affect the dam. The weather near Clear Lake is highly variable. The Dam Commission has divided the months into two two-month blocks of similar weather. The months within each block have the same probabilities for weather, which are assumed independent of one another. In each month of the first block, they assign a probability of ½ to having a natural 100-mm increase in water levels, and probabilities of ¼ to having a 50-mm decrease or a 250-mm increase in water levels. All these figures correspond to natural changes in water level without dam releases. In each month of the second block, they assign a probability of ½ to having a natural 150-mm increase in water levels, and probabilities of ¼ to having a 50-mm increase or a 350-mm increase in water levels. If a flood occurs, then dammage is assessed at 10,000 MU (monetary units) per mm above flood level. A wate level too low leads to costly importation of water. These costs are 5000 MU per mm less than 250 mm below flood stage. The commission first considers an overall goal of minimizing expected costs. They also consider minimizing the probability of violating the maximum and minimum water levels. Consider both objectives and formulate this problem.*

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Notations

x(t,s): level of water in the dam at the end of period t for scenario s

y(t,s) mm of water released normally at the end of the period t for scenario s

z(t,s): mm of imported water at the end of period t for scenario s

fw(t,s): mm of flood water at the end of period t for scenario s

t: Time periods

s: Scenarios

alfa(t,s) and beta(t,s) are binary variables

alfa: If there is flood, alfa is 1. Otherwise 0. (max 5mm of flood can be neglected.)

beta: If we import water, beta is 1. Otherwise 0. (max 5mm of water importation can be neglected.)

t = {1, 2, 3, 4} 1 is for January, 2 is for February, 3 is for March, 4 is for April

p: Precipitation levels for each month

p = {low, medium, high}

Reservoir level change data for each season:

t p	Low	Medium	High
Jan	-50	100	250
Feb	-50	100	250
Mar	50	150	350
Apr	50	150	350

P(x): Probability of x

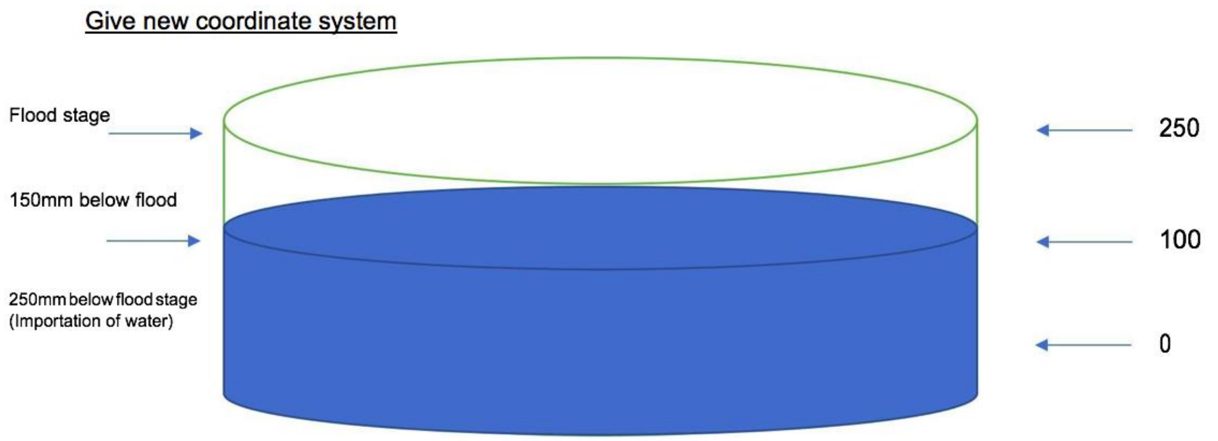
P(low) = %25

P(medium) = %50

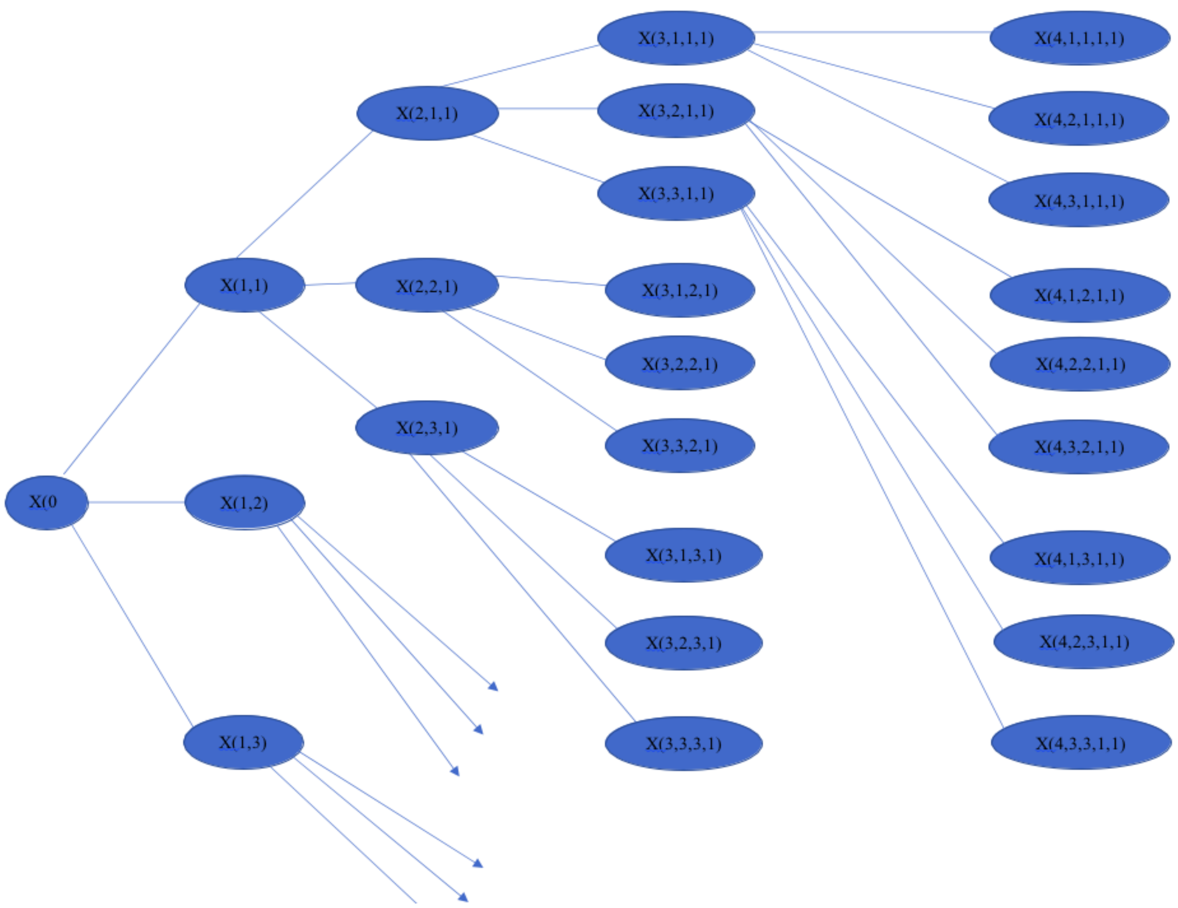
P(high) = %25

Assumptions

In order to make understanding and programming easier, we have developed a new coordinate system as shown below (First stage):



Mathematical Model Equilibrium constraints



x(t,s)= x(t-1,s) + scenario(t,s) – y(t,s)+ z(t,s) – fw(t,s)

The water levels at the end of period 1

$x(1,1) = x(0) + (-50) - y(1,1) + z(1,1) - fw(1,1)$
$x(1,2) = x(0) + (100) - y(1,2) + z(1,2) - fw(1,2)$
$x(1,3) = x(0) + (250) - y(1,3) + z(1,3) - fw(1,3)$

The water levels at the end of period 2

$x(2,1,1) = x(1,1) + (-50) - y(2,1,1) + z(2,1,1) - fw(2,1,1)$
$x(2,2,1) = x(1,1) + (100) - y(2,2,1) + z(2,2,1) - fw(2,2,1)$
$x(2,3,1) = x(1,1) + (250) - y(2,3,1) + z(2,3,1) - fw(2,3,1)$

$x(2,1,2) = x(1,2) + (-50) - y(2,1,2) + z(2,1,2) - fw(2,1,2)$
$x(2,2,2) = x(1,2) + (100) - y(2,2,2) + z(2,2,2) - fw(2,2,2)$
$x(2,3,2) = x(1,2) + (250) - y(2,3,2) + z(2,3,2) - fw(2,3,2)$

$x(2,1,3) = x(1,3) + (-50) - y(2,1,3) + z(2,1,3) - fw(2,1,3)$
$x(2,2,3) = x(1,3) + (100) - y(2,2,3) + z(2,2,3) - fw(2,2,3)$
$x(2,3,3) = x(1,3) + (250) - y(2,3,3) + z(2,3,3) - fw(2,3,3)$

The water levels at the end of period 3

$x(3,1,1,1)= x(2,1,1) + (50) - y(3,1,1,1) + z(3,1,1,1) - fw(3,1,1,1)$
$x(3,2,1,1)= x(2,1,1) + (150) - y(3,2,1,1) + z(3,2,1,1) - fw(3,2,1,1)$
$x(3,3,1,1)= x(2,1,1) + (350) - y(3,3,1,1) + z(3,3,1,1) - fw(3,3,1,1)$

$x(3,1,2,1)= x(2,2,1) + (50) - y(3,1,2,1) + z(3,1,2,1) - fw(3,1,2,1)$
$x(3,2,2,1)= x(2,2,1) + (150) - y(3,2,2,1) + z(3,2,2,1) - fw(3,2,2,1)$
$x(3,3,2,1)= x(2,2,1) + (350) - y(3,3,2,1) + z(3,3,2,1) - fw(3,3,2,1)$

$x(3,1,3,1)= x(2,3,1) + (50) - y(3,1,3,1) + z(3,1,3,1) - fw(3,1,3,1)$
$x(3,2,3,1)= x(2,3,1) + (150) - y(3,2,3,1) + z(3,2,3,1) - fw(3,2,3,1)$
$x(3,3,3,1)= x(2,3,1) + (350) - y(3,3,3,1) + z(3,3,3,1) - fw(3,3,3,1)$

$x(3,1,1,2)= x(2,1,2) + (50) - y(3,1,1,2) + z(3,1,1,2) - fw(3,1,1,2)$
$x(3,2,1,2)= x(2,1,2) + (150) - y(3,2,1,2) + z(3,2,1,2) - fw(3,2,1,2)$
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$x(3,1,3,2)= x(2,3,2) + (50) - y(3,1,3,2) + z(3,1,3,2) - fw(3,1,3,2)$

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$x(3,1,1,3)= x(2,1,3) + (50) - y(3,1,1,3) + z(3,1,1,3) - fw(3,1,1,3)$
$x(3,2,1,3)= x(2,1,3) + (150) - y(3,2,1,3) + z(3,2,1,3) - fw(3,2,1,3)$
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$x(3,1,3,3)= x(2,3,3) + (50) - y(3,1,3,3) + z(3,1,3,3) - fw(3,1,3,3)$
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$x(3,3,3,3)= x(2,3,3) + (350) - y(3,3,3,3) + z(3,3,3,3) - fw(3,3,3,3)$

The water levels at the end of period 4

$x(4,1,1,1,1)= x(3,1,1,1) + (50) - y(4,1,1,1,1) + z(4,1,1,1,1) - fw(4,1,1,1,1)$
$x(4,2,1,1,1)= x(3,1,1,1) + (150) - y(4,2,1,1,1) + z(4,2,1,1,1) - fw(4,2,1,1,1)$
$x(4,3,1,1,1)= x(3,1,1,1) + (350) - y(4,3,1,1,1) + z(4,3,1,1,1) - fw(4,3,1,1,1)$

$x(4,1,2,1,1)= x(3,2,1,1) + (50) - y(4,1,2,1,1) + z(4,1,2,1,1) - fw(4,1,2,1,1)$
$x(4,2,2,1,1)= x(3,2,1,1) + (150) - y(4,2,2,1,1) + z(4,2,2,1,1) - fw(4,2,2,1,1)$
$x(4,3,2,1,1)= x(3,2,1,1) + (350) - y(4,3,2,1,1) + z(4,3,2,1,1) - fw(4,3,2,1,1)$

$x(4,1,3,1,1)= x(3,3,1,1) + (50) - y(4,1,3,1,1) + z(4,1,3,1,1) - fw(4,1,3,1,1)$
$x(4,2,3,1,1)= x(3,3,1,1) + (150) - y(4,2,3,1,1) + z(4,2,3,1,1) - fw(4,2,3,1,1)$
$x(4,3,3,1,1)= x(3,3,1,1) + (350) - y(4,3,3,1,1) + z(4,3,3,1,1) - fw(4,3,3,1,1)$

$x(4,1,1,2,1)= x(3,1,2,1) + (50) - y(4,1,1,2,1) + z(4,1,1,2,1) - fw(4,1,1,2,1)$
$x(4,2,1,2,1)= x(3,1,2,1) + (150) - y(4,2,1,2,1) + z(4,2,1,2,1) - fw(4,2,1,2,1)$
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$x(4,3,2,2,1)= x(3,2,2,1) + (350) - y(4,3,2,2,1) + z(4,3,2,2,1) - fw(4,3,2,2,1)$

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$x(4,2,1,1,2)= x(3,1,1,2) + (150) - y(4,2,1,1,2) + z(4,2,1,1,2) - fw(4,2,1,1,2)$
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$x(4,1,1,1,3)= x(3,1,1,3) + (50) - y(4,1,1,1,3) + z(4,1,1,1,3) - fw(4,1,1,1,3)$
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x(4,1,1,2,3)= x(3,1,2,3) + (50) – y(4,1,1,2,3) + z(4,1,1,2,3) – fw(4,1,1,2,3)
x(4,2,1,2,3)= x(3,1,2,3) + (150) – y(4,2,1,2,3) + z(4,2,1,2,3) – fw(4,2,1,2,3)
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x(4,2,3,2,3)= x(3,3,2,3) + (150) – y(4,2,3,2,3) + z(4,2,3,2,3) – fw(4,2,3,2,3)
x(4,3,3,2,3)= x(3,3,2,3) + (350) – y(4,3,3,2,3) + z(4,3,3,2,3) – fw(4,3,3,2,3)

x(4,1,1,3,3)= x(3,1,3,3) + (50) – y(4,1,1,3,3) + z(4,1,1,3,3) – fw(4,1,1,3,3)
x(4,2,1,3,3)= x(3,1,3,3) + (150) – y(4,2,1,3,3) + z(4,2,1,3,3) – fw(4,2,1,3,3)
x(4,3,1,3,3)= x(3,1,3,3) + (350) – y(4,3,1,3,3) + z(4,3,1,3,3) – fw(4,3,1,3,3)

x(4,1,2,3,3)= x(3,2,3,3) + (50) – y(4,1,2,3,3) + z(4,1,2,3,3) – fw(4,1,2,3,3)
x(4,2,2,3,3)= x(3,2,3,3) + (150) – y(4,2,2,3,3) + z(4,2,2,3,3) – fw(4,2,2,3,3)
x(4,3,2,3,3)= x(3,2,3,3) + (350) – y(4,3,2,3,3) + z(4,3,2,3,3) – fw(4,3,2,3,3)

x(4,1,3,3,3)= x(3,3,3,3) + (50) – y(4,1,3,3,3) + z(4,1,3,3,3) – fw(4,1,3,3,3)
x(4,2,3,3,3)= x(3,3,3,3) + (150) – y(4,2,3,3,3) + z(4,2,3,3,3) – fw(4,2,3,3,3)
x(4,3,3,3,3)= x(3,3,3,3) + (350) – y(4,3,3,3,3) + z(4,3,3,3,3) – fw(4,3,3,3,3)

Binary Constraint for Flood

5 * alfa(t,s) ≤ fw(t,s) ≤ 99999 * alfa(t,s)

Binary Constraint for Importation

5 * beta(t,s) ≤ z(t,s) ≤ 99999 * beta(t,s)

Releasable water Constraint

$y(t,s) \leq 200$

Dam Limit Constraint

$x(t,s) \leq 250$

Non-Negativity Constraints

All variables ≥ 0

Objective Function 1

Min z = 10000 *(fw(1,1) * 0.25 + fw(1,2) * 0.50 + … + fw(4,2,3,3,3) * 0.50 * 0.25 * 0.25 * 0.25 + fw(4,3,3,3,3) * 0.25* 0.25* 0.25* 0.25) + 5000*(z(1,1) * 0.25 + z(1,2) * 0.50 + … + z(4,2,3,3,3) * 0.50 * 0.25 * 0.25 * 0.25 + z(4,3,3,3,3) * 0.25* 0.25* 0.25* 0.25)

or

Min z = (Sum of all fw multiplied by each previous fw's probability) *10000 + (Sum of all z multiplied by each previous z's probability) *5000

Objective Function 2

Min z = (alfa(1,1) * 0.25 + alfa(1,2) * 0.50 + … + alfa(4,2,3,3,3) * 0.50 * 0.25 * 0.25 * 0.25+ alfa(4,3,3,3,3) * 0.25* 0.25* 0.25* 0.25) + (beta(1,1) * 0.25 + beta (1,2) * 0.50 + … + beta (4,2,3,3,3) * 0.50 * 0.25 * 0.25 * 0.25 + beta (4,3,3,3,3) * 0.25* 0.25* 0.25* 0.25)

or

Min z = (Sum of all alfa multiplied by each previous alfa's probability) + (Sum of all beta multiplied by each previous beta's probability)

Note: All equations are for all t and all s.

Note: More information about model can be found in Dam_Control.gms