

Lecture 3: Association Rule Mining

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AGENDA

01	Association	Rules –	A Priori	algorithm
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02 R Exercise

- According to the existence of target (Y) variable
 - √ Supervised learning vs. Unsupervised learning

Supervised Learning

A given dataset X & Y

	Var. 1	Var. 2	 Var. d		Υ
Ins. 1			 		
Ins. 2			 	y = f(x)	
Ins. N			 		

Unsupervised Learning

A given dataset X Var. 1 Var. 2 Var. d								
lns. 1								
Ins. 2								
Ins. N								

Semi-supervised Learning

A given dataset X & Y

	Var. 1	Var. 2	 Var. d		Υ
Ins. 1			 		
Ins. 2			 	y = f(x)	
Ins. N			 		
•••			 		
Ins. M			 		

Unsupervised Learning

$$\mathcal{X} = \{\mathbf{x}_i | i = 1, ..., n, \ \mathbf{x}_i \in \mathbb{R}^d\}$$

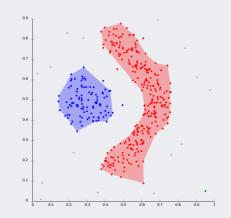
Unsupervised learning

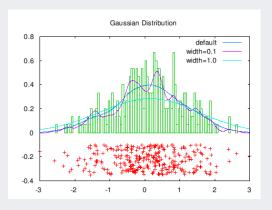
- Explores intrinsic characteristics.
- Estimates underlying distribution.
- Density estimation, clustering, novelty detection, etc.

Unsupervised Learning

A given dataset X

	Var. 1	Var. 2	 Var. d
Ins. 1			
Ins. 2			
Ins. N			











Supervised Learning

$$\mathcal{X} = \{\mathbf{x}_i | i = 1, ..., n, \ \mathbf{x}_i \in \mathbb{R}^d\}$$

$$y = f(x)$$

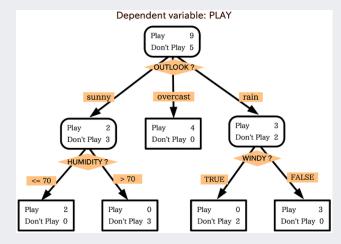
Supervised learning

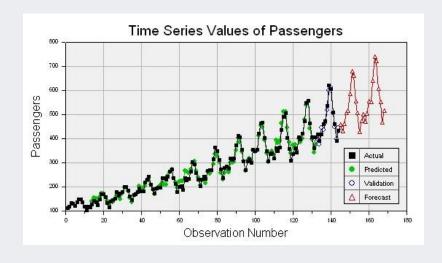
- Finds relations between X and Y.
- Estimate the underlying function y = f(x).
- Classification, regression.

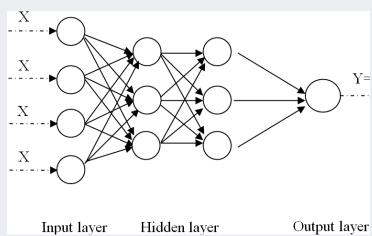
$$\mathcal{Y} = \{y_i | i = 1, ..., n, y_i = f(\mathbf{x}_i)\}$$

Supervised Learning

	Var. 1	Var. 2	 Var. d		Υ
Ins. 1			 		
Ins. 2		**	 ••	y = f(x)	
Ins. N			 		

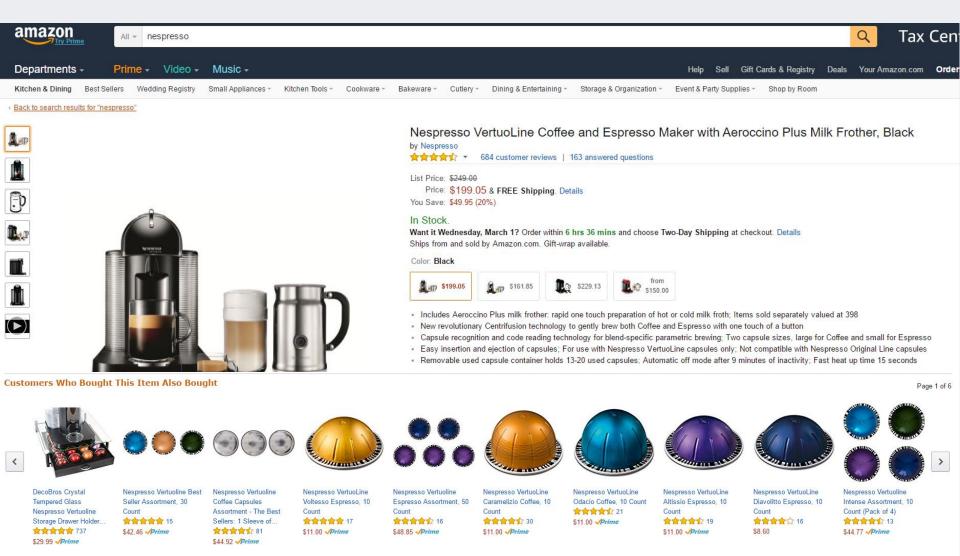












Also known as "Market Basket Analysis"



Wall Mart (USA)





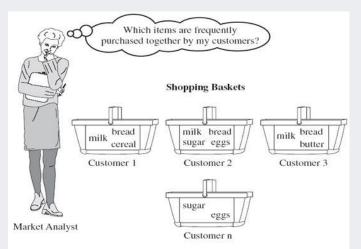
E-Mart (Korea)

• Goal:

- √ Produce rules that define "what goes with what"
- √ "If X was purchased, then Y was also purchased"

Features

- ✓ Rows are transactions
- ✓ Used in recommendation systems "Our records show that you bought X, thus you may also like Y"
- ✓ Also called "affinity analysis" or "market basket analysis"



- Dataset for association rule mining
 - √ Each transaction is represented as a record
 - ✓ Two representations are possible: (1) item list and (2) item matrix

Tid	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

	FEES	Beer	Diapers	Milk	Bread	Tld
0	0	0	0	1	1	1
0	1	1	1	0	1	2
1	0	1	1	1	0	3
0	0	1	1	1	1	4
1	0	0	1	1	1	5
	0	_ 5	1 1	1 1	(400)	4

• A toy example: a tiny retail market data

Transaction	Item 1	Item 2	Item 3	Item 4
1		맞라 있는	を記され	
2	맞라 있면 는	MATERIAL STATE		
3	맛라 있면 는	Gowern		
4		맞라 (취) 있면 (보기)	WE THE STATE OF TH	
5		Couled		
6	맛라 있면 는	Courtes		
7	망라 있면 느	9 (1)		
8		만라 (취) 있면 (보기)	lanced	第20
9		맞라 <u>**</u> 있면 보기	Calcia	
10	S			

Terminology

- ✓ Antecedent "IF" part
- √ Consequent "THEN" part
- √ Item set the items comprising the antecedent or consequent
- √ Antecedent and consequent are disjoint (have no items in common)

Generating rules

- √ Many rules are possible (e.g., for transaction 1)
 - If egg is bought, then noddle is also bought
 - If egg and noddle are bought, then tuna is also bought
 - If tuna is bought, then egg is also bought, etc.

Performance Measures for the rule A \rightarrow B

Support

$$support(A) = P(A)$$

- ✓ Used to find the frequent item sets
- Confidence

$$confidence(A \rightarrow B) = \frac{P(A, B)}{P(A)}$$

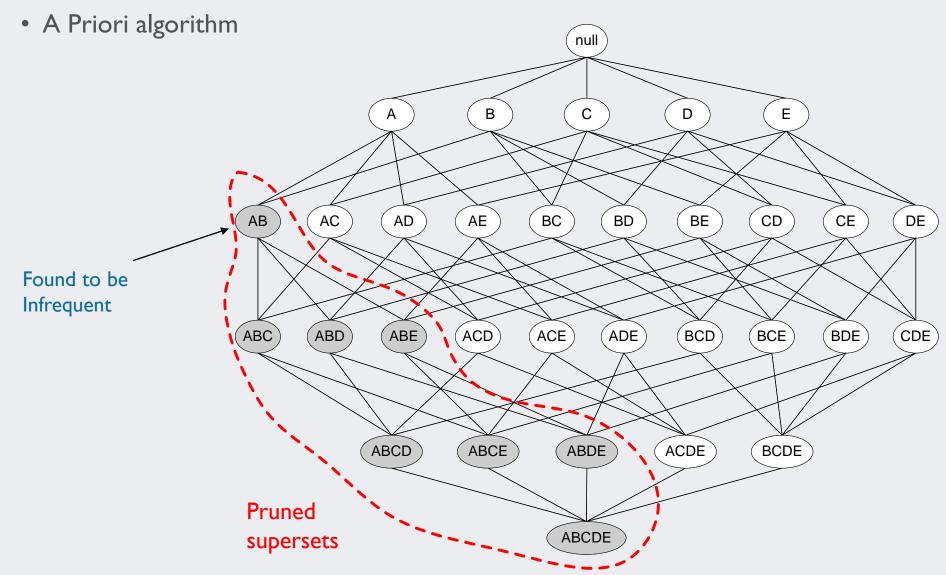
- ✓ Used to generate meaningful rules
- Lift

$$lift(A \to B) = \frac{P(A, B)}{P(A) \cdot P(B)}$$

✓ Used to determine the usefulness of generated rules

- How to generate an effective association rules?
 - ✓ Ideally, create all possible combinations of items and see what rules are effective and what rules are not.
 - √ Computation time grows exponentially as the number of items increases.
- Brute-force approach
 - ✓ List all possible association rules
 - ✓ Compute the support and confidence for each rule
 - ✓ Prune rules that fail the minsup and minconf threshold.
 - √ Computationally prohibitive!

- A priori algorithm
 - ✓ Consider only "frequent item sets"
 - √ "support"
 - Criterion for item set frequency P(A)
 - #(%) of transactions that include both the antecedent and the consequent
 - Support for the item set {egg, noodle} is 4 out of transactions, or 40%
 - ✓ Support of an itemset never exceeds the support of its subsets, which is known as anti-monotone property of support.



- Generating frequent item sets
 - ✓ Users set a minimum support criterion: e.g. 2 transactions or 20%

Transaction	Item 1	Item 2	Item 3	Item 4
1		Par S	5원 원 등원 참기	
2	맞작 (P) 이번 는	ज़्री हो		
3	Pri S	General		
4		맞라 있면 ***	M is	
5		Coulott		
6	맞라 를 있면 는	Courselle		
7	DO TO	of vi		
8		만라 (취) 한면	Couloth	(日本) (日本) (日本) (日本) (日本) (日本) (日本) (日本)
9		만라 (취) 있면 (보	(con lesta)	
10	S			

- Generating frequent item sets
 - ✓ Generate the list of one-item sets that meets the support criterion
 - Support {noodle} = 8/10 = 80%
 - Support {egg} = 5/10 = 50%
 - Support {cola} = 5/10 = 50%
 - Support {rice} = 3/10 = 30%
 - Support {tuna} = 2/10 = 20%
 - Support {onion} = 1/10 = 10%
 - ✓Onion is removed because it does not meet the minimum support criterion

- Generating frequent item sets
 - ✓ Use the life of one-item sets to generate list of two-item sets that meet the support criterion

	noodle	egg	cola	rice	tuna
noodle		40%	40%	20%	20%
egg			30%	0%	20%
cola				0%	10%
rice					0%
tuna					

^{√ {}noodle, egg}, {noodle, cola}, {noodle, rice}, {noodle, tuna}, {egg, cola}, {egg, tuna} are
frequent two-item sets

- Generating frequent item sets
 - ✓ Use the list of two-item sets to generate the three-item sets.
 - ✓ Continue up through k-item sets.

Set-size	Item 1	Item 2	Item 3	•••	Item 6
1	noodle				
1	egg				
1	cola				
1	rice				
1	tuna				
2	noodle	egg			
2	noodle	cola			
2	noodle	rice			
•••	•••	•••			

- A Priori algorithm
 - ✓ Let k=I
 - √ Generate frequent itemsets of length I
 - √ Repeat until no new frequent itemsets are identified
 - Generate length (k+1) candidate itemsets from length k frequent itemsets
 - Prune candidate itemsets containing subsets of length k that are infrequent
 - Count the support of each candidate by scanning the DB
 - Eliminate candidates that are infrequent, leaving only those that are frequent

Confidence

- ✓ The % of antecedent transactions that also have the consequent item set
- √ E.g. "if noddle is purchased, then egg is also purchased"

support
$$(noodle) = P(noodle) = \frac{8}{10}$$
, support $(egg) = P(egg) = \frac{5}{10}$

$$confidence(noodle \rightarrow egg) = \frac{P(noodle, egg)}{P(noodle)} = \frac{4/10}{8/10} = 0.5(50\%)$$

- ✓ Benchmark confidence: transactions with consequent (P(egg), support(egg)) of all transactions
- ✓ If the confidence of (noodle \rightarrow egg) is smaller then the support of egg, this rule is useless

• Lift

√ Confidence/(benchmark confidence)

$$lift(noodle \rightarrow egg)$$

$$= \frac{confidence(noodle \rightarrow egg)}{\text{support}(egg)} = \frac{\frac{P(noodle, egg)}{P(noodle)}}{P(egg)} = \frac{P(noodle, egg)}{P(noodle) \times P(egg)}$$

$$=\frac{\frac{4}{10}}{\frac{8}{10}\times\frac{5}{10}}=1$$

- \checkmark If lift = 1, then the antecedent and the consequents are statistically independent
- ✓ If lift > I, then the rule is useful in finding consequent item sets

Generated rules

- ✓ Set the support to 20%.
- ✓ Set the confidence to 70%.

Rule #	Antecedent (a)	Consequent	Support	Confidence	Lift
1	tuna=>	egg, noodle	2	100	2.5
2	tuna=>	egg	2	100	2
3	noodle, tuna=>	egg	2	100	2
4	rice=>	noodle	3	100	1.25
5	egg, tuna=>	noodle	2	100	1.25
6	tuna=>	noodle	2	100	1.25
7	cola=>	noodle	5	80	1
8	egg=>	noodle	5	80	1

Summary

- ✓ Produce rules on associations between items from a database of transactions
- √ Widely used in recommender systems
- ✓ Most popular method is A-priori algorithm
- √ To reduce computation, consider only "frequent" item sets (=support)
- ✓ Performance is measured by confidence and lift

AGENDA

- 01 Association Rules A Priori algorithm
- 02 R Exercise

Package "arules" & "arulesViz

Package 'arules'

October 2, 2016

Version 1.5-0 **Date** 2016-09-23

Title Mining Association Rules and Frequent Itemsets

Description Provides the infrastructure for representing, manipulating and analyzing transaction data and patterns (frequent itemsets and association rules). Also provides interfaces to C implementations of the association mining algorithms Apriori and Eclat by C. Borgelt.

Classification/ACM G.4, H.2.8, I.5.1

URL http://lyle.smu.edu/IDA/arules

BugReports https://github.com/mhahsler/arules/issues

Package 'arulesViz'

October 3, 2016

Version 1.2-0 Date 2016-10-02

Title Visualizing Association Rules and Frequent Itemsets

Author Michael Hahsler and Sudheer Chelluboina

Maintainer Michael Hahsler <mhahsler@lyle.smu.edu>

Depends arules (>= 1.4.1), grid

Imports scatterplot3d, vcd, seriation, igraph (>= 1.0.0), graphics, methods, utils, grDevices, stats, colorspace, DT, plotly

Suggests graph, Rgraphviz, iplots

Description

Extends package arules with various visualization techniques for association rules and itemsets. The package also includes several interactive visualizations for rule exploration.

License GPL-3

URL http://lyle.smu.edu/IDA/arules/

BugReports https://github.com/mhahsler/arulesViz/issues

Titanic Data

```
1 ▼ # Association Rules -----
 2 # arules and arulesviz packages install
 3 install.packages("arules", dependencies = TRUE)
   install.packages("arulesViz", dependencies = TRUE)
 6 library(arules)
 7 library(arulesviz)
 8 library(wordcloud)
10 # Load titanic data set
11 titanic <- read.delim("titanic.txt", dec=",")</pre>
12 str(titanic)
13 head(titanic)
> # Load titanic data set
> titanic <- read.delim("titanic.txt", dec=",")
> str(titanic)
'data.frame': 1313 obs. of 5 variables:
          : Factor w/ 1310 levels "Abbing, Mr Anthony",..: 22 25 26 27 24 31 45 46 50 54 ...
$ PClass : Factor w/ 3 levels "1st","2nd","3rd": 1 1 1 1 1 1 1 1 1 1 ...
          : Factor w/ 75 levels "0.17", "0.33",...: 28 18 30 24 5 48 66 39 60 73 ...
 $ Age
           : Factor w/ 2 levels "female", "male": 1 1 2 1 2 2 1 2 1 2 ...
 $ Survived: int 1 0 0 0 1 1 1 0 1 0 ...
> head(titanic)
                                          Name PClass Age
                                                              Sex Survived
                  Allen, Miss Elisabeth Walton
1
                                                  1st
                                                        29 female
                   Allison, Miss Helen Loraine
2
                                                  1st
                                                       2 female
           Allison, Mr Hudson Joshua Creighton
                                                  1st
                                                        30
                                                             male
4 Allison, Mrs Hudson JC (Bessie Waldo Daniels)
                                                  1st 25 female
                                                                         0
5
                 Allison, Master Hudson Trevor
                                                  1st 0.92
                                                             male
                                                                         1
6
                            Anderson, Mr Harry
                                                  1st 47
                                                             male
                                                                         1
```

Data Preprocessing

✓ Categorize a numeric variable, remove NA, etc.

```
# Remove "Name" column and group "Age" column
   titanic_ar <- titanic[,2:5]
    titanic_ar$Age = as.character(titanic_ar$Age)
17
18
   c_idx <- which(as.numeric(titanic_ar$Age) < 20)</pre>
    a_idx <- which(as.numeric(titanic_ar$Age) >= 20)
19
20
    na_idx <- which(is.na(titanic_ar$Age))</pre>
21
22
    titanic_ar$Age[c_idx] <- "Child"
    titanic_ar$Age[a_idx] <- "Adult"
23
    titanic_ar$Age[na_idx] <- "Unknown"
24
25
26 # Convert the attribues to factor
    titanic_ar$Age <- as.factor(titanic_ar$Age)</pre>
27
    titanic_ar$Survived <- as.factor(titanic_ar$Survived)</pre>
28
```

	PClass	Age	Sex	Survived
1	1st	Adult	female	1
2	1st	Child	female	0
3	1st	Adult	male	0
4	1st	Adult	female	0
5	1st	Child	male	1
6	1st	Adult	male	1
7	1st	Adult	female	1
8	1st	Adult	male	0
9	1st	Adult	female	1
10	1st	Adult	male	0

Find rules (default setting)

```
# Rule generation by Apriori algorithm with default settings
rules <- apriori(titanic_ar)
inspect(rules)</pre>
```

```
> rules <- apriori(titanic_ar)
parameter specification:
 confidence minval smax arem aval original Support support minlen maxlen target
                      1 none FALSE
                                              TRUE
                                                       0.1
                                                                1
                                                                      10 rules FALSE
algorithmic control:
filter tree heap memopt load sort verbose
    0.1 TRUE TRUE FALSE TRUE
apriori - find association rules with the apriori algorithm
version 4.21 (2004.05.09)
                                                Christian Borgelt
                                 (c) 1996-2004
set item appearances ...[0 item(s)] done [0.00s].
set transactions ...[10 item(s), 1313 transaction(s)] done [0.00s].
sorting and recoding items ... [10 item(s)] done [0.00s].
creating transaction tree ... done [0.00s].
checking subsets of size 1 2 3 4 done [0.00s].
writing ... [16 rule(s)] done [0.00s].
creating S4 object ... done [0.00s].
```

Find rules (default setting)

```
> inspect(rules)
                                  support confidence
   1hs
                   rhs
                => {Survived=0} 0.4364052 0.8059072 1.226137
1 {PClass=3rd}
                => {Survived=0} 0.5399848 0.8331375 1.267566
2 {Sex=male}
3 {Survived=0}
                => {Sex=male}
                                0.5399848 0.8215527 1.267566
4 {PClass=2nd,
   Sex=male}
                => {Survived=0} 0.1127190 0.8554913 1.301576
5 {PClass=2nd.
   Survived=0}
                => {Sex=male}
                                0.1127190 0.9192547 1.418309
6 {PClass=1st,
                => {Survived=1} 0.1020564 0.9370629 2.734141
    Sex=female}
7 {Sex=female,
    Survived=0}
                => {PClass=3rd} 0.1005331 0.8571429 1.582881
8 {PClass=3rd,
   Age=Unknown} => {Survived=0} 0.2536177 0.8473282 1.289156
9 {Age=Unknown,
   Sex=male}
                 => {Survived=0} 0.2566641 0.8798956 1.338706
10 {Age=Unknown,
    Survived=0}
                => {Sex=male}
                                0.2566641 0.8023810 1.237986
11 {Age=Adult,
   Sex=male}
                => {Survived=0} 0.2482864 0.8253165 1.255667
12 {Age=Adult,
   Survived=0}
                => {Sex=male}
                                0.2482864 0.8693333 1.341286
13 {PClass=3rd,
   Sex=male}
                => {Survived=0} 0.3358720 0.8837675 1.344596
14 {PClass=3rd,
    Age=Unknown,
    Sex=male}
                => {Survived=0} 0.1957350 0.9081272 1.381658
15 {PClass=3rd,
   Age=Adult.
    Sex=male}
                => {Survived=0} 0.1142422 0.8670520 1.319165
16 {PClass=3rd,
   Age=Adult.
    Survived=0}
                => {Sex=male}
                                0.1142422 0.8064516 1.244267
```

Find rules (customized setting)

```
> inspect(rules)
                                  support confidence
                                                         lift
  1hs
                   rhs
1 {PClass=2nd.
   Sex=male}
                => {Survived=0} 0.1127190 0.8554913 1.301576
2 {PClass=1st.
   Sex=female} => {Survived=1} 0.1020564 0.9370629 2.734141
3 {PClass=3rd.
   Age=Unknown} => {Survived=0} 0.2536177 0.8473282 1.289156
4 {Age=Unknown,
               => {Survived=0} 0.2566641 0.8798956 1.338706
   Sex=male}
5 {Age=Adult,
               => {Survived=0} 0.2482864 0.8253165 1.255667
   Sex=male}
6 {PClass=3rd,
               => {Survived=0} 0.3358720 0.8837675 1.344596
   Sex=male}
7 {PClass=3rd,
   Age=Unknown,
   Sex=male}
               => {Survived=0} 0.1957350 0.9081272 1.381658
8 {PClass=3rd,
   Age=Adult,
   Sex=male}
               => {Survived=0} 0.1142422 0.8670520 1.319165
```

• Visualize the rules

0.84

0.1

0.15



0.2

support

0.25

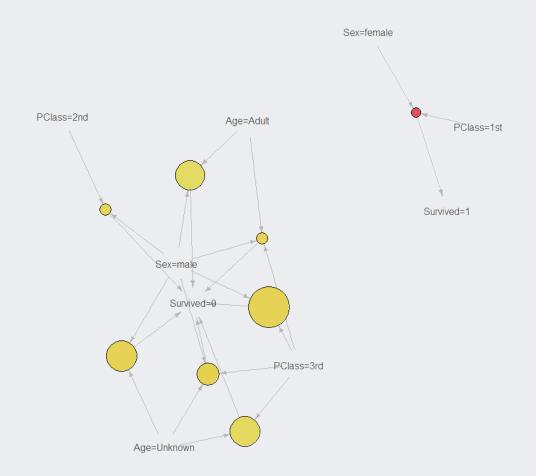
0.3

- 1.5

• Visualize the rules

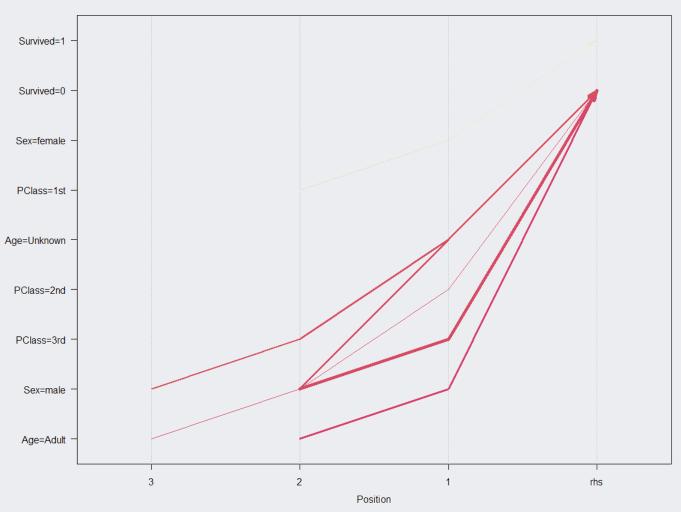
Graph for 8 rules

size: support (0.102 - 0.336) color: lift (1.256 - 2.734)



• Visualize the rules

Parallel coordinates plot for 8 rules



Groceries shopping data

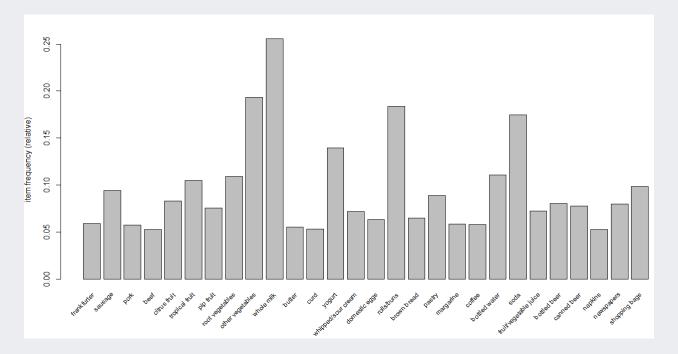
```
44  # Load transaction data "Groceries"
45  data("Groceries")
46  summary(Groceries)
47  str(Groceries)
48  inspect(Groceries)
```

```
> summary(Groceries)
transactions as itemMatrix in sparse format with
9835 rows (elements/itemsets/transactions) and
169 columns (items) and a density of 0.02609146
most frequent items:
     whole milk other vegetables
                                      rolls/buns
                                                             soda
                                                                                           (Other)
                                                                           yogurt
                            1903
                                            1809
                                                             1715
                                                                             1372
                                                                                             34055
element (itemset/transaction) length distribution:
sizes
                      5
                           6
                                           10
                                                11
                                                     12
                                                            13
2159 1643 1299 1005 855 645 545 438 350 246 182 117
                                                            78
                                                                77
                                                                     55
                                                                               29
           28 29
                    32
  Min. 1st Qu. Median
                          Mean 3rd Qu.
 1.000 2.000 3.000
                       4.409 6.000 32.000
includes extended item information - examples:
      labels level2
1 frankfurter sausage meet and sausage
     sausage sausage meet and sausage
3 liver loaf sausage meet and sausage
```

- Groceries shopping data
 - ✓ Item inspection

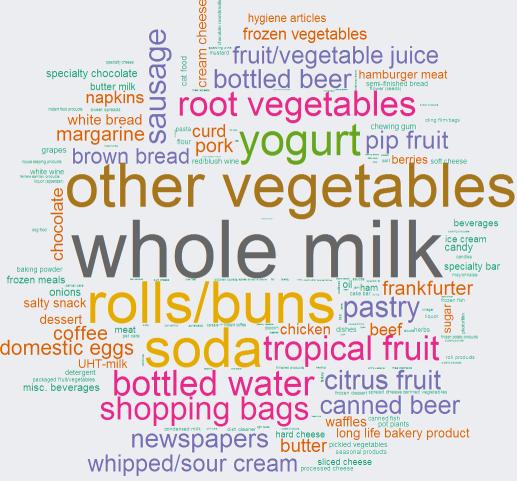
```
# Item inspection
itemName <- itemLabels(Groceries)
itemCount <- itemFrequency(Groceries)*9835

col <- brewer.pal(8, "Dark2")
wordcloud(words = itemName, freq = itemCount, min.freq = 1, scale = c(10, 0.2), col = col , random.order = FALSE)
itemFrequencyPlot(Groceries, support = 0.05, cex.names=0.8)</pre>
```



• Groceries shopping data

✓ Item inspection



Find and visualize rules

```
# Rule generation by Apriori
rules <- apriori(Groceries, parameter=list(support=0.001, confidence=0.5))
rules

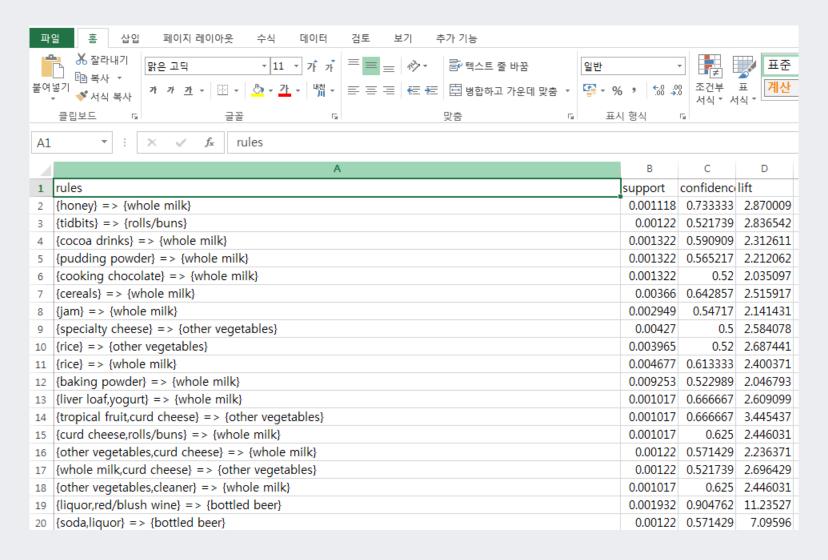
# List the first three rules with the highest lift values
inspect(head(sort(rules, by="lift"),3))

# Save the rules in a text file
write.csv(as(rules, "data.frame"), "Groceries_rules.csv", row.names = FALSE)

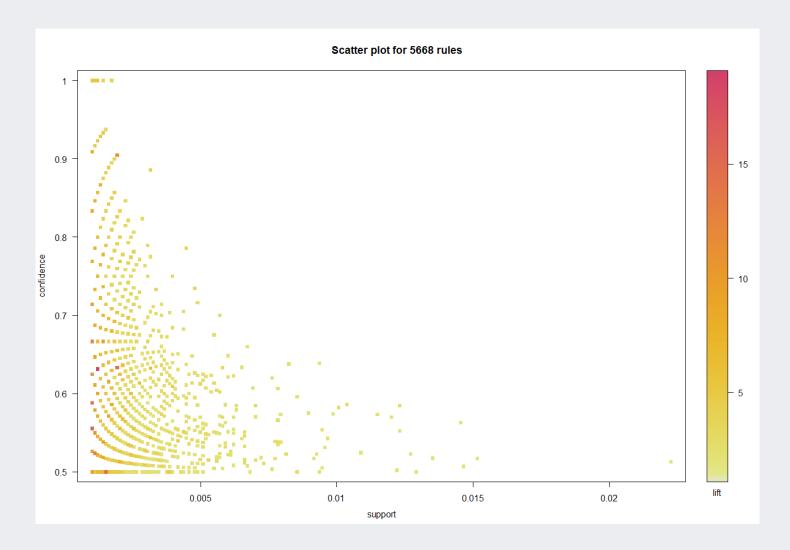
# Plot the rules
plot(rules)
plot(rules, method="grouped")
```

```
> inspect(head(sort(rules, by="lift"),3))
                                                  support confidence
                                                                         lift
  1hs
                             rhs
1 {Instant food products,
                          => {hamburger meat} 0.001220132  0.6315789  18.99565
   soda}
2 {soda.
                          => {salty snack}
   popcorn}
                                              0.001220132 0.6315789 16.69779
3 {flour,
   baking powder}
                         => {sugar}
                                              0.001016777 0.5555556 16.40807
```

Find and visualize rules



• Find and visualize rules



Find and visualize rules

