

PGL₂ Webs on the Torus, the Punctured Torus, and the T-Shirt

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Abstract

The Kauffman bracket skein algebra (KBSA) of an oriented surface Σ is defined as the free algebra spanned by diagrams of unoriented framed links in Σ modulo the Kauffman bracket relations with multiplication defined by stacking diagrams using over crossings. One particular modification of this construction is obtained by instead considering the free algebra spanned by trivalent graphs embedded in Σ modulo a different set of relations. This algebra is called the graph skein algebra of Σ . In this paper we describe bases for the graph skein algebras of the torus, the punctured torus, and the T-shirt. Additionally, we fully describe multiplication in the graph skein algebra of the torus, and show that the graph skein algebras of the punctured torus and T-shirt embed into their respective KBSAs. We show that this fails for the graph skein algebra of the torus, which is the first instance of this failing in the literature.