## PGL<sub>2</sub> Webs on the Torus, the Punctured Torus, and the T-Shirt

Cole Ellison<sup>1</sup>, Mark Sandey<sup>2</sup>, and Andrew Sylvester<sup>1</sup>

<sup>1</sup>University of California, Santa Barbara <sup>2</sup>University of California, Riverside

## Abstract

The Kauffman bracket skein algebra (KBSA) of a 3-manifold M—defined as the free algebra generated by unoriented framed links in M modulo the Kauffman relations—is generally an object of interest in the study of quantum topology. One particular modification to this construction is to consider instead the free algebra generated by trivalent graphs embedded in M modulo a different set of relations. This algebra is called the graph skein algebra of M. In this paper we describe bases and multiplication in the graph skein algebras of the torus, the punctured torus, and the tee-shirt. We also prove that the graph skein algebras of the punctured torus and tee-shirt embed into their respective KBSAs, but that the torus does not. Notably, this the torus is the first known example for which this occurs.