PGL₂ Webs on the Torus, the Punctured Torus, and the T-Shirt

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Abstract

The Kauffman bracket skein algebra (KBSA) of a 3-manifold M—defined as the free algebra generated by unoriented framed links in M modulo the Kauffman relations— is generally an object of interest in the study of quantum topology. One particular modification to this construction is to consider instead the free algebra generated by trivalent graphs embedded in M modulo a different set of relations. This algebra is called the graph skein algebra of M. In this paper we describe bases for the graph skein algebras of the torus, the punctured torus, and the tee-shirt. Additionally, we fully describe mutliplication in the graph skein algebra of the torus, and show that the algebras of the punctured torus and tee-shirt embed into their respective KBSAs. This is not the case for the graph skein algebra of the torus, and is the first known example for which this holds.