```
In[*]:= Clear[x, a, b, c, d, equation, roots]
        (*Define the coefficients*)
       a = 1;
       b = -2;
       c = -3;
       d = 4;
        (*Define the equation*)
       equation = a * x^3 + b * x^2 + c * x + d;
        (*Solve the equation numerically using NSolve*)
       roots = NSolve[equation == 0, x];
        (*Display the roots*)
       roots
Out[ • ]=
       \{\,\{\,x\,\to\, \text{-1.56155}\,\}\,\,,\,\,\{\,x\,\to\,\text{1.}\,\}\,\,,\,\,\{\,x\,\to\,\text{2.56155}\,\}\,\}
 In[*]:= Clear[x, a, b, c, d, equation, roots]
       (*Define the coefficients*)
       a = 1;
       b = -2;
       c = -3;
       d = 4;
       (*Define the equation*)
       equation = a * x^3 + b * x^2 + c * x + d;
        (*Loop to check roots for each ten numbers in the range[-100,100]*)
       rootsList = {};
       For[start = -100, start ≤ 100, start = start + 10,
         roots = NSolve[{equation == 0, start ≤ x < start + 10}, x];
         rootsList = Join[rootsList, roots];]
        (*Display the roots*)
       rootsList
Out[ • ]=
       \{\{x \rightarrow -1.56155\}, \{x \rightarrow 1.\}, \{x \rightarrow 2.56155\}\}
```

```
Clear[x, a, b, c, d, equation, roots]
(*Define the coefficients*)
(*Define the equation*)
equation = a * x^3 + b * x^2 + c * x + d;
(*Loop to check roots for each ten numbers in the range[-100,100]*)
rootsList = {};
For[start = -100, start ≤ 100, start = start + 10,
 roots = NSolve[{equation == 0, start ≤ x < start + 10}, x];
 rootsList = Join[rootsList, roots];]
(*Display the roots*)
rootsList
FindRoot \left[ \left\{ \alpha \times - \beta y - \gamma z + \psi x^2 - \delta y z, -\alpha \times + \beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \right\} \right]
   \xi - \alpha x + \beta y + \gamma z - \eta \text{ Abs}[y] x , {{x, 1000}, {y, 1000}, {z, 1000}}}
```

```
In[*]:= Clear[x, a, b, c, d, equation, roots]
        equation = x^3 - 6 * x^2 + 11 x + 6
        FindRoot[equation, {x, 10}]
Out[ • ]=
        6 + 11 x - 6 x^2 + x^3
Out[ • ]=
        \{x \rightarrow -0.434841\}
```

```
In[*]:= Clear[x, a, b, c, d, equation, roots]
       For [i = -10, i < 10, i++]
        For [j = -10, j < 10, j++]
       FindRoot[\{x^3 * y + 4 * y^2 - 3 x y, x y^4 - 8 y^3 + 7 x^2 y - 8\}, \{\{x, i\}, \{y, j\}\}\}
Out[ • ]=
        \{x \rightarrow 2.47105, y \rightarrow -1.91881\}
        \{x \rightarrow 2.4710451213615308^{\circ}, y \rightarrow -1.9188060683963832^{\circ}\}\
 In[*]:= Clear[x, a, b, c, d, equation, roots]
        (*Define the coefficients*)
       a = 1;
       b = -2;
       c = -3;
       d = 4;
        (*Define the equation*)
        equation = a * x^3 + b * x^2 + c * x + d;
        (*Loop to check roots for each ten numbers in the range[-100,100]*)
        rootsList = {};
       For[start = -100, start ≤ 100, start = start + 10,
         roots = NSolve[{equation == 0, start ≤ x < start + 10}, x];
         rootsList = Join[rootsList, roots];]
        (*Display the roots*)
        rootsList
Out[ • ]=
        \{\,\{\,x \to -1.56155\}\,\,,\,\,\{\,x \to 1.\,\}\,\,,\,\,\{\,x \to 2.56155\}\,\}
```

Out[ • ]=

```
In[ • ]:=
      Clear[x, a, b, c, d, equation, roots]
      equation = x^3 * y + 4 * y^2 - 3 xy;
      equation1 = xy^4 - 8y^3 + 7x^2y - 8;
      rootsList = {};
      For[start = -10, start ≤ 10, start = start + 1, roots = NSolve[
          {equation == 0, start \le x < \text{start} + 1, equation1 == 0, start \le y \le \text{start} + 1}, x, y];
        rootsList = Join[rootsList, roots];]
      rootsList
       ••• NSolve: 警告: y 不是一个有效的域指定. 假定它是一个需要消除的变量.
       ••• NSolve: 警告: y 不是一个有效的域指定. 假定它是一个需要消除的变量.
       ••• NSolve: 警告: y 不是一个有效的域指定. 假定它是一个需要消除的变量.
       ··· General: 在本次计算中, NSolve::bdomv 的进一步输出将被抑制. ①
Out[ • ]=
       {}
       ••• SetDelayed: (-8 + xy^4 + 7x^2y - 8y^3)[x_1, y_1] 中的标签 Plus 被保护. ①
       ··· Table: 迭代器 {x, range_x} 没有适当的边界. 🕡
       ••• Table: 迭代器 {x, range_x} 没有适当的边界. 🕡
```

Table[ $\{x, y, equation1[x, y]\}, \{x, range_x\}, \{y, range_y\}$ ]

(\*This is for absolute value y,and z have different situation\*)

```
ClearAll
             \alpha = 0.05;
             \beta = 0.01;
             \gamma = 0.05;
             \delta = 0.02;
             \epsilon = 0;
             \xi = 0.01;
             \eta = 0;
             \theta = 1.0;
             \psi = -0.1;
  In[\circ]:= FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2 - \delta y z, -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x z, \xi - \alpha x + \beta y + \gamma z - \eta y x \},
                \{\{x, 1000\}, \{y, 1000\}, \{z, 1000\}\}\]
Out[ • ]=
              \{x \rightarrow 0.289824, y \rightarrow 0.0972025, z \rightarrow 0.0985556\}
              \{x \to 0.28982442362707644^{\circ}, y \to 0.09720252647933221^{\circ}, z \to 0.0985555845431781^{\circ}\}
  In[*]:= FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2 - \delta y z, -\alpha x + \beta y + \gamma z + \theta y^2 z + \varepsilon x z, \xi - \alpha x + \beta y + \gamma z + \eta y x \}
                \{\{x, 1000\}, \{y, 1000\}, \{z, 1000\}\}\]
              ••• FindRoot: 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal
                       指定的容差范围内,但是无法使优化目标函数的值减小得足够多. 您可能需要多于 MachinePrecision
                      位的工作精度以满足这些容差. ①
Out[ • ]=
             \{x \rightarrow -0.31527, y \rightarrow 0.012177, z \rightarrow -0.512961\}
  In[ • ]:=
             \mathsf{FindRoot}\big[\big\{\alpha\,\mathsf{x}\,-\,\beta\,\mathsf{y}\,-\,\gamma\,\mathsf{z}\,+\,\psi\,\mathsf{x}^2\,-\,\delta\,\mathsf{y}\,\mathsf{z}\,,\,\,-\,\alpha\,\mathsf{x}\,+\,\beta\,\mathsf{y}\,+\,\gamma\,\,\mathsf{z}\,+\,\theta\,\mathsf{y}^2\,+\,\varepsilon\,\mathsf{x}\,\mathsf{z}\,,\,\,\xi\,-\,\alpha\,\mathsf{x}\,+\,\beta\,\mathsf{y}\,+\,\gamma\,\mathsf{z}\,-\,\eta\,\,\mathsf{y}\,\mathsf{x}\,\big\}\,,
                \{\{x, 1000\}, \{y, 1000\}, \{z, 1000\}\}\]
Out[ • ]=
              \{x \rightarrow 0.292069, y \rightarrow 0.0884856, z \rightarrow 0.100216\}
  In[ • ]:=
             \mathsf{FindRoot} \big[ \big\{ \alpha \, \mathsf{x} - \beta \, \mathsf{y} - \gamma \, \mathsf{z} + \psi \, \mathsf{x}^2 - \delta \, \mathsf{y} \, \mathsf{z} \,, \, -\alpha \, \mathsf{x} + \beta \, \mathsf{y} + \gamma \, \, \mathsf{z} + \theta \, \mathsf{y}^2 - \varepsilon \, \mathsf{x} \, \mathsf{z} \,, \, \, \xi - \alpha \, \mathsf{x} + \beta \, \mathsf{y} + \gamma \, \mathsf{z} + \eta \, \mathsf{y} \, \mathsf{x} \, \big\} \,,
                \{\{x, 1000\}, \{y, 1000\}, \{z, 1000\}\}\}
```

```
\{x \rightarrow -0.3063541197571247^{\circ}, y \rightarrow 0.11317880799639801^{\circ}, z \rightarrow -0.49431708725750717^{\circ}\}
            (*This is for the special case 2,
           maybe this one also should have four situation *)
           \mathsf{FindRoot}\big[\big\{\alpha\,\mathsf{x}\,-\,\beta\,\mathsf{y}\,-\,\gamma\,\mathsf{z}\,+\,\psi\,\mathsf{x}^2\,-\,\delta\,\mathsf{y}\,\mathsf{z}\,,\,\,-\,\alpha\,\mathsf{x}\,+\,\beta\,\mathsf{y}\,+\,\gamma\,\,\mathsf{z}\,+\,\theta\,\mathsf{y}^2\,-\,\varepsilon\,\mathsf{x}\,\mathsf{z}\,,\,\,\xi\,-\,\alpha\,\mathsf{x}\,+\,\beta\,\mathsf{y}\,+\,\gamma\,\mathsf{z}\,+\,\eta\,\,\mathsf{y}\,\mathsf{x}\,\big\}\,,
             \{\{x, 1000\}, \{y, 1000\}, \{z, 1000\}\}\]
Out[ • ]=
           \{x \rightarrow -0.306354, y \rightarrow 0.113179, z \rightarrow -0.494317\}
  In[ • ]:=
           \alpha = 0.05;
           \beta = 0.01;
           \gamma = 0.05;
           \delta = 0;
           \epsilon = 0.03;
           \zeta = 0.01;
           \eta = 0;
           \theta = 1.0;
           \psi = -0.1;
  In[ • ]:=
           FindRoot [\{\alpha \times -\beta y - \gamma z + \psi \times^2, -\alpha \times +\beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \xi - \alpha \times +\beta y + \gamma z\},
             \{x, 100\}, \{y, 100\}, \{z, 100\}\]
           ••• FindRoot: 在 {x, y, z} = {100., 100., 100.} 处, 函数值
                   \zeta} 不是由数字组成的维度为 \{3\} 的列表.
```

```
In[\bullet]:= FindRoot[\{\alpha \times -\beta \times -\gamma \times +\psi \times^2, -\alpha \times +\beta \times +\gamma \times +\theta \times^2 -\epsilon \times Abs[x], \xi-\alpha \times +\beta \times +\gamma \times Z\},
                                   \{x, 100\}, \{y, 100\}, \{z, 100\}\}
                            Clear [\alpha, \beta, \gamma, \epsilon, \xi, \theta, \psi, x, y, z]
                            ClearAll
                            FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2, -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x \text{ Abs}[z], \xi - \alpha x + \beta y + \gamma z\},
                                  \{x, -1\}, \{y, 100\}, \{z, 100\}
                              ··· FindRoot: 在 {x, y, z} = {100., 100., 100.} 处, 函数值
                                                 \zeta} 不是由数字组成的维度为 {3} 的列表.
Out[ • ]=
                             FindRoot \left[\left\{\alpha \ x - \beta \ y - \gamma \ z + \psi \ x^2, -\alpha \ x + \beta \ y + \gamma \ z + \theta \ y^2 - \epsilon \ x \ \text{Abs} \left[z\right], \zeta - \alpha \ x + \beta \ y + \gamma \ z\right\}
                                  \{x, 100\}, \{y, 100\}, \{z, 100\}\}
Out[ • ]=
                             ClearAll
                              ••• FindRoot: 在 {x, y, z} = {-1., 100., 100.} 处, 函数值
                                                 \{-1. \alpha - 100. \beta - 100. \gamma + 1. \psi, 1. \alpha + 100. \beta + 100. \gamma + 100. \epsilon + 10000. \theta, 1. \alpha + 100. \beta + 100. \gamma + \zeta\}
                                                 不是由数字组成的维度为 {3} 的列表.
Out[ • ]=
                             \label{eq:findRoot} \text{FindRoot} \left[ \left\{ \alpha \ \mathbf{x} - \beta \ \mathbf{y} - \gamma \ \mathbf{z} + \psi \ \mathbf{x}^2 \right. \right. \\ \left. - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} + \Theta \ \mathbf{y}^2 - \epsilon \ \mathbf{x} \ \mathsf{Abs} \left[ \mathbf{z} \right] \right. \\ \left. \left. \right\} \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} + \gamma \ \mathbf{z} \right) \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \right. \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left. \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \right] \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \\ \left( - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right) \left[ - \alpha \ \mathbf{x} \right] \left[ - \alpha \ \mathbf{x} \right] \\ \left[ - \alpha \ \mathbf{x} + \beta \ \mathbf{y} \right] \left[ - \alpha \ \mathbf{x} \right
                                   \{x, -1\}, \{y, 100\}, \{z, 100\}
     ln[\circ]:= For [i = -10, i < 10, i++; For [j = -10, j < 10, j++;
                                       Print[FindRoot[\{x^3 * y + 4 * y^2 - 3 x y, x y^4 - 8 y^3 + 7 x^2 y - 8\}, \{x, 1\}, \{y, j\}],
                                             ", i= ", i, ", j=", j]
                                  ]]
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, i = -9
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -8
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -7
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -6
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -5
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -4
                              \{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -9, j = -3
                              ••• FindRoot: 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal
                                                 指定的容差范围内,但是无法使优化目标函数的值减小得足够多. 您可能需要多于 MachinePrecision
                                                位的工作精度以满足这些容差. ①
                              \{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -9, j = -2
```

••• FindRoot: 无法在 100 次迭代中收敛到要求的准确度或者精度. ①

$$\{x \to 1.45676, y \to 0.395532\}, i = -9, j = -1$$

••• FindRoot: 在点 {x, y} = {1., 0.} 碰到奇异雅克比. 尝试扰动初始点. 🕡

$$\{x \to 1., y \to 0.\}, i = -9, j = 0$$

••• FindRoot: 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal 指定的容差范围内,但是无法使优化目标函数的值减小得足够多. 您可能需要多于 MachinePrecision 位的工作精度以满足这些容差. ①

$$\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -9, j=1$$

- **FindRoot:** 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal 指定的容差范围内,但是无法使优化目标函数的值减小得足够多. 您可能需要多于 MachinePrecision 位的工作精度以满足这些容差. ①
- ··· General: 在本次计算中,FindRoot::Istol 的进一步输出将被抑制. ①

$$\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -9, j = 2$$

$$\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = -9, j = 3$$

$$\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -9, j = 4$$

$$\{x \to 1.53585, y \to 0.422711\}, i = -9, j = 5$$

••• FindRoot: 无法在 100 次迭代中收敛到要求的准确度或者精度. ①

$$\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -9, j = 6$$

$$\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -9, j = 7$$

$$\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -9, j = 8$$

$$\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = -9, j = 9$$

$$\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -9, j = 10$$

$$\{\,x\,\rightarrow\,2\,\text{.47105}\,\text{, }y\,\rightarrow\,-\,1\,\text{.91881}\,\}\,\text{, }i=\,-\,8\,\text{, }j=-\,9$$

$$\{\,x\,\rightarrow\,2.47105\,,\;y\,\rightarrow\,-1.91881\,\}$$
 , i=  $-\,8\,,\;j=-\,8$ 

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -8, j = -7$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -8, j = -6$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -8, j = -5$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -8, j = -4$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -8, j = -3$$

$$\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -8, j = -2$$

- ••• FindRoot: 无法在 100 次迭代中收敛到要求的准确度或者精度. 🕡
- ••• General: 在本次计算中,FindRoot::cvmit 的进一步输出将被抑制. 🕡

$$\{x \to 1.45676, y \to 0.395532\}, i = -8, j = -1$$

••• FindRoot: 在点 {x, y} = {1., 0.} 碰到奇异雅克比. 尝试扰动初始点. 🕡

```
\{x \rightarrow 1., y \rightarrow 0.\}, i = -8, j = 0
\{x \to 1.52276, y \to 0.418185\}, i = -8, j = 1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -8, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = -8, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -8, j = 4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = -8, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -8, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -8, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -8, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = -8, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -8, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -7, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -7, j = -2
\{x \to 1.45676, y \to 0.395532\}, i = -7, j = -1
```

- ••• FindRoot: 在点 {x, y} = {1., 0.} 碰到奇异雅克比. 尝试扰动初始点. 🕡
- ••• General: 在本次计算中, FindRoot::jsing 的进一步输出将被抑制. ①

$$\{x \to 1., y \to 0.\}$$
,  $i = -7$ ,  $j = 0$   
 $\{x \to 1.52276, y \to 0.418185\}$ ,  $i = -7$ ,  $j = 1$   
 $\{x \to 1.53155, y \to 0.421222\}$ ,  $i = -7$ ,  $j = 2$   
 $\{x \to 1.59568, y \to 0.443617\}$ ,  $i = -7$ ,  $j = 3$   
 $\{x \to 1.53993, y \to 0.424128\}$ ,  $i = -7$ ,  $j = 4$   
 $\{x \to 1.53585, y \to 0.422711\}$ ,  $i = -7$ ,  $j = 5$   
 $\{x \to 1.51819, y \to 0.416607\}$ ,  $i = -7$ ,  $j = 6$   
 $\{x \to 1.51775, y \to 0.416456\}$ ,  $i = -7$ ,  $j = 7$ 

$$\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -7, j = 8$$

$$\{x \to 1.60327, y \to 0.446293\}, 1 = -7, J = 6$$

$$\{\,x\,\rightarrow\,\text{1.56941, y}\,\rightarrow\,\text{0.434397}\,\}\,\,\text{, i=}\,\,-7\,\text{, j=9}$$

$$\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -7, j = 10$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -9$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -8$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -7$$

$$\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -6$$
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -5$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -4$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -6, j = -3$ 
 $\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -6, j = -2$ 
 $\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = -6, j = -1$ 
 $\{x \rightarrow 1., y \rightarrow 0.\}, i = -6, j = 0$ 
 $\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -6, j = 1$ 
 $\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -6, j = 2$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -6, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -6, j = 6$ 
 $\{x \rightarrow 1.51819, y \rightarrow 0.416456\}, i = -6, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -6, j = 7$ 
 $\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -6, j = 9$ 
 $\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -6, j = 10$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -9$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -7$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -5, j = -6$ 
 $\{x \rightarrow 1.53585, y \rightarrow 0.443617\}, i = -5, j = -2$ 
 $\{x \rightarrow 1.53585, y \rightarrow 0.443617\}, i = -5, j = 3$ 
 $\{x \rightarrow 1.53276, y \rightarrow 0.418185\}, i = -5, j = 3$ 
 $\{x \rightarrow 1.53293, y \rightarrow 0.422122\}, i = -5, j = 3$ 
 $\{x \rightarrow 1.53293, y \rightarrow 0.422122\}, i = -5, j = 3$ 
 $\{x \rightarrow 1.53393, y \rightarrow 0.424128\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 
 $\{x \rightarrow 1.51775, y \rightarrow 0.416607\}, i = -5, j = 6$ 

 $\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = -5, j = 9$ 

```
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -5, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -4, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -4, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = -4, j = -1
\{x \to 1., y \to 0.\}, i = -4, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -4, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -4, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = -4, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -4, j = 4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = -4, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -4, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -4, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -4, j = 8
\{x \to \text{1.56941, } y \to \text{0.434397}\}, i = -4, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -4, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -8
\{x \to 2.47105, y \to -1.91881\}, i = -3, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -3, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -3, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = -3, j = -1
\{x \to 1., y \to 0.\}, i = -3, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -3, j = 1
\{x \to 1.53155, y \to 0.421222\}, i = -3, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = -3, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -3, j = 4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = -3, j = 5
```

```
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -3, j = 6
\{x \to 1.51775, y \to 0.416456\}, i = -3, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -3, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = -3, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -3, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -2, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -2, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = -2, j = -1
\{x \to 1., y \to 0.\}, i = -2, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -2, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -2, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = -2, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -2, j=4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = -2, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -2, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -2, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -2, j = 8
\{x \to 1.56941, y \to 0.434397\}, i = -2, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -2, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = -1, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = -1, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = -1, j = -1
\{x \to 1., y \to 0.\}, i = -1, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = -1, j=1
```

```
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = -1, j = 2
\{x \to 1.59568, y \to 0.443617\}, i = -1, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = -1, j = 4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = -1, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = -1, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = -1, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = -1, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = -1, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = -1, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 0, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 0, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 0, j = -1
\{x \rightarrow 1., y \rightarrow 0.\}, i = 0, j = 0
\{x \to 1.52276, y \to 0.418185\}, i= 0, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = 0, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 0, j = 3
\{x \rightarrow \text{1.53993, } y \rightarrow \text{0.424128}\}, i = 0, j = 4
\{\,x\,\rightarrow\,\text{1.53585}\,,\;y\,\rightarrow\,\text{0.422711}\,\}\,\,,\,\,\,\,i\,\text{=}\,\,\,\text{0}\,\,,\,\,\,\,j\,\text{=}\,\text{5}
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 0, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 0, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 0, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 0, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 0, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 1, j = -3
```

$$\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 1, j = -2$$
 $\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 1, j = -1$ 
 $\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 1, j = -1$ 
 $\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 1, j = 1$ 
 $\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = 1, j = 2$ 
 $\{x \rightarrow 1.53958, y \rightarrow 0.443617\}, i = 1, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 1, j = 4$ 
 $\{x \rightarrow 1.53885, y \rightarrow 0.422711\}, i = 1, j = 5$ 
 $\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 1, j = 6$ 
 $\{x \rightarrow 1.518775, y \rightarrow 0.416456\}, i = 1, j = 7$ 
 $\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 1, j = 8$ 
 $\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 1, j = 9$ 
 $\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 1, j = 10$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -9$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -7$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -6$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -5$ 
 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 2, j = -3$ 
 $\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 2, j = -2$ 
 $\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 2, j = -1$ 
 $\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = 2, j = 2$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 2, j = 3$ 
 $\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 3, j = 9$ 
 $\{x \rightarrow 1.60076, y \rightarrow 0.434397\}, i = 2, j = 9$ 
 $\{x$ 

 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 3, j = -7$ 

```
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 3, j = -6
\{\,x\,\rightarrow\,2.47105\,,\ y\,\rightarrow\,-\,1.91881\,\}\,\,,\ i=\,3\,,\ j=-\,5\,
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i=3, j=-4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 3, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i= 3, j=-2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i= 3, j=-1
\{x \to 1., y \to 0.\}, i=3, j=0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 3, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i= 3, j=2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i= 3, j=3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 3, j = 4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = 3, j = 5
\{x \rightarrow \text{1.51819, } y \rightarrow \text{0.416607}\}, i = 3, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 3, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 3, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 3, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 3, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 4, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i=4, j=-3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 4, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 4, j = -1
\{x \to 1., y \to 0.\}, i = 4, j = 0
\{\, x \, \to \, \text{1.52276} \,, \,\, y \, \to \, \text{0.418185} \,\} \,\,, \  \, \text{i= 4, j=1}
\{x \to 1.53155, y \to 0.421222\}, i= 4, j=2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 4, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 4, j=4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = 4, j = 5
\{x \to 1.51819, y \to 0.416607\}, i = 4, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 4, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i= 4, j=8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 4, j = 9
```

```
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 4, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i=5, j=-9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 5, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 5, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 5, j = -1
\{x \to 1., y \to 0.\}, i = 5, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 5, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i=5, j=2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 5, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 5, j = 4
\{x \to 1.53585, y \to 0.422711\}, i = 5, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 5, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 5, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 5, j = 8
\{x \to 1.56941, y \to 0.434397\}, i= 5, j=9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 5, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 6, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 6, j = -8
\{\,x\,\rightarrow\,2.47105\,,\ y\,\rightarrow\,-\,1.91881\,\}\,\,,\ i=\,6\,,\ j=-\,7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 6, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 6, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 6, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i=6, j=-3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i=6, j=-2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 6, j = -1
\{x \to 1., y \to 0.\}, i = 6, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i=6, j=1
\{x \to 1.53155, y \to 0.421222\}, i = 6, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 6, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 6, j = 4
```

 $\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = 6, j = 5$ 

```
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 6, j = 6
\{x \to 1.51775, y \to 0.416456\}, i = 6, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 6, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 6, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 6, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 7, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i=7, j=-2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 7, j = -1
\{x \to 1., y \to 0.\}, i = 7, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 7, j = 1
\{x \to 1.53155, y \to 0.421222\}, i = 7, j = 2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 7, j = 3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 7, j = 4
\{x \to 1.53585, y \to 0.422711\}, i= 7, j=5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 7, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 7, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 7, j = 8
\{x \to 1.56941, y \to 0.434397\}, i = 7, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 7, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 8, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 8, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i=8, j=-7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i= 8, j=-6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 8, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 8, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i= 8, j=-3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i= 8, j=-2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 8, j = -1
\{x \to 1., y \to 0.\}, i = 8, j = 0
\{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 8, j=1
```

```
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = 8, j=2
\{x \to 1.59568, y \to 0.443617\}, i= 8, j=3
\{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i= 8, j=4
\{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i = 8, j = 5
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 8, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i= 8, j=7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i= 8, j=8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 8, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 8, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -4
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 9, j = -3
\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i = 9, j = -2
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 9, j = -1
\{x \rightarrow 1., y \rightarrow 0.\}, i=9, j=0
\{x \to 1.52276, y \to 0.418185\}, i= 9, j=1
\{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i = 9, j=2
\{x \rightarrow 1.59568, y \rightarrow 0.443617\}, i = 9, j = 3
\{x \rightarrow \text{1.53993, } y \rightarrow \text{0.424128}\}, i = 9, j = 4
\{\,x\,\rightarrow\,\text{1.53585}\,,\;y\,\rightarrow\,\text{0.422711}\,\}\,\,,\,\,\,\,i\,\text{=}\,\,\,9\,\,,\,\,\,\,j\,\text{=}\,5\,\,
\{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i = 9, j = 6
\{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 9, j = 7
\{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i = 9, j = 8
\{x \rightarrow 1.56941, y \rightarrow 0.434397\}, i = 9, j = 9
\{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i = 9, j = 10
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i= 10, j=-9
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 10, j = -8
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 10, j = -7
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 10, j = -6
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 10, j = -5
\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i = 10, j = -4
```

 $\{x \rightarrow 2.47105, y \rightarrow -1.91881\}, i= 10, j=-3$ 

```
\{x \rightarrow 1.45676, y \rightarrow 0.395532\}, i = 10, j = -1
        \{x \rightarrow 1., y \rightarrow 0.\}, i=10, j=0
        \{x \rightarrow 1.52276, y \rightarrow 0.418185\}, i = 10, j = 1
        \{x \rightarrow 1.53155, y \rightarrow 0.421222\}, i= 10, j=2
        \{x \to 1.59568, y \to 0.443617\}, i = 10, j = 3
        \{x \rightarrow 1.53993, y \rightarrow 0.424128\}, i = 10, j = 4
        \{x \rightarrow 1.53585, y \rightarrow 0.422711\}, i= 10, j=5
        \{x \rightarrow 1.51819, y \rightarrow 0.416607\}, i= 10, j=6
        \{x \rightarrow 1.51775, y \rightarrow 0.416456\}, i = 10, j = 7
        \{x \rightarrow 1.60327, y \rightarrow 0.446295\}, i= 10, j=8
        \{x \to 1.56941, y \to 0.434397\}, i= 10, j=9
        \{x \rightarrow 1.62076, y \rightarrow 0.452481\}, i= 10, j=10
 ln[*]:= FindRoot[{x^3*y+4*y^2-3xy,xy^4-8y^3+7x^2y-8}, {x, 9}, {y, 7}]
        ••• FindRoot: 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal
             指定的容差范围内,但是无法使优化目标函数的值减小得足够多, 您可能需要多干 MachinePrecision
             位的工作精度以满足这些容差. ①
Out[ • ]=
        \{x \rightarrow 1.58031, y \rightarrow 0.438212\}
 ln[*]:= xyrule = FindRoot[{x^3*y+4*y^2-3xy, xy^4-8y^3+7x^2y-8}, {x, 1.5}, {y, .42}]
        ••• FindRoot: 线搜索把步长降低到由 AccuracyGoal 和 PrecisionGoal
             指定的容差范围内,但是无法使优化目标函数的值减小得足够多. 您可能需要多于 MachinePrecision
             位的工作精度以满足这些容差. ①
Out[ • ]=
        \{x \rightarrow 1.50907, y \rightarrow 0.413465\}
 ln[\circ]:= \{x^3 * y + 4 * y^2 - 3 x y, x y^4 - 8 y^3 + 7 x^2 y - 8\} /. xyrule
Out[ • ]=
        \{0.232882, -1.93029\}
 In[ • ]:=
       Clear[x, a, b, c, d, y, equation, roots]
 In[.] = equation = x^3y + 2x^2 + 9xy;
        equation2 = 4y^4 - 5xy^3 + 9xy;
       For [i = -5, i < 5, i++;
         xyrule = FindRoot[{equation, equation2}, {x, 1}, {y, i}] x
             rasiduals = {equation, equation2} /. xyrule x
               Print[xyrule, "", rasiduals, "x = 1", ", " "y = ", i]
       1
```

 $\{x \rightarrow -0.42134, y \rightarrow -0.826914\}, i= 10, j=-2$ 

```
\left\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\right\} / .
     Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /.
                      Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\})
                                            4\,y^4\big\} \ / \, . \ \text{Null} \, \left( \left\{ 2\,x^2 + 9\,x\,y + x^3\,y \, , \, 9\,x\,y - 5\,x\,y^3 + 4\,y^4 \right\} \, / \, . \, \, \, \text{Null} \, \left( \left\{ 2\,x^2 + 9\,x\,y + x^3\,y \, , \, y + x^3\,y \, \right) \right\} \, .
                                                          9\;x\;y\;-\;5\;x\;y^3\;+\;4\;y^4\,\big\}\;\;/\;\text{.}\;\;\text{Null}\;\left(\left\{2\;x^2\;+\;9\;x\;y\;+\;x^3\;y\,\text{,}\;\;9\;x\;y\;-\;5\;x\;y^3\;+\;4\;y^4\right\}\;\;/\;\text{.}\;\;
                                                               9 \times y - 5 \times y^3 + 4 y^4 /. {Null (x \rightarrow 4.00203 \times 10^{-40}),
                                                                                   Null (y \rightarrow 2.39972 \times 10^{-8}) \}))))))) rasidualsx = 1, y = -4
••• ReplaceAll: \{\text{Null}(\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} / .
                         Null\left(\{Times[\ll 2\gg] + Times[\ll 3\gg] + Times[\ll 2\gg], Times[\ll 3\gg] + Times[\ll 3\gg] + Times[\ll 2\gg]\right\}/.
                                    Null (\{\ll 2\gg\}/. \text{ Times}[\ll 2\gg]))
           既不是替换规则列表,也不是一个有效的分派表,因此无法用来替换. ①
••• Set: rasiduals \{x \to 2.28457 \times 10^{-40}, y \to -2.42626 \times 10^{-8} \} 中的标签 Times 被保护. ①
••• ReplaceAll: \{\text{Null}(x \to 4.00203 \times 10^{-40}), \text{Null}(y \to 2.39972 \times 10^{-8})\}
           既不是替换规则列表,也不是一个有效的分派表,因此无法用来替换. ①
••• ReplaceAll: \{\text{Null}(\{2 \times 2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} / \{\text{Null}(x \rightarrow 4.00203 \times 10^{-40}), \text{Null}(y \rightarrow 2.39972 \times 10^{-8})\}\}\}
           既不是替换规则列表,也不是一个有效的分派表,因此无法用来替换. 0
··· General: 在本次计算中,ReplaceAll::reps 的进一步输出将被抑制. ①
\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
     Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right) /. Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right) /.
                      Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\} /. Null (\{2x^2 + 9xy + x^3y, 9xy - 14y^4\} /.
                                            5 \times y^3 + 4 y^4 /. Null (\{2 \times^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                               Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 4 y^4\} /.
                                                                     5 \times y^3 + 4 y^4 /. Null (\{2 \times^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /. Null
                                                                           \left(\left\{2\;x^{2}+9\;x\;y+x^{3}\;y\text{, }9\;x\;y-5\;x\;y^{3}+4\;y^{4}\right\}\right. Null \left(\left\{2\;x^{2}+9\;x\;y+x^{3}\;y\text{, }\right.\right)
                                                                                           9 x y - 5 x y ^3 + 4 y ^4\big\} /. 
 \left\{ \text{Null } \left(x \rightarrow \text{4.00203} \times \text{10}^{-\text{40}}\right) \text{, Null} \right.
                                                                                              (y \rightarrow 2.39972 \times 10^{-8})))))))) rasidualsx = 1, y = -3
••• Set: rasiduals \{x \to 8.34682 \times 10^{-40}, y \to -2.47133 \times 10^{-8}\} 中的标签 Times 被保护. ①
\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
     Null\left(\left\{2\,\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,\,Null\left(\left\{2\,\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,\,y^{4}\right\}\,\,/\,.\,\,Null\left(\left\{2\,x^{2}\,+\,9\,x\,\,y\,+\,x^{3}\,y\,,\,\,9\,x\,\,y\,-\,5\,x\,\,y^{3}\,+\,4\,y^{4}\right\}\,\right)
                      Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right) /. Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right)
                                            4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                          9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                                               9 \times y - 5 \times y^{3} + 4 y^{4} /. Null (\{2 x^{2} + 9 \times y + x^{3} y, 9 \times y - 5 \times y^{3} + 4 y^{4}\}
                                                                                              4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                                                 \left\{ \text{Null } \left( x \rightarrow 4.00203 \times 10^{-40} \right), \text{ Null } \left( y \rightarrow 2.39972 \times 10^{-40} \right) \right\}
                                                                                                               10^{-8}) }) ) ) ) ) ) ) ) rasidualsx = 1, y = -2
```

**… Set:** rasiduals  $\{x \to 1.11492 \times 10^{-38}, y \to -3.02672 \times 10^{-8}\}$  中的标签 Times 被保护. **①** 

## ••• General: 在本次计算中, Set::write 的进一步输出将被抑制. ①

```
\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
   Null\left(\left\{2\,{x}^{2}+9\,x\,y+{x}^{3}\,y,\,9\,x\,y-5\,x\,{y}^{3}+4\,{y}^{4}\right\}\right./.\ Null\left(\left\{2\,{x}^{2}+9\,x\,y+{x}^{3}\,y,\,9\,x\,y-5\,x\,{y}^{3}+4\,{y}^{4}\right\}\right./.
               Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                              4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                        9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                           Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, y^4\})
                                                         9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 + y^4 \}
                                                                4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                  Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                        \left\{ \text{Null } \left( x \to 4.00203 \times 10^{-40} \right), \, \text{Null } \left( y \to 2.39972 \times 10^{-40} \right) \right\}
                                                                            \left\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\right\} / .
   Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /.
               Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 2 y^4\}
                              4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                        9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                           Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, y^4\})
                                                         9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 + y^4 \}
                                                                4y^{4}} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                  Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                        Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                            \left\{ \text{Null } \left( x \to \text{4.00203} \times \text{10}^{-\text{40}} \right) \text{, Null } \left( y \to \text{2.39972} \times \text{10}^{-\text{40}} \right) \right\}
                                                                            \left\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\right\} / .
   Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /.
               Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                              4\ y^{4}\,\big\}\ /\text{. Null}\,\left(\left\{ 2\ x^{2}+9\ x\ y+x^{3}\ y\text{, }9\ x\ y-5\ x\ y^{3}+4\ y^{4}\right\} \ /\text{. Null}\,\left(\left\{ 2\ x^{2}+9\ x\ y+x^{3}\ y\text{, }9x^{2}+4y^{4}\right\} \right) \right)
                                        9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                           Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, y^4\})
                                                         9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 \}
                                                                4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                  Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                        Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                            Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                            \left\{ \text{Null } \left( x \rightarrow 4.00203 \times 10^{-40} \right), \text{ Null } \left( y \rightarrow 2.39972 \times 10^{-40} \right) \right\}
```

```
\left\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\right\} / .
         Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /.
                                     Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 2y^4\})
                                                                          4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                                                 9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, y + x^3
                                                                                                                                           9 \times y - 5 \times y^{3} + 4 y^{4} /. Null (\{2 x^{2} + 9 \times y + x^{3} y, 9 \times y - 5 \times y^{3} + y^{3} + y^{4}\}
                                                                                                                                                             4 y^{4} /. Null (\{2 x^{2} + 9 x y + x^{3} y, 9 x y - 5 x y^{3} + 4 y^{4}\} /.
                                                                                                                                                                  Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                                                                                Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                                                                                         Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                                                                                         Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                                                                                         \left\{ \text{Null } \left( x \to 4.00203 \times 10^{-40} \right) \text{, Null } \left( y \to 2.39972 \times 10^{-40} \right) \right\}
                                                                                                                                                                                         10^{-8})}))))))))))))rasidualsx = 1, y = 2
\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
         Null\left(\left\{2\,x^{2}+9\,x\,y+x^{3}\,y,\,9\,x\,y-5\,x\,y^{3}+4\,y^{4}\right\}\right./.\ Null\left(\left\{2\,x^{2}+9\,x\,y+x^{3}\,y,\,9\,x\,y-5\,x\,y^{3}+4\,y^{4}\right\}\right./.
                                     Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\})
                                                                          4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                                                 9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, y + x^3
                                                                                                                                           9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 + y^4 \}
                                                                                                                                                             4 y^4} /. Null ({2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4} /.
                                                                                                                                                                  Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                                                                                Null \left(\left\{2\;x^2\,+\,9\;x\;y\,+\,x^3\;y\,,\;9\;x\;y\,-\,5\;x\;y^3\,+\,4\;y^4\right\}\right. / .
                                                                                                                                                                                         Null (2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4) /.
                                                                                                                                                                                         Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                                                                                         Null (2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4) /.
                                                                                                                                                                                         {Null (x \rightarrow 4.00203 \times 10^{-40}), Null (y \rightarrow 2.39972 \times 10^{-40})
```

```
\left\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\right\} / .
      Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                        Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 2y^4\})
                                                 4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                                                      Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /. Null (\{2 x^2 + 9 x y + x^3 y, y + x^3
                                                                                            9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 \times x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 + y^4 \}
                                                                                                        4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                                                           Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                     Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           Null (2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4) /.
                                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           \left\{ \text{Null } \left( x \rightarrow 4.00203 \times 10^{-40} \right), \text{ Null } \left( y \rightarrow 2.39972 \times 10^{-40} \right) \right\}
                                                                                                                           \left\{2\;x^2\,+\,9\;x\;y\,+\,x^3\;y\,\text{, }9\;x\;y\,-\,5\;x\;y^3\,+\,4\;y^4\right\}\;/\;\text{.}
     Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y\,,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right) /. Null \left(\left\{2\,x^2+9\,x\,y+x^3\,y\,,\,9\,x\,y-5\,x\,y^3+4\,y^4\right\}\right) /.
                        Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\})
                                                 4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                9 \times y - 5 \times y^3 + 4 y^4} /. Null (\{2 \times^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + 4 y^4\} /.
                                                                      Null (\{2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4\}) /. Null (\{2x^2 + 9xy + x^3y, y^4\})
                                                                                            9 \times y - 5 \times y^3 + 4 y^4 /. Null (\{2 x^2 + 9 \times y + x^3 y, 9 \times y - 5 \times y^3 + y^3 \}
                                                                                                        4y^{4} /. Null (\{2x^{2} + 9xy + x^{3}y, 9xy - 5xy^{3} + 4y^{4}\} /.
                                                                                                           Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                     Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                           Null (2x^2 + 9xy + x^3y, 9xy - 5xy^3 + 4y^4) /.
                                                                                                                           Null (2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4) /.
                                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           Null (\{2 x^2 + 9 x y + x^3 y, 9 x y - 5 x y^3 + 4 y^4\} /.
                                                                                                                           \left\{ \text{Null } \left( x \to \text{4.00203} \times \text{10}^{-\text{40}} \right) \text{, Null } \left( y \to \text{2.39972} \times \right. \right.
                                                                                                                           10^{-8}) }) ) ) ) ) ) ) ) ) ) ) ) rasidualsx = 1, y = 5
```

In[ • ]:=

FindRoot[ $\{x^3y + 2x^2 + 9xy, 4y^4 - 5xy^3 + 9xy\}, \{x, 1\}, \{y, 4\}$ ]

```
Out[ • ]=
              \{x \rightarrow 1.68081 \times 10^{-39}, y \rightarrow 2.7359 \times 10^{-8}\}
```

```
In[\circ]:= For[i=-5, i<5, i++;
           xyrule = FindRoot[\{x^3y + 2x^2 + 9xy, 4y^4 - 5xy^3 + 9xy\}, \{x, 1\}, \{y, 4.6\}]
          ]
  ln[*]:= (*3.12-3.27*)
           (*We give the value for x and loop in y and z, first we find the roots,
          then we verify the loop is working and give 2 known roots. *)
  In[ • ]:=
           (*This is for subcause2*)
  In[*]:= ClearAll
Out[ • ]=
          ClearAll
  In[ • ]:=
          xyrule = FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2, -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x Abs[z], \xi - \alpha x + \beta y + \gamma z\},
              \{x, 1\}, \{y, -5\}, \{z, -5\}
Out[ • ]=
          \{x \rightarrow 0.316228, y \rightarrow -0.102138, z \rightarrow 0.136655\}
  In[\bullet]:= \left\{ \alpha \times -\beta y - \gamma z + \psi \times^2, -\alpha \times +\beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \xi -\alpha \times +\beta y + \gamma z \right\} /. \text{ xyrule}
          \left\{-8.67362 \times 10^{-19}, -6.93889 \times 10^{-18}, -8.67362 \times 10^{-19}\right\}
  In[\circ]:= For[i = -5, i < 5, i++;
            xyrule = FindRoot [\{\alpha x - \beta y - \gamma z + \psi x^2,
                 -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x \text{ Abs}[z], \ \xi - \alpha x + \beta y + \gamma z\}, \ \{x, 1\}, \ \{y, i\}, \ \{z, i\}];
              \{\alpha \times \beta y - \gamma z + \psi x^2, -\alpha \times \beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \xi - \alpha \times \beta y + \gamma z\} /. xyrule;
            Print[xyrule, "", rasiduals, "x =1", ",", "y= ", "z = ", i]
```

```
\{x \rightarrow 0.316228, y \rightarrow -0.102138, z \rightarrow 0.136655\}
                                \{0., -1.73472 \times 10^{-18}, -1.73472 \times 10^{-18}\}x = 1, y = z = -4
                            \{x \rightarrow 0.316228, y \rightarrow -0.102138, z \rightarrow 0.136655\}
                                \left\{0., -5.37764 \times 10^{-17}, -1.73472 \times 10^{-18}\right\} x = 1, y = z = -3
                            \{x \rightarrow 0.316228, y \rightarrow -0.102138, z \rightarrow 0.136655\}
                                \{0., -1.73472 \times 10^{-18}, -1.73472 \times 10^{-18}\}x = 1, y = z = -2
                            \{x \rightarrow 0.316228, y \rightarrow -0.102138, z \rightarrow 0.136655\}
                                \{0., -1.73472 \times 10^{-18}, -1.73472 \times 10^{-18}\}x =1,y= z = -1
                            \{x \rightarrow 0.316228, y \rightarrow 0.101505, z \rightarrow 0.0959267\}
                                \{1.73472 \times 10^{-18}, -2.60209 \times 10^{-18}, -3.46945 \times 10^{-18}\} x =1,y= z = 0
                             \{x \rightarrow 0.316228, y \rightarrow 0.101505, z \rightarrow 0.0959267\} \{0., -8.67362 \times 10^{-19}, -1.73472 \times 10^{-18}\} x = 1, y = z = 1, y = z = 1, z = 1
                            \{x \rightarrow 0.316228, y \rightarrow 0.101505, z \rightarrow 0.0959267\} \{0., -1.73472 \times 10^{-18}, -1.73472 \times 10^{-18}\} x = 1, y = z = 2, y = 10^{-18}, y =
                            \{x \rightarrow 0.316228, y \rightarrow 0.101505, z \rightarrow 0.0959267\}
                               \{-8.67362 \times 10^{-19}, 1.1189 \times 10^{-16}, -8.67362 \times 10^{-19}\}x = 1, y = z = 3
                            \{x \rightarrow 0.316228, y \rightarrow 0.101505, z \rightarrow 0.0959267\} \{0., -1.73472 \times 10^{-18}, -1.73472 \times 10^{-18}\} x = 1, y = z = 4
                            \{x \to 0.316228, y \to 0.101505, z \to 0.0959267\}\{0., 0., -1.73472 \times 10^{-18}\}x = 1, y = z = 5
     In[ • ]:=
                            (*This is for x = -1*)
                           ClearAll
Out[ • ]=
                            ClearAll
     ln[\bullet]:= xyrule = FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2, -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x Abs[z], \xi - \alpha x + \beta y + \gamma z\},
                                     \{x, -1\}, \{y, -5\}, \{z, -5\}
Out[ • ]=
                            \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
    In[\,\circ\,]:=\left\{\alpha\,x\,-\,\beta\,y\,-\,\gamma\,z\,+\,\psi\,\,x^2\,,\,\,-\,\alpha\,x\,+\,\beta\,y\,+\,\gamma\,z\,+\,\theta\,\,y^2\,-\,\varepsilon\,\,x\,\,\text{Abs}\,[\,z\,]\,\,,\,\,\,\mathcal{E}\,-\,\alpha\,\,x\,+\,\beta\,\,y\,+\,\gamma\,\,z\,\right\}\,\,\text{/. xyrule}
Out[ • ]=
                            \{0., -1.38778 \times 10^{-17}, 6.93889 \times 10^{-18}\}
                           For [i = -5, i < 5, i++;
                                xyrule = FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2,
                                               -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x \text{ Abs}[z], \ \xi - \alpha x + \beta y + \gamma z\}, \ \{x, -1\}, \ \{y, i\}, \ \{z, i\}\};
                                 rasiduals =
                                      Print[xyrule, "", rasiduals, "x =-1", ",", "y= ", "z = ", i ]
```

Out[ • ]=

Out[ • ]=

 $\{0.05 - 0.2 x, -0.01, -0.05\},$ 

```
\{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{0., -3.46945 \times 10^{-18}, -6.93889 \times 10^{-18}\}x = -1, y = z = -4
         \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{0., 4.82253 \times 10^{-16}, 6.93889 \times 10^{-18}\}x = -1, y = z = -3
         \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{0., 1.04083 \times 10^{-17}, 6.93889 \times 10^{-18}\} x =-1,y= z = -2
         \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{0., -3.46945 \times 10^{-18}, -6.93889 \times 10^{-18}\} x =-1,y= z = -1
         \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{3.46945 \times 10^{-18}, -4.51028 \times 10^{-17}, 3.46945 \times 10^{-18}\} x =-1,y= z = 0
          \{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
            \{0., -3.46945 \times 10^{-18}, -6.93889 \times 10^{-18}\} x = -1, y = z = 1
          \{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
           \{3.46945 \times 10^{-18}, 0., 3.46945 \times 10^{-18}\}x = -1, y = z = 2
          \{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
           \{0., -3.46945 \times 10^{-18}, -6.93889 \times 10^{-18}\} x = -1, y = z = 3
          \{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
           \{0., -3.46945 \times 10^{-18}, -6.93889 \times 10^{-18}\}x = -1, y = z = 4
         \{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
            \{3.46945 \times 10^{-18}, 2.42861 \times 10^{-17}, 3.46945 \times 10^{-18}\} x =-1,y= z = 5
          (*This is for April 2, defined a 3x3 matrics for Jacobian, x y, z*)
In[*]:= ClearAll
In[•]:= ClearAll
         \mathsf{mat} = \left\{ \{ \alpha + 2 \, \psi \, \mathsf{x} \,,\, -\beta \,,\, -\gamma \} \,,\, \left\{ -\alpha - \varepsilon \, \mathsf{Abs} \, [\, \mathsf{z} \,] \,,\, \beta + 2 \, \theta \, \mathsf{y} \,,\, \gamma - \varepsilon \,\, \frac{\mathsf{z}}{\mathsf{Abs} \, [\, \mathsf{z} \,]} \, \mathsf{x} \right\} ,\, \left\{ -\alpha \,,\, \beta \,,\, \gamma \right\} \right\}
         mat // MatrixForm
         ClearAll
```

 $\left\{-0.05-0.01\,\text{Abs[z], 0.01+2.y, 0.05}-\frac{0.01\,x\,z}{\text{Abs[z]}}\right\}, \; \left\{-0.05, \, 0.01, \, 0.05\right\}\right\}$ 

```
In[*]:= \begin{pmatrix} 0.05 - 0.2 \times & -0.01 & -0.05 \\ -0.05 - 0.01 & Abs[z] & 0.01 + 2. & y & 0.05 - \frac{0.01 \times z}{Abs[z]} \\ -0.05 & 0.1 & 0.05 \end{pmatrix}
          Det[mat]
Out[ • ]=
          \{0.05 - 0.2 x, -0.01, -0.05\},
           \left\{-0.05 - 0.01 \, Abs[z], \, 0.01 + 2. \, y, \, 0.05 - \frac{0.01 \, x \, z}{Abs[z]}\right\}, \, \left\{-0.05, \, 0.1, \, 0.05\right\}
Out[ • ]=
          0. + 1.35525 \times 10^{-20} \text{ x} - 0.02 \text{ x y} - \frac{0.00002 \text{ x}^2 \text{ z}}{\text{Abs[z]}}
 In[\circ]:= For[i = -5, i < 5, i++;
            xyrule = FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2,
                 -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x \text{ Abs}[z], \ \xi - \alpha x + \beta y + \gamma z\}, \ \{x, -1\}, \ \{y, i\}, \ \{z, i\}];
             \{\alpha \times \beta y - \gamma z + \psi \times^2, -\alpha \times \beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \xi - \alpha \times \beta y + \gamma z\} /. xyrule;
            rasiduals = {mat} /.xyrule;
            Print[xyrule, "", rasiduals, "x =-1", ",", "y= ", "z = ", i] x
             Print[xyrule, "", rasiduals, "x =-1", ",", "y= ", "z = ", i ]
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -4
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -4
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
            \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -3
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -3
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
            \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -2
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
           \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
           x = -1, y = z = -2
          \{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
            \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
```

x = -1, y = z = -1

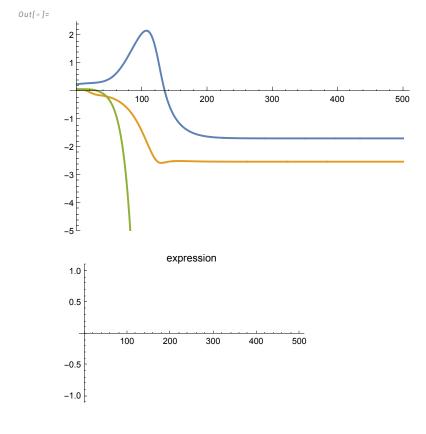
```
\{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = -1
\{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 0
\{x \rightarrow -0.316228, y \rightarrow -0.0917911, z \rightarrow -0.49787\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0549787, -0.173582, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 0
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 1
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 1
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 2
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 2
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 3
\{\,x\,\rightarrow\,-\,\text{0.316228}\,\text{, }y\,\rightarrow\,\text{0.0911586}\,\text{, }z\,\rightarrow\,-\,\text{0.534459}\,\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 3
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 4
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 4
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 5
\{x \rightarrow -0.316228, y \rightarrow 0.0911586, z \rightarrow -0.534459\}
 \{\{\{0.113246, -0.01, -0.05\}, \{-0.0553446, 0.192317, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
 x = -1, y = z = 5
```

```
In[ • ]:=
       For [i = -5, i < 5, i++;
         xyrule = FindRoot[\{\alpha x - \beta y - \gamma z + \psi x^2,
              -\alpha x + \beta y + \gamma z + \theta y^2 - \epsilon x \text{ Abs}[z], \ \xi - \alpha x + \beta y + \gamma z\}, \ \{x, 1\}, \ \{y, i\}, \ \{z, i\}\};
         rasiduals =
          \{\alpha \times -\beta y - \gamma z + \psi \times^2, -\alpha \times +\beta y + \gamma z + \theta y^2 - \epsilon \times Abs[z], \xi -\alpha \times +\beta y + \gamma z\} /. xyrule;
         rasiduals = {mat} /.xyrule;
         Print[xyrule, "", rasiduals, "x =1", ",", "y= ", "z = ", i] x
          Print[xyrule, "", rasiduals, "x =1", ",", "y= ", "z = ", i]
       (*This is for the plot*)
       ClearAll
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -4
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -4
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -3
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -3
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -2
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -2
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -1
       \{x \rightarrow 0.316228, y \rightarrow -0.106322, z \rightarrow 0.137492\}
        \{\{-0.0132456, -0.01, -0.05\}, \{-0.0513749, -0.202644, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = -1
       \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = 0
       \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
        \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
        x = 1, y = z = 0
```

```
\{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 1
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 1
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 2
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 2
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 3
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 3
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 4
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 4
         \{\, x \, \rightarrow \, \text{0.316228} \, , \, \, y \, \rightarrow \, \text{0.104425} \, , \, \, z \, \rightarrow \, \text{0.0953428} \, \}
          \{\{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 5
         \{x \rightarrow 0.316228, y \rightarrow 0.104425, z \rightarrow 0.0953428\}
          \{\{-0.0132456, -0.01, -0.05\}, \{-0.0509534, 0.218849, 0.0468377\}, \{-0.05, 0.01, 0.05\}\}\}
          x = 1, y = z = 5
Out[ • ]=
         ClearAll
 In[.] = \alpha = .05;
         \beta = .01;
         \gamma = .05;
         \delta = .02;
         \epsilon = .03;
         \xi = .01;
         \eta = .05;
         \theta = 1;
         \psi = -.1;
```

```
s = NDSolve[\{x'[t] = \alpha x[t] + \beta y[t] - \gamma z[t] + \psi x[t]^2 - \delta z[t] \times y[t],
                y'[t] = \beta y[t] - \alpha x[t] + \gamma z[t] + \theta y[t]^2 - \epsilon Abs[z[t]] \times x[t],
                z'[t] = g + \gamma z[t] - \alpha x[t] + \beta y[t] - \eta x[t] \times Abs[y[t]],
                x[0] = 0.2753, y[0] = 0.0283, z[0] = 0.011\}, \{x, y, z\}, \{t, 500\}]
Out[ • ]=
          \Big\{\Big\{x \to InterpolatingFunction\Big| \hspace{0.1in} \boxplus \hspace{0.1in} \Big\}
              \textbf{y} \rightarrow \textbf{InterpolatingFunction} \Big[
              z \rightarrow InterpolatingFunction
```

Plot[Evaluate[ $\{x[t], y[t], z[t]\} /.s], \{t, 0, 500\}, PlotRange \rightarrow \{2.5, -5\}]$  $Plot[\{x[t], y[t], z[t]\}, \{t, 0, 500\}, PlotLabel \rightarrow "expression"]$ 



In[•]:= ClearAll Out[ • ]=

ClearAll