

CL Tutorial 5

all line separators are double lines, i am unaware of how to typeset double lines

Exercise 1

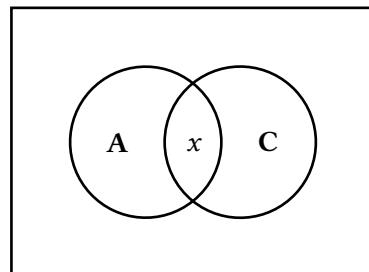


Figure 1: $a \not\models \neg c$

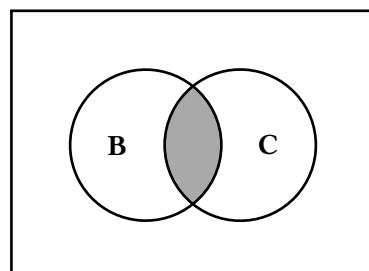


Figure 2: $b \models \neg c$

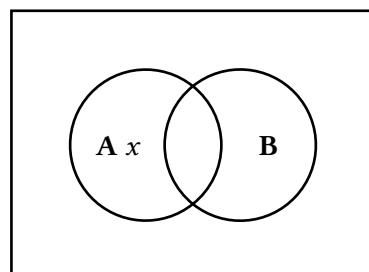


Figure 3: $a \not\models b$

$$\begin{aligned} & \therefore \frac{a \not\models \neg c \quad b \models \neg c}{a \not\models b} \\ & \equiv \frac{b \models \neg c \quad a \not\models \neg c}{a \not\models b} \\ & \equiv \text{celantes} \end{aligned}$$

Exercise 2

$$\begin{aligned}
 \frac{a \models b \quad b \models c}{a \models c} &\equiv \frac{a \models \neg b \quad \neg b \models c}{a \models c} && \text{variable swap} \\
 &\equiv \frac{a \models \neg b \quad \neg b \models \neg c}{a \models \neg c} && \text{variable swap} \\
 &\equiv \frac{a \models \neg b \quad \neg \neg c \models \neg \neg b}{a \models \neg c} && \text{contrapone sequent} \\
 &\equiv \frac{a \models \neg b \quad c \models b}{a \models \neg c} && \text{double negation} \\
 &\equiv \frac{a \models \neg b \quad a \not\models \neg c}{c \not\models b} && \text{contrapone rules}
 \end{aligned}$$

Exercise 3

$$\begin{array}{c}
 \frac{}{p \wedge q \models p \wedge q} I \\
 \frac{}{p, q \models p \wedge q} \wedge L \\
 \frac{}{\models p \wedge q, \neg p, \neg q} \neg R, \neg R \\
 \frac{}{\models (p \wedge q) \vee \neg p \vee \neg q} \vee R, \vee R
 \end{array}$$

Exercise 4

$$\begin{array}{c}
 \frac{}{\Gamma, q \models \Delta} I \quad \frac{}{\Gamma \models p, \Delta} \neg L \\
 \frac{\Gamma, q \models \Delta \quad \Gamma \models p, \Delta}{\Gamma, p \rightarrow q \models \Delta} \rightarrow L \\
 \frac{}{\Gamma, q \vee \neg p \models \Delta} \text{def} \\
 \therefore \\
 \frac{\Gamma, q \models \Delta \quad \Gamma \models p, \Delta}{\Gamma, p \rightarrow q \models \Delta} \rightarrow L
 \end{array}$$

Figure 5: $\neg L$

$$\begin{array}{c}
 \frac{\Gamma, p \models q, \Delta}{\Gamma \models q, \neg p, \Delta} \neg R \\
 \frac{\Gamma \models q, \neg p, \Delta}{\Gamma \models q \vee \neg p, \Delta} \vee R \\
 \frac{\Gamma \models q \vee \neg p, \Delta}{\Gamma \models p \rightarrow q, \Delta} \text{def} \\
 \therefore
 \end{array}$$

$$\begin{array}{c}
 \frac{\Gamma, p \models q, \Delta}{\gamma, \models p \rightarrow q, \Delta} \rightarrow R
 \end{array}$$

Figure 6: $\neg R$