

CL Tutorial 5

all line separators are double lines, i am unaware of how to typeset double lines

Exercise 1

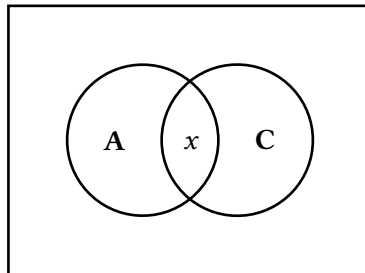


Figure 1: $a \not\models \neg c$

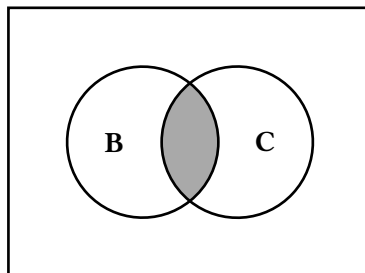


Figure 2: $b \models \neg c$

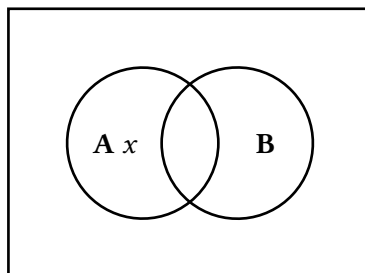


Figure 3: $a \not\models b$

$$\begin{aligned}
 & \therefore \frac{a \not\models \neg c \quad b \models \neg c}{a \not\models b} \\
 & \equiv \frac{b \models \neg c \quad a \not\models \neg c}{a \not\models b} \\
 & \equiv \text{celantes}
 \end{aligned}$$

Exercise 2

$$\begin{aligned}
\frac{a \models b \quad b \models c}{a \models c} &\equiv \frac{a \models \neg b \quad \neg b \models c}{a \models c} && \text{variable swap} \\
&\equiv \frac{a \models \neg b \quad \neg b \models \neg c}{a \models \neg c} && \text{variable swap} \\
&\equiv \frac{a \models \neg b \quad \neg \neg c \models \neg \neg b}{a \models \neg c} && \text{contrapone sequent} \\
&\equiv \frac{a \models \neg b \quad c \models b}{a \models \neg c} && \text{double negation} \\
&\equiv \frac{a \models \neg b \quad a \not\models \neg c}{c \not\models b} && \text{contrapone rules}
\end{aligned}$$

Exercise 3

$$\begin{aligned}
&\frac{}{p \wedge q \models p \wedge q} I \\
&\frac{}{p, q \models p \wedge q} \wedge L \\
&\frac{}{p, q \models p \wedge q} \neg R, \neg R \\
&\frac{}{\models p \wedge q, \neg p, \neg q} \vee R, \vee R \\
&\frac{}{\models (p \wedge q) \vee \neg p \vee \neg q}
\end{aligned}$$

Exercise 4

$$\begin{aligned}
&\frac{}{\Gamma, q \models \Delta} I \quad \frac{}{\Gamma \models p, \Delta} I \\
&\frac{}{\Gamma, q \models \Delta} \neg L \quad \frac{}{\Gamma, \neg p \models \Delta} \neg L \\
&\frac{}{\Gamma, q \vee \neg p \models \Delta} \vee L \\
&\frac{}{\Gamma, p \rightarrow q \models \Delta} \text{def} \\
&\vdots \\
&\frac{\Gamma, q \models \Delta \quad \Gamma \models p, \Delta}{\Gamma, p \rightarrow q \models \Delta} \rightarrow L \\
&\text{Figure 5: } \neg L
\end{aligned}$$

$$\begin{aligned}
&\frac{}{\Gamma, p \models q, \Delta} \neg R \\
&\frac{}{\Gamma \models q, \neg p, \Delta} \neg R \\
&\frac{}{\Gamma \models q \vee \neg p, \Delta} \vee R \\
&\frac{}{\Gamma \models p \rightarrow q, \Delta} \text{def} \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
&\frac{}{\Gamma, p \models q, \Delta} \rightarrow R \\
&\frac{}{\gamma, \models p \rightarrow q, \Delta} \\
&\text{Figure 6: } \neg R
\end{aligned}$$