

CL Tut 4

Ex1

$$\begin{array}{c}
 \frac{}{a \wedge b \vdash a \wedge b} \wedge R \\
 \frac{a \wedge b \vdash a \wedge b}{a, b \vdash a \wedge b} \wedge L \\
 \frac{a, b, \neg(a \wedge b) \vdash}{\neg(a \wedge b) \vdash \neg a, \neg b} \neg R, \neg R \\
 \frac{\neg(a \wedge b) \vdash \neg a, \neg b}{\neg(a \wedge b) \vdash \neg a \vee \neg b} \vee R
 \end{array}$$

Ex2:

$$\begin{array}{c}
 \frac{}{\Gamma, x \vdash x, \Delta} \mid \quad \frac{}{\Gamma, x \vdash x, \Delta} \mid \quad \frac{}{\Gamma, y \vdash y, \Delta} \mid \quad \frac{}{\Gamma, z \vdash z, \Delta} \mid \\
 \frac{}{x, x \vee z \vdash x, y \wedge z} \mid \quad \frac{y, x \vdash x, y \wedge z}{y, x \vee z \vdash x, y \wedge z} \vee L \quad \frac{y, z \vdash x, y}{y, z \vdash x, y \wedge z} \wedge R \\
 \frac{x \vee y, x \vee z \vdash x, y \wedge z}{(x \vee y) \wedge (x \vee z) \vdash x \vee (y \wedge z)} \wedge L, \vee R
 \end{array}$$

Ex3:

$$\begin{array}{c}
 \frac{}{x \vdash x, \Delta} \mid \quad \frac{}{\Gamma, y \vdash y, \Delta} \mid \\
 \frac{x \vdash x, \neg y, z}{x \vdash x \wedge y, \neg y, z} \wedge R \quad \frac{x, y \vdash y, z}{x \vdash y \wedge y, z} \wedge R \quad \frac{z \vdash z, \Delta}{z \vdash x \wedge y, \neg y, z} \wedge R \\
 \frac{x \vdash x \wedge y, \neg y, z}{x \vee z \vdash x \wedge y, \neg y \vee z} \vee R \quad \frac{z \vdash x \wedge y, \neg y \vee z}{z \vdash x \wedge y, \neg y \vee z} \vee R \\
 \frac{x \vee z \vdash x \wedge y, \neg y \vee z}{\vdash (x \wedge y), \neg(x \vee z), (\neg y \vee z)} \neg R \\
 \frac{\vdash (x \wedge y), \neg(x \vee z), (\neg y \vee z)}{\vdash (x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))} \vee R, \vee R
 \end{array}$$