CL Tutorial 8

Exercise 1

Logical Expression:

$$(\neg a \land b) \lor (a \land \neg b)$$

Tseytin Transformation

substitutions:

$$\begin{aligned}
w \leftrightarrow \neg a &\equiv (\neg a \lor \neg w) \land (w \lor a) \\
x \leftrightarrow \neg b &\equiv (\neg b \lor \neg x) \land (x \lor b) \\
y \leftrightarrow w \land b &\equiv (w \lor \neg y) \land (b \lor \neg y) \land (y \lor \neg w \lor \neg b) \\
z \leftrightarrow a \land x &\equiv (\neg z \land a) \land (\neg z \lor x) \land (z \lor \neg a \lor \neg x) \\
r \leftrightarrow y \lor z &\equiv (r \lor \neg y) \land (r \lor \neg z) \land (\neg r \lor y \lor z)
\end{aligned}$$

conjoining, for $r = 1$ (satisfiable):

$$\begin{aligned}
(\neg a \lor \neg w) \land (w \lor a) \land (\neg b \lor \neg x) \land (x \lor b) \\
\land (w \lor \neg y) \land (b \lor \neg y) \land (y \lor \neg w \lor \neg b) \\
\land (\neg z \lor a) \land (\neg z \lor x) \land (z \lor \neg a \lor \neg x) \\
\land (1 \lor \neg y) \land (1 \lor \neg z) \land (\neg 1 \lor y \lor z)
\end{aligned}$$

simplifying:

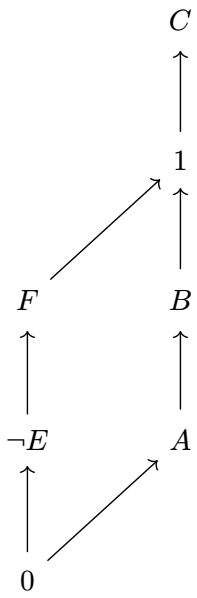
$$\begin{aligned}
(\neg a \lor \neg w) \land (w \lor a) \land (\neg b \lor \neg x) \land (x \lor b) \\
\land (w \lor \neg y) \land (b \lor \neg y) \land (y \lor \neg w \lor \neg b) \\
\land (\neg z \land a) \land (\neg z \lor x) \land (z \lor \neg a \lor \neg x) \\
\land (y \lor z)
\end{aligned}$$

Exercise 2

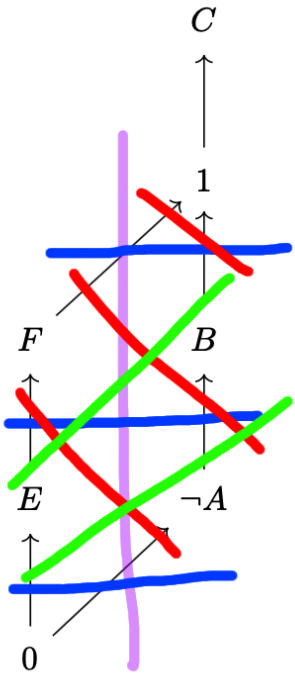
Expression:

$$(E \vee F) \wedge (\neg A \vee B) \wedge C$$
$$\equiv (\neg E \to F) \wedge (A \to B) \wedge C$$

Diagram:



Cuts:



3 blue + 3 red + 2 green + 1 purple

=> 9 satisfying assignments