

CL Tut 4

Ex 1

$$\begin{array}{c}
 \frac{\overline{a \wedge b \vdash a \wedge b}}{a, b \vdash a \wedge b} \text{ AR} \\
 \frac{a, b \vdash a \wedge b}{\neg(a \wedge b) \models \neg a, \neg b} \text{ \neg L} \\
 \frac{\neg(a \wedge b) \models \neg a, \neg b}{\neg(a \wedge b) \models \neg a \vee \neg b} \text{ \neg R, \neg R} \\
 \frac{\neg(a \wedge b) \models \neg a \vee \neg b}{\neg(a \wedge b) \models \neg a \vee \neg b} \text{ VR}
 \end{array}$$

Ex 2:

$$\begin{array}{c}
 \frac{\Gamma, x \vdash x, \Delta}{\Gamma, x \vee z \vdash x, y \wedge z} \quad \frac{\Gamma, x \vdash x, \Delta}{y, x \vdash x, y \wedge z} \\
 \frac{x, x \vee z \vdash x, y \wedge z \quad y, x \vee z \vdash x, y \wedge z}{x \vee y, x \vee z \vdash x, y \wedge z} \text{ \vee L} \\
 \frac{x \vee y, x \vee z \vdash x, y \wedge z}{(x \vee y) \wedge (x \vee z) \vdash x \vee (y \wedge z)} \text{ \wedge L, VR}
 \end{array}$$

Ex 3:

$$\begin{array}{c}
 \frac{\Gamma, y \vdash y, \Delta}{x \vdash x, \Delta} \quad \frac{\Gamma, y \vdash y, \Delta}{x, y \vdash y, z} \\
 \frac{x \vdash x, \gamma y, z \quad x, y \vdash y, z}{x \vdash x \wedge y, \gamma y, z} \text{ AR} \quad \frac{x \vdash x \wedge y, \gamma y, z}{z \vdash z, \Delta} \\
 \frac{x \vdash x \wedge y, \gamma y, z}{x \vdash x \wedge y, \gamma y \vee z} \text{ VR} \quad \frac{z \vdash z, \Delta}{z \vdash x \wedge y, \gamma y, z} \text{ VR} \\
 \frac{x \vdash x \wedge y, \gamma y \vee z \quad z \vdash x \wedge y, \gamma y, z}{x \vee z \vdash x \wedge y, \gamma y \vee z} \text{ \vee L} \\
 \frac{x \vee z \vdash x \wedge y, \gamma y \vee z}{\models (x \wedge y), \neg(x \vee z), (\neg y \vee z)} \text{ \neg R} \\
 \frac{\models (x \wedge y), \neg(x \vee z), (\neg y \vee z)}{\models (x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))} \text{ VR, VR}
 \end{array}$$