Quantum Mechanics for Mathematicians: Problem Set 9

Due Monday, April 15

Problem 1: Compute the following commutators of elements of the Lie algebra of the Poincaré group

$$[K_l, P_m]$$

where the K_l ($l = 1, 2, 3$) generate boosts in the Lorentz subgroup, and P_m ($m = 0, 1, 2, 3$) generate translations.

Problem 2:

- For the real scalar field theory, find the momentum operator acting on the quantum field theory state space, in terms of the scalar quantum field. Show that this gives the expected expression in terms of a sum of the number operators for each momentum mode and the corresponding momentum.
- Repeat the same calculation for the complex scalar field theory.

Problem 3: In class we showed that taking two real free scalar fields, one could make a theory with $SO(2)$ symmetry, and we found the charge operator Q that gives the action of the Lie algebra of $SO(2)$ on the state space of this theory. Instead, consider two complex free scalar fields, and show that this theory has a $U(2)$ symmetry. Find the four operators that give the Lie algebra action for this symmetry on the state space, in terms of a basis for the Lie algebra of $U(2)$. Note that this is the field content and symmetry of the Higgs sector of the standard model (where the difference is that the theory is not free, but interacting, and has a lowest energy state not invariant under the symmetry).