Representation Theory PSET 1

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Proposition 1. *Consider the category of topological abelian groups with continuous homomorphisms between them. Show this category is additive but not abelian.*

Proof. $\square$

Proposition 2. *Let $S \to R$ be a homomorphism of rings. It induces the restriction and induction functors between the corresponding categories of modules, Res :* **R-Mod** $\to$ **S-Mod** *and Ind :* **S-Mod** $\to$ **R-Mod***, where Ind $A = R \otimes_S A$ and Res B is the module B viewed as an S-module. Show that these functors are adjoint in the sense that* $\mathrm{Hom}_R(\mathit{Ind}\ A, B) = \mathrm{Hom}_S(A, \mathit{Res}\ B)$.

Proposition 3. *Show that every module in category $\mathcal{O}$ is finitely generated as a $\mathcal{U}\mathfrak{n}_-$-module.*

Proposition 4. *For $V_1, V_2 \in \mathit{Obj}\ \mathcal{O}$, consider the tensor product $V_1 \otimes V_2$. Show that it is in $\mathcal{O}$ is one of the factors is finite-dimensional, but not in general.*

Proposition 5. *Consider the action of $GL(3)$ on polynomials of degree d in x_1, x_2, x_3. Resolve this representation by Verma modules.*