Differentiable Manifolds Problem Set 4

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Problem 1

Let $\pi : E \to M$ be a smooth vector bundle of rank k over a smooth manifold M. Suppose that $\{U_\alpha\}_{\alpha \in A}$ is an open cover of M, and for each $\alpha \in A$ we are given a smooth local trivialization $\Phi_\alpha : \pi^{-1}(U_\alpha) \to U_\alpha \times \mathbb{R}^k$ of E. For each $\alpha, \beta \in A$ such that $U_\alpha \cap U_\beta \neq \varnothing$, let $\tau_{\alpha\beta} : U_\alpha \cap U_\beta \to GL(k, \mathbb{R})$ be the transition function defined by $\Phi_\alpha \circ \Phi_\beta^{-1}(p, v) = (p, \tau_{\alpha\beta}(p)v)$.