Physics 6047
Problem Set 5, due 2/28/13
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1. In class, we worked out the scattering amplitude for $2 \to 2$ scattering of (real) scalar ϕ particles by using the LSZ formula, which can be expressed as:

$$
{}_{\rm out}\langle k_1' k_2' | k_1 k_2 \rangle_{\rm in} - \text{"}1\text{"} = i^4 ({k_1'}^2 + m^2)({k_2'}^2 + m^2)({k_1}^2 + m^2)({k_2}^2 + m^2)
$$
$$
\int d^4 x_1' d^4 x_2' d^4 x_1 d^4 x_2 \, e^{i(-k_1' x_1' - k_2' x_2' + k_1 x_1 + k_2 x_2)} \langle 0 | T \phi(x_1') \phi(x_2') \phi(x_1) \phi(x_2) | 0 \rangle \quad (1)
$$

We computed the four point function (written in shorthand as $\langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 \rangle$) in a somewhat circuitous manner: by first computing the generating functional and then taking its derivatives. Here, I ask you to compute the four point function (and the scattering amplitude) in a more direct way i.e. without going through the generating functional. Specifically, as argued in class, the only part of the four point function that contributes to non-trivial scattering is its connected part i.e.

$$
\langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 \rangle_c = \langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 \rangle - \langle \phi_{1'} \phi_{2'} \rangle \langle \phi_1 \phi_2 \rangle - \langle \phi_{1'} \phi_1 \rangle \langle \phi_{2'} \phi_2 \rangle - \langle \phi_{1'} \phi_2 \rangle \langle \phi_{2'} \phi_1 \rangle \qquad (2)
$$

Compute the lowest order contribution to this connected four point function (i.e. $O(g^2)$) by noting that:

$$
\begin{aligned}
\langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 \rangle &= \int D\phi \, \phi_{1'} \phi_{2'} \phi_1 \phi_2 \, e^{iS_{\rm free} + iS_{\rm int.}} \\
&\sim \int D\phi \, \phi_{1'} \phi_{2'} \phi_1 \phi_2 \left[1 + iS_{\rm int.} + (iS_{\rm int.})^2/2 + ... \right] e^{iS_{\rm free}} \\
&\sim \langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 \rangle_{\rm free} + \langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 (iS_{\rm int.}) \rangle_{\rm free} + \frac{1}{2} \langle \phi_{1'} \phi_{2'} \phi_1 \phi_2 (iS_{\rm int.})^2 \rangle_{\rm free} + ... , \quad (3)
\end{aligned}
$$

where $S_{\rm free}$ is the free scalar action $S_{\rm free} = \int d^4 x [-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2]$, and $S_{\rm int.}$ contains the interactions. For this problem, you need only focus on $S_{\rm int.} = \int d^4 x \frac{g}{3!} \phi^3$. After you obtain the (connected) four-point function, *compute* the $2 \to 2$ scattering amplitude by using the LSZ formula. Check that it agrees with what was derived in the class.

For problems 2, 3 and 4 below, if you can guess the correct Feynman rules, you can skip steps and directly write down the answers. But if you are unsure, for instance whether symmetry factors cancel or whether you have included all the relevant diagrams, by all means derive from scratch, using the generating functional, or the method outlined above.

2. Work out the 2-to-2 tree scattering amplitude for the $\frac{1}{4!}\lambda\phi^4$ theory of the last problem set. By 'tree', we mean no loops. In this context, it's the amplitude that involves only one interaction vertex, with incoming momenta k_1 and k_2, and outgoing momenta k_1' and k_2'.

3. Srednicki problem 10.5.

4. Srednicki problem 11.4