Physics 6047
Problem Set 6, due 3/7/13
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1. In class, we did not work out in detail the loop contribution and the wave-function/mass-renormalization contribution to the 2 point function. *Show that*:

$$\frac{1}{2}(ig)^2 \int d^d x d^d y \frac{1}{i}\Delta(x_1 - x)\frac{1}{i}\Delta(x_2 - y)\left[\frac{1}{i}\Delta(x-y)\right]^2 =$$
$$\int \frac{d^d k}{(2\pi)^d} e^{ik\cdot(x_1 - x_2)} \left[\left(\frac{1}{i}\tilde{\Delta}(k^2)\right)^2 \frac{1}{2}(ig)^2 \int \frac{d^d \ell}{(2\pi)^d}\frac{1}{i}\tilde{\Delta}(\ell^2)\frac{1}{i}\tilde{\Delta}((\ell+k)^2)\right] . \quad (1)$$

This is from the loop term. Here, $\tilde{\Delta}(k^2)/i$ is the Fourier transform of the free propagator i.e. $\tilde{\Delta}(k^2) = 1/(k^2 + m^2 - i\epsilon)$.

Show that:

$$-i \int d^d x \frac{1}{i}\Delta(x_1 - x)\left[-A\Box_x + Bm^2\right]\frac{1}{i}\Delta(x_2 - x) =$$
$$\int \frac{d^d k}{(2\pi)^d} e^{ik\cdot(x_1 - x_2)} \left[\left(\frac{1}{i}\tilde{\Delta}(k^2)\right)^2 (-i)(Ak^2 + Bm^2)\right] . \quad (2)$$

This is from the wave-function/mass counter-terms.

The above justifies why we say the lowest order contributions to the 1PI amplitude $i\Pi(k^2)$ is:

$$i\Pi(k^2) = \frac{1}{2}(ig)^2 \int \frac{d^d \ell}{(2\pi)^d}\frac{1}{i}\tilde{\Delta}(\ell^2)\frac{1}{i}\tilde{\Delta}((\ell+k)^2) - i(Ak^2 + Bm^2) . \quad (3)$$

2. Srednicki 13.1. Note that this is not a perturbative calculation. No Feynman diagram necessary.

3. Srednicki 10.4.

4. We need the following trick when evaluating the loop integral. Show that:

$$\frac{1}{AB} = \int_0^1 dx\,[xB + (1-x)A]^{-2} \quad (4)$$

One way to proceed goes like this. Use the fact that $1/A = \int_0^\infty e^{-tA}dt$, and $1/B = \int_0^\infty e^{-t'B}dt'$. Choose an appropriate change of variable $(t, t') \to (x, y)$ such that you can integrate over y and be left with an integral over x of the form in eq. (4).