Notes on Modules

Nilay Kumar

Last updated: July 7, 2013

0 Modules

Let A be a ring. An A-**module** is an Abelian group M with a multiplication map $A \times M \to M$, written $(f, m) \mapsto fm$ that satisfies

(i) $f(m \pm n) = fm \pm fn$;

(ii) $(f + g)m = fm + gm$;

(iii) $(fg)m = f(gm)$;

(iv) $1_A m = m$

for all $f, g \in A$ and $m, n \in M$. We call a subset $N \subset M$ a **submodule** if $fm + gn \in N$ for all $f, g \in A$ and $m, n \in N$. A **homomorphism** is a map $t : M \to N$ of A-modules that is A-linear, in the obvious sense that $t(fm + gn) = ft(m) + gt(n)$ for all $f, g \in A$ and $m, n \in M$.

1 Exact Sequences