DIFFERENTIABLE MANIFOLDS HOMEWORK 9

THIS PROBLEM SET WILL NOT BE COLLECTED OR GRADED.
INSTEAD, SOLUTIONS WILL BE EMAILED TO YOU LATER.

(1) Suppose M is an n-dimensional embedded submanifold of $\mathbb{R}^{2n+1}$. Show that there is a $v \in \mathbb{R}^{2n+1}$ such that $\pi_v|_M$ is a smooth immersion into $\mathbb{R}^{2n}$. (see problem 6.2 on page 147 of [1])
(2) Exercise 6.1 on page 126 of [1]
(3) Problem 6.1 on page 147 of [1].
(4) Show that the Möbius band is not orientable by showing that any 2-form must vanish at a point.
(5) Problem 15-1 on page 397 of [1].
(6) Problem 15-12 on page 398 of [1].
(7) Problem 16-2 on page 434 of [1].
(8) Let ω_1 be the $(n-1)$ form on $\mathbb{R}^n$ defined by

$$\omega_1 = \sum_{i=1}^{n} (-1)^{(i-1)} x^i dx^1 \wedge \cdots \wedge \widehat{dx^i} \wedge \cdots \wedge dx^n$$

   (a) Show that the restriction of ω_1 to S^{n-1} is an orientation form.
   (b) Equip S^{n-1} with the orientation as the boundary of B^n. Use Stoke's Theorem to compute the integral $\int_{S^{n-1}} \omega_1$.
   (c) Let $\omega_2 = |x|^{-n}\omega_1$. Show that ω_2 is closed but not exact on $\mathbb{R}^n - \{0\}$.

References

[1] Lee, John M. *Introduction to Smooth Manifolds,* Second edition, Graduate Texts in Mathematics, 218, Springer-Verlag, New York, 2012.

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