

LIE GROUPS AND REPRESENTATIONS, FALL 2013

Problem Set 3

Due Monday, October 21

Problem 1: Kirillov Problem 4.5

Problem 2: (a) Show that

$$\pi : t \rightarrow \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

gives a representation of the group \mathbf{R} on \mathbf{C}^2 .

(b) Find all subrepresentations.

(c) Show this representation is not unitary, that is is reducible, but not completely reducible.

Problem 3: Kirillov Problem 4.7

Problem 4: Kirillov Problem 4.10

Problem 5: Prove that the Frobenius-Schur indicator

$$\frac{1}{|G|} \sum_{g \in G} \chi_V(g^2)$$

for an irreducible representation V over \mathbf{R} takes three possible values: $-1, 0, 1$. For each of these cases, what is

$$\text{Hom}_G(V, V)$$

Note: this is not covered in Kirillov, and you should try consulting some other sources to learn more about this. One place that has a good discussion is the textbook of Fulton and Harris, in section 3.5.