Commutative Algebra: Problem Set 7

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Problem 2

Let us find a basis for $L(D)$ when $D = 2v_0 + 3v_1$. Recall the definition

$$L(D) = \left\{ f \in K^\times \mid (f) + D \geq 0 \right\}.$$

Since the principal divisor of f is just defined as $(f) = \sum_v v(f)v$, we see that the condition on $f \in L(2v_0 + 3v_1)$ allows poles of up to order 2 at 0 and poles of up to order 3 at 1. It is easy to see, then, that the following set of rational functions satisfy the condition:

$$\left\{ 1, \frac{1}{x}, \frac{1}{x^2}, \frac{1}{x-1}, \frac{1}{(x-1)^2}, \frac{1}{(x-1)^3} \right\}.$$

Any product of these can be decomposed by partial fractions into these basis elements. It is clear that these are in fact a basis for $L(D)$ as a k-vector space (the dimension is 6, which falls into the bound we proved in class as $\deg D = 5$).

Next consider $D = 2v_0 + 2v_\infty$. Here we are allowed poles at 0 of up to order 2 and poles at infinity up to order 2. The basis is then

$$\left\{ 1, x, x^2, \frac{1}{x}, \frac{1}{x^2} \right\}.$$

For exactly the same reasons as above, we see that this gives us a basis for $L(D)$ as a k-vector space (again, the bound checks out).

Problem 3

Problem 4