DIFFERENTIABLE MANIFOLDS HOMEWORK 2

DUE FEB. 19 , 2013 IN CLASS

(1) Prove that a subspace of a Hausdorff and second countable topological space is itself Hausdorff and second countable.

(2) Construct a partition of unity for S^2 subject to the open cover which consists of the two stereographical coordinate charts (U, ϕ) and (V, ψ) in problem 1 of Homework 1. Thus, find smooth functions u and v on S^2 such that
   (a) supp $u \subset U$ and supp $v \subset V$.
   (b) $0 \leq u \leq 1$, $0 \leq v \leq 1$ and $u + v = 1$ everywhere on S^2.

(3) Let $SL(n)$ be the set of all $n \times n$ matrices with determinant 1. Show that $SL(n)$ is a differentiable manifold. You may assume Problem 7-4 on page 171 of [1].

(4) Consider a map $F : \mathbb{R}^4 \to \mathbb{R}^2$ defined by
$$F(x, y, s, t) = (x^3 + y, x^3 + y^2 + s^2 + t^2 + y).$$
Show that $(0, 1)$ is a regular value of F and that the level set $F^{-1}((0, 1))$ is diffeomorphic to S^2. Find the subset of $F^{-1}((0, 1))$ on which x and y can be solved as smooth functions of s and t.

(5) Problem 2-3 on page 48 of [1].

(6) Read Lemma 2.26 (Extension Lemma for Smooth Functions) on page 45 of [1] and do Exercise 2.27 on the same page.

References

[1] Lee, John M. *Introduction to Smooth Manifolds,* Second edition, Graduate Texts in Mathematics, 218, Springer-Verlag, New York, 2012.