

DIFFERENTIABLE MANIFOLDS HOMEWORK 8

DUE APRIL 25, 2013 IN CLASS

- (1) Let m, n, k be positive integers with $k \leq \min\{m, n\}$. Show that an $m \times n$ matrix has rank k if and only if there exists a $k \times k$ submatrix with nonzero determinant.
- (2) Let $M(m \times n)$ be the space of real $m \times n$ matrices as a differentiable manifold and $M_k(m \times n)$ be the set consists of matrices with rank k . Show that $M_k(m \times n)$ is an embedded submanifold of $M(m \times n)$ with codimension $(m - k) \times (n - k)$.
- (3) Show that any closed subset of a compact space is compact and any compact subset of a Hausdorff space is closed.
- (4) 5-4 on page 123 of [1]
- (5) 5-5 on page 123 of [1].
- (6) 5-11 on page 123 of [1].

REFERENCES

- [1] Lee, John M. *Introduction to Smooth Manifolds*, Second edition, Graduate Texts in Mathematics, 218, Springer-Verlag, New York, 2012.