Representation Theory Notes

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Last updated: June 18, 2013

Definition 1. A **representation** of a finite group G on a finite-dimensional complex vector space V is a homomorphism $\rho : G \to GL(V)$ of G to the group of automorphisms of V. A map ϕ between two representations V and W of G is a vector space map $\phi : V \to W$ such that $g \cdot \phi = \phi \cdot g$ and is called a **morphism** of representations.

A **subrepresentation** of a representation of V is a vector subspace W of V which is invariant under G. A representation V is called irreducible if there is no proper nonzero invariant subspace W of V.

If V and W are representations, the direct sum $V \oplus W$ and the tensor product $V \otimes W$ are also representations. These are given by $g(v, w) = (gv, gw)$ and $g(v \otimes w) = gv \otimes gw$.