PROBLEM SET II

1. Let $a_1, \cdots, a_{2N+1}$ be $2N+1$ pairwise distinct points in $\mathbf{C}$.

(a) Construct the Riemann surface $\hat{\Sigma}$ of the function $w = \sqrt{\prod_{j=1}^{2N+1}(z-a_j)}$. What is the topology of this surface?

(b) Determine all the zeroes and poles of the two functions z and w on $\hat{\Sigma}$.

(c) Consider the forms $\omega_k = z^k \frac{dz}{w}$. For which values of k are the forms ω_k holomorphic on $\hat{\Sigma}$?

2. Same questions as in the previous problem with the function $w = \sqrt{\prod_{j=1}^{2N}(z-a_j)}$.

3. Consider the Riemann surface $\hat{\Sigma}$ of the function $w = \sqrt{z(z-1)(z-\lambda)}$, where λ is a complex number different from 0 and 1. Let Q be an arbitrary point on $\hat{\Sigma}$. Construct a meromorphic form $\omega_Q(p)$ on $\hat{\Sigma}$ with exactly a double pole at $p = Q$. (Distinguish carefully between the case where Q is or is not a branch point $0, 1, \infty$.)