7.1-Linearization:

Initial conditions: $\theta_0 = 0, \alpha_0 = 0, \dot{\theta}_0 = 0, \dot{\alpha}_0 = 0$

$z^T = [\,\theta, \alpha, \dot{\theta}, \dot{\alpha}, \ddot{\theta}, \ddot{\alpha}] = [0\ 0\ 0\ 0\ 0\ 0]$

$$f(z) = \left(m_p L_r^2 + \frac{1}{4} m_p L_p^2 - \frac{1}{4} m_p L_p^2 \cos(\alpha)^2 + J_r\right)\ddot{\theta} - \left(\frac{1}{2} m_p L_p L_r cos(\alpha)\right)\ddot{\alpha}$$

$$+ \left(\frac{1}{2} m_p L_p^2 \sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha} + (\frac{1}{2} m_p L_p L_r sin(\alpha))\ \dot{\alpha}^2$$

$$\left.\frac{\partial f(z)}{\partial \ddot{\theta}}\right|_{z=z_0} = m_p L_r^2 + \frac{1}{4} m_p L_p^2 - \frac{1}{4} m_p L_p^2 \cos(0)^2 + J_r = m_p L_r^2 + J_r$$

$$\left.\frac{\partial f(z)}{\partial \ddot{\alpha}}\right|_{z=z_0} = -\frac{1}{2} m_p L_p L_r cos(0) = -\frac{1}{2} m_p L_p L_r$$

$$\left.\frac{\partial f(z)}{\partial \dot{\theta}}\right|_{z=z_0} = \left.\frac{\partial f(z)}{\partial \dot{\alpha}}\right|_{z=z_0} = \left.\frac{\partial f(z)}{\partial \theta}\right|_{z=z_0} = \left.\frac{\partial f(z)}{\partial \alpha}\right|_{z=z_0} = f(z_0) = 0$$

First Equation: $(m_p L_r^2 + J_r)\ddot{\theta} - \frac{1}{2} m_p L_p L_r \ddot{\alpha} = \tau - B_r \dot{\theta}$

$$f(z) = -\left(\frac{1}{2} m_p L_p L_r cos(\alpha)\right)\ddot{\theta} + \left(J_p + \frac{1}{4} m_p L_p^2\right)\ddot{\alpha} - \left(\frac{1}{4} m_p L_p^2 \sin(\alpha)\cos(\alpha)\right)\dot{\theta}^2 - \left(\frac{1}{2} m_p L_p g sin(\alpha)\right)$$

$$\left.\frac{\partial f(z)}{\partial \ddot{\theta}}\right|_{z=z_0} = -\frac{1}{2} m_p L_p L_r$$

$$\left.\frac{\partial f(z)}{\partial \ddot{\alpha}}\right|_{z=z_0} = J_p + \frac{1}{4} m_p L_p^2$$

$$\left.\frac{\partial f(z)}{\partial \alpha}\right|_{z=z_0} = -\frac{1}{2} m_p L_p g$$

Second equation: $-\frac{1}{2} m_p L_p L_r \ddot{\theta} + \left(J_p + \frac{1}{4} m_p L_p^2\right)\ddot{\alpha} - \frac{1}{2} m_p L_p g \alpha = -B_p \dot{\alpha}$

7.2-State space:

$$\begin{bmatrix} m_p L_r^2 + J_r & -\frac{1}{2} m_p L_p L_r \\ -\frac{1}{2} m_p L_p L_r & J_p + \frac{1}{4} m_p L_p^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{a} \end{bmatrix} = \begin{bmatrix} \tau - B_r \dot{\theta} \\ \frac{1}{2} m_p L_p g \alpha - B_p \dot{\alpha} \end{bmatrix}$$

$$J_T = \left(m_p L_r^2 + J_r\right)\left(J_p + \frac{1}{4} m_p L_p^2\right) - \frac{1}{4} {m_p}^2 {L_p}^2 {L_r}^2 = J_p m_p L_r^2 + J_r J_p + \frac{1}{4} J_p m_p L_p^2$$

$$\ddot{\theta} = \frac{1}{J_T}\left(J_p + \frac{1}{4} m_p L_p^2\right)(\tau - B_r \dot{\theta}) + \frac{1}{2J_T} m_p L_p L_r \left(\frac{1}{2} m_p L_p g \alpha - B_p \dot{\alpha}\right)$$

$$\ddot{a} = \frac{1}{2J_T} m_p L_p L_r (\tau - B_r \dot{\theta}) + \frac{1}{J_T}\left(m_p L_r^2 + J_r\right)\left(\frac{1}{2} m_p L_p g \alpha - B_p \dot{\alpha}\right)$$

$$x_1 = \theta, \qquad x_2 = \alpha, \qquad x_3 = \dot{\theta}, \qquad x_4 = \dot{\alpha}$$

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{1}{J_T}\left(-\left(J_p + \frac{1}{4} m_p L_p^2\right) B_r x_3 - \frac{1}{2} m_p L_p L_r B_p x_4 + \frac{1}{4} {m_p}^2 {L_p}^2 L_r g x_2 + \left(J_p + \frac{1}{4} m_p L_p^2\right) u\right)$$

$$\dot{x}_4 = \frac{1}{J_T}\left(\frac{1}{2} m_p L_p L_r B_r x_3 - \left(m_p L_r^2 + J_r\right) B_p x_4 + \frac{1}{2} m_p L_p g \left(m_p L_r^2 + J_r\right) x_2 + \frac{1}{2} m_p L_p L_r u\right)$$

$$A = \frac{1}{J_T} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4} {m_p}^2 L_p^2 L_r g & -\left(J_p + \frac{1}{4} m_p L_p^2\right) B_r & -\frac{1}{2} m_p L_p L_r B_p \\ 0 & \frac{1}{2} m_p L_p g (m_p L_r^2 + J_r) & \frac{1}{2} m_p L_p L_r B_r & -\left(m_p L_r^2 + J_r\right) B_p \end{bmatrix}$$

$$B = \frac{1}{J_T} \begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4} m_p L_p^2 \\ \frac{1}{2} m_p L_p L_r \end{bmatrix}$$

7.3-Open loop poles:

$$A = \frac{1}{J_T}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}{m_p}^2 L_p^2 L_r g & -\left(J_p + \frac{1}{4} m_p L_p^2\right) B_r & -\frac{1}{2} m_p L_p L_r B_p \\ 0 & \frac{1}{2} m_p L_p g (m_p L_r^2 + J_r) & \frac{1}{2} m_p L_p L_r B_r & -(m_p L_r^2 + J_r) B_p \end{bmatrix}$$

$$B = \frac{1}{J_T}\begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4} m_p L_p^2 \\ \frac{1}{2} m_p L_p L_r \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

MATLAB: eig (A) = -17.1, 8.34, -2.87, 0