

N-Body Approach to the Traveling Salesman Problem (*TSP*)

Clayton Tobin

Johnny Seay

Dr. Bryant Wyatt

Dr. Jesse Crawford

Tarleton State University
Math Department



Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?



5 Cities

Number of routes to check

$$\frac{1}{2}(4)(3)(2)(1)$$

$$= \frac{1}{2}(5-1)!$$

=

$$= 12 \text{ routes}$$



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25 Cities

Number of routes to check

$$= \frac{1}{2}(25-1)!$$

=

$$\frac{1}{2}(24)(23)\cdots(3)(2)(1)$$

$$= 310,224,200,866,619,719,680,000$$

routes

$$\cong 310 \text{ sextillion routes}$$

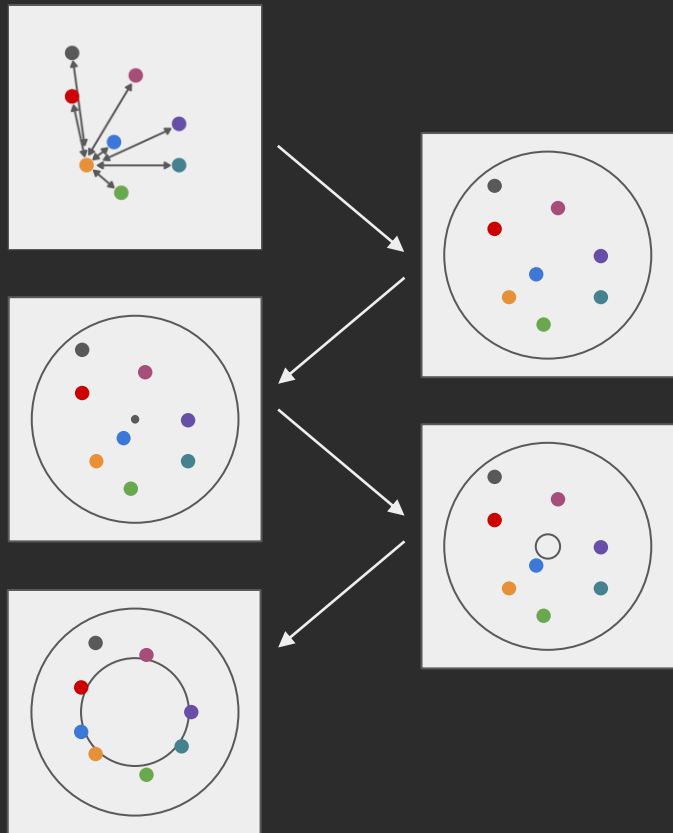
Our Approach

Attach an attractive-repulsive force to each pair of cities.

Create an outer circle surrounding the cities.

Insert smaller circle at the center of the outer circle, with an initial radius of 0.

Increase the inner circle radius over time until it touches the outer circle.

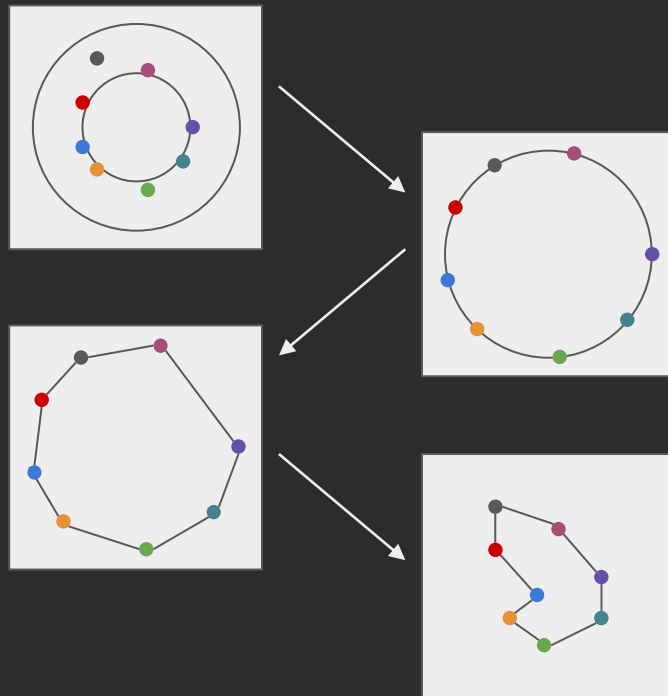
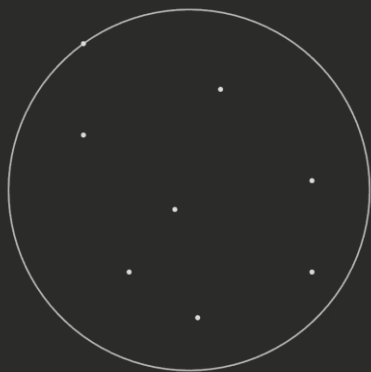


Our Approach *(cont.)*

The circles act as walls that push on the cities.

The cities will end up in a circle.

Obtain a path by observing only the order in which the cities lie on the circle.

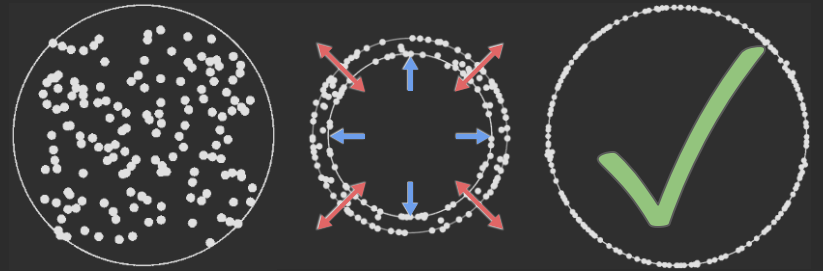


Pressure

As the area between the two walls approaches zero, the total force within the system goes beyond the limits of the numerical scheme.

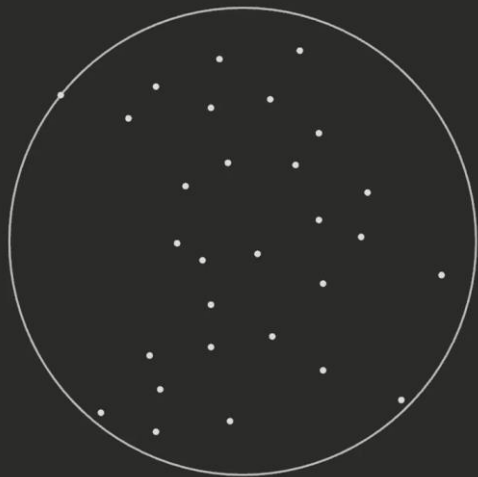
Keeping the pressure on the outer circle within some range helped.

When the pressure on the outer circle gets out of the range, its radius either grows or shrinks to get the pressure back in range.



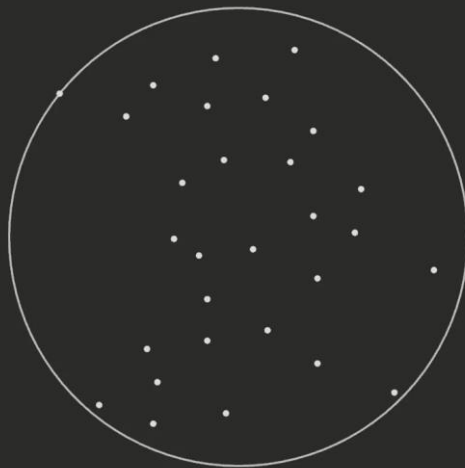
Pressure Example

Without Adjusting for Pressure



n-body path: 10723.421875
percent difference: 15.412926%

Adjusting for Pressure



n-body path: 10055.721680
percent difference: 8.226672%

dataset: *bayg29*
optimal path: 9291.352570

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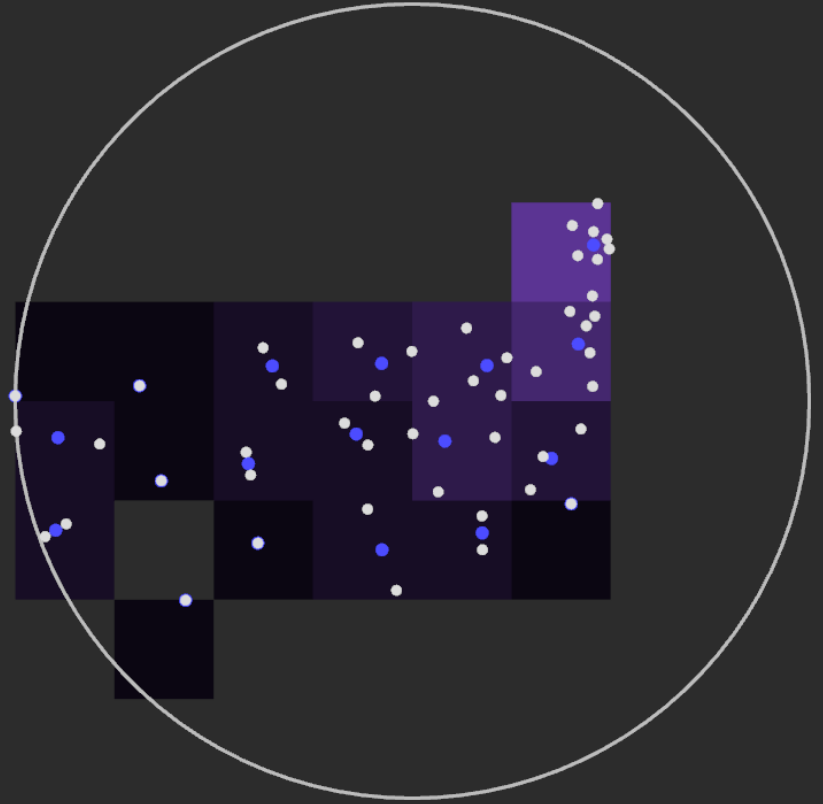
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Density

The cities can sometimes get pushed/pulled to one side, or they may not spread out uniformly.

Dense areas can occasionally lead to unpredictable (and sometimes undesirable) results.

Inserting additional circles in dense areas can lead to better results.



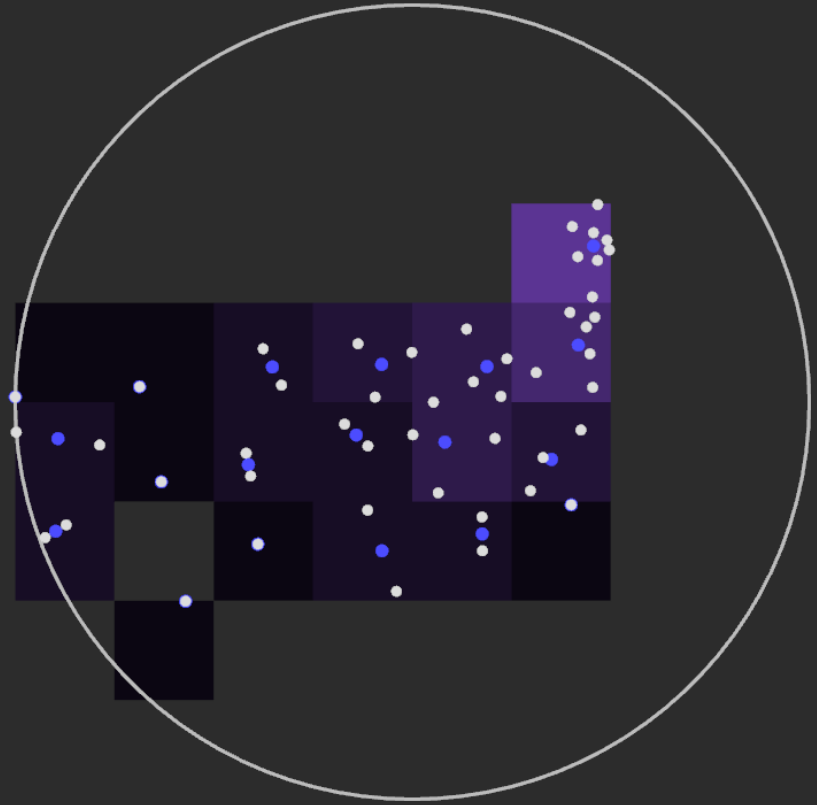
Density

The space is divided into cells.

Density is given by the number of cities within that cell.

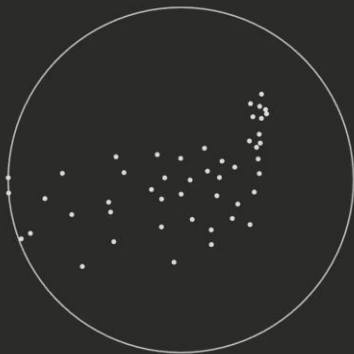
The lighter the cell, the more dense it is.

The purple dot is the centroid of the cities within that cell. Will be used to initially insert additional circles.



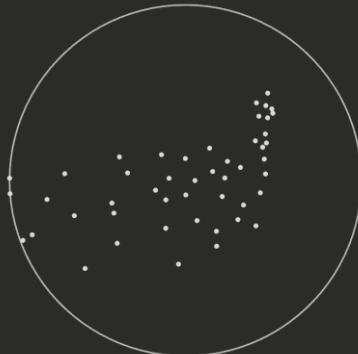
Example Runs

Without Pressure And Density



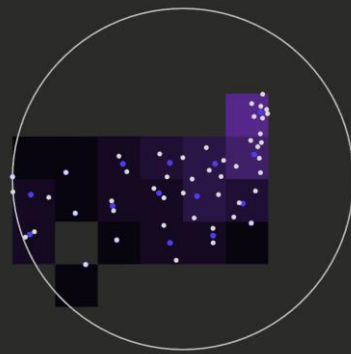
n-body path: 39104.851562
percent difference: 16.648348%

Adjusting for Pressure



n-body path: 36967.234375
percent difference: 10.271914%

Accounting for Density



n-body path: 35725.371094
percent difference: 6.567481%

dataset: *att48*
optimal path: 33523.708

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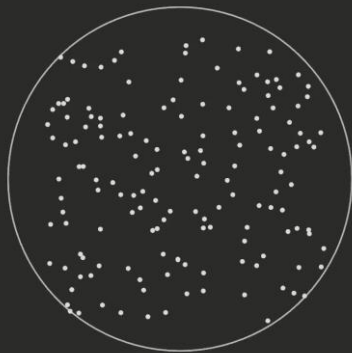


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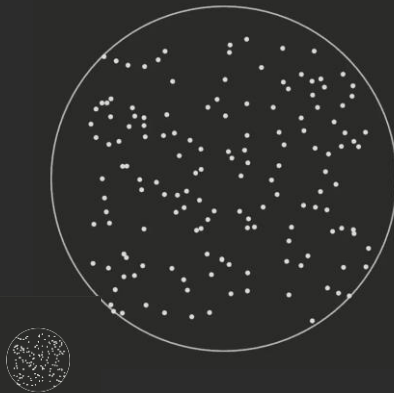
Are Bubbles Always Better?

Without Pressure And Density



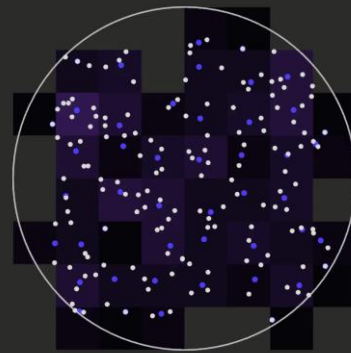
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Accounting for Density



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Sources

[6] Retrieved from https://en.wikipedia.org/wiki/Grumman_LLV

[7] Retrieved from <https://medium.com/@aramis720/science-and-religion-and-the-controversy-over-the-tmt-telescope-on-mauna-kea-16bee51345d5>

[8] Retrieved from <https://microboard.com/mpi-manufacturing-equipment/>

[9 - 12] Retrieved from https://en.wikipedia.org/wiki/Travelling_salesman_problem

[13] Retrieved from <https://royalsocietypublishing.org/doi/10.1098/rsos.180396>

Acknowledgements

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Contact Info

johnny.seay@go.tarleton.edu

clayton.tobin@go.tarleton.edu

Thank You for your time.

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