QUESTION 22/11 SOLUTION (Cantele Alberto/Cancelliere Biagio)

Using C.S. conditions, we need to find **primal** and **dual** of the investment problem:

Primal:

$$\max(\sum_{i=1}^{n} c_i y_i)$$

$$\sum_{i=1}^{n} a_i y_i \leq b \qquad (\tau)$$

$$y_i \leq 1 \quad i = 1, \dots, n \quad (\mu_i)$$
$$y_i \geq 0 \quad i = 1, \dots, n$$

Dual:

$$\min(\tau b + \sum_{i=1}^{n} \mu_i)$$

$$a_i \tau + \mu_i \geq c_i \quad i = 1, \dots, n$$

$$\tau \geq 0, \mu_i \geq 0 \quad i = 1, \dots, n$$

C.S. conditions:

$$\tau\left(\sum_{i=1}^{n} a_i y_i - b\right) = 0$$

$$\mu_i(y_i - 1) = 0 \qquad i = 1, \dots, n$$
$$y_i(a_i\tau + \mu_i - c_i) = 0 \quad i = 1, \dots, n$$

Assuming that $0 < f_H < 1$ from the optimal solution $\bar{y} = [1, \dots, 1, f_H, 0, \dots 0]$, then focus on $[f_H, \dots, 0] \Rightarrow y_h < 1$.

So if $\mu_k(y_k - 1)$ then $\mu_k = 0$ assuming $k = H, \dots, n$

and if $y_H(a_H\tau + \mu_H - C_H) = 0 \Rightarrow \tau = \frac{C_H}{Q_H}$ $assuming\ y_H \geq 0, \mu_H = 0$

Then focus on $[1, \dots, f_H]$, if $y_J(a_J\tau + \mu_J - C_J) = 0 \Rightarrow \mu_J = C_J - a_J\tau \quad J = 1, \dots, h-1$

for each $J < h \quad \Rightarrow \frac{C_J}{a_J} \geq \frac{C_H}{a_H}$