

How do you use evidence to make a decision?

Readings for today

- Gold, J. I., & Shadlen, M. N. (2007). The neural basis of decision making. *Annu. Rev. Neurosci.*, 30, 535-574.

Topics

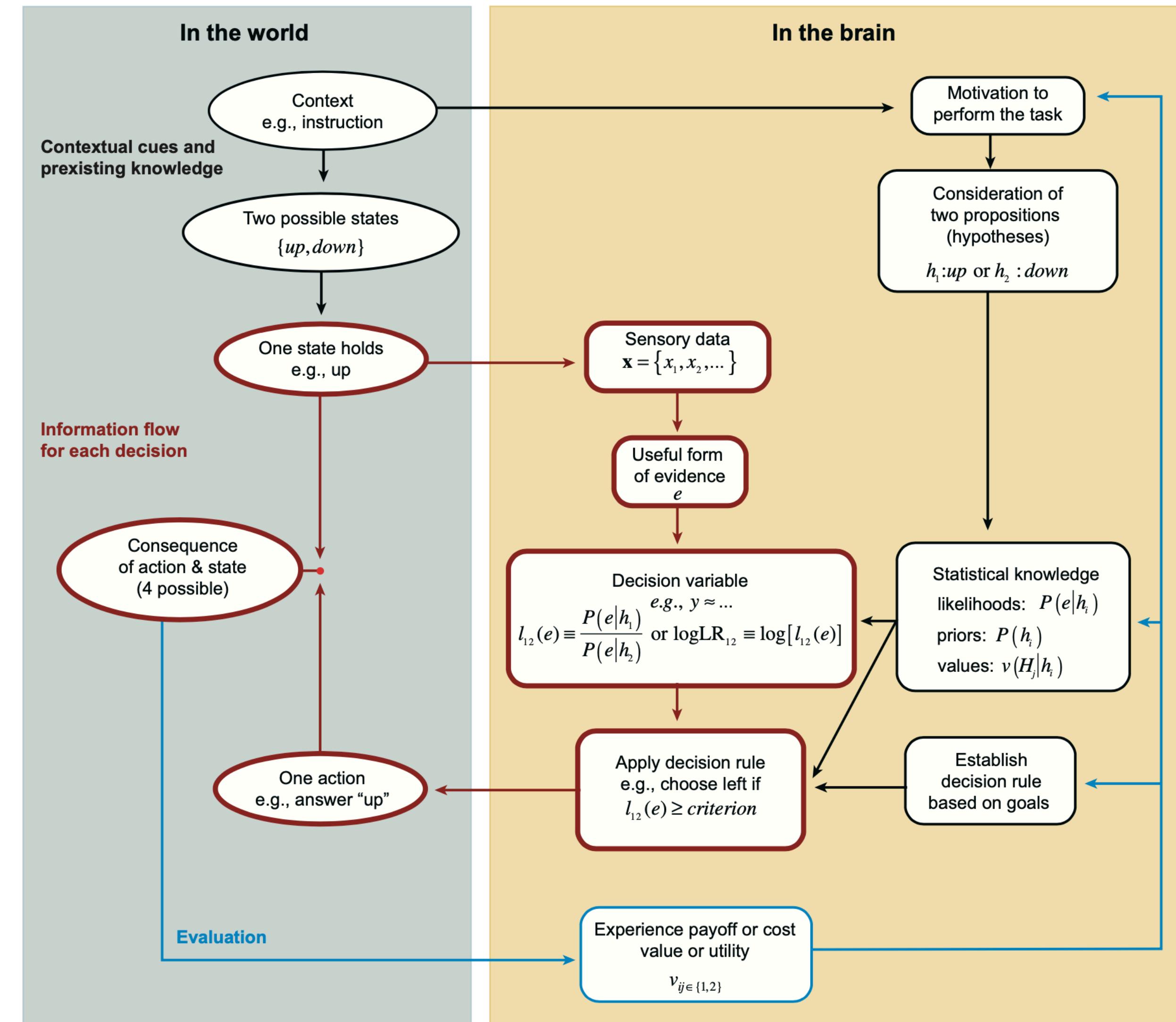
- Information accumulation
- The drift diffusion model
- The “accumulator” agent

Information accumulation

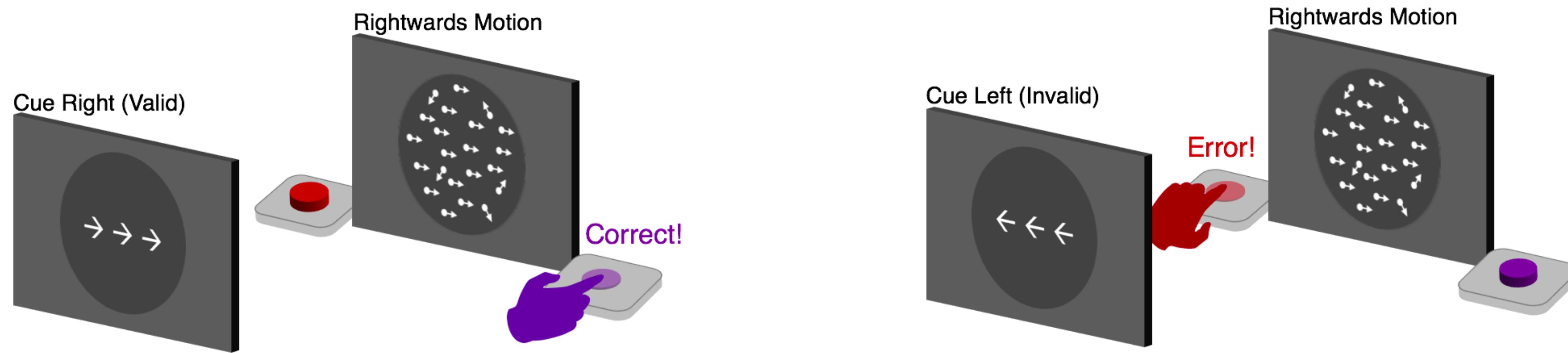
Evidence Accumulation



Elements of making a decision



Cued dot motion task



Serial evaluation of evidence

Sequential analysis framework

$$e_0 \rightarrow f_0(e_0) \Rightarrow \text{Stop}_{\text{or}}$$



$$e_1 \rightarrow f_1(e_0, e_1) \Rightarrow \text{Stop}_{\text{or}}$$

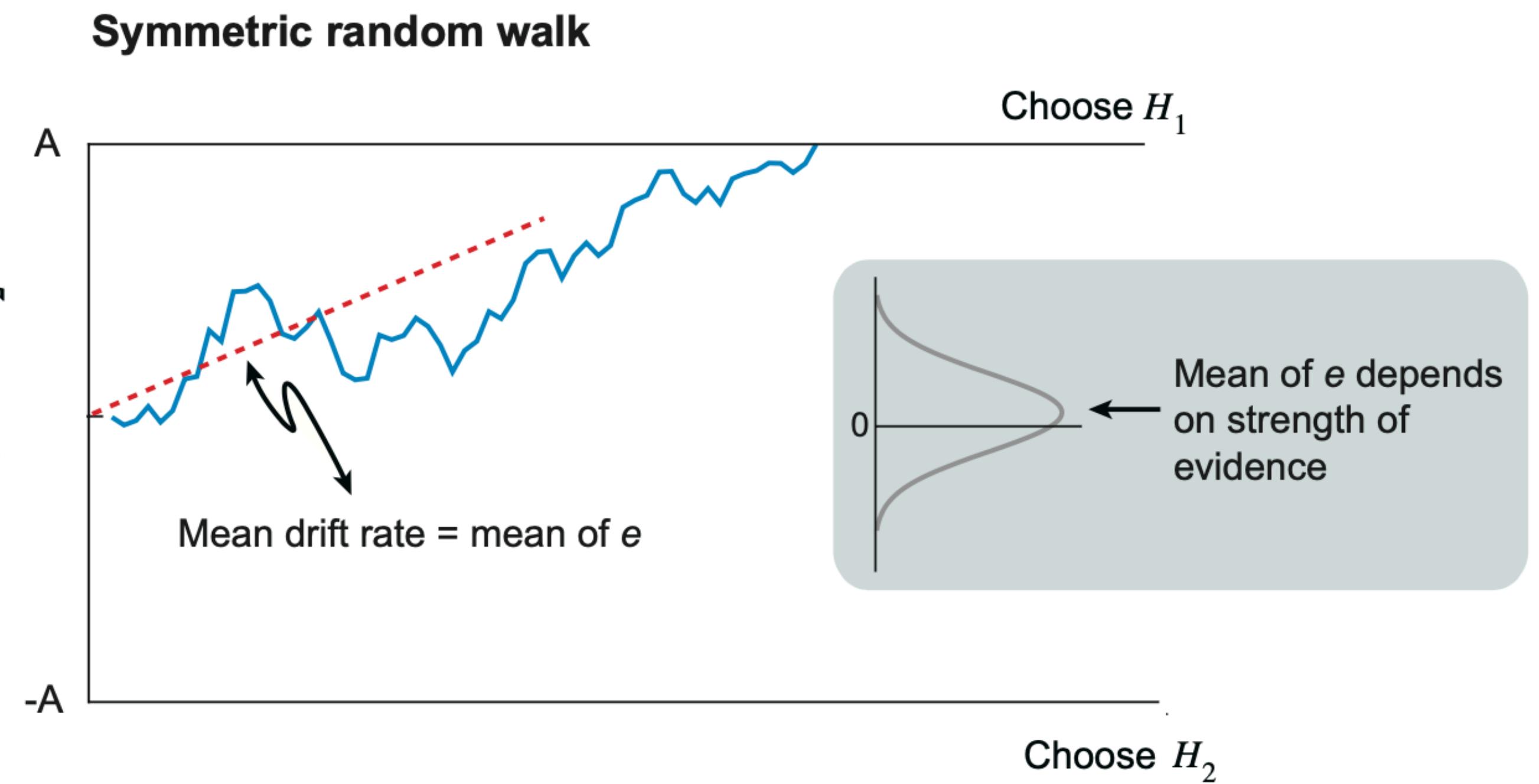


$$e_2 \rightarrow f_1(e_0, e_1, e_2) \Rightarrow \text{Stop}_{\text{or}}$$

⋮

$$e_n \rightarrow f_1(e_0, e_1, \dots, e_n) \Rightarrow \dots$$

Accumulated
evidence
for h_1 over h_2



Log-likelihood ratio (logLR) test

$$\begin{aligned}\log LR_{12} &\equiv \log \frac{P(e_1, e_2, \dots, e_n | h_1)}{P(e_1, e_2, \dots, e_n | h_2)} \\ &= \sum_{i=1}^n \log \frac{P(e_i | h_1)}{P(e_i | h_2)}.\end{aligned}$$

Given n samples of evidence (e), what is the likelihood that the evidence was generated under one of two competing hypotheses (h)?

Problem: When have you accumulated enough evidence to make a decision?

The sequential probability ratio test (SPRT)

1. On each sample i , evaluate logLR

$$w_i = \log \left(\frac{P(e_i | h_1)}{P(e_i | h_2)} \right)$$

2. Sum all logLR tests up to current observation

$$y_n = \sum_{i=1}^n w_i$$

3. Determine stopping rule

$$y_n \geq \log \frac{1-\alpha}{\alpha}, \text{ then select } h_1$$

$$y_n \leq \log \frac{\beta}{1-\beta}, \text{ then select } h_2$$

$$\log \frac{\beta}{1-\beta} \leq y_n \leq \log \frac{1-\alpha}{\alpha}, \text{ continue sampling}$$

α = type I error

β = type II error

The drift diffusion model

The Drift Diffusion Model (DDM)

θ : evidence (decision) variable.

$\theta(0) = z \rightarrow$ initialized with value z at $t = t_{er}$

Continuous time for (Stochastic Diff Eq)

$$d\theta = vdt + \sigma dW$$

dW : Wiener noise process

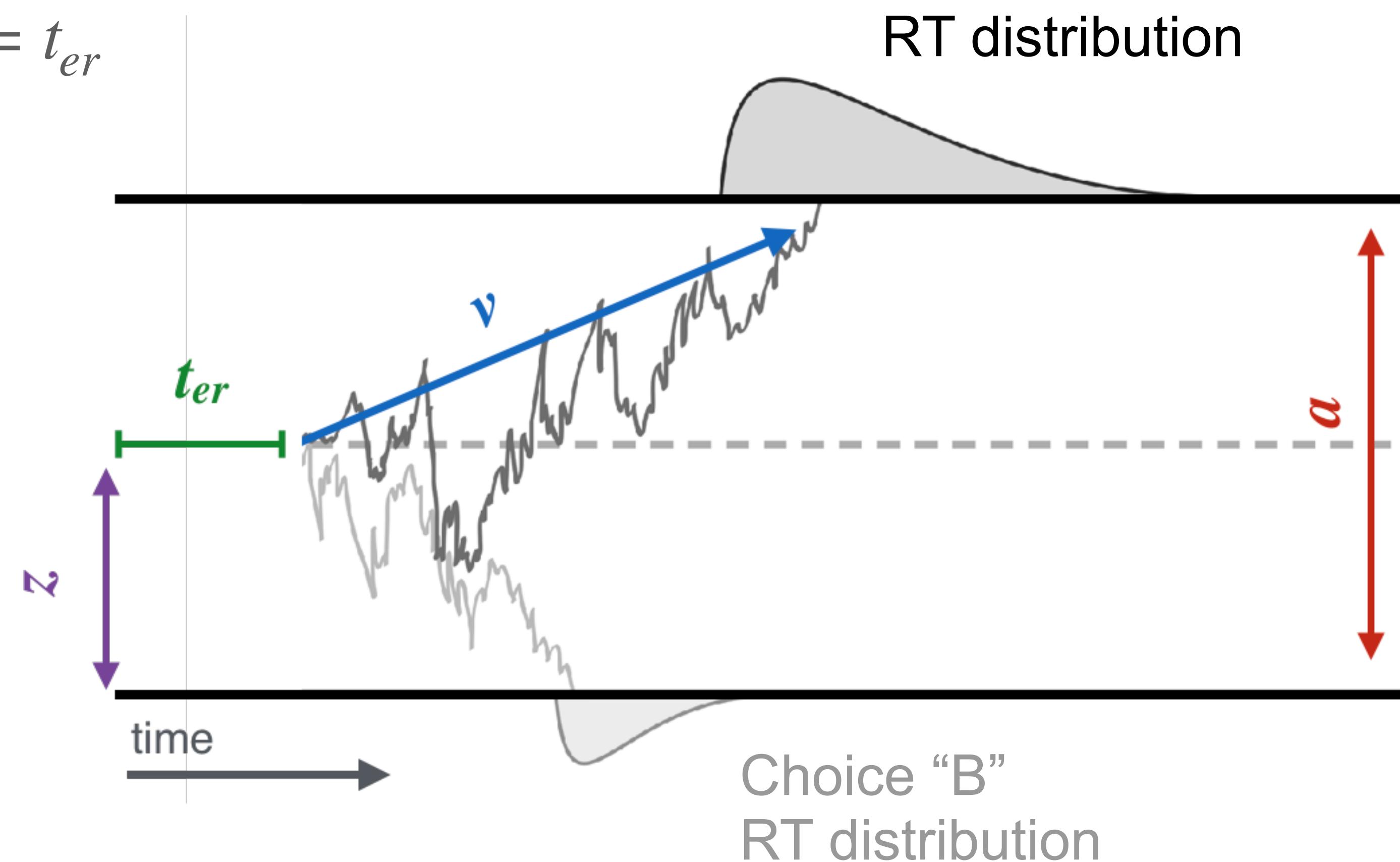
σ : diffusion constant (noise scalar)

Discrete time form

$$\theta(t + \Delta t) = \theta(t) + v\Delta t + \sigma\sqrt{\Delta t}\epsilon(t)$$

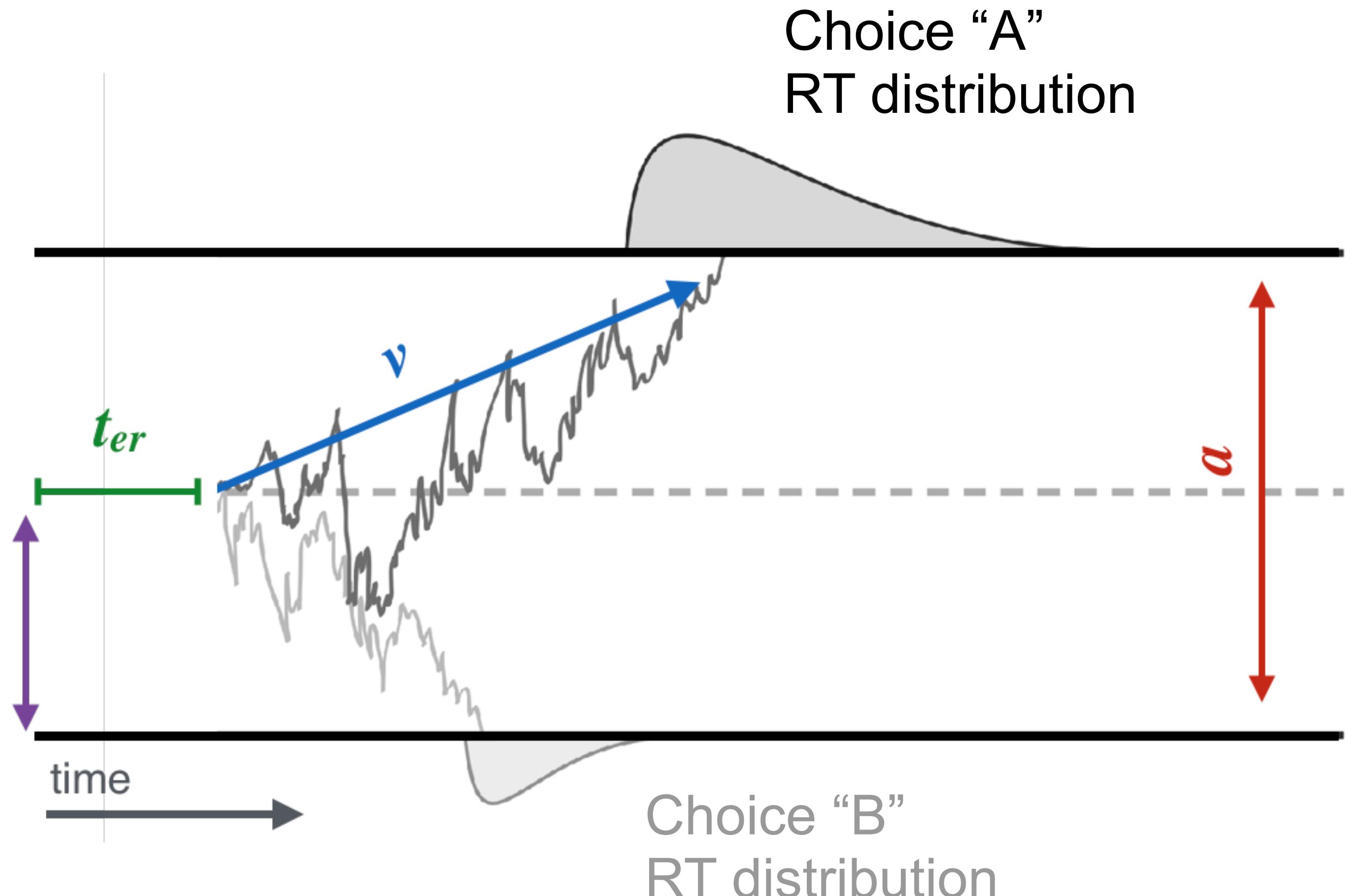
$\epsilon \sim N(0,1)$: Gaussian noise

Δt : timestep



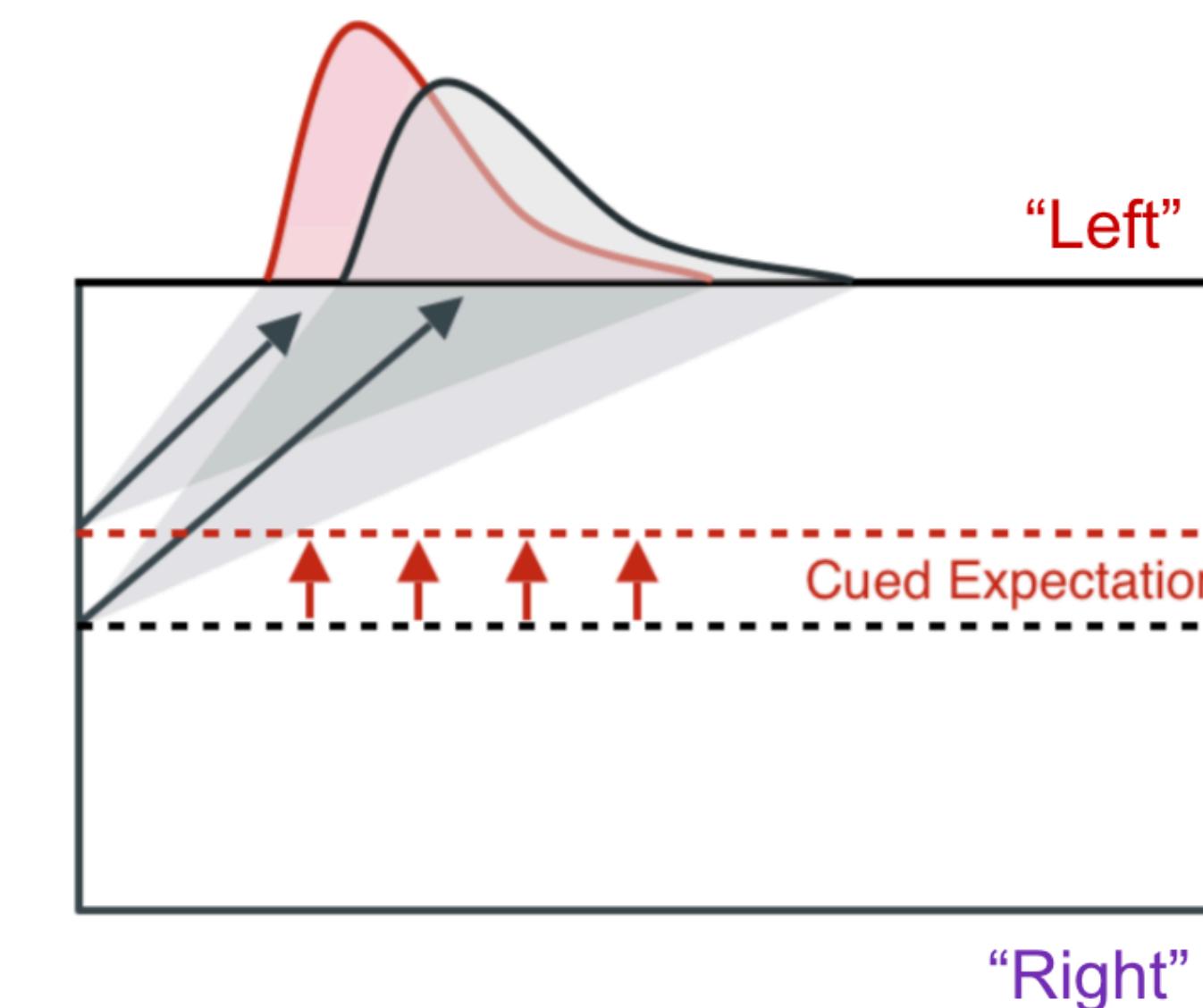
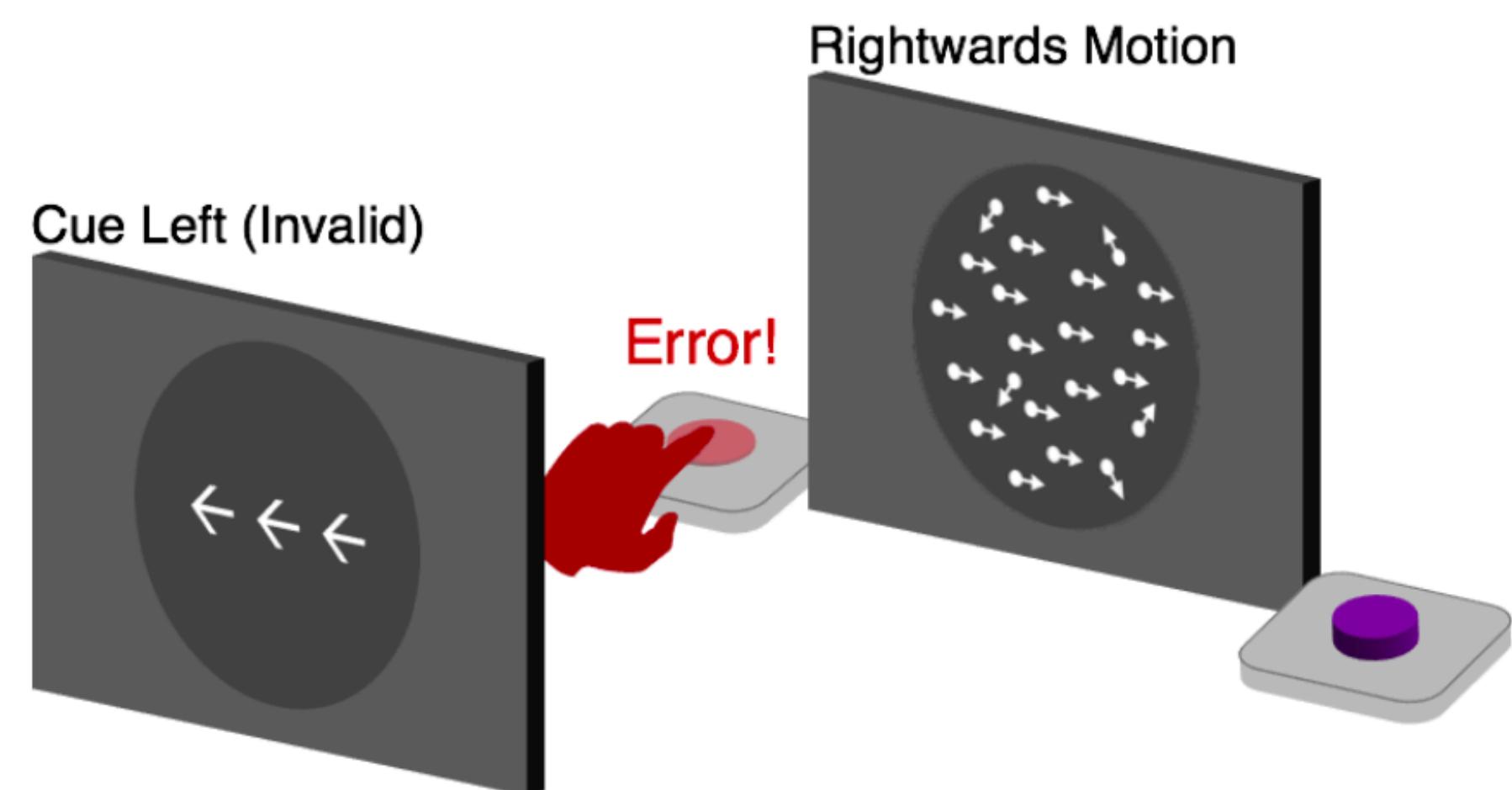
Links between variables & task parameters

- boundary height (a): evidence criterion
 - reflects internal strategy or priorities of agent
 - liberal strategy or prioritize speed (reduce a)
 - cautious strategy or prioritize accuracy (raise a)
- drift-rate (v): relative strength of evidence for two alt. choices
 - typically depends on environment (externally controlled)
 - higher motion coherence => increase in v
 - greater difference in associated value of alt. actions => increase in v
- starting-point (z): initial value of evidence, bias
 - reflects internal biases or expectations of agent
 - anticipate left motion, move z closer to “left” decision boundary
 - anticipate right motion, move z closer to “right” decision boundary
- non-decision time (tr) sensory and motor delays:
 - traditionally viewed as a “nuisance” parameter (i.e., capturing internal and/or external variables related to sensory or motor but *not* decision computation)



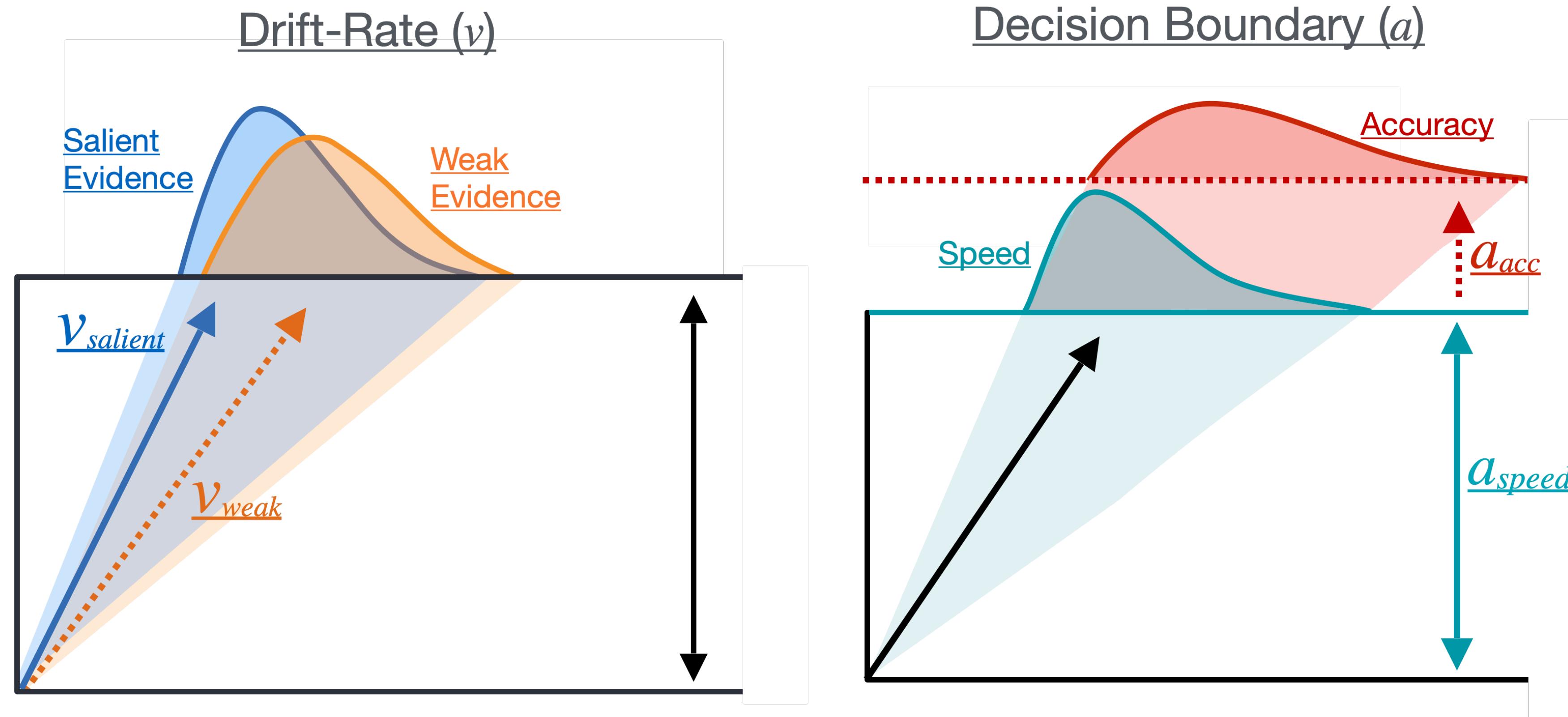
Two critical properties of the DDM

1. Parameters are *identifiable* (e.g., recoverable) from behavioral data (RT distributions and choice accuracy)
2. Parameters are reliably modulated by specific task variables, capturing sensible links between internal decision mechanisms and external task environmental demands



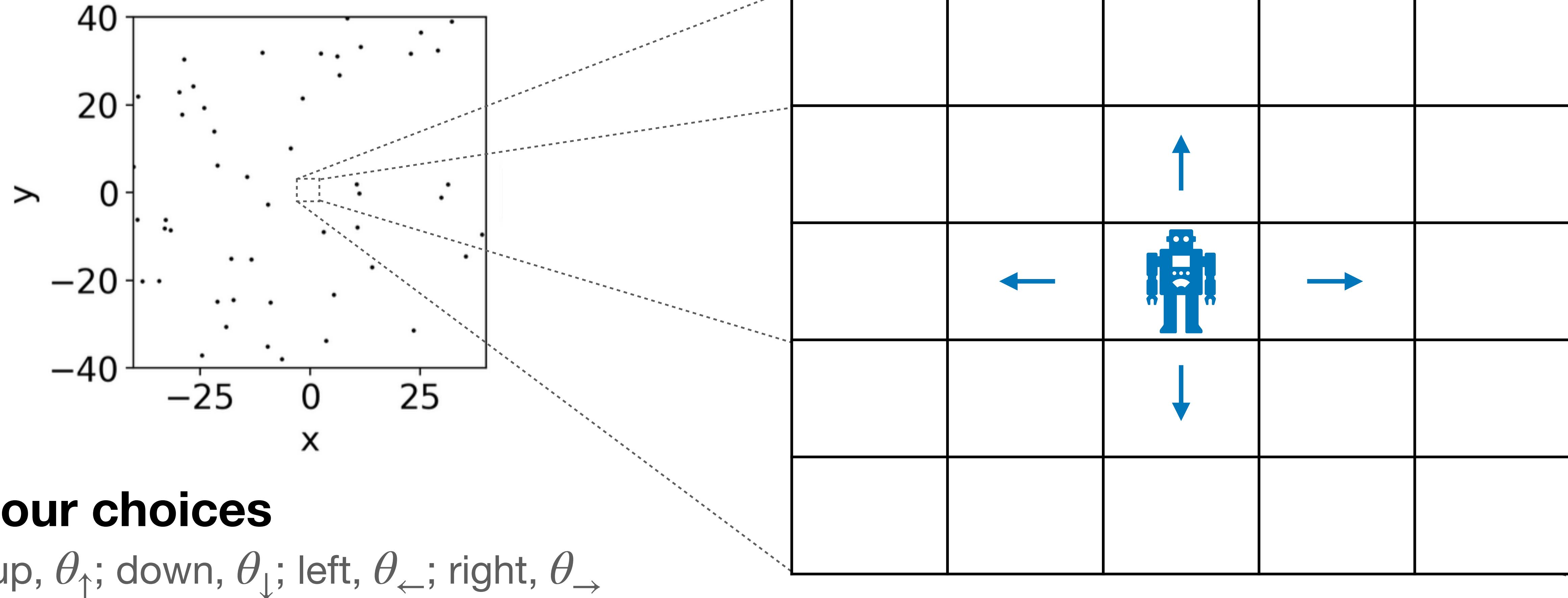
Parameter identifiability

Example: How do we distinguish between behavioral effects caused by a change in drift-rate (v) and change in boundary height (a)?



The “accumulator” agent

Moving to a grid world



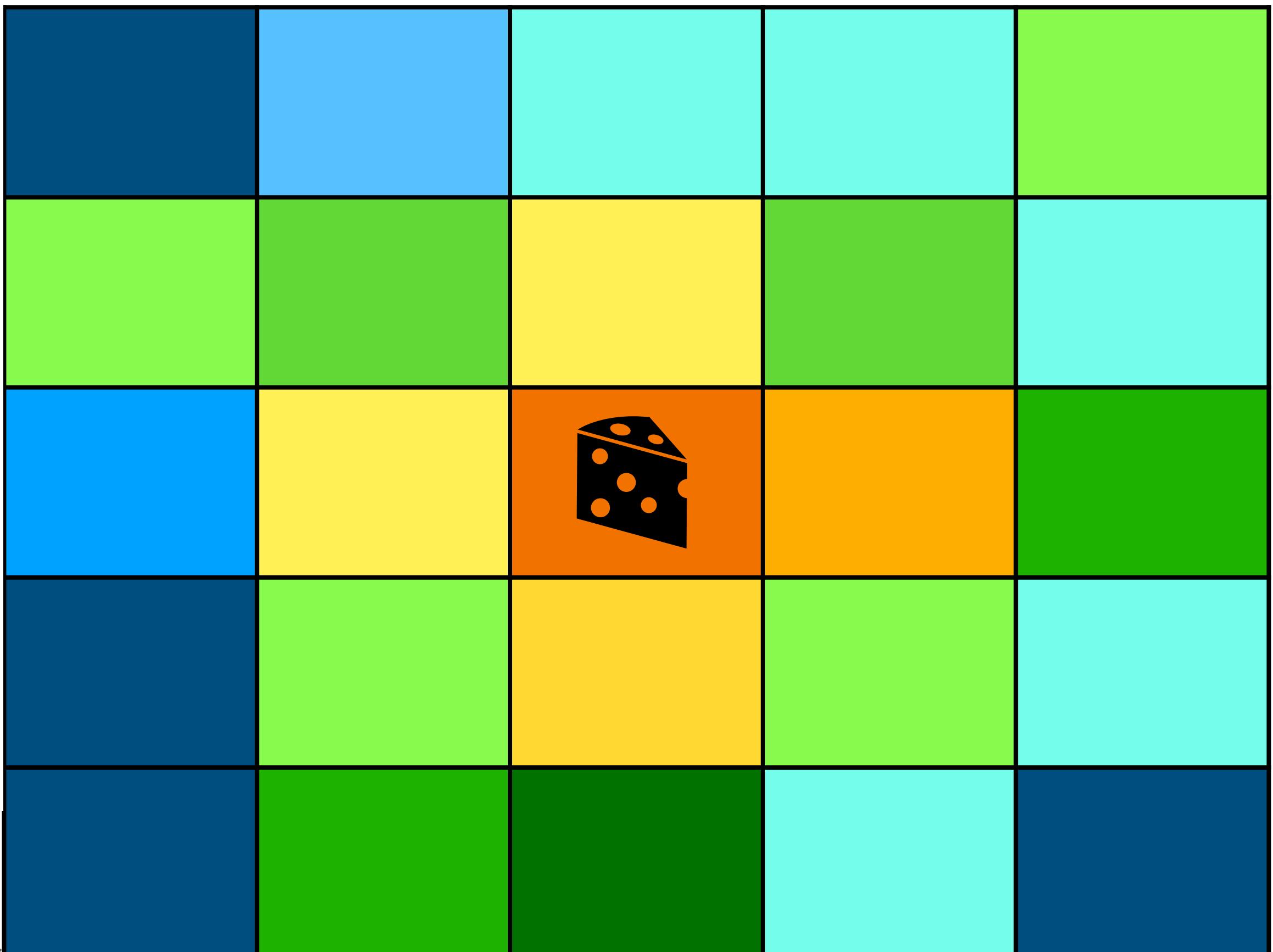
The “accumulator” algorithm

Algorithm 3 Accumulator

```
1: Set  $n_{max}$  number steps
2: Set drift rate,  $\nu$ 
3: Set boundary height,  $a$ 
4: Set accumulation noise,  $\sigma$ 
5: Set initial direction:  $\theta_0 \sim U(1, 4)$ 
6: for  $step = 1, \dots, n_{max}$  do
7:   Sample gradient:  $\Delta o = o_s - o_{s-1}$ 
8:   Update evidence:  $e_s = \nu\Delta o + \sigma\epsilon_s$ 
9:   if  $e_t > a$  then
10:    Select new direction:  $\theta_s \sim U(1, 4)$ 
11:   else
12:    Continue with old direction:  $\theta_s = \theta_{s-1}$ 
13:   end if
14:   Move 1 step in  $\theta_s$  direction
15: end for
```

Goal

Accumulate evidence to “decide”
whether to tumble in a new direction



Take home message

- At any given moment, the ratio of likelihoods is the optimal method for evaluating two hypotheses (when probabilities are known).
- Accumulation of evidence optimally occurs via a sequential probability ratio test (SPRT), which follows an information diffusion process.
- Models like the drift diffusion model reflect operationalizations of the SPRT that can be fit to empirical data.

Lab 4: Evidence accumulation

URL: https://coaxlab.github.io/BIX-book/notebooks/lab4-evidence_accumulation.html

The screenshot shows a web browser window with the URL https://coaxlab.github.io/BIX-book/notebooks/lab4-evidence_accumulation.html in the address bar. The main content is titled "Lab 4 - Evidence Accumulation".

Left Sidebar (Dark Mode):

-
- Biologically Intelligent eXploration (BIX)
- Getting started
 - Introduction to python
- Labs
 - Lab 1- Information theory
 - Lab 2 - Random exploration
 - Lab 3 - Chemotaxis
 - Lab 4 - Evidence Accumulation**
 - Lab 5 - CBGT pathways
 - Lab 6 - Predator-prey dynamics
 - Lab 7 - Foraging

Main Content Area:

Lab 4 - Evidence Accumulation

This lab has two main components designed to go over the process of evidence accumulation.

1. Visualize the drift-diffusion model and see how parameters change evidence accumulation.
2. Introduce an exploratory agent that accumulates sensory evidence , and explore how the parameters of the DDM influence the performance of the agent

Note that this lab re-uses agents we had in the last lab, but explores them in much more detail.

Section - Setup

First let's set things up for the two parts of the lab.

Install ADMCode and explorationlib libraries

Install the code for running the DDM simulations (ADMCode) and the evidence accumulation agents (explorelib)

```
# Install explorationlib & gym-maze
!pip install --upgrade git+https://github.com/coaxlab/explorationlib.git
!pip install --upgrade git+https://github.com/MattChanTK/gym-maze.git
!pip install celluloid # for the gifs
```