What is the value of information?

Readings for today

- Sheridan, T. B. (1995). Reflections on information and information value. IEEE transactions on systems, man, and cybernetics, 25(1), 194-196.

Topics

- Information vs. rewards
- Theories of information value
- Overlapping neural representations

Information vs. rewards

Information vs. rewards

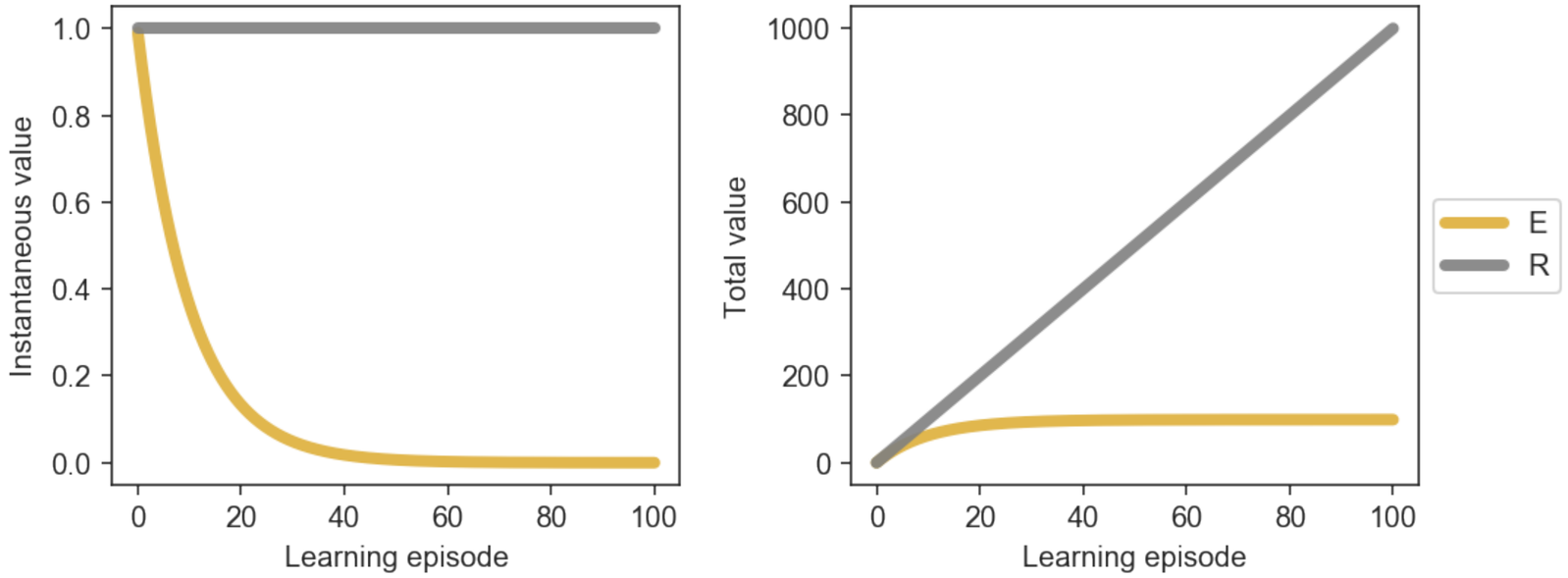
Rewards

- Always increasing (e.g., 4 potato chips > 2 potato chips > 1 potato chip)
- Rate limited (e.g., sharing a chip equally between 2 people means they each have 50% of a chip)
- Externally represented (e.g., you can count how many potato chips someone physically has).

Information

- Depends on what you already know (e.g., the value of someone telling you an answer on an exam is low if you already have the key)
- Rate unlimited (e.g., telling someone the capital of New Mexico does not reduce the fact in your own mind)
- Internally represented (e.g., difficult to quantify facts in a memory… particularly human).

Information vs. rewards



- Information value decays with experience in stable environments.
- Reward value value does not.

Theories of information value

Information or information value

Information (Shannon 1949)

$$H = \sum_i p(x_i) log_2 [\frac{1}{p(x_i)}] = - \sum_i p(x_i) log_2 p(x_i)$$

- x : event
- i : state
- $p(x_i)$: probability of event i

Average uncertainty (entropy) about state i *before* message i is sent.

(Sheridan 1995)

Information or information value

Information (Shannon 1949)

$$H = \sum_i p(x_i) log_2 [\frac{1}{p(x_i)}] = -\sum_i p(x_i) log_2 p(x_i)$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action

Information value (Howard 1966)

$$V^*_{avg} = V_{avg} - V'_{avg}$$

$$= \sum_i p(x_i) \{ \max_j [V(u_j | x_i)] \} - \max_j \{ \sum_i p(x_i) V(u_j | x_i) \}$$

Gain in taking best action vs. gain in taking action in ignorance of each specific x_i

(Sheridan 1995)

Information *and* information value

Information (Shannon 1949) plus cost

$$H^*_{avg} = -C \sum_i p(x_i) log_2 p(x_i)$$

Information value (Sheridan 1995)

$$V^*_{net} = V^*_{avg} - H^*_{avg}$$

$$= \sum_i p(x_i) \{ \max_j [V(u_j | x_i)] \}$$

$$- \max_j \{ \sum_i p(x_i) V(u_j | x_i) \} + C \sum_i p(x_i) log_2 [p(x_i)]$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action
- C : cost per bit

(Sheridan 1995)

Information *and* information value

Information value (Sheridan 1995)

$$V^*_{net} = \underbrace{V^*_{avg}}_{\text{best vs overall}} - \underbrace{H^*_{avg}}_{\text{info cost}}$$

$$V^*_{net} = \begin{cases} > 0, & \text{seek info} \\ \leq 0, & \text{do not} \end{cases}$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action
- C : cost per bit

(Sheridan 1995)

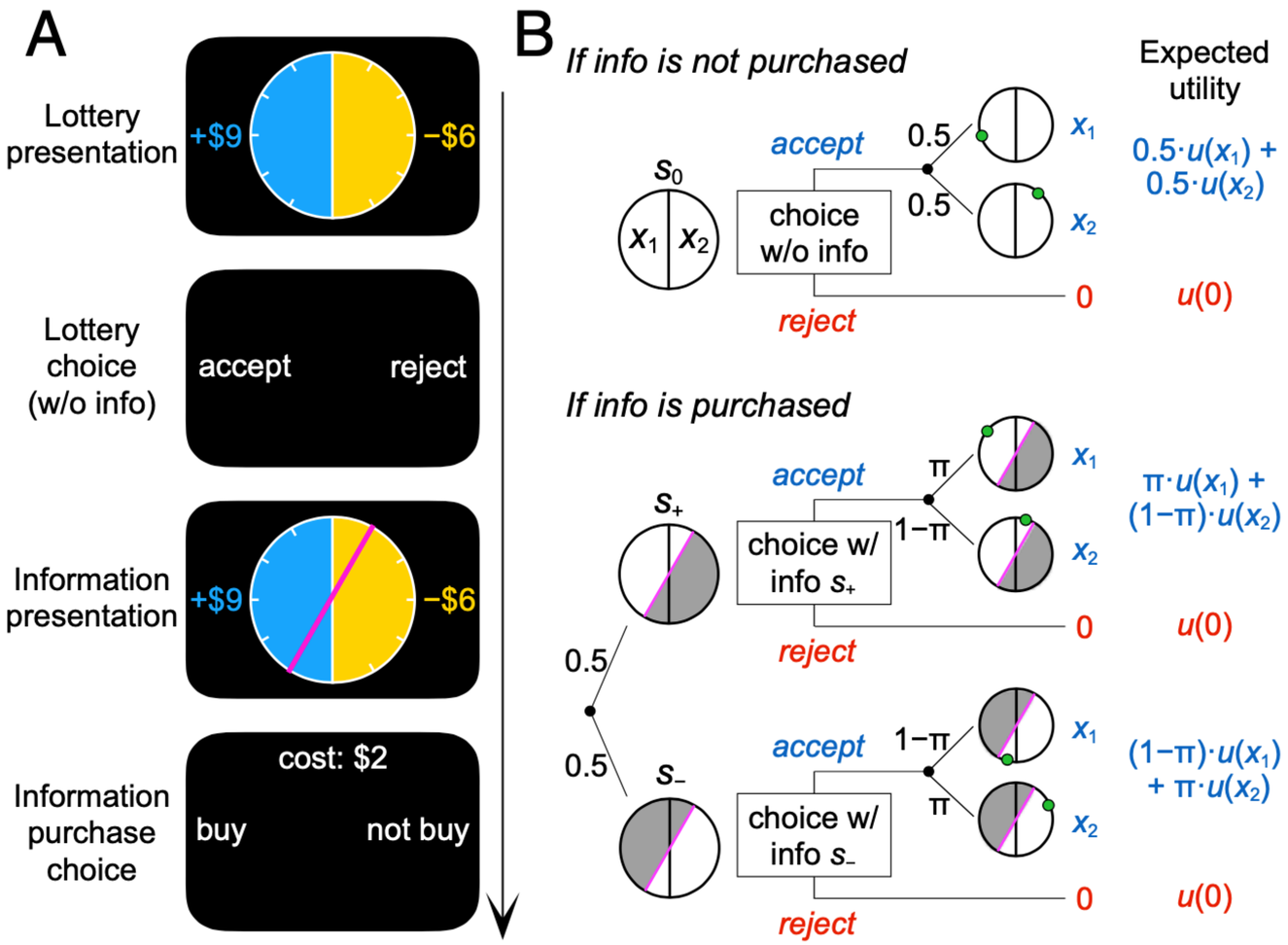
Overlapping neural representations

Subjective value of information

Sometimes the value of an action is driven by subjective preferences as well as the value of information itself.

(Kobayashi & Hsu, 2019)

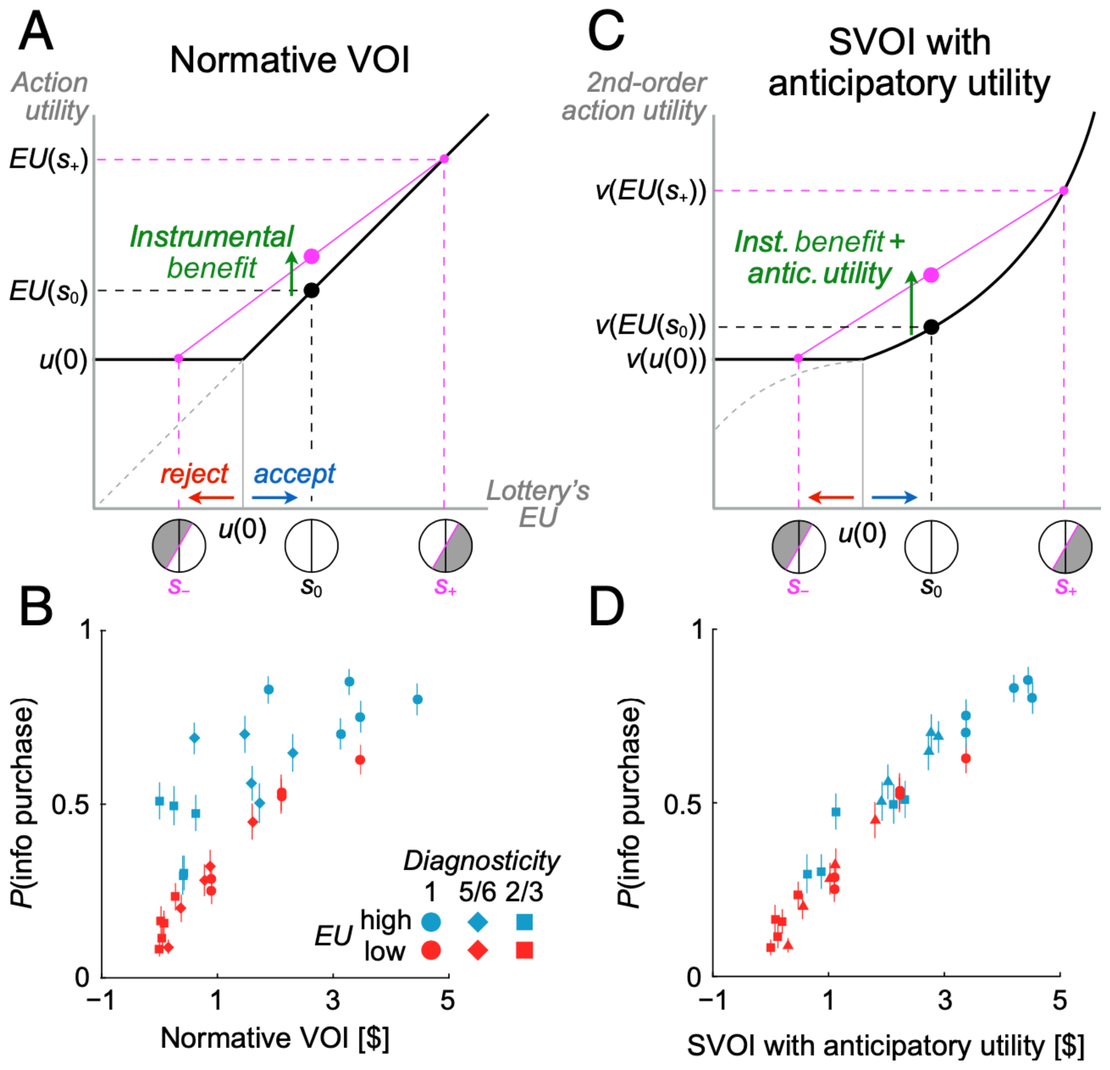
Subjective value of information



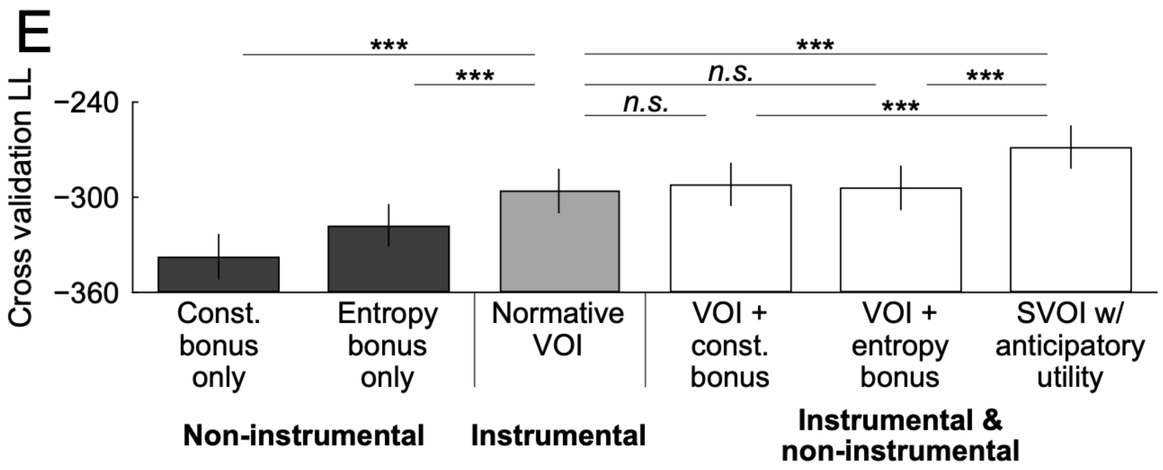
- Presented with roulette-style lottery.
- Initial accept/reject decision.
- Given option to buy information about choice.
- If purchased, participant can change initial accept/reject decision.

(Kobayashi & Hsu, 2019)

Normative vs. subjective value of information

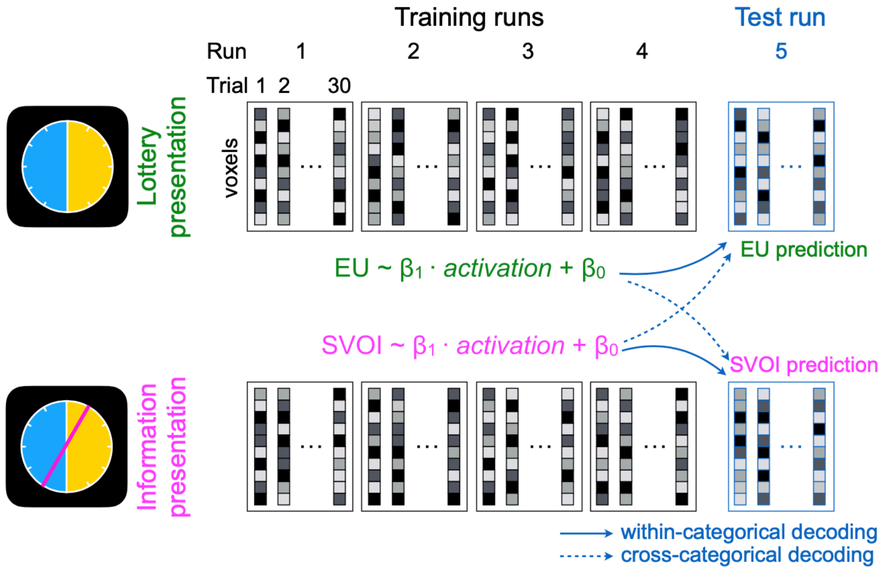


- Normative information value models underestimate the instrumental benefit of information.
- The subjective value of information (SVOI) model (VOI + sunk cost term) better accounts for behavior



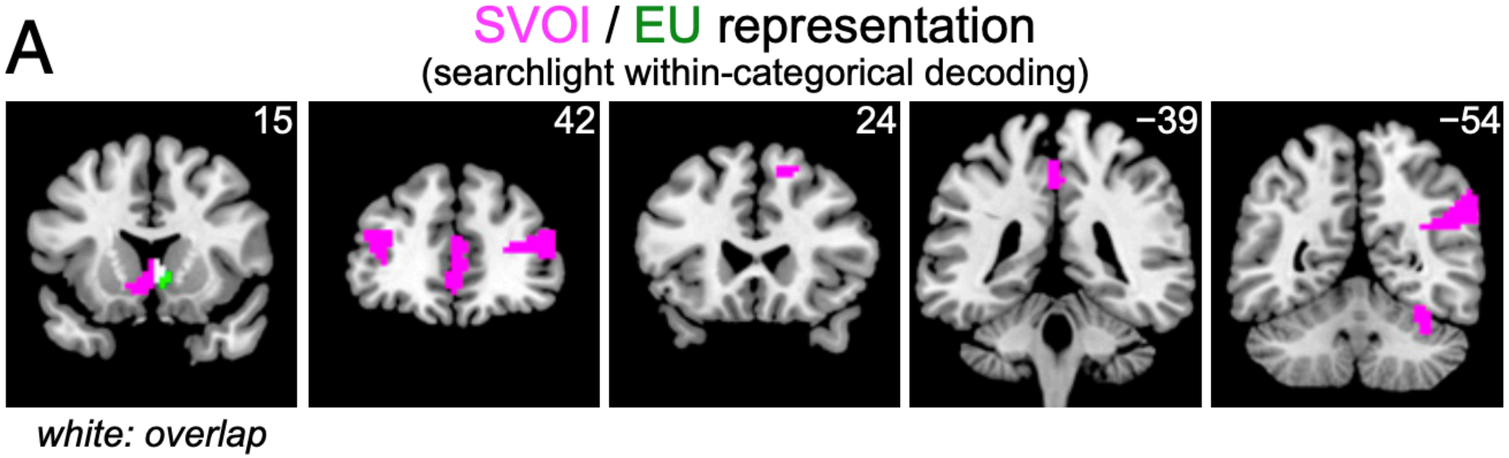
(Kobayashi & Hsu, 2019)

Neural representations

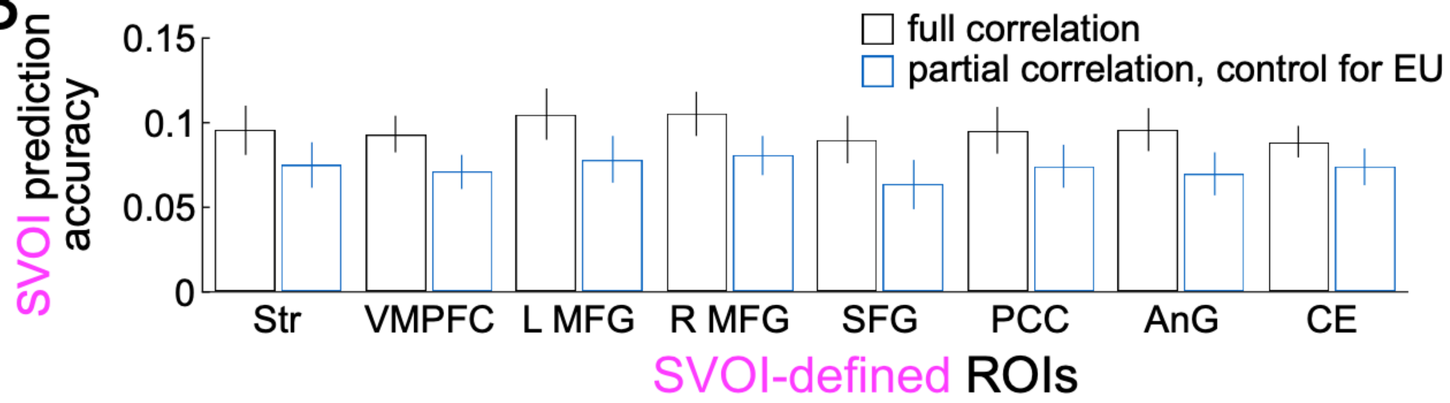


Overlapping regions of representation (reward & information)

A

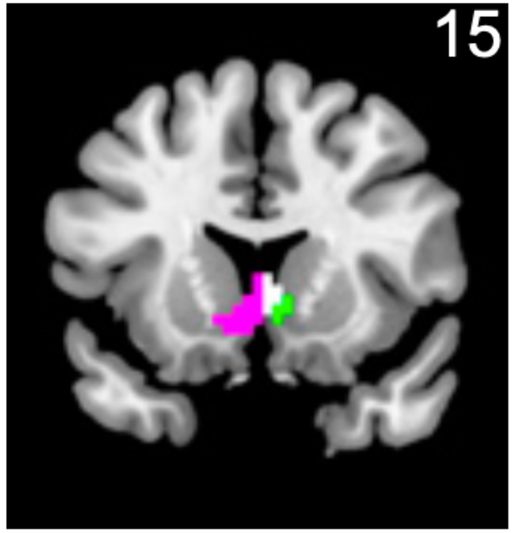


B



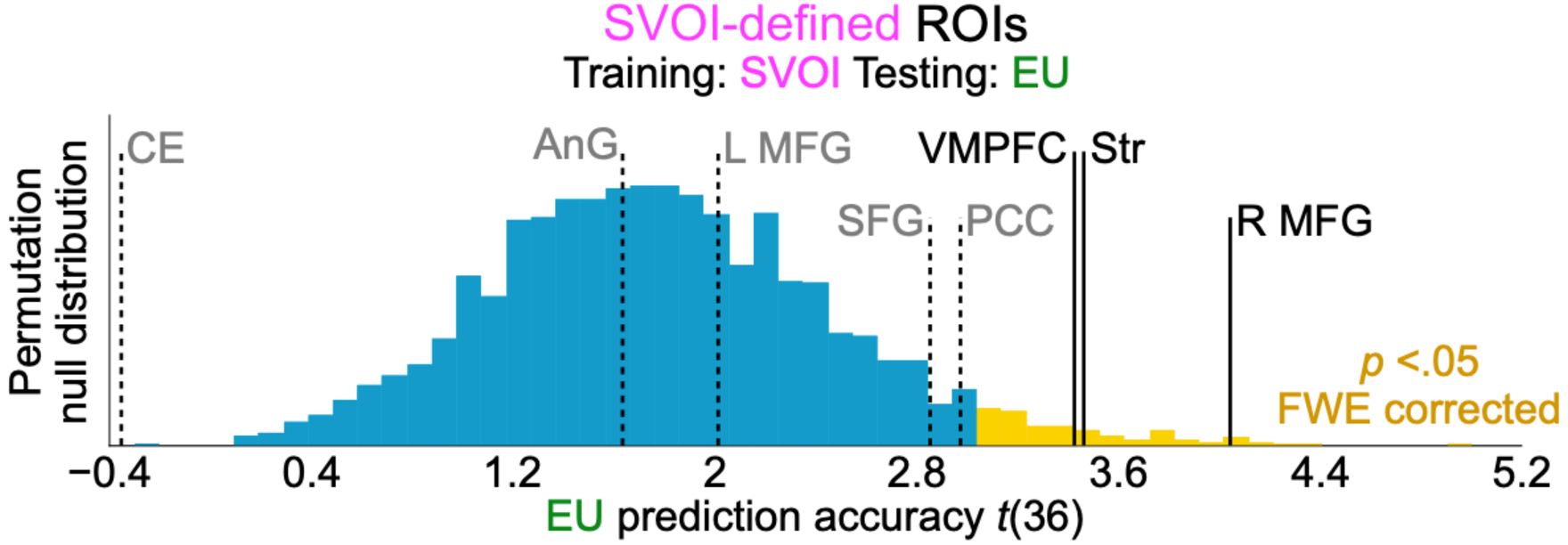
(Kobayashi & Hsu, 2019)

Mixture of representation



white: overlap

Ventral striatum (part of the “reward circuit”) shows a mixture of representations for expected reward and subjective information value



(Kobayashi & Hsu, 2019)

Take home message

- Sheridan proposes that the intrinsic value of information is the difference between the best expected value for given action estimates against the cost of gaining information.
- Consistent with a key assumption of Sheridan's hypothesis, representation of expected utility and subjective value of information share overlapping neural representations in the brain.

Small group discussions

Information value (Sheridan 1995)

$$V^*_{net} = \underbrace{V^*_{avg}}_{\text{best vs overall}} - \underbrace{H^*_{avg}}_{\text{info cost}}$$

$$= (V_{avg} - V'_{avg}) - H^*_{avg}$$

$$= \sum_i p(x_i)\{\max_j [V(u_j|x_i)]\}$$

$$- \max_j \{ \sum_i p(x_i) V(u_j|x_i)\} + C \sum_i p(x_i) log_2[p(x_i)]$$

Small group project

1. Translate Sheridan's information value into non-technical English.
2. Present a "real world" example of this estimation process in practice.