What is the value of information?

Readings for today

- Sheridan, T. B. (1995). Reflections on information and information value. IEEE transactions on systems, man, and cybernetics, 25(1), 194-196.
- Kobayashi, K., & Hsu, M. (2019). Common neural code for reward and information value. Proceedings of the National Academy of Sciences, 116(26), 13061-13066.

Information vs. rewards

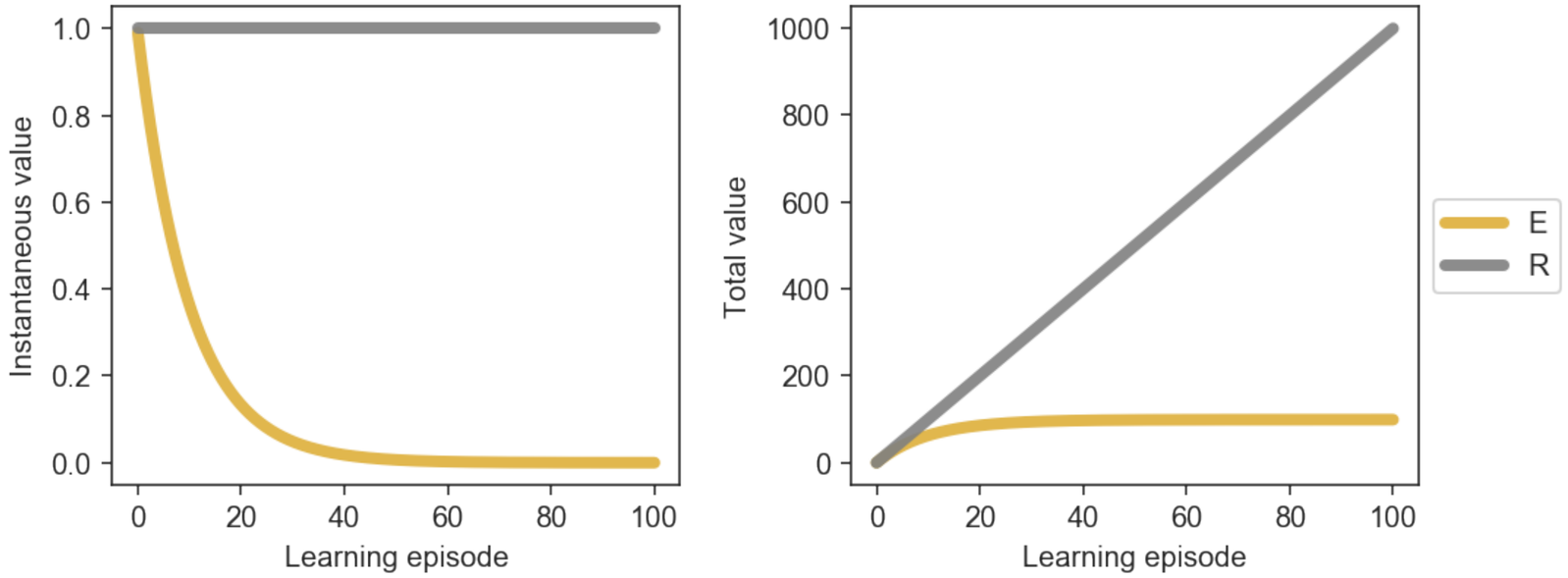
Rewards

- Always increasing (e.g., 4 potato chips > 2 potato chips > 1 potato chip)
- Rate limited (e.g., sharing a chip equally between 2 people means they each have 50% of a chip)
- Externally represented (e.g., you can count how many potato chips someone physically has).

Information

- Depends on what you already know (e.g., the value of someone telling you an answer on an exam is low if you already have the key)
- Rate unlimited (e.g., telling someone the capital of New Mexico does not reduce the fact in your own mind)
- Internally represented (e.g., difficult to quantify facts in a memory… particularly human).

Information vs. rewards



- Information value decays with experience in stable environments.
- Reward value value does not.

Information or information value

Information (Shannon 1949)

$$H = \sum_i p(x_i) log_2 [\frac{1}{p(x_i)}] = - \sum_i p(x_i) log_2 p(x_i)$$

- x : event
- i : state
- $p(x_i)$: probability of event i

Average uncertainty (entropy) about state i *before* message i is sent.

(Sheridan 1995)

Information or information value

Information (Shannon 1949)

$$H = \sum_i p(x_i) log_2 [\frac{1}{p(x_i)}] = -\sum_i p(x_i) log_2 p(x_i)$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action

Information value (Howard 1966)

$$V^*_{avg} = V_{avg} - V'_{avg}$$

$$= \sum_i p(x_i) \{ \max_j [V(u_j | x_i)] \} - \max_j \{ \sum_i p(x_i) V(u_j | x_i) \}$$

Gain in taking best action vs. gain in taking action in ignorance of each specific x_i

(Sheridan 1995)

Information *and* information value

Information (Shannon 1949) plus cost

$$H^*_{avg} = -C \sum_i p(x_i) log_2 p(x_i)$$

Information value (Sheridan 1995)

$$V^*_{net} = V^*_{avg} - H^*_{avg}$$

$$= \sum_i p(x_i) \{ \max_j [V(u_j | x_i)] \}$$

$$- \max_j \{ \sum_i p(x_i) V(u_j | x_i) \} + C \sum_i p(x_i) log_2 [p(x_i)]$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action
- C : cost per bit

(Sheridan 1995)

Information *and* information value

Information value (Sheridan 1995)

$$V^*_{net} = \underbrace{V^*_{avg}}_{\text{best vs overall}} - \underbrace{H^*_{avg}}_{\text{info cost}}$$

$$V^*_{net} = \begin{cases} > 0, & \text{seek info} \\ \leq 0, & \text{do not} \end{cases}$$

- x : event
- i : state
- $p(x_i)$: probability of event i
- u: action
- C : cost per bit

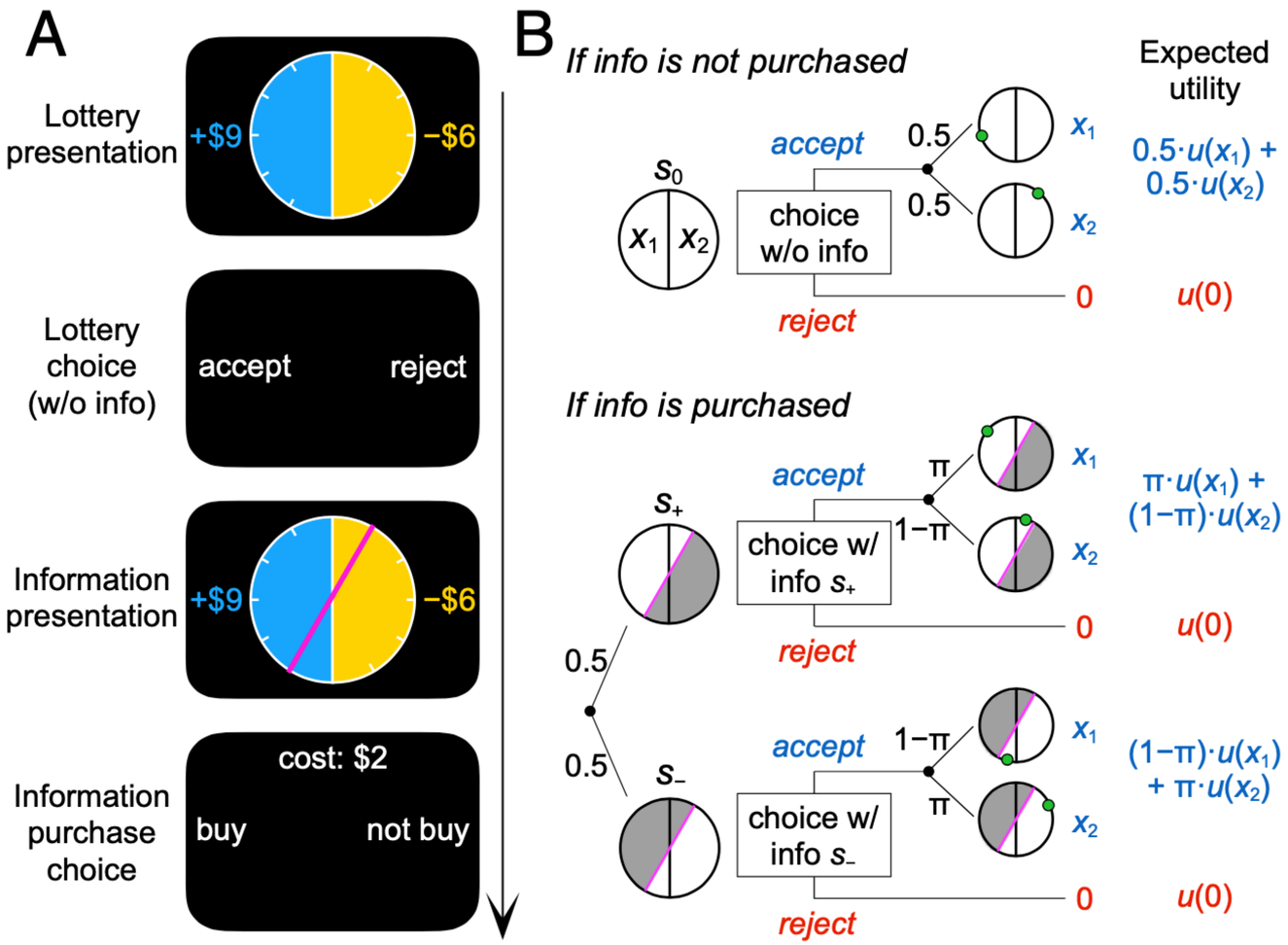
(Sheridan 1995)

Subjective value of information

Sometimes the value of an action is driven by subjective preferences as well as the value of information itself.

(Kobayashi & Hsu, 2019)

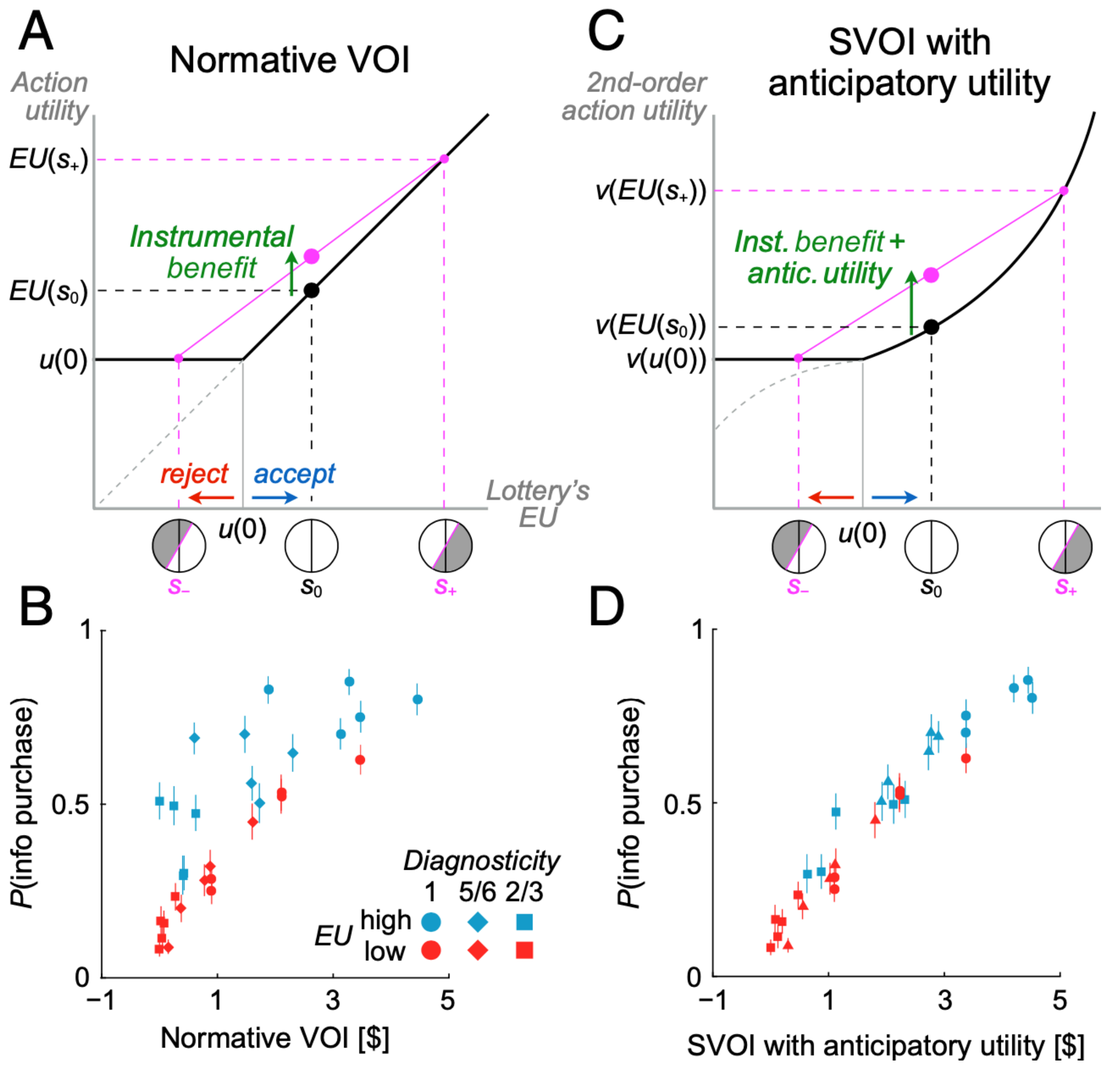
Subjective value of information



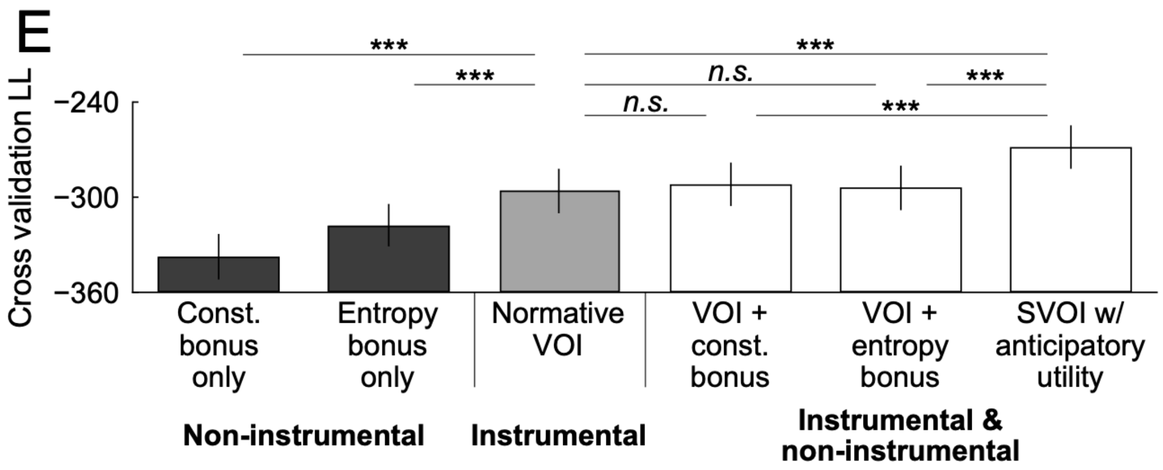
- Presented with roulette-style lottery.
- Initial accept/reject decision.
- Given option to buy information about choice.
- If purchased, participant can change initial accept/reject decision.

(Kobayashi & Hsu, 2019)

Normative vs. subjective value of information

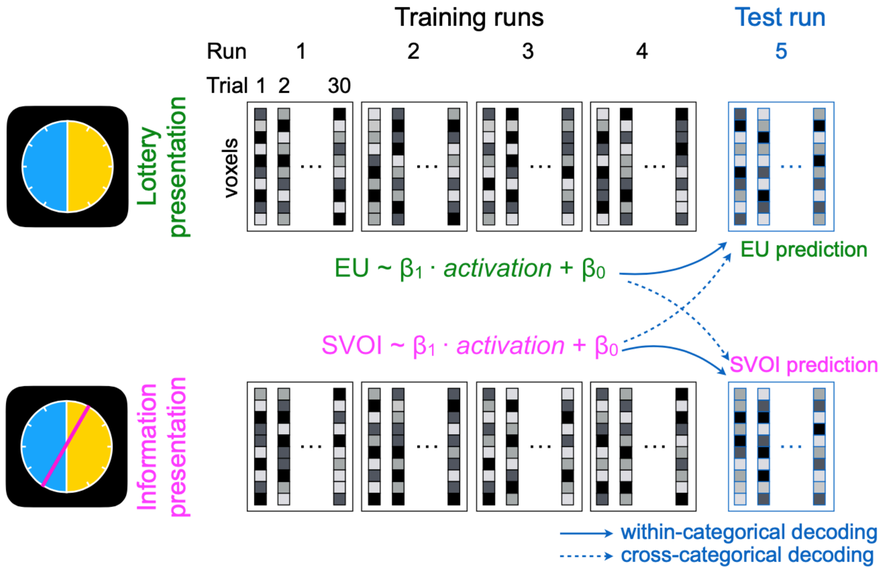


- Normative information value models underestimate the instrumental benefit of information.
- The subjective value of information (SVOI) model (VOI + sunk cost term) better accounts for behavior



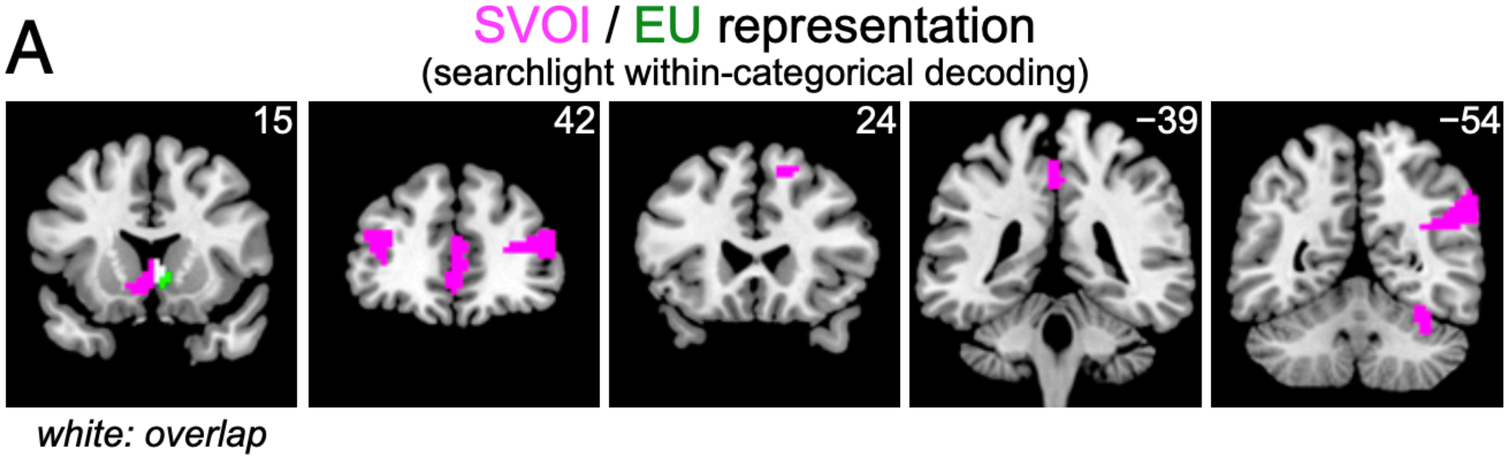
(Kobayashi & Hsu, 2019)

Neural representations

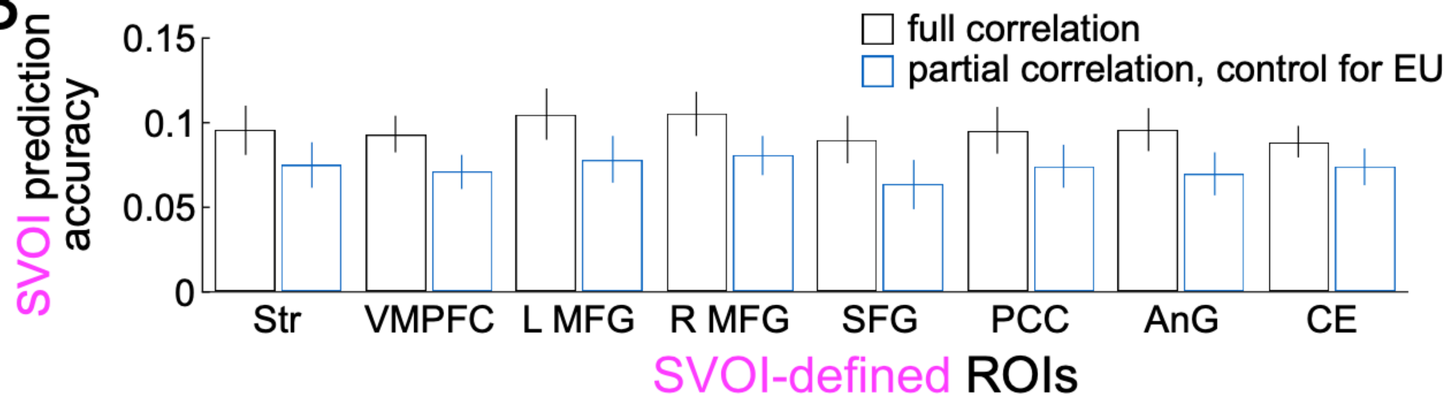


Overlapping regions of representation (reward & information)

A

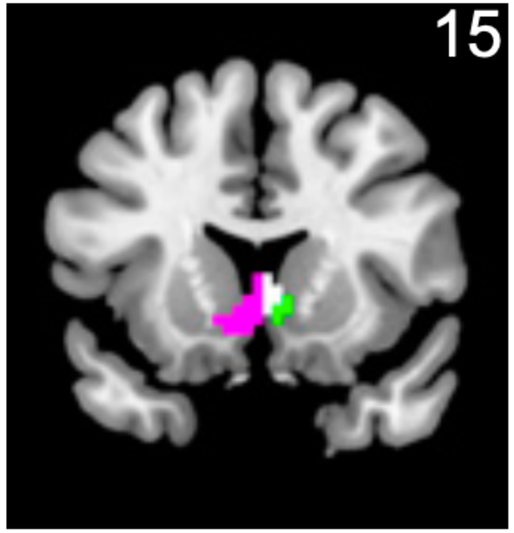


B



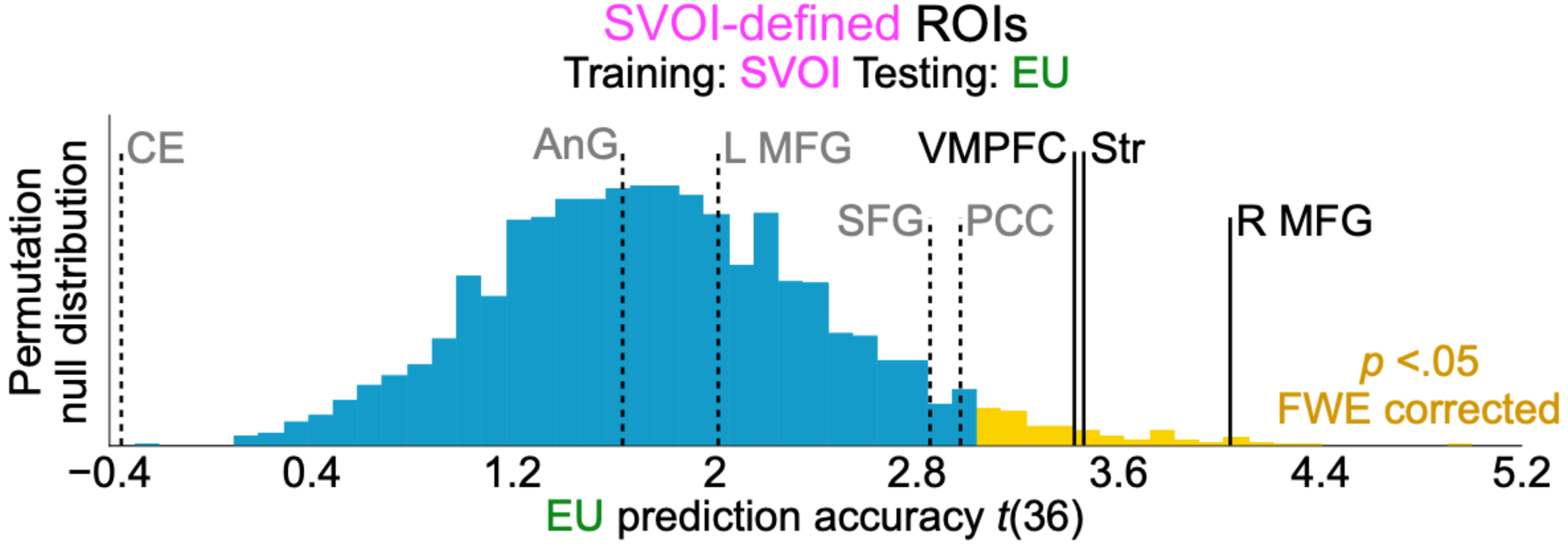
(Kobayashi & Hsu, 2019)

Mixture of representation



white: overlap

Ventral striatum (part of the “reward circuit”) shows a mixture of representations for expected reward and subjective information value



(Kobayashi & Hsu, 2019)

Take home message

- Sheridan proposes that the intrinsic value of information is the difference between the best expected value for given action estimates against the cost of gaining information.
- Consistent with a key assumption of Sheridan's hypothesis, representation of expected utility and subjective value of information share overlapping neural representations in the brain.

Small group discussions

Information value (Sheridan 1995)

$$V^*_{net} = \underbrace{V^*_{avg}}_{\text{best vs overall}} - \underbrace{H^*_{avg}}_{\text{info cost}}$$

$$= (V_{avg} - V'_{avg}) - H^*_{avg}$$

$$= \sum_i p(x_i)\{\max_j [V(u_j|x_i)]\}$$

$$- \max_j \{ \sum_i p(x_i) V(u_j|x_i)\} + C \sum_i p(x_i) log_2[p(x_i)]$$

Small group project

1. Translate Sheridan's information value into non-technical English.
2. Present a "real world" example of this estimation process in practice.