Is randomness an effective strategy (for exploration)?

Readings for today

- Berg, H. C. (Ed.). (2004). E. coli in Motion. New York, NY: Springer New York.

Topics

- Brownian motion
- The structure of simple random walks

Brownian motion

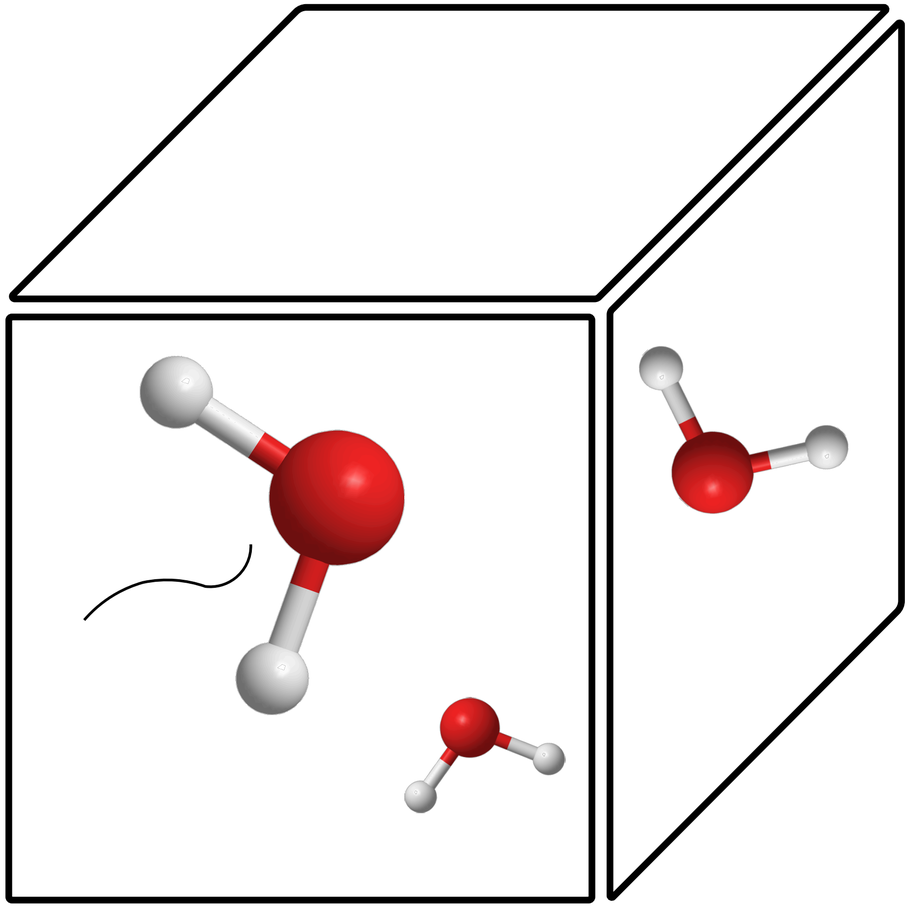
What is the complete way to explore?

Complete Exploration:

the process of taking every available action in a given context at least once.

Gas particles filling an empty volume of space will settle to being evenly (uniformly) distributed throughout the space.

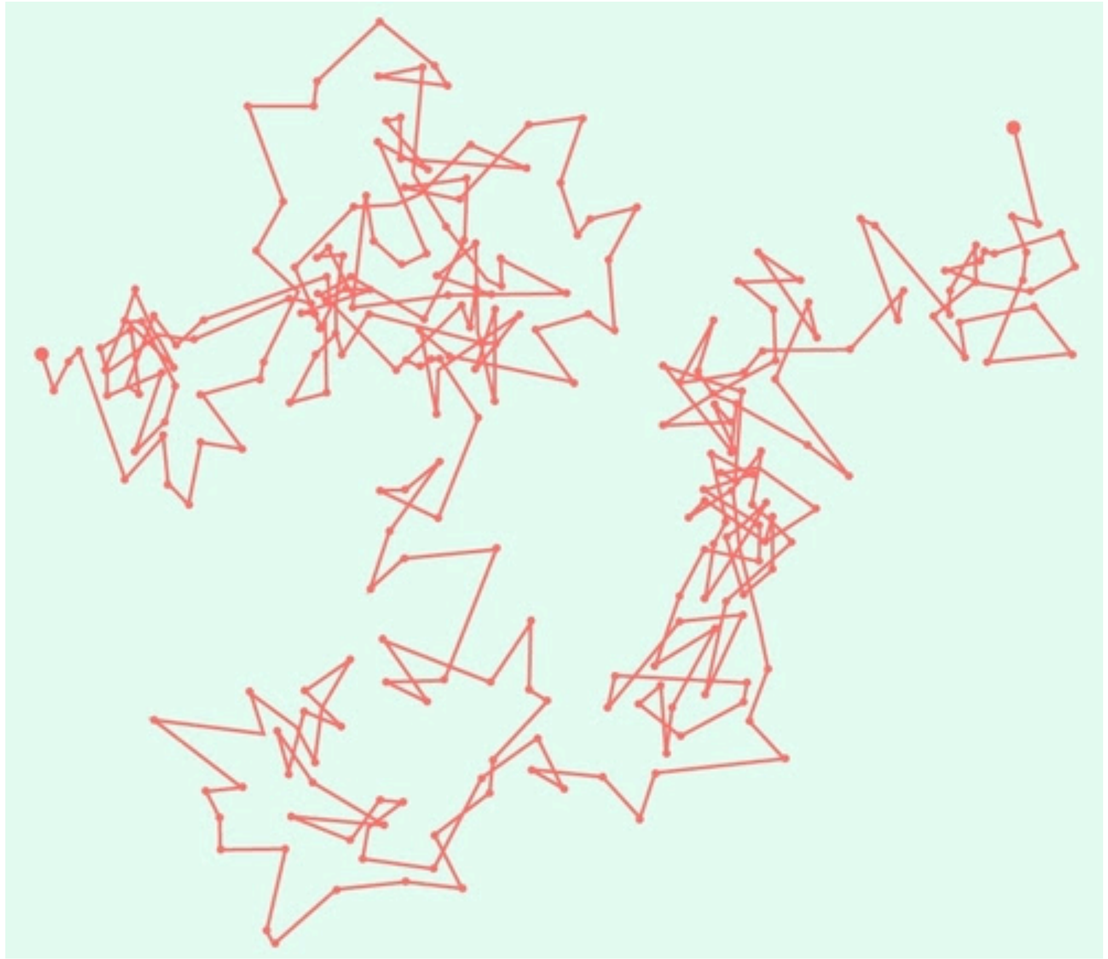
Complete exploration via randomness



Brownian motion

https://www.youtube.com/watch?v=R5t-oA796to

Brownian motion



"While examining the form of these particles immersed in water, I observed many of them evidently in motion; their motion consisting not only of a change of place in teh fluid, manifested by alterations in their relative positions, but also not infrequently of a change of form in the particle itself; a contraction or curvature taking place repeatedly about the middle of one side, accompanied by a corresponding swelling or convexity on the opposite side of the particle."

- Robert Brown (1827)

https://www.youtube.com/watch?v=R5t-oA796to

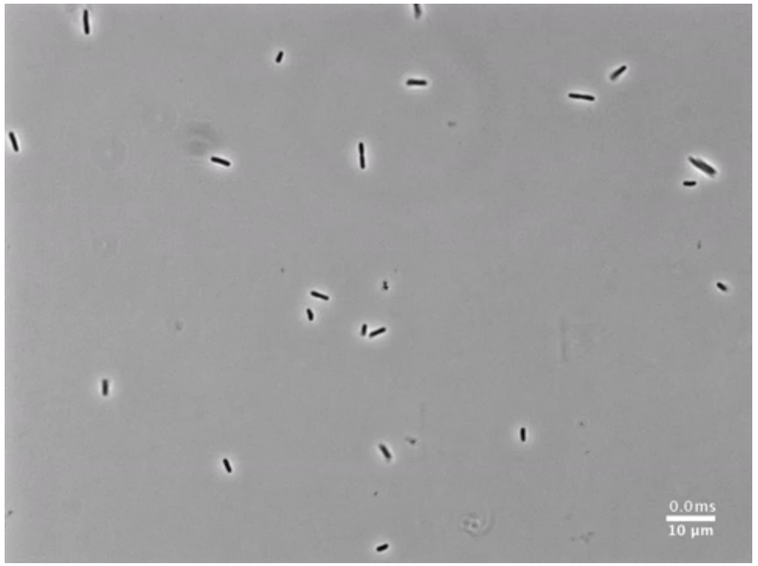
History of a concept

A brief history of random walks:

- **1827**: Robert Brown’s discovery while observing pollen in water.
- **1877**: Thorvald Thiele & Louis Bachelier statistical analysis approach laid the framework for describing this motion mathematically.
- **1905**: Albert Einstein & Marian Smoluchowski’s theoretical explanations, providing a mathematical model linking Brownian motion to molecular theory.
- **1905**: Karl Pearson first uses the term “random walk” in a letter to Nature.
- **1906**: Pearson publishes “A Mathematical Theory of Random Migration” showing how random walks can explain biological movement.
- **1908**: Jean Perrin's experiments confirmed Einstein’s theory, providing evidence for the existence of atoms.

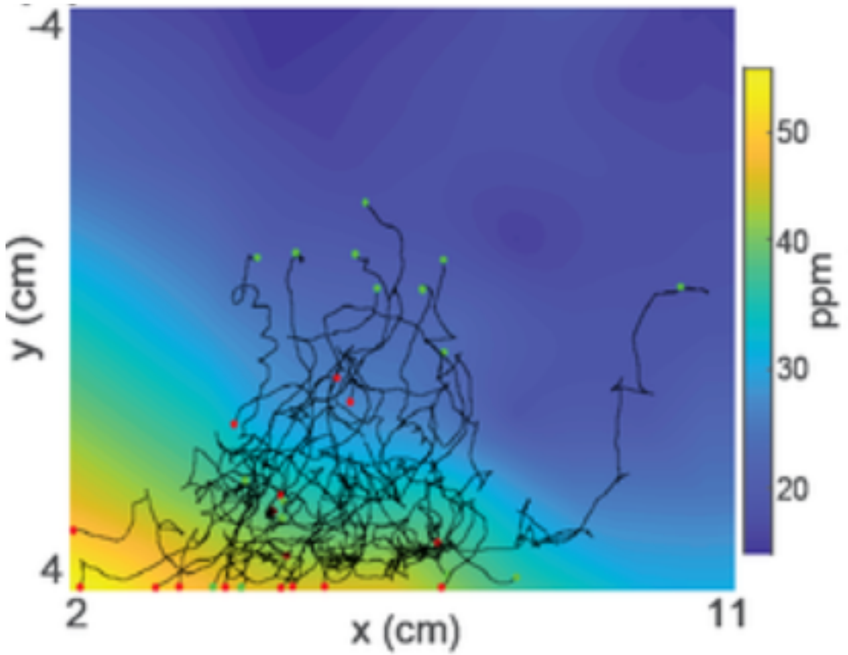
Random walks in biology

E. Coli bacteria

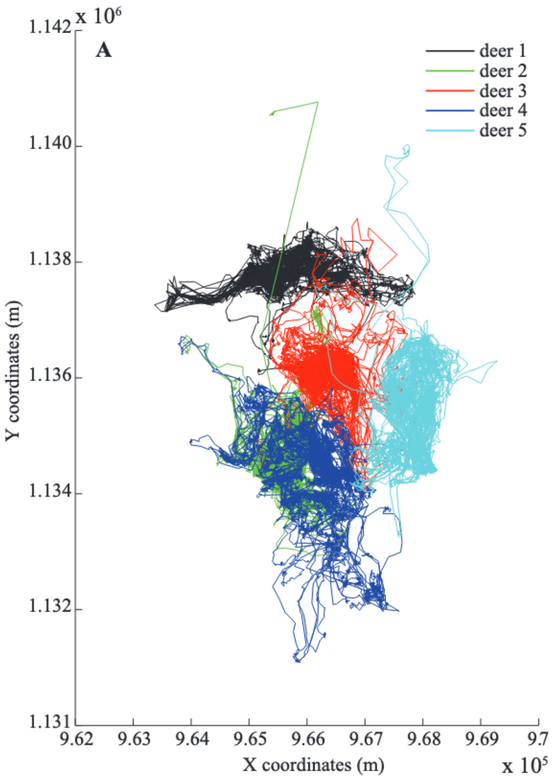


https://www.youtube.com/watch?v=CldjFTSr4fY

C. Elegan flatworms



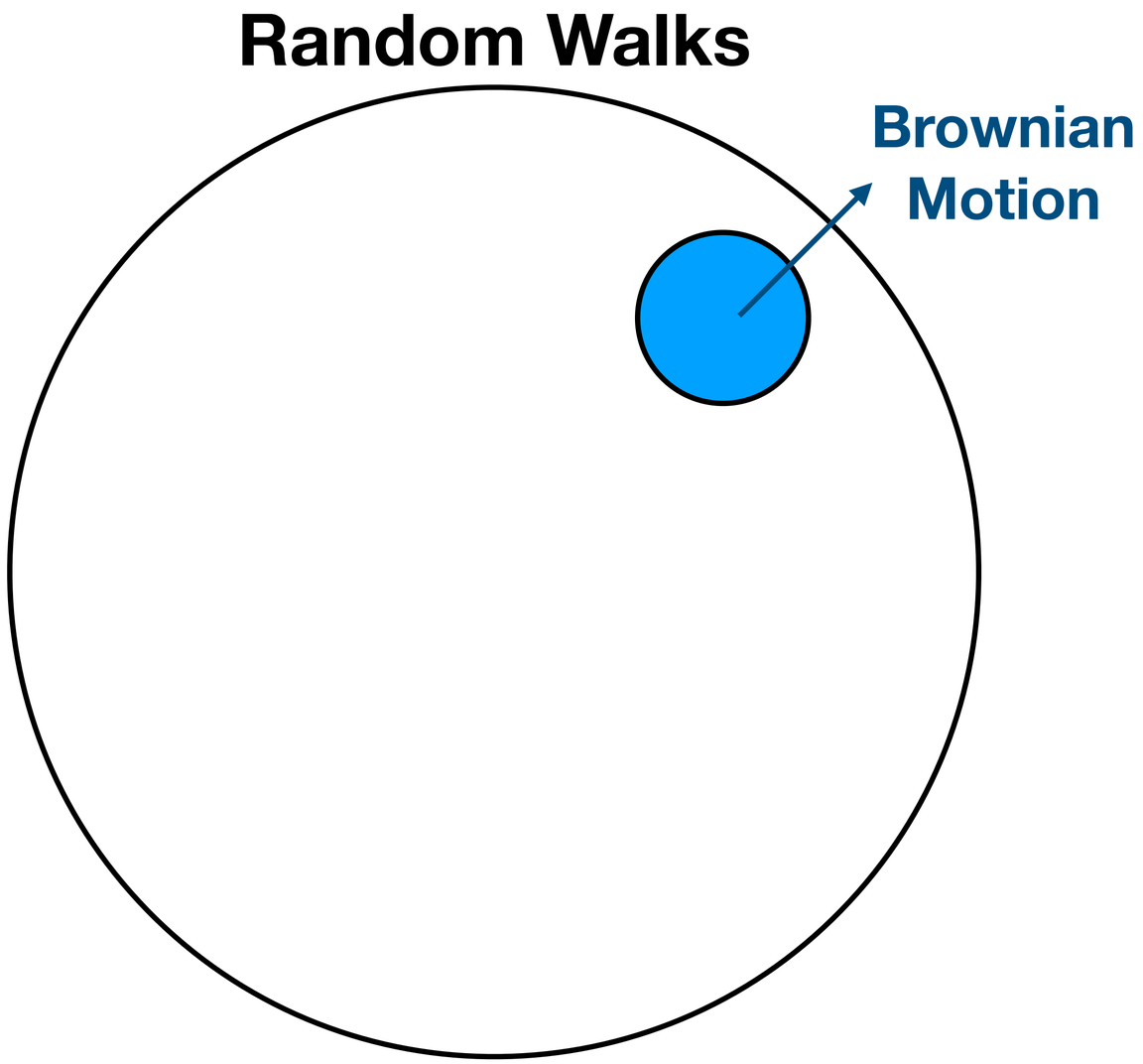
Red Deer



(Berthelot et al. 2020)

The structure of simple random walks

Random walks vs. Brownian motion



Random Walk

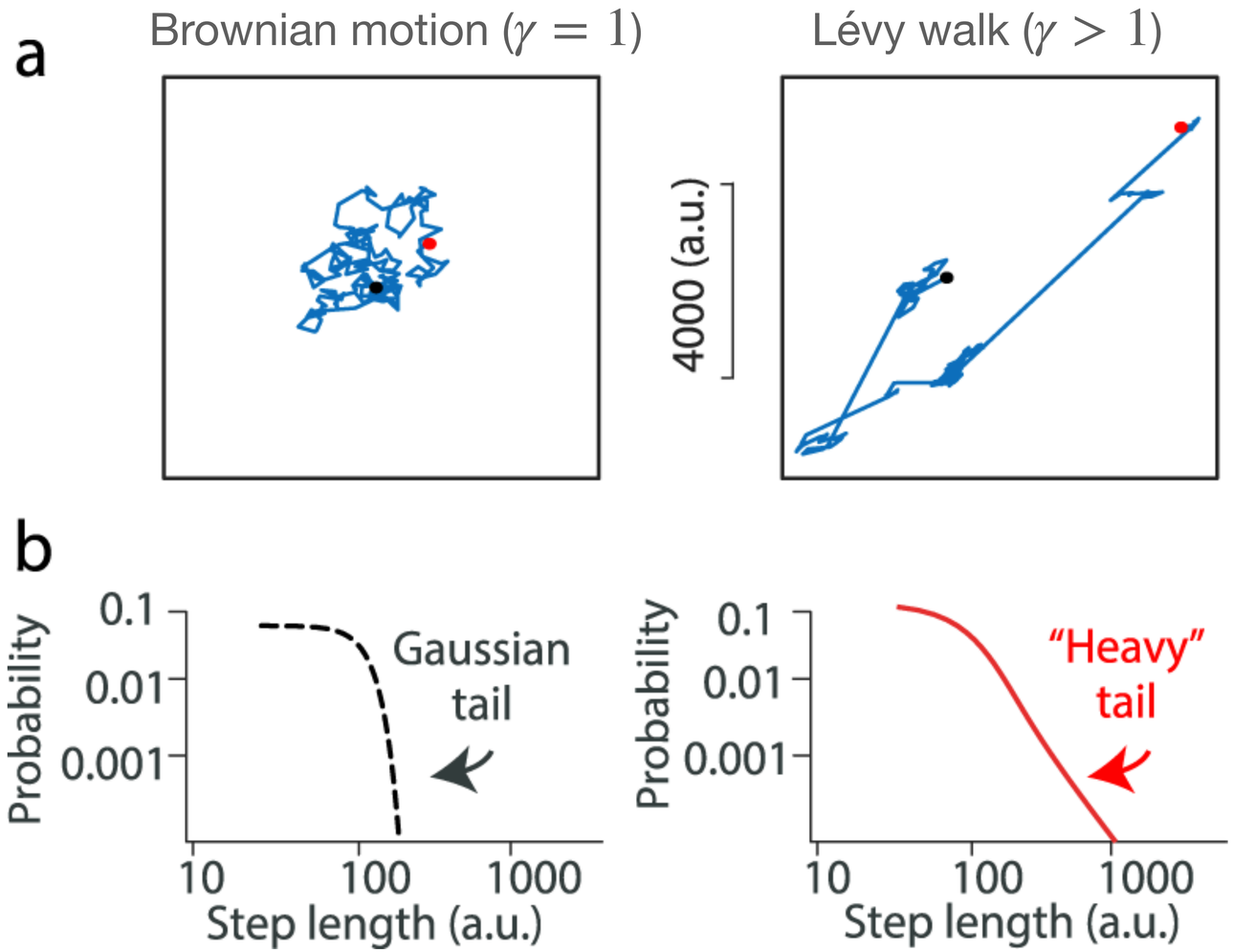
A mathematical description of any process in which an agent, be it a particle or animal, takes a sequence of steps whose direction and length are determined by random probabilities.

Brownian Motion

A subset of random walks where the probability density function of visited positions best matches a Gaussian distribution

Scaling laws

The shape of the probability distribution that best describes the trajectories determines the type of random walks being described.



(Liu et al. 2021)

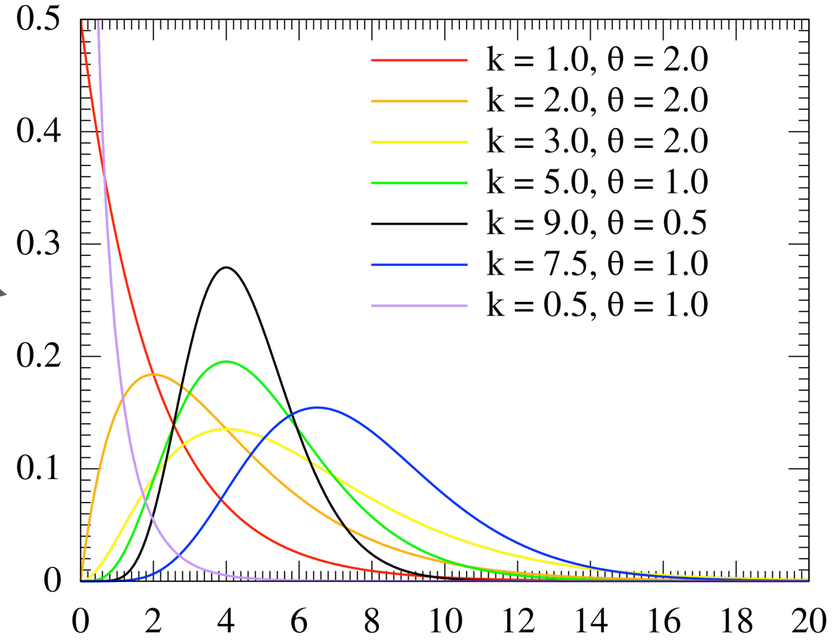
Random (Brownian) walk in 2d space

Angle $\rightarrow \theta_i \sim U(-\pi, \pi)$

Step length $\rightarrow l_i \sim Gamma(k, \theta)$

$$x_i = x_{i-1} + l_i \cos(\theta_i)$$
$$y_i = y_{i-1} + l_i \sin(\theta_i)$$

$$p(x) = x^{k-1} \frac{e^{\frac{-x}{\theta}}}{\theta^k \Gamma(k)}$$



https://en.wikipedia.org/wiki/Gamma_distribution

Take home message

- Brownian motion is simple enough to implement complete exploration in the right contexts.
- Brownian movement is part of a larger class of models known as *random walks.*
- Random walks exist throughout biology and reflect the earliest example of exploration strategies.

Lab time!

https://coaxlab.github.io/BIX-book/notebooks/lab1-randomsearch.html