Can information be its own signal?

Readings for today

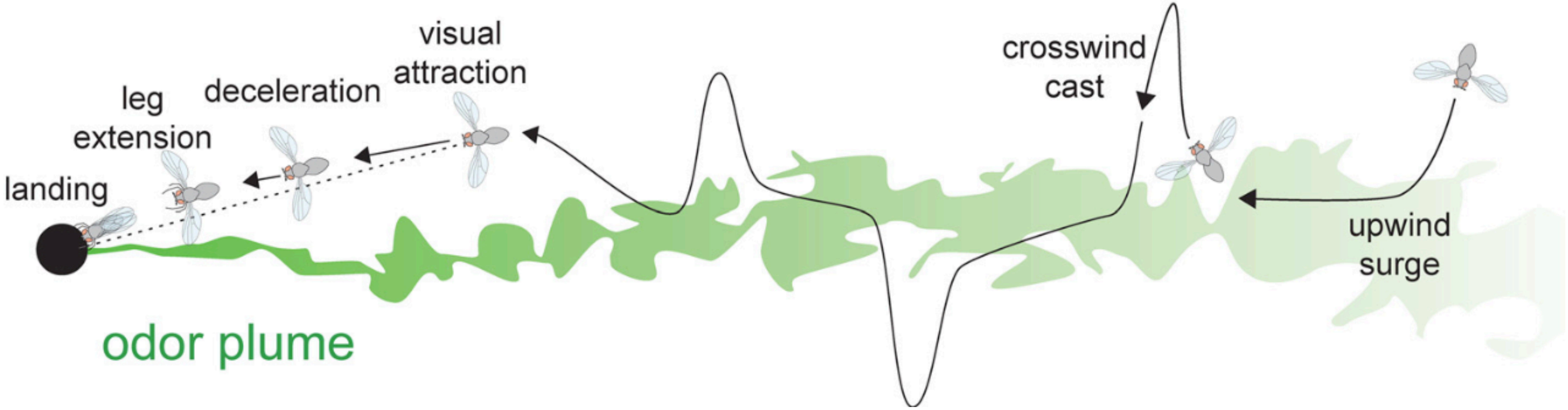
- Vergassola, M., Villermaux, E., & Shraiman, B. I. (2007). ‘Infotaxis’ as a strategy for searching without gradients. Nature, 445(7126), 406-409.

Topics

- Infotaxis by entropy reduction
- Curiosity-driven search

Infotaxis by entropy reduction

Recall the problem of the plume



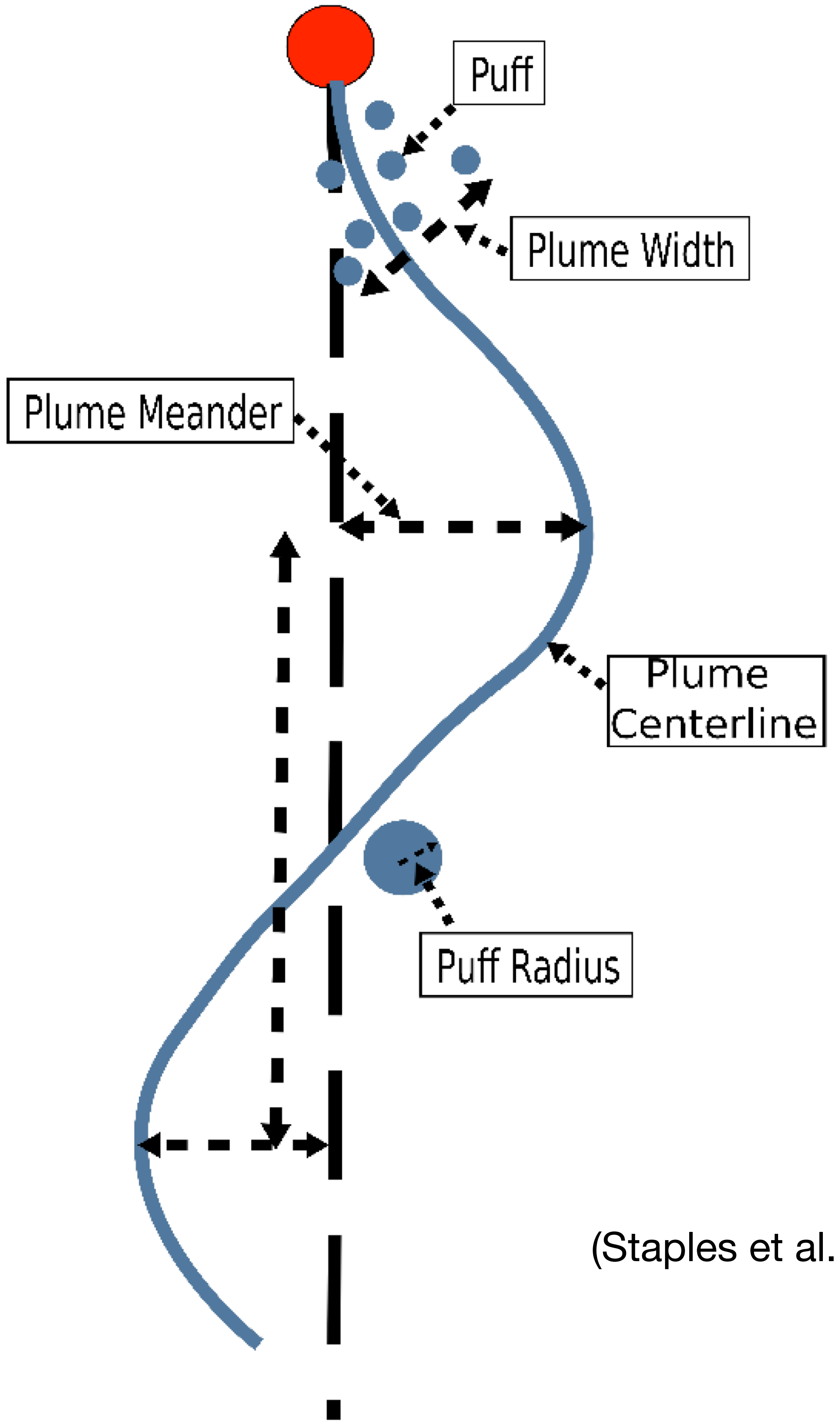
(Baker et al, 2018)

Search for information

Infotaxis

A search strategy that organisms or algorithms use to find a source of interest by optimizing the acquisition of information from the environment, often in situations where the source is intermittently detectable or has a sparse distribution.

A balance between exploring areas of uncertainty and exploiting areas where the source has been previously detected.



(Staples et al. 2023)

(Vergassola et al. 2007)

Infotaxis algorithm

Shannon entropy

detection probability

$$H(s) = -\sum_{\mathbf{x}} p(\boldsymbol{x}) \log_2 \boxed{p(\boldsymbol{x})}$$

belief state $s = [\boldsymbol{x}^a, p(\boldsymbol{x})]$

Information gain

with action a in belief state s

$$G(s,a) = H(s) - H(s \mid a)$$

How much uncertainty is reduced by taking action a

Expected entropy

upon taking action a in belief state s

$$H(s \mid a) = \sum_{s'} \mathrm{P}\left(s' \mid s, a\right) H(s')$$

successor state

Infotaxis policy

Select the action a that maximizes the expected information gain in belief s

$$\pi^{info}(s) = \arg\min_{a} \sum_{s'} \mathrm{P}\left(s' \mid s, a\right) H(s'),$$

(Vergassola et al. 2007; Loisy and Eloy, 2022)

Maximizing information gain

Information gain is entropy reduction

$$G(s,a) = \underbrace{H(s)}_{\text{overall entropy}} - \underbrace{H(s \mid a)}_{\text{entropy after action } a} \longrightarrow H(s \mid a) < H(s)$$

search trajectory

$$\underbrace{\Delta H(r_i \to \boxed{r_j})}_{\text{change in entropy}} = \underbrace{P_t(r_j)[-H]}_{\text{finding the source}} + \underbrace{[1 - P_t(r_j)][\rho_0(r_j)\Delta H_0 + \rho_1(r_j)\Delta H_1 + \ldots]}_{\text{alternative case when source is not found}}$$

Note: In Vergassola et al. 2007, S is used as the symbol for entropy. Here we use the traditional H

(Vergassola et al. 2007)

Infotaxis policy in more detail

Infotaxis policy

$$\pi^{info}(s) = \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) H\left(s'\right)$$
$$= \arg\max_a \; G(s, a)$$
$$= \arg\min_a \; H(s \mid a)$$

At each time step, the searcher chooses the direction that **locally maximizes the expected rate of information acquisition**. Entropy decreases faster closer to the source because cues arrive at a faster rate.

Optimal source-tracking policy

$$\pi^*(s) = \arg\min_\pi \mathbb{E}_{p_0,\pi}[T] \longrightarrow T = \text{search duration}$$
$$= \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) \left[1 + \boxed{v^*\left(s'\right)}\right]$$

Optimal value

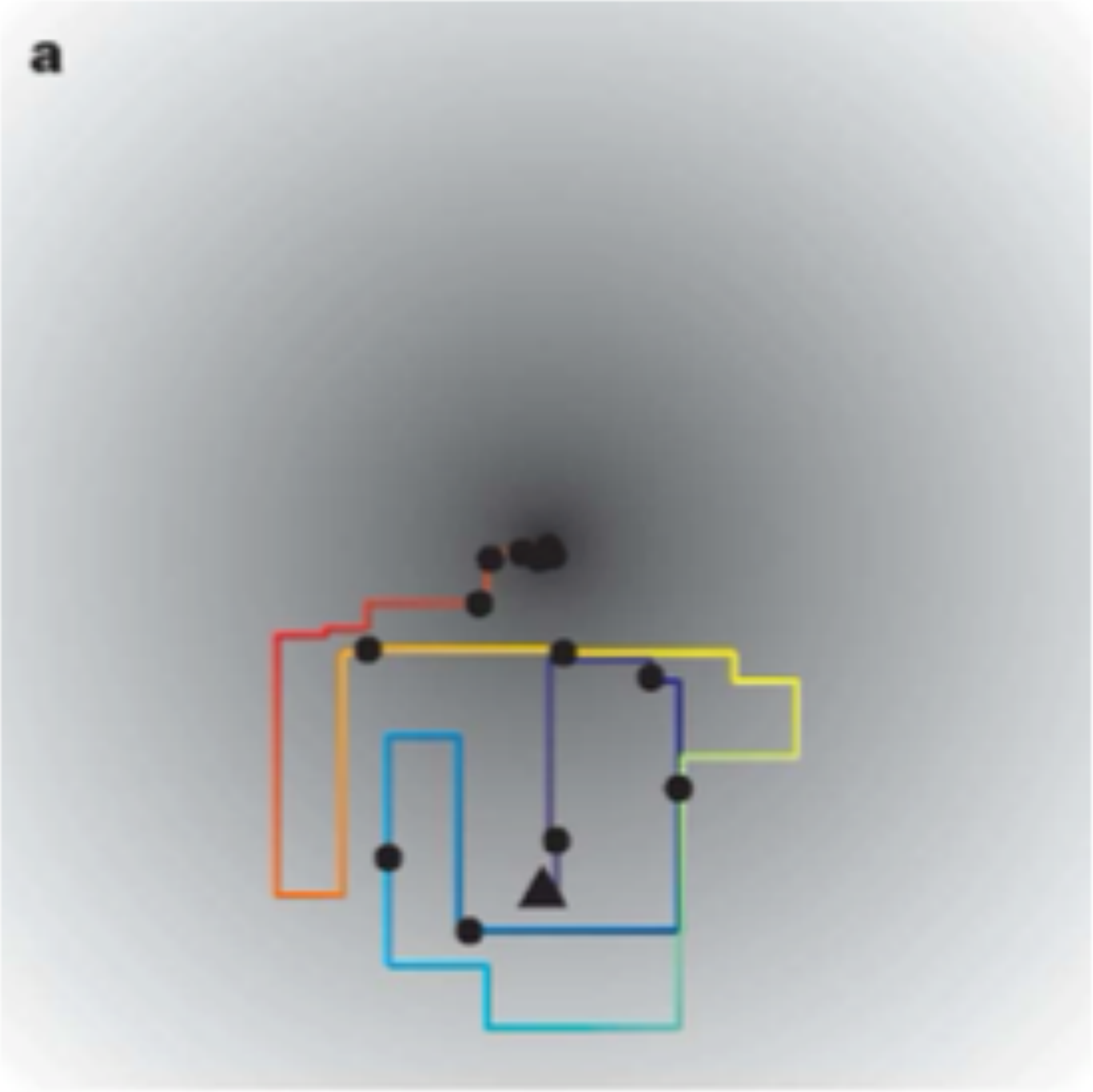
Optimal Infotaxis policy

$$\pi^*(s) = 1 + \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) v^*\left(s'\right)$$

(Vergassola et al. 2007; Loisy and Eloy, 2022)

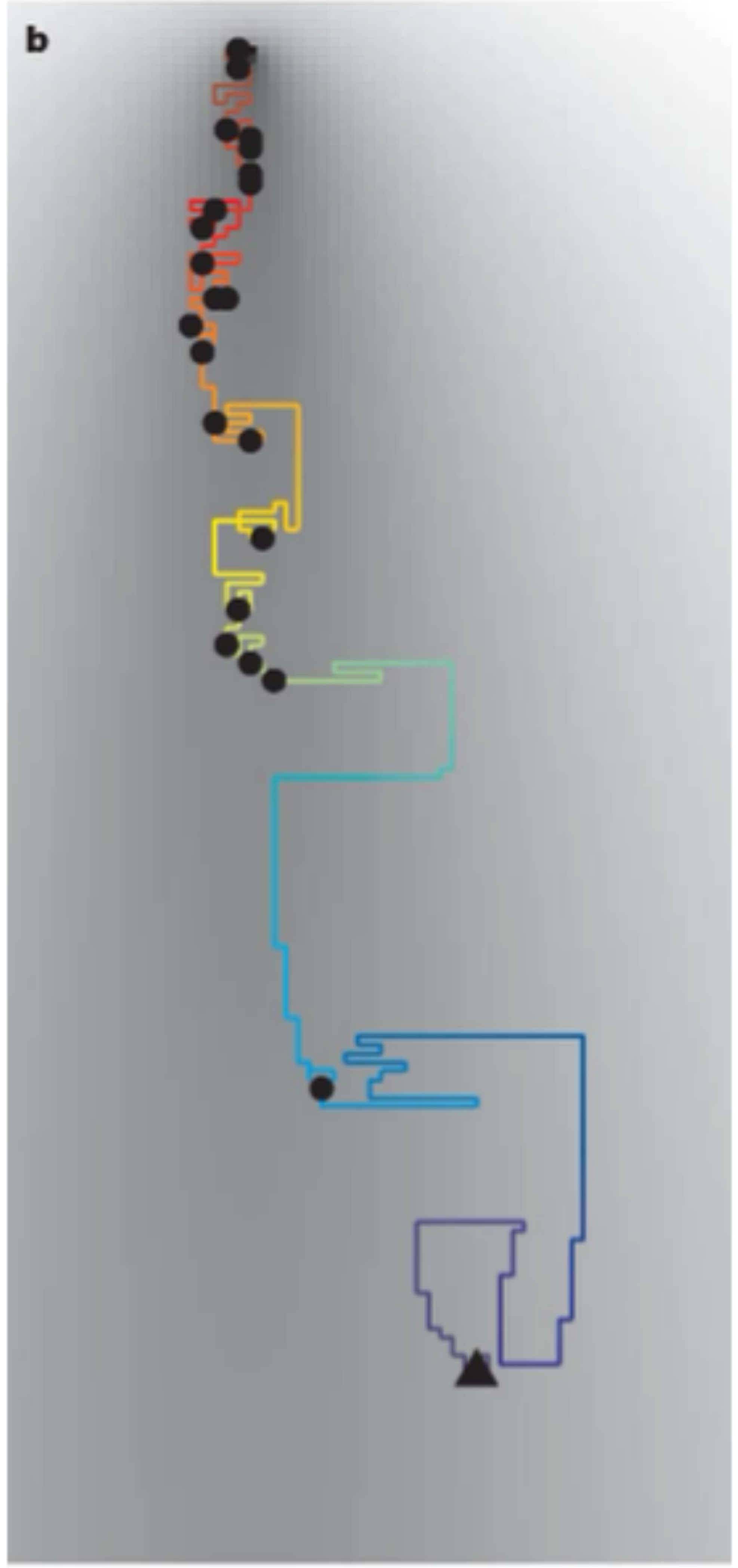
Efficiency of infotaxis

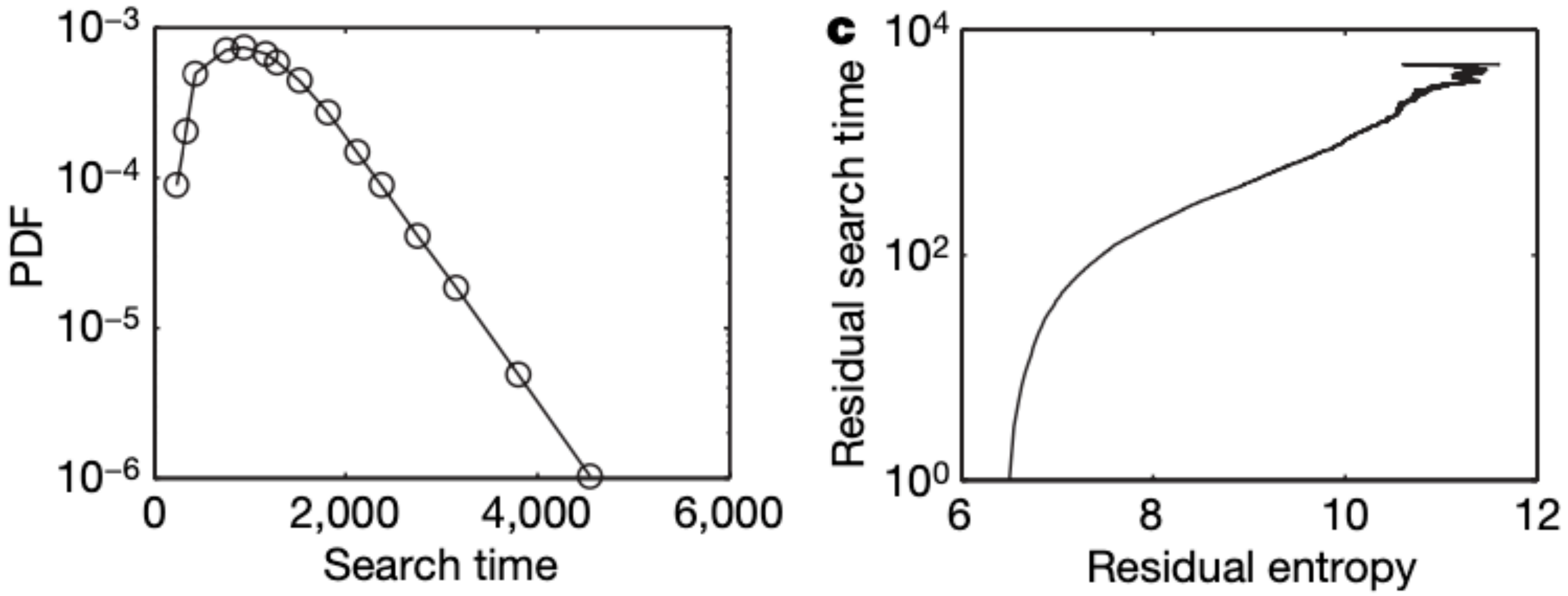
Without wind



● Scent detection (hit)

With wind

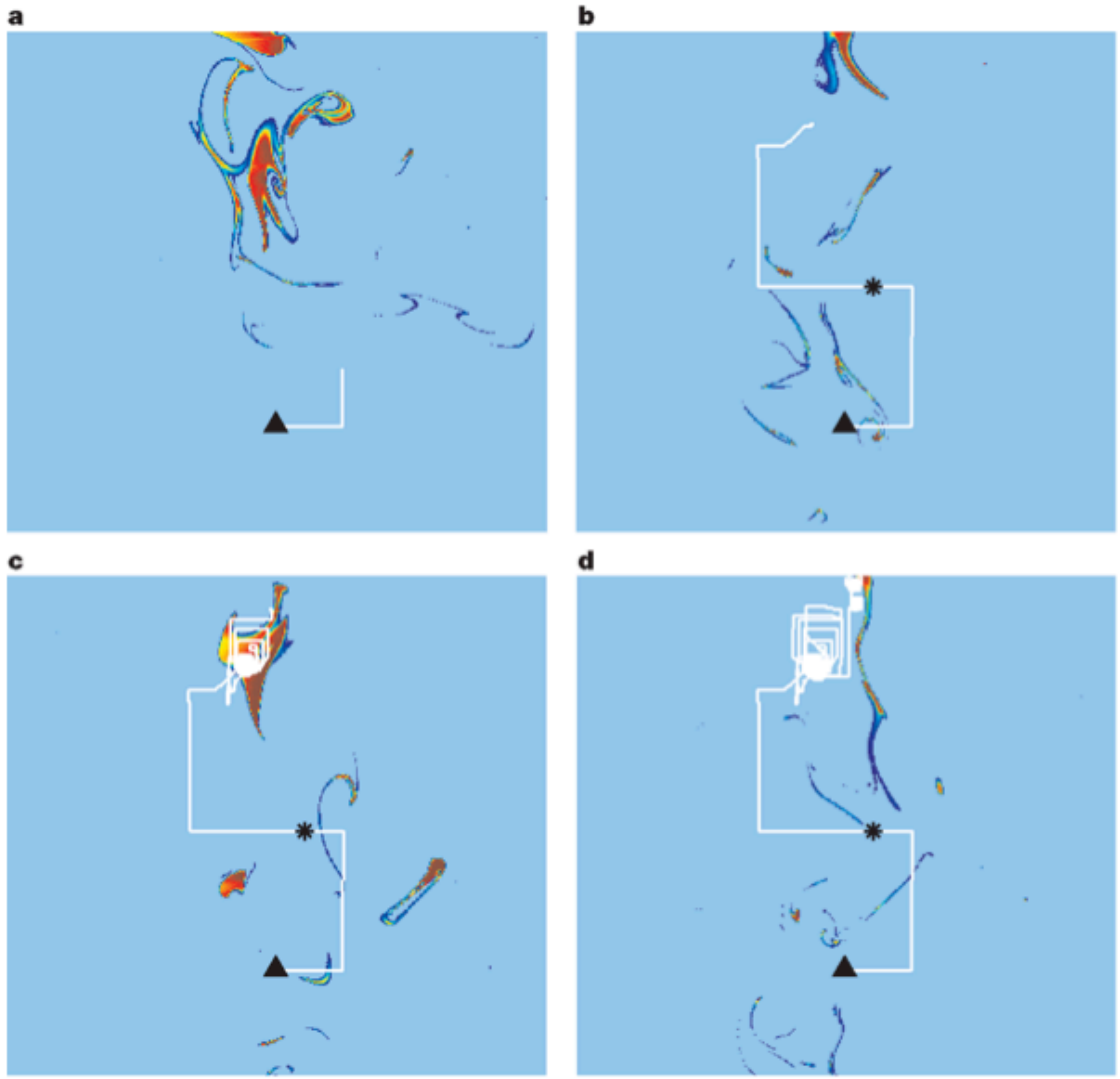




Infotaxis allows for a fast and efficient search in high entropy environments by tracking the information obtained from sparse signal detection events.

(Vergassola et al. 2007)

Robustness of infotaxis

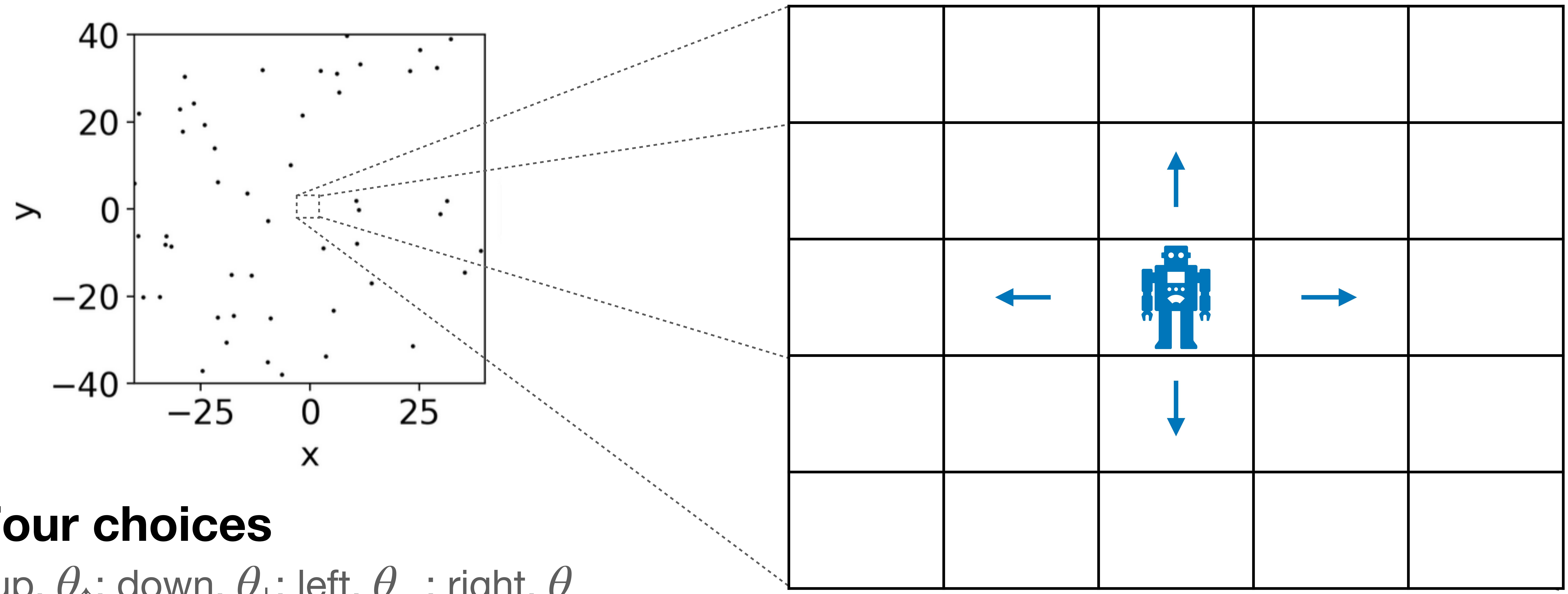


Infotaxis allows is effective even in dynamic environments where the spatial distribution of signal varies with time.

(Vergassola et al. 2007)

Curiosity-driven search

Information seeking valentino (Info)



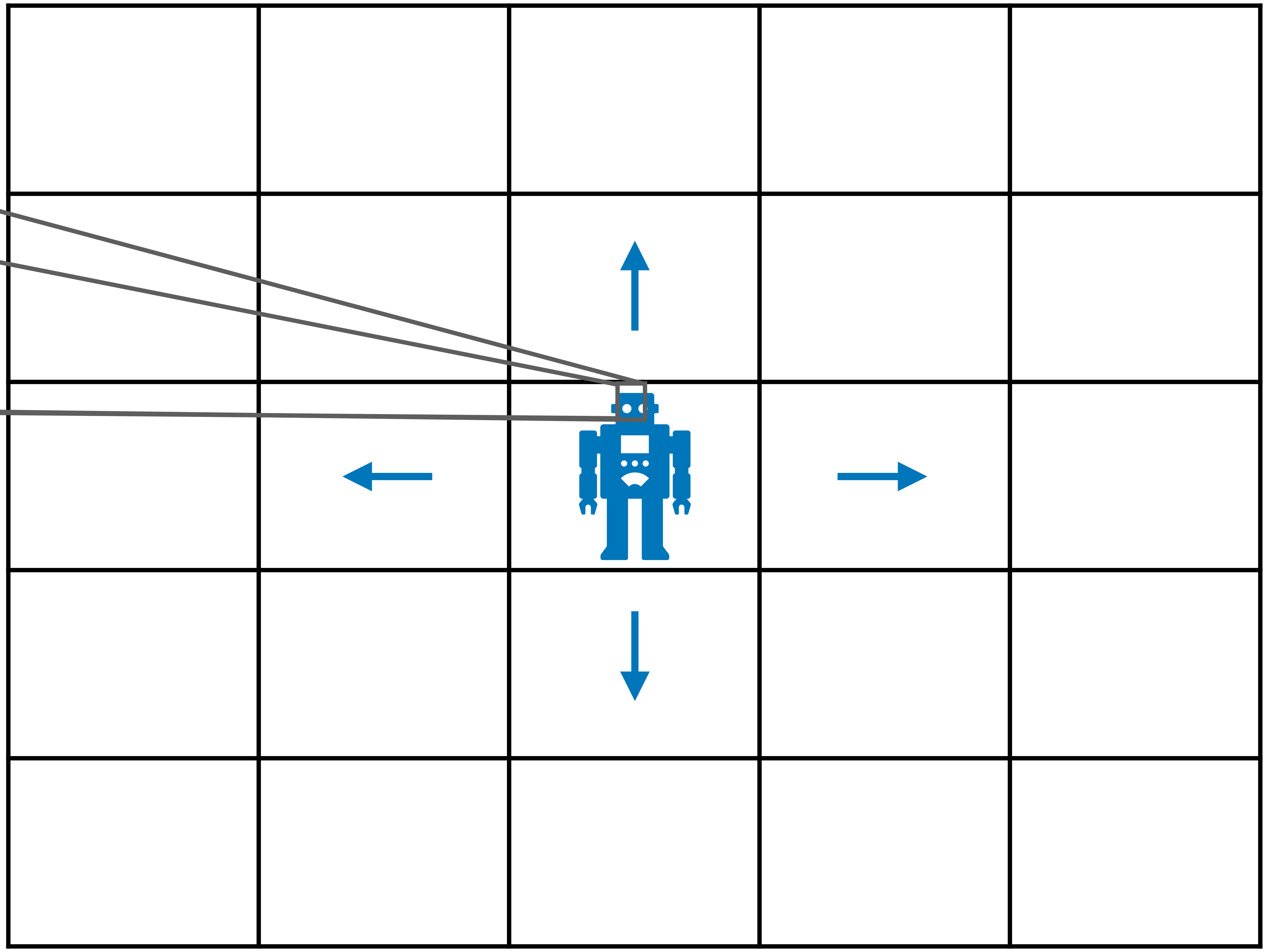
Four choices

up, $\theta_{\uparrow}$; down, $\theta_{\downarrow}$; left, $\theta_{\leftarrow}$; right, $\theta_{\rightarrow}$

Information seeking valentino (Info)

Information Memory E

1/n	1/n	1/n	1/n	1/n
1/n	1/n	1/n	1/n	1/n
1/n	1/n	1/n	1/n	1/n
1/n	1/n	1/n	1/n	1/n
1/n	1/n	1/n	1/n	1/n



- Give Info a spatial memory, E,of the information contained at each position of the grid
- Initialize E as a uniform distribution at first, giving each position equal amounts of information.
- Update the information at each point in the grid.

The “info” algorithm

Algorithm 4 Info

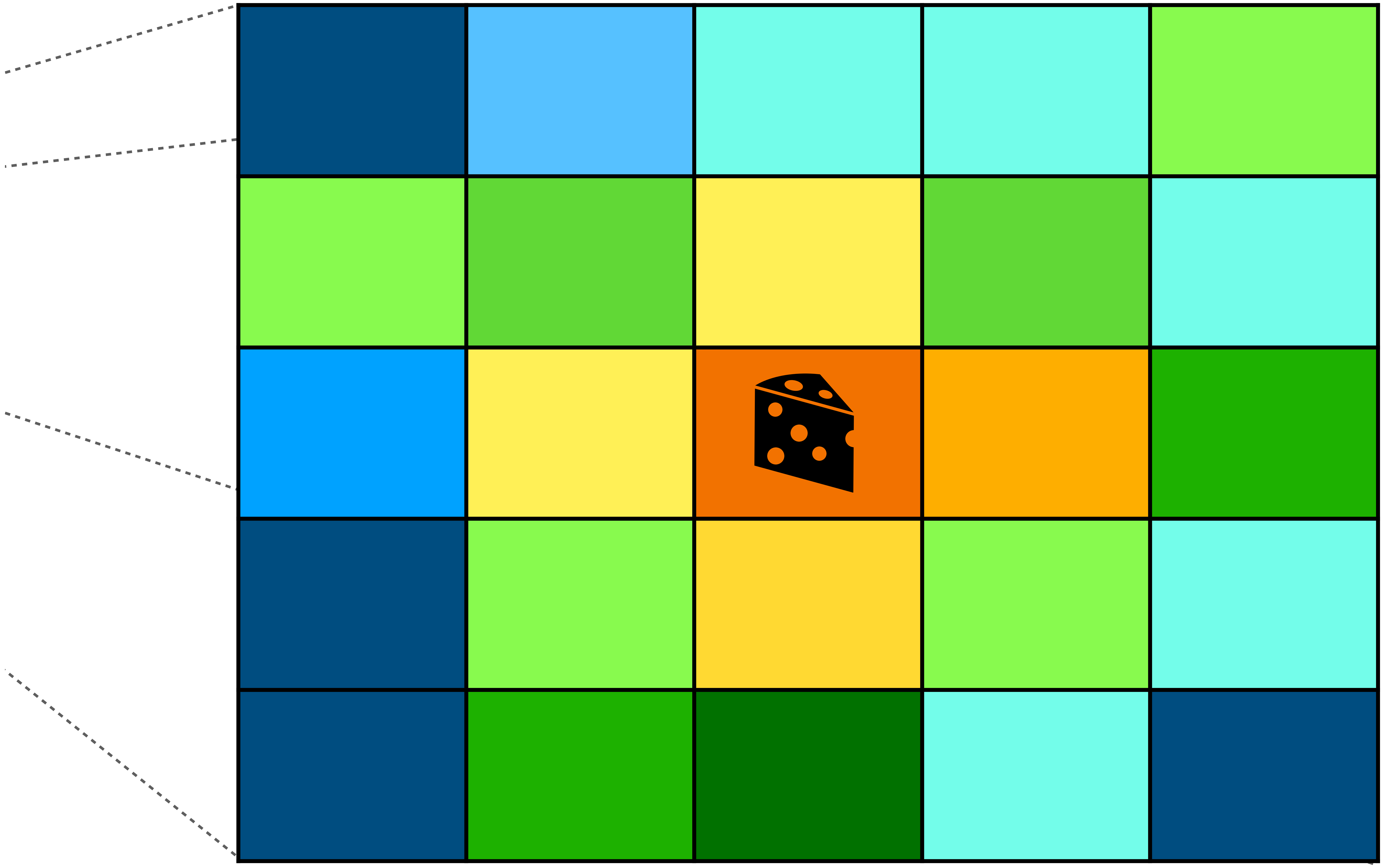
```
Set n_max number steps
Set a information threshold
Determine n_pos number of positions on the grid
Initialize grid memory ∀i, j E(i, j) = 1/n_pos
Set probability of tumble when Δo > 0 as ρ+
Set probability of tumble when Δo ≤ 0 as ρ−
for step = 1, . . . , n_max do
    Sample gradient: Δo = o_s − o_{s−1}
    Sample state: η_t ∼ U(0, 1)

    if Δo > 0 then
        hit = 1
    else
        hit = 0
    end if

    Get old info state: p(i, j) = E(i, j)
    Determine info gain: ΔE = D_KL(p(i, j), hit)

    if (ΔE > a and η_s > ρ+) or (ΔE ≤ a and η_s > ρ−) then
        Select direction: θ_s ∼ U(1, 4)
    else
        Select direction: θ_s = θ_{s−1}
    end if

    Change memory: E(i, j) = hit
    Move 1 step in θ_s direction
end for
```



Take home message

- Infotaxis uses the information gained from sparsely occurring events as a policy to determine search.
- Infotaxis allow for fast and efficient search behaviors in dynamic, low-signal environments.

Lab 8: Infotaxis

URL: https://coaxlab.github.io/BIX-book/notebooks/lab7-infotaxis.html