Can information be its own signal?

Readings for today

- Vergassola, M., Villermaux, E., & Shraiman, B. I. (2007). ‘Infotaxis’ as a strategy for searching without gradients. Nature, 445(7126), 406-409.

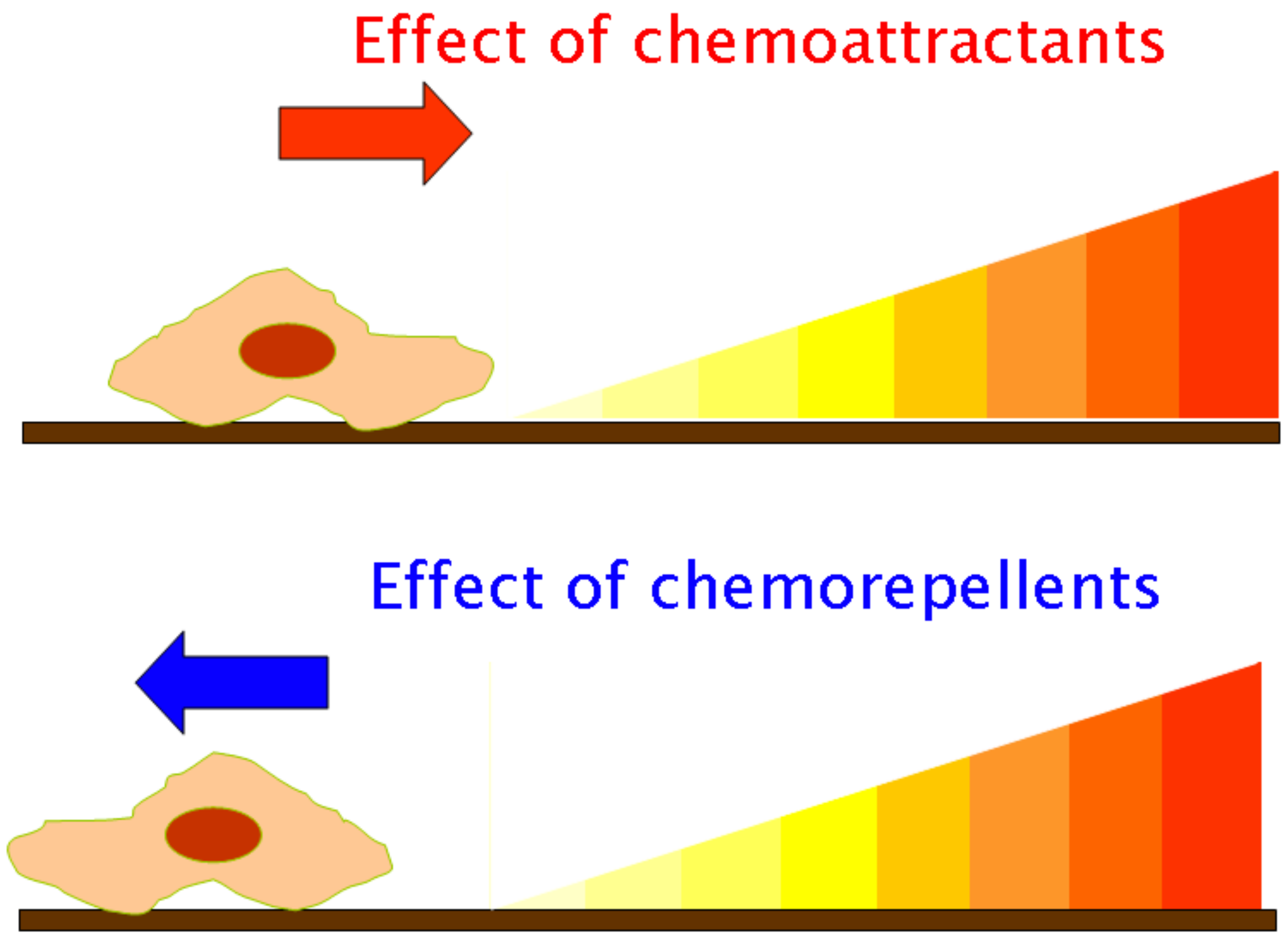
Topics

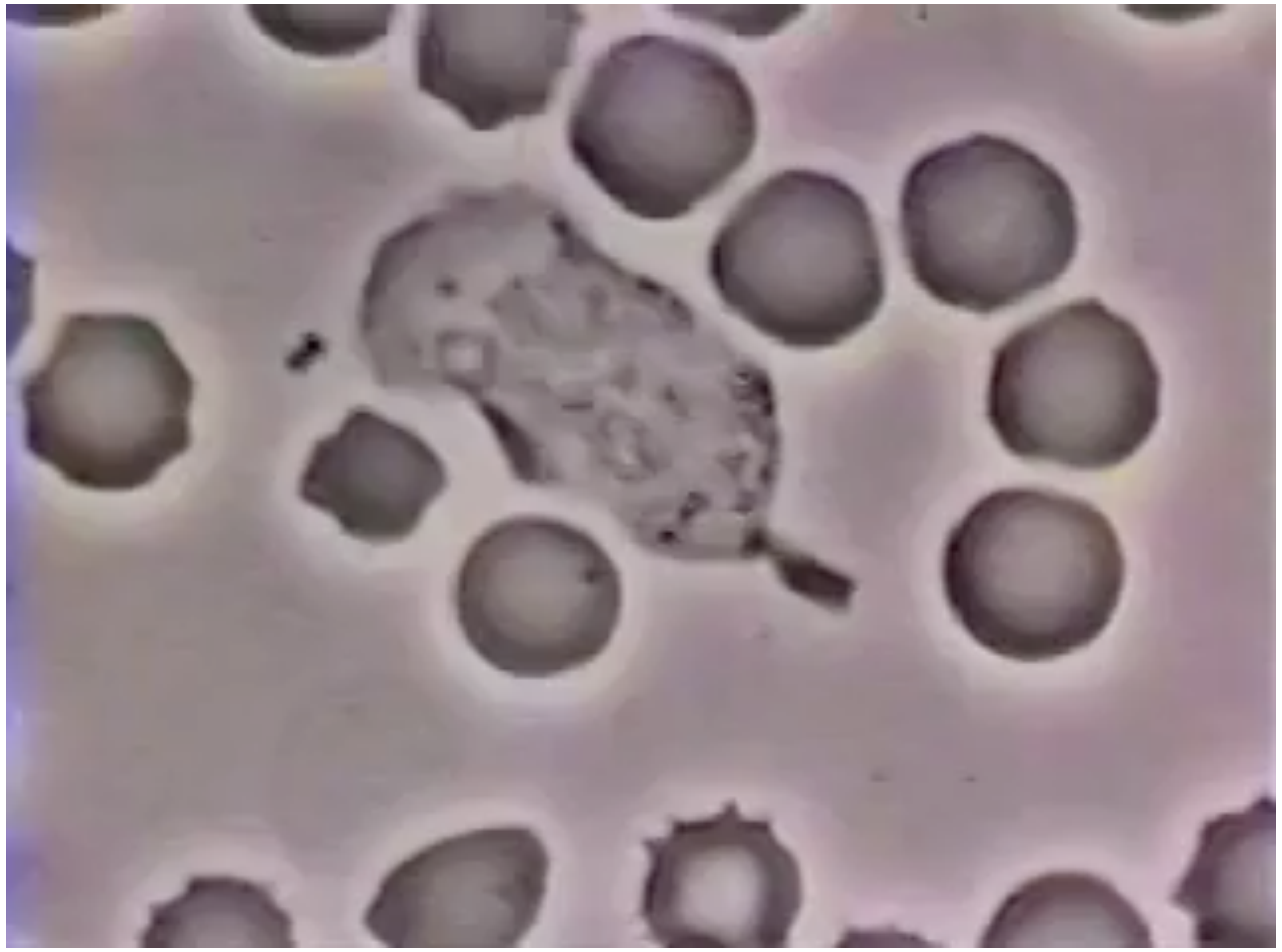
- Review of chemotaxis
- Infotaxis by entropy reduction

Review of chemotaxis

Chemotaxis

Movement in response to a chemical stimulus.

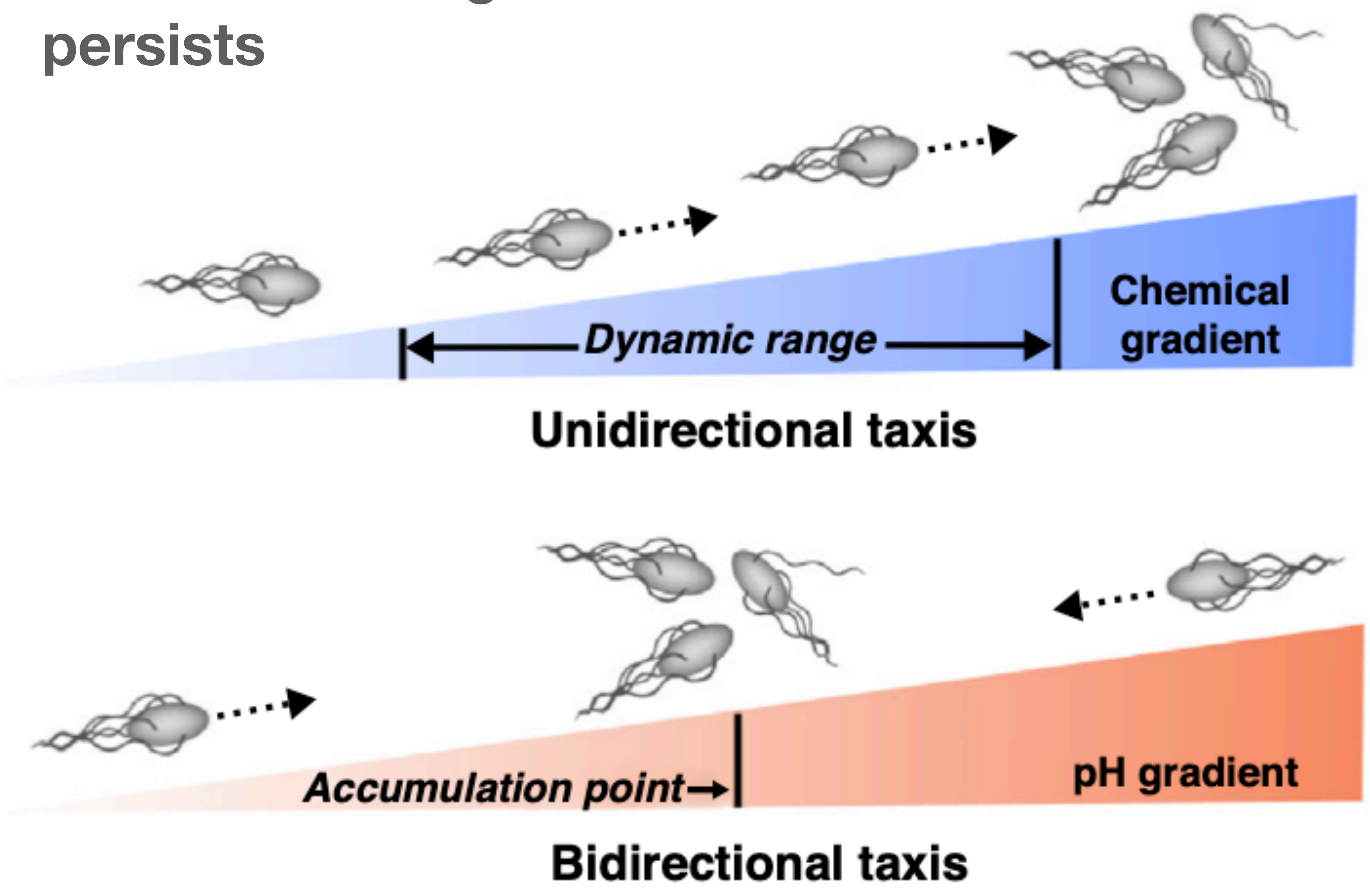




https://routledgetextbooks.com/textbooks/9780815344506/videos.php

The chemosensing algorithm

Keep moving in the same direction if the gradients persists



Calculate gradient

olfactory scent magnitude

$$\nabla o = o_t - o_{t-1}$$

Accumulate evidence

accumulation noise

$$e_t = \gamma * \nabla o + \eta$$

accumulation rate

Make decision

$$\theta_t = \begin{cases} \theta_{t-1}, & \text{if } e_t < a \\ U(-\pi, \pi), & \text{if } e_t \geq a \end{cases}$$

movement angle

random turn

(Bi & Source, 2018)

The limitations of chemotaxis

The problem of dilution

In many natural environments, the dispersal of a chemical will lead to a fast reduction in the signal-to-noise and loss of the gradient altogether.

The dilute limit

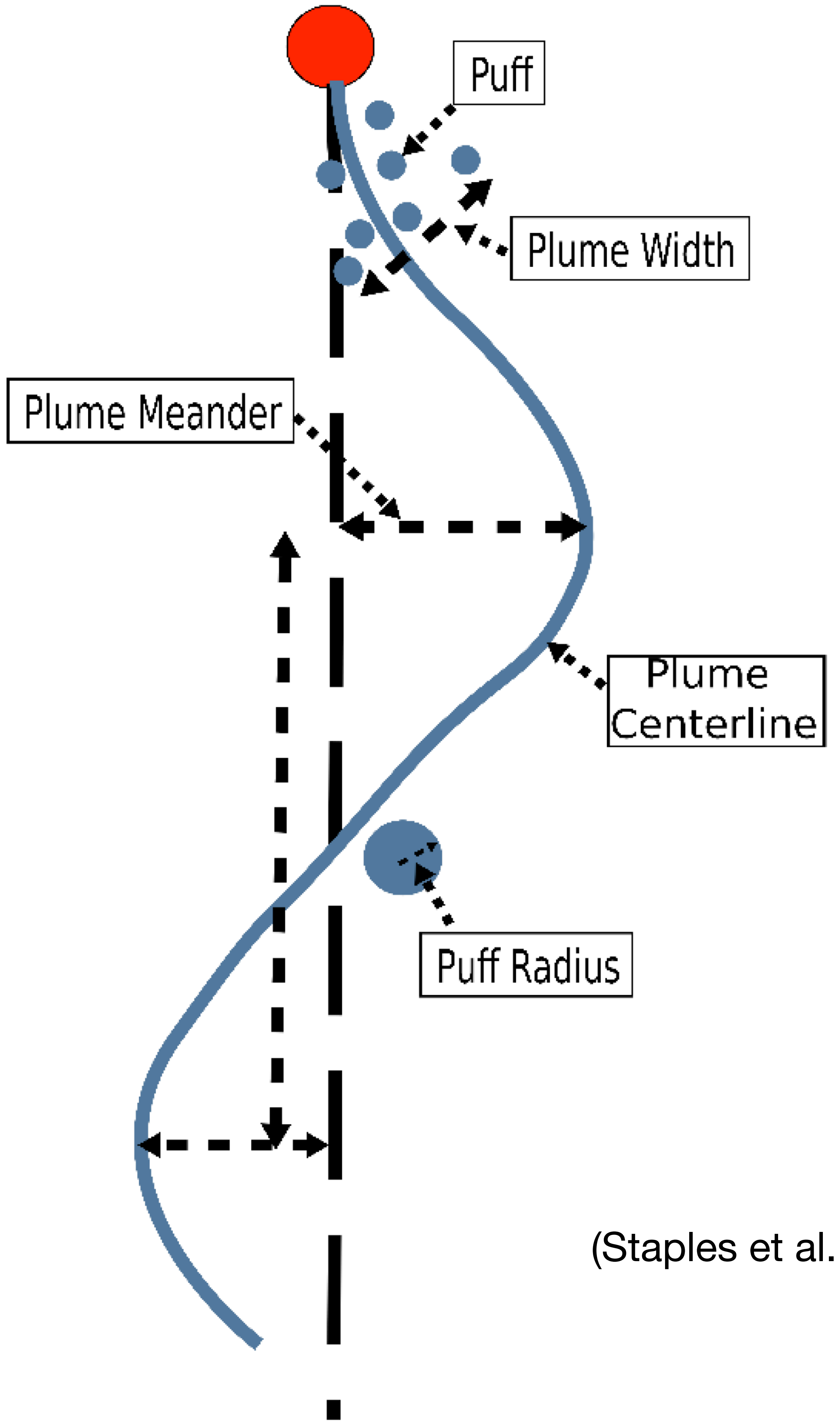
Infotaxis by entropy reduction

Search for information

Infotaxis

A search strategy that organisms or algorithms use to find a source of interest by optimizing the acquisition of information from the environment, often in situations where the source is intermittently detectable or has a sparse distribution.

A balance between exploring areas of uncertainty and exploiting areas where the source has been previously detected.



(Staples et al. 2023)

(Vergassola et al. 2007)

Infotaxis algorithm

Shannon entropy

detection probability

$$H(s) = -\sum_{\mathbf{x}} p(\boldsymbol{x}) \log_2 \boxed{p(\boldsymbol{x})}$$

belief state $s = \left[\boldsymbol{x}^a, p(\boldsymbol{x})\right]$

Information gain

with action a in belief state s

$$G(s, a) = H(s) - H(s \mid a)$$

How much uncertainty is reduced by taking action a

Expected entropy

upon taking action a in belief state s

$$H(s \mid a) = \sum_{s'} \Pr\left(s' \mid s, a\right) H\left(s'\right)$$

successor state

Infotaxis policy

Select the action a that maximizes the expected information gain in belief s

$$\pi^{info}(s) = \arg\min_{a} \sum_{s'} \Pr\left(s' \mid s, a\right) H\left(s'\right),$$

(Vergassola et al. 2007; Loisy and Eloy, 2022)

Maximizing information gain

Information gain is entropy reduction

$$\underbrace{G(s,a)}_{} = \underbrace{H(s)}_{\text{overall entropy}} - \underbrace{H(s \mid a)}_{\text{entropy after action } a} \longrightarrow H(s \mid a) < H(s)$$

search trajectory

$$\underbrace{\Delta H(r_i \to \boxed{r_j})}_{\text{change in entropy}} = \underbrace{P_t(r_j)[-H]}_{\text{finding the source}} + \underbrace{[1 - P_t(r_j)][\rho_0(r_j)\Delta H_0 + \rho_1(r_j)\Delta H_1 + \ldots]}_{\text{alternative case when source is not found}}$$

Note: In Vergassola et al. 2007, S is used as the symbol for entropy. Here we use the traditional H

(Vergassola et al. 2007)

Infotaxis policy in more detail

Infotaxis policy

$$\pi^{info}(s) = \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) H\left(s'\right)$$
$$= \arg\max_a \; G(s, a)$$
$$= \arg\min_a \; H(s \mid a)$$

At each time step, the searcher chooses the direction that **locally maximizes the expected rate of information acquisition**. Entropy decreases faster closer to the source because cues arrive at a faster rate.

Optimal source-tracking policy

$$\pi^*(s) = \arg\min_\pi \mathbb{E}_{p_0,\pi}[T] \longrightarrow T = \text{search duration}$$
$$= \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) \left[1 + \boxed{v^*\left(s'\right)}\right]$$

Bellman optimality

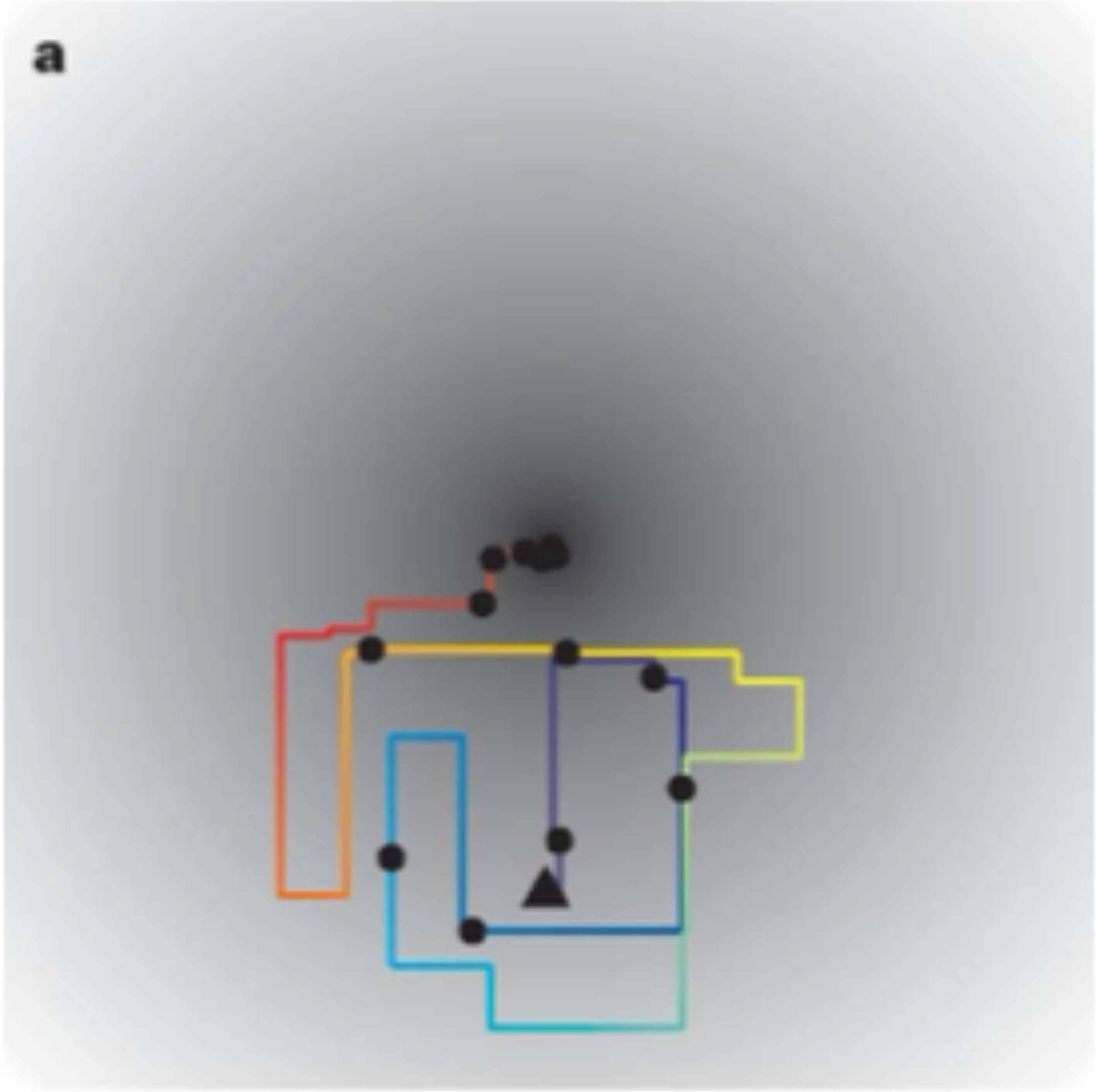
Optimal Infotaxis policy

$$\pi^*(s) = 1 + \arg\min_a \sum_{s'} \Pr\left(s' \mid s, a\right) v^*\left(s'\right)$$

(Vergassola et al. 2007; Loisy and Eloy, 2022)

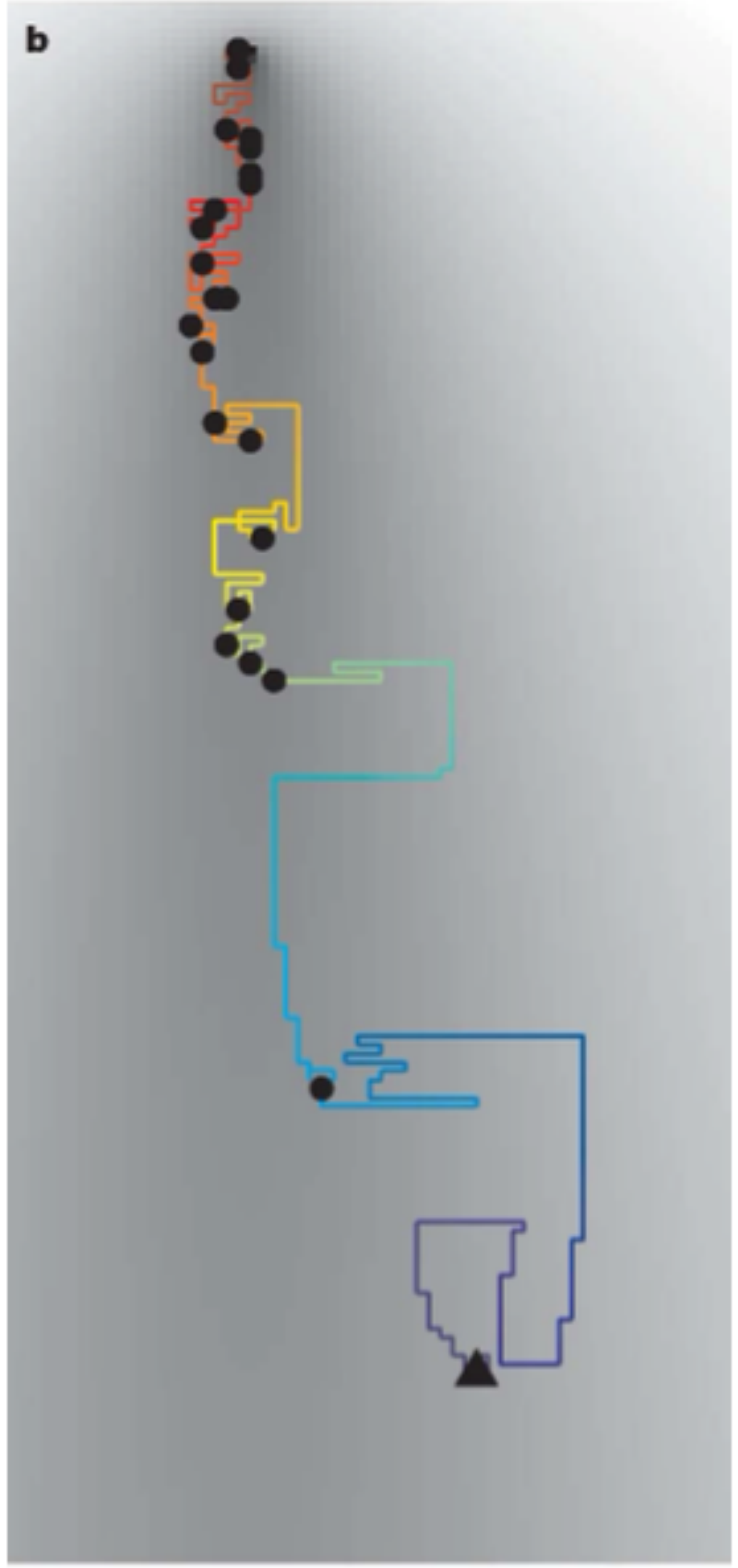
Efficiency of infotaxis

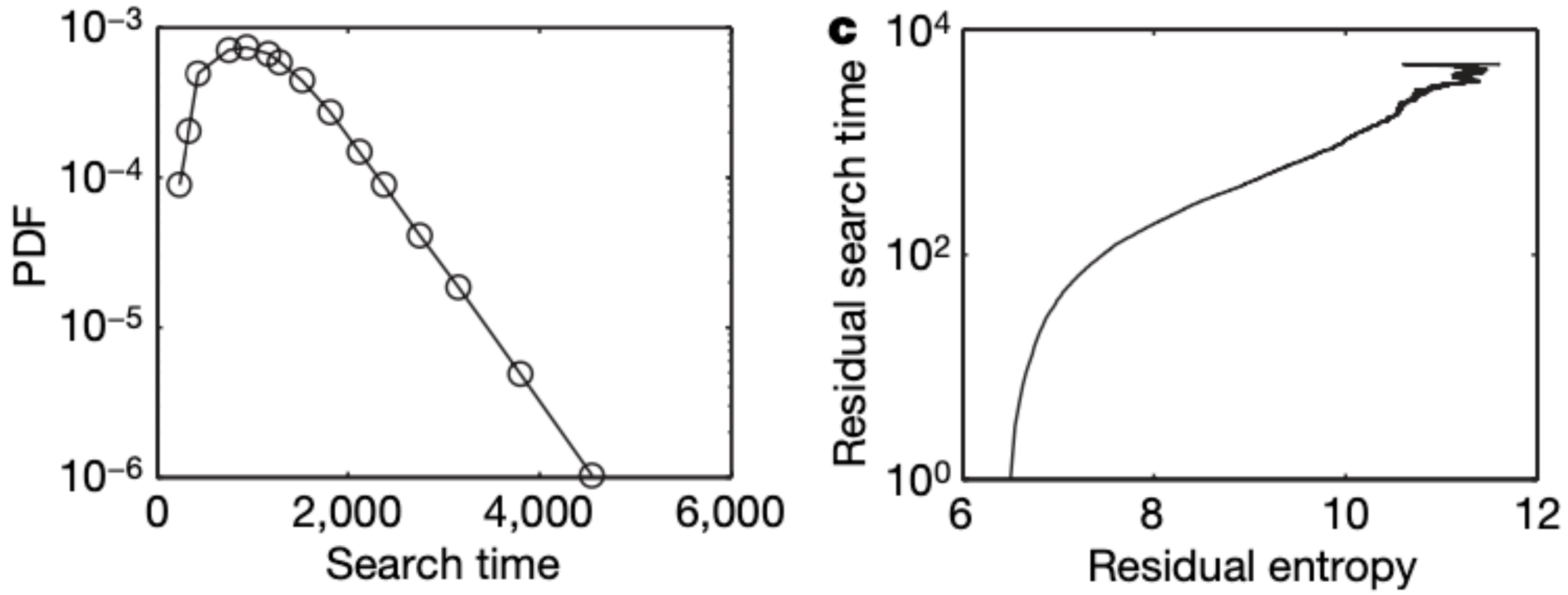
Without wind



● Scent detection (hit)

With wind

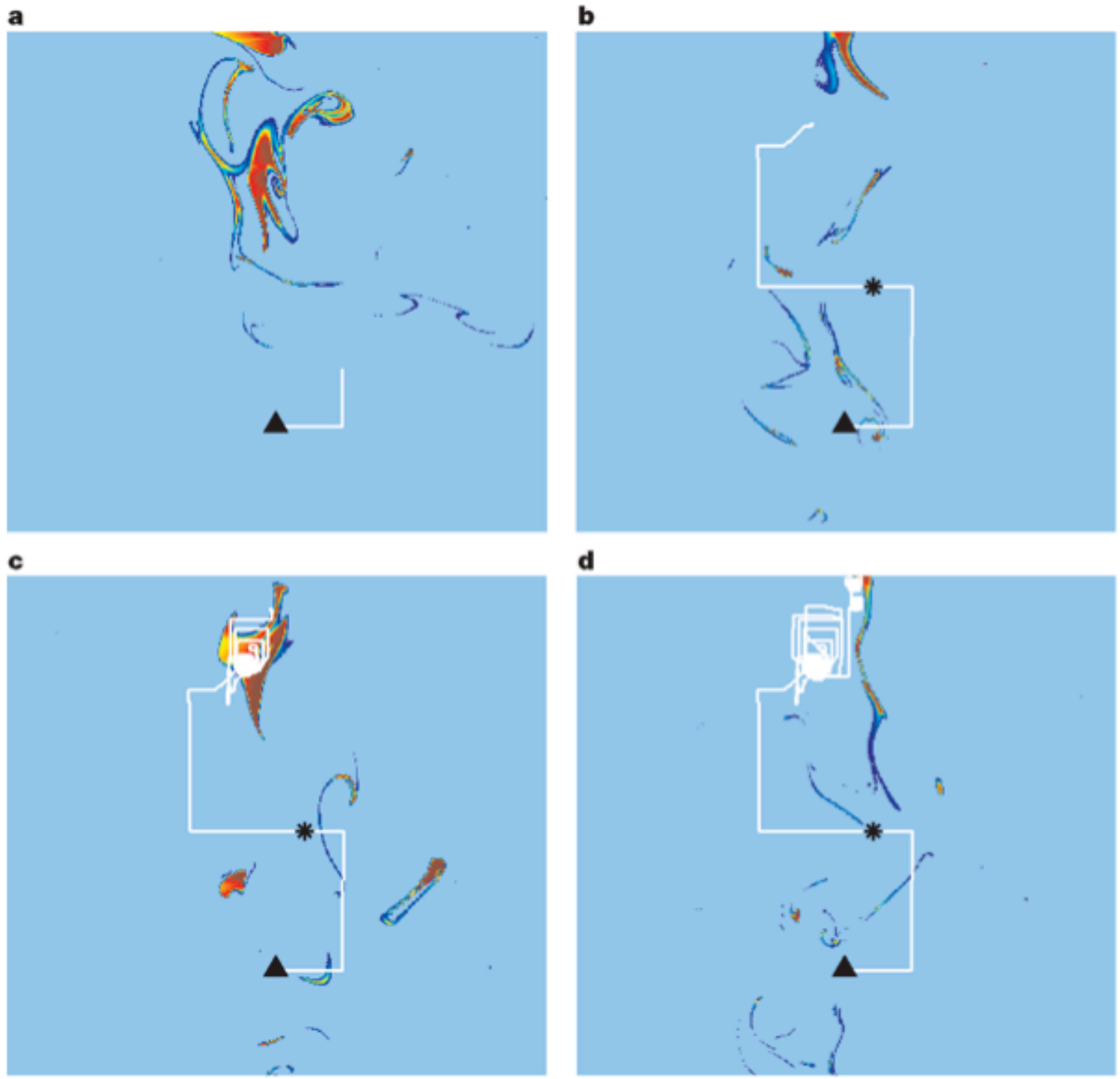




Infotaxis allows for a fast and efficient search in high entropy environments by tracking the information obtained from sparse signal detection events.

(Vergassola et al. 2007)

Robustness of infotaxis



Infotaxis allows is effective even in dynamic environments where the spatial distribution of signal varies with time.

(Vergassola et al. 2007)

Take home message

- Chemotaxis can fail when the signal becomes so sparse that the gradient disappears.
- Infotaxis uses the information gained from sparsely occurring events as a policy to determine search.
- Infotaxis allow for fast and efficient search behaviors in dynamic, low-signal environments.

Lab 8: Infotaxis

URL: https://coaxlab.github.io/BIX-book/notebooks/lab8-infotaxis.html