How do you use evidence to make a decision?

Readings for today

- Gold, J. I., & Shadlen, M. N. (2007). The neural basis of decision making. Annu. Rev. Neurosci., 30, 535-574.

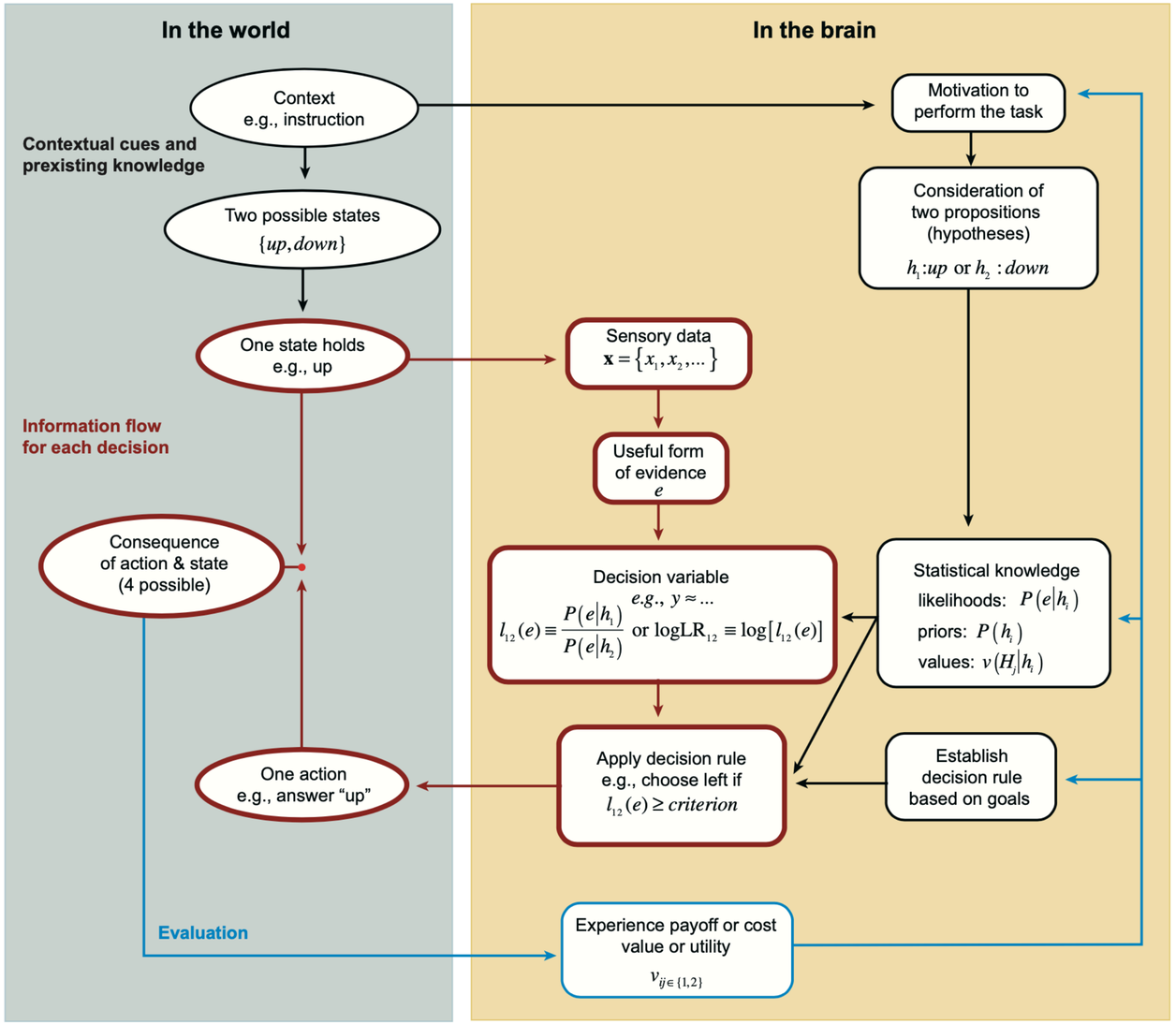
Topics

- Information accumulation
- The drift diffusion model
- The “accumulator” agent

Information accumulation

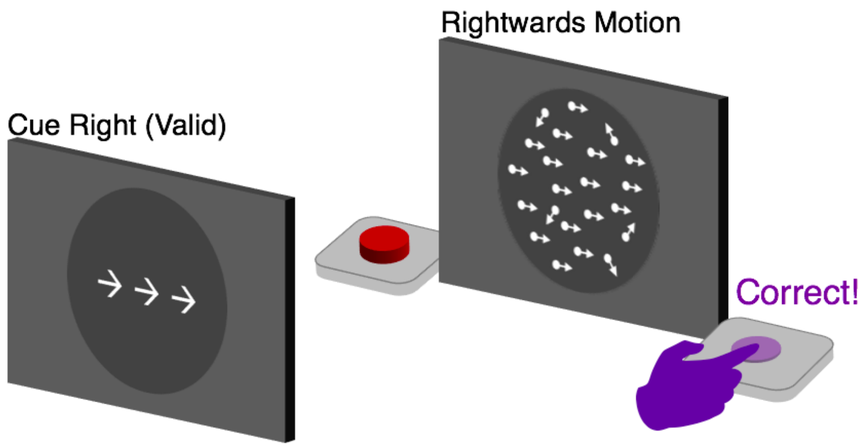
Evidence Accumulation

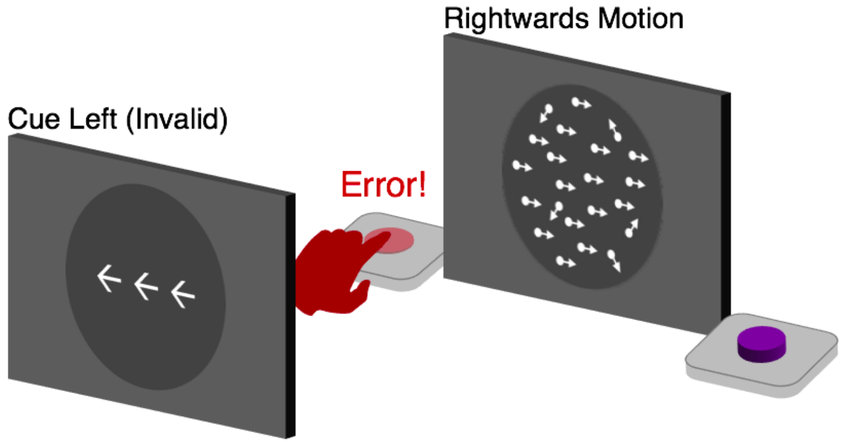
Elements of making a decision



(Gold & Shadlen, 2007)

Cued dot motion task





(Dunovan & Verstynen, 2016)

Serial evaluation of evidence

Sequential analysis framework

$$e_0 \rightarrow f_0(e_0) \Rightarrow \underset{or}{Stop}$$

$$\swarrow$$

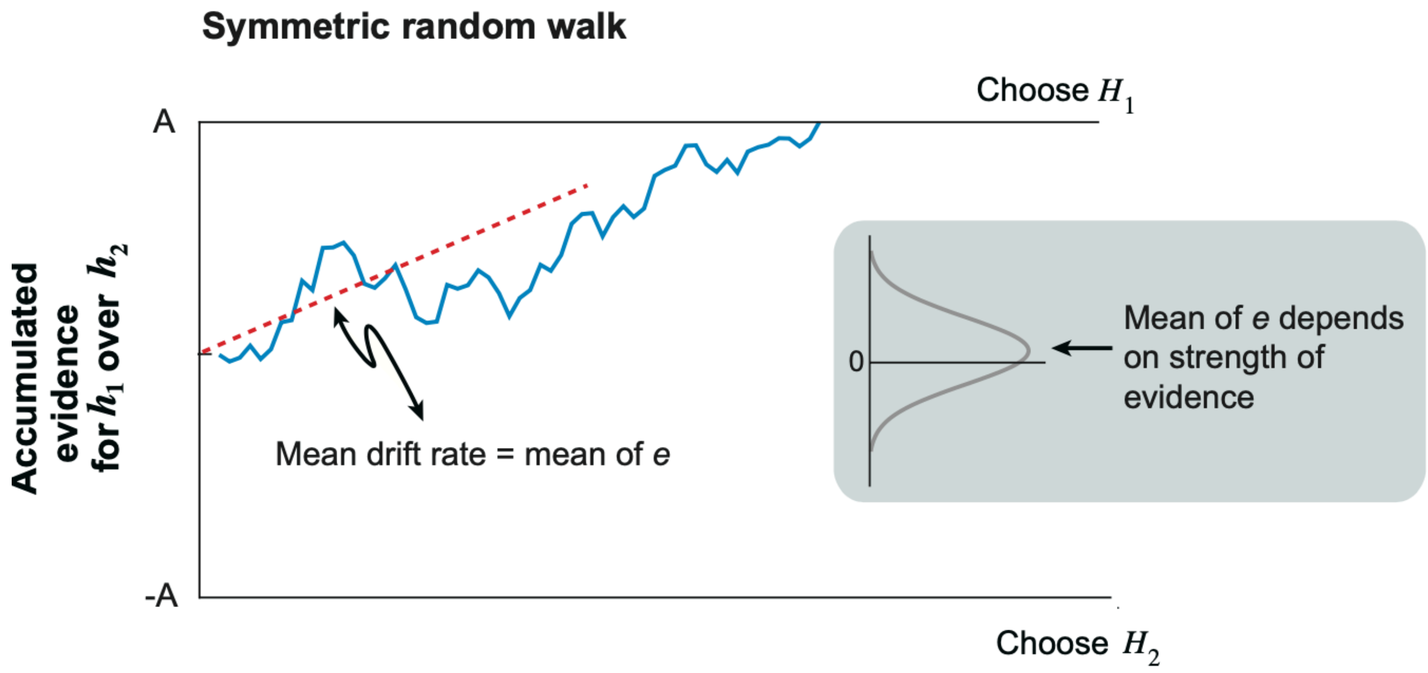
$$e_1 \rightarrow f_1(e_0, e_1) \Rightarrow \underset{or}{Stop}$$

$$\swarrow$$

$$e_2 \rightarrow f_1(e_0, e_1, e_2) \Rightarrow \underset{or}{Stop}$$

$$\vdots$$

$$e_n \rightarrow f_1(e_0, e_1, \ldots, e_n) \Rightarrow \ldots$$



(Gold & Shadlen, 2007)

Log-likelihood ratio (logLR) test

$$\log LR_{12} \equiv \log \frac{P(e_1, e_2, \ldots, e_n|h_1)}{P(e_1, e_2, \ldots, e_n|h_2)}$$
$$= \sum_{i=1}^{n} \log \frac{P(e_i|h_1)}{P(e_i|h_2)}.$$

Given *n* samples of evidence (*e*), what is the likelihood that the evidence was generated under one of two competing hypotheses (*h*)?

Problem: When have you accumulated enough evidence to make a decision?

(Gold & Shadlen, 2007)

The sequential probability ratio test (SPRT)

1. On each sample *i*, evaluate logLR

$$w_i = \log\left(\frac{P(e_i \mid h_1)}{P(e_i \mid h_2)}\right)$$

2. Sum all logLR tests up to current observation

$$y_n = \sum_{i=1}^{n} w_i$$

3. Determine stopping rule

α = type I error

β = type II error

$$y_n \geq \log\frac{1-\alpha}{\alpha}, \text{ then select } h_1$$

$$y_n \leq \log\frac{\beta}{1-\beta}, \text{ then select } h_2$$

$$\log\frac{\beta}{1-\beta} \leq y_n \leq \log\frac{1-\alpha}{\alpha}, \text{ continue sampling}$$

(Gold & Shadlen, 2007)

The drift diffusion model

The Drift Diffusion Model (DDM)

θ : evidence (decision) variable.

$\theta(0) = z \rightarrow$ initialized with value z at $t = t_{er}$

Continuous time for (Stochastic Diff Eq)

$d\theta = vdt + \sigma dW$

dW : Wiener noise process

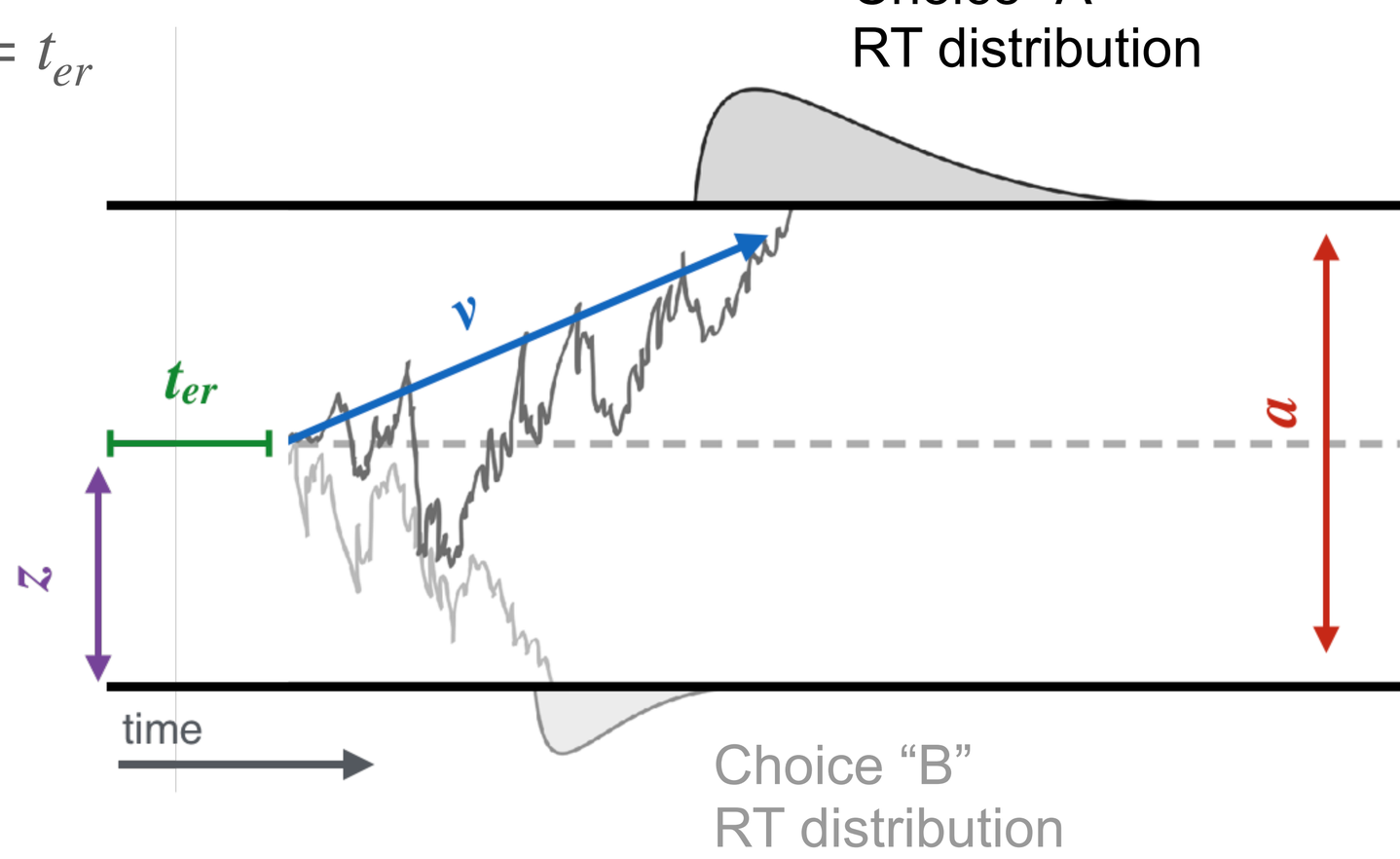
σ : diffusion constant (noise scalar)

Discrete time form

$\theta(t + \Delta t) = \theta(t) + v\Delta t + \sigma\sqrt{\Delta t}\epsilon(t)$

$\epsilon \sim N(0,1)$: Gaussian noise

Δt : timestep



(Ratcliff 1978)

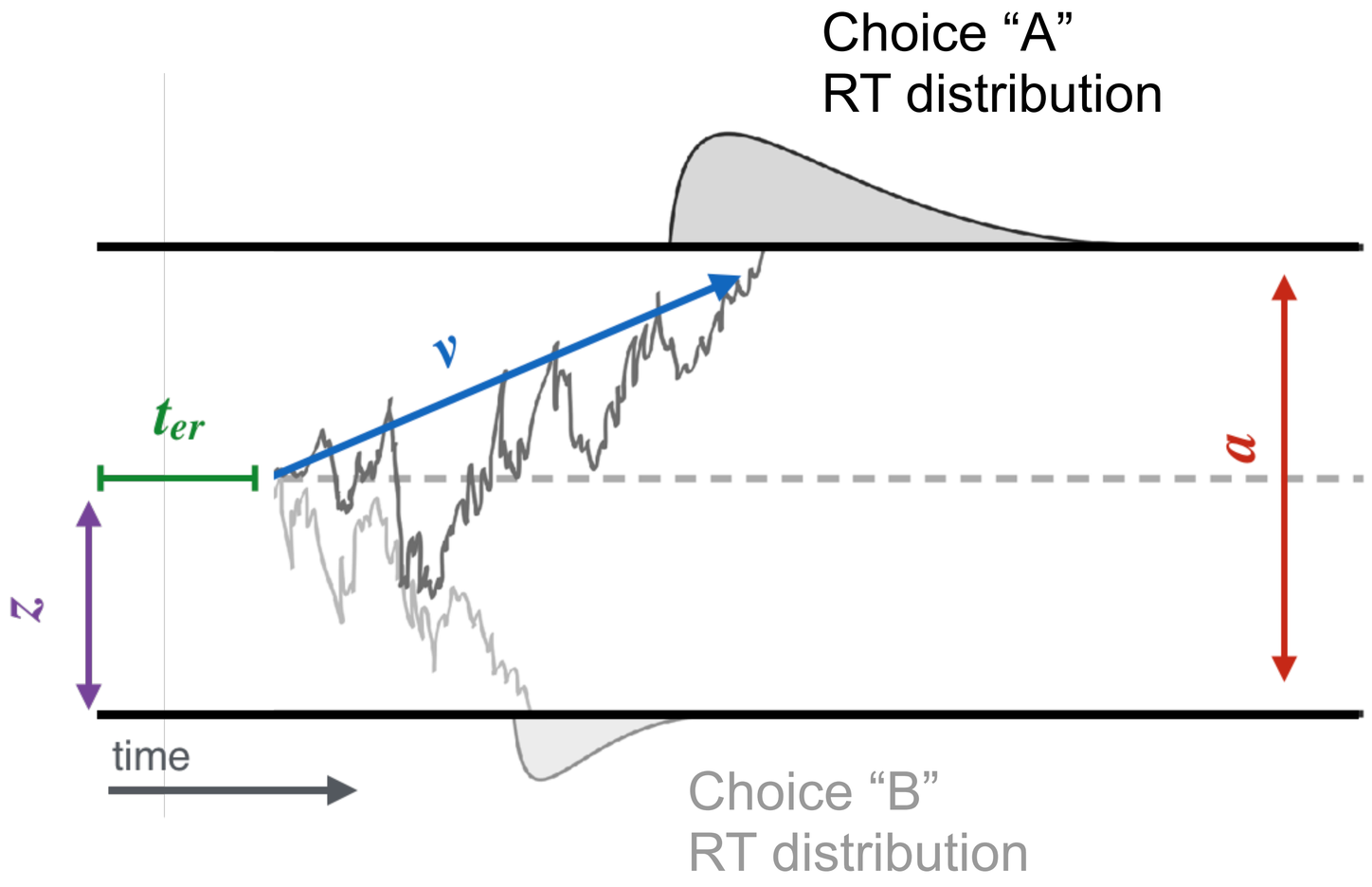
Links between variables & task parameters

- boundary height (***a***): evidence criterion
  - reflects internal strategy or priorities of agent
    - liberal strategy or prioritize speed (reduce ***a***)
    - cautious strategy or prioritize accuracy (raise ***a***)

- drift-rate (***v***): relative strength of evidence for two alt. choices
  - typically depends on environment (externally controlled)
    - higher motion coherence => increase in ***v***
    - greater difference in associated value of alt. actions => increase in ***v***

- starting-point (***z***): initial value of evidence, bias
  - reflects internal biases or expectations of agent
    - anticipate left motion, move ***z*** closer to “left” decision boundary
    - anticipate right motion, move ***z*** closer to “right” decision boundary

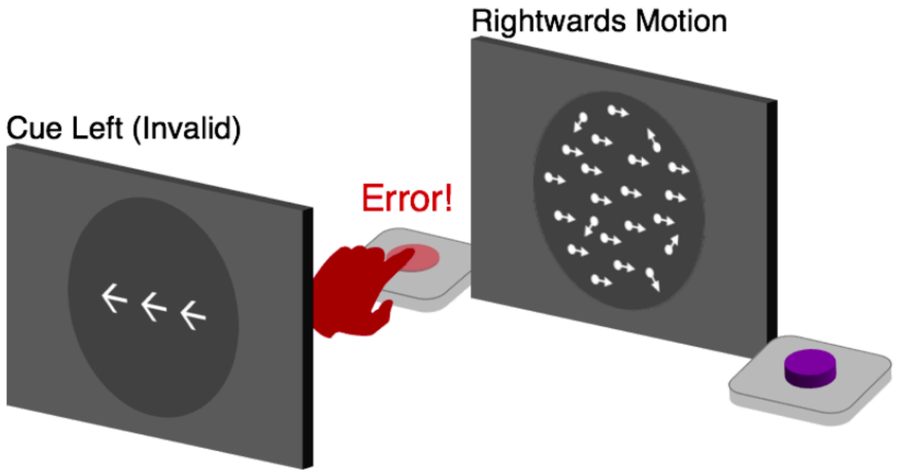
- non-decision time (***tr***) sensory and motor delays:
  - traditionally viewed as a “nuisance” parameter (i.e., capturing internal and/or external variables related to sensory or motor but *not* decision computation)

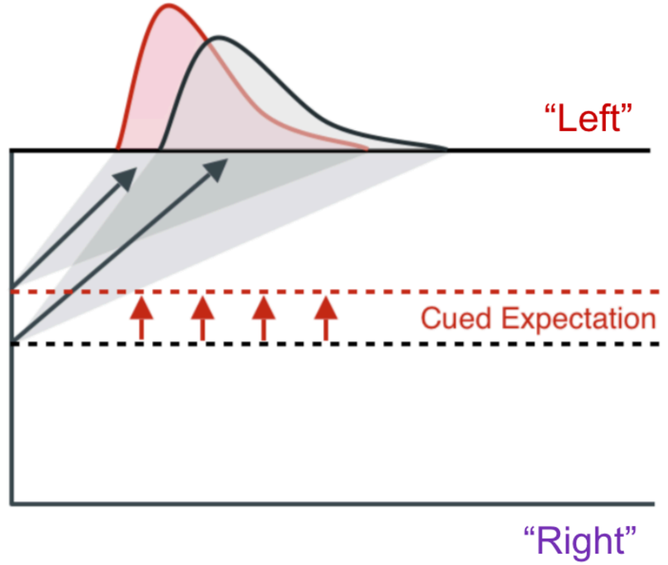


Two critical properties of the DDM

1. Parameters are ***identifiable*** (e.g., recoverable) from behavioral data (RT distributions and choice accuracy)

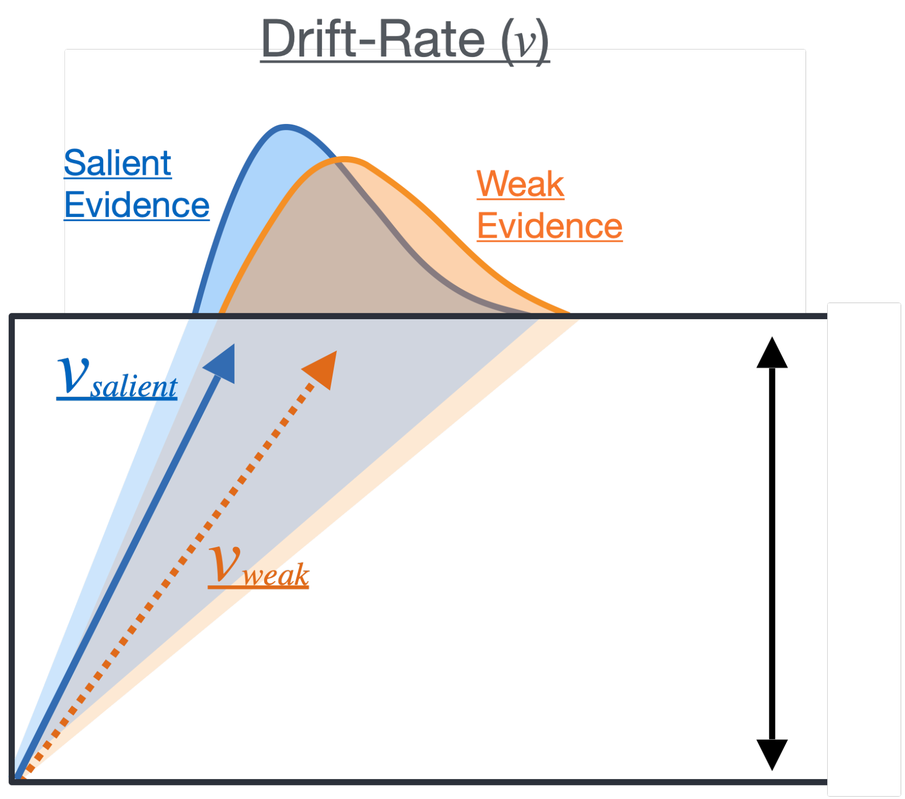
2. Parameters are reliably modulated by specific task variables, capturing sensible links between internal decision mechanisms and external task environmental demands

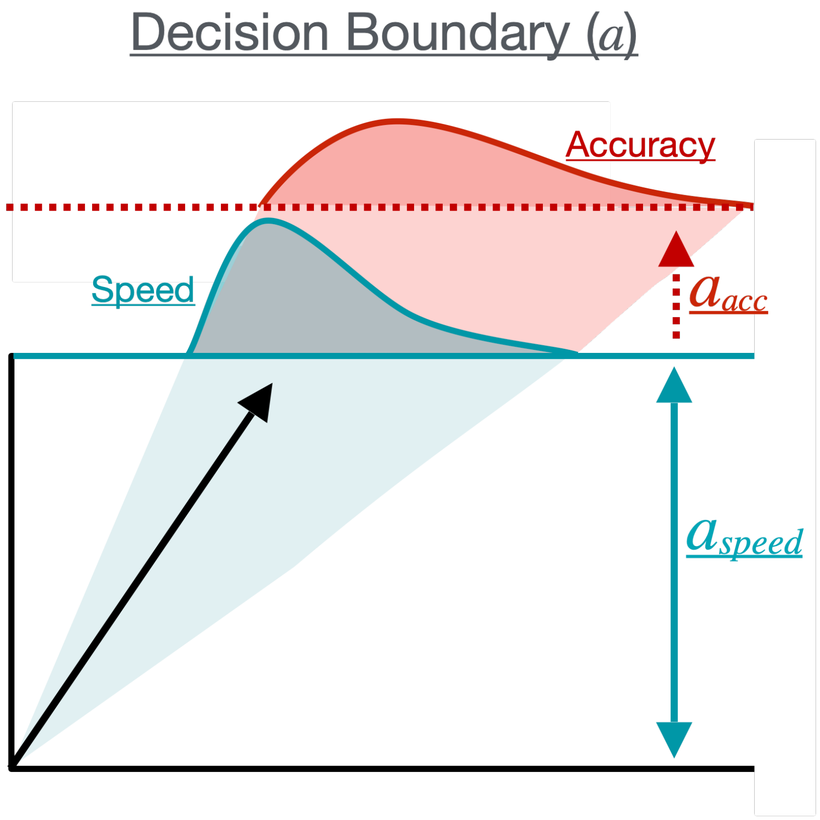




Parameter identifiability

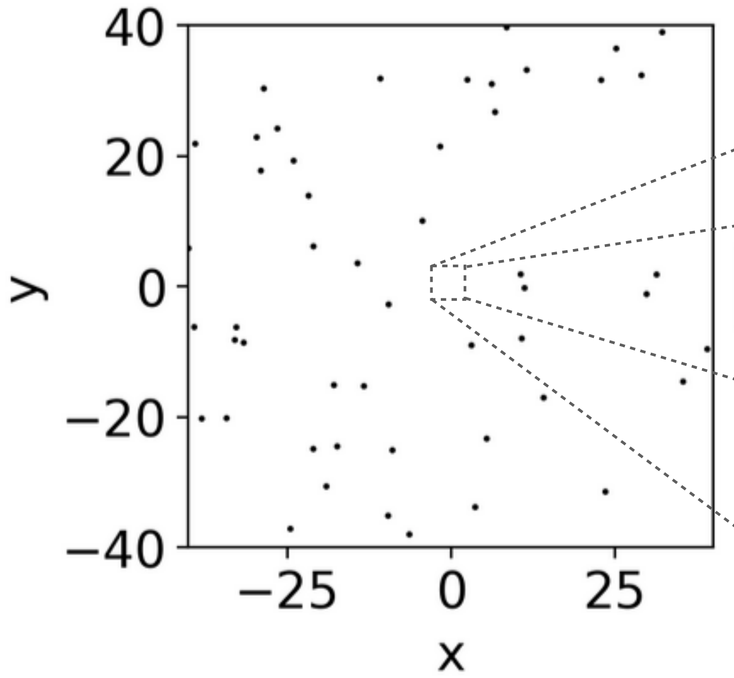
Example: How do we distinguish between behavioral effects caused by a change in drift-rate (v) and change in boundary height (a)?

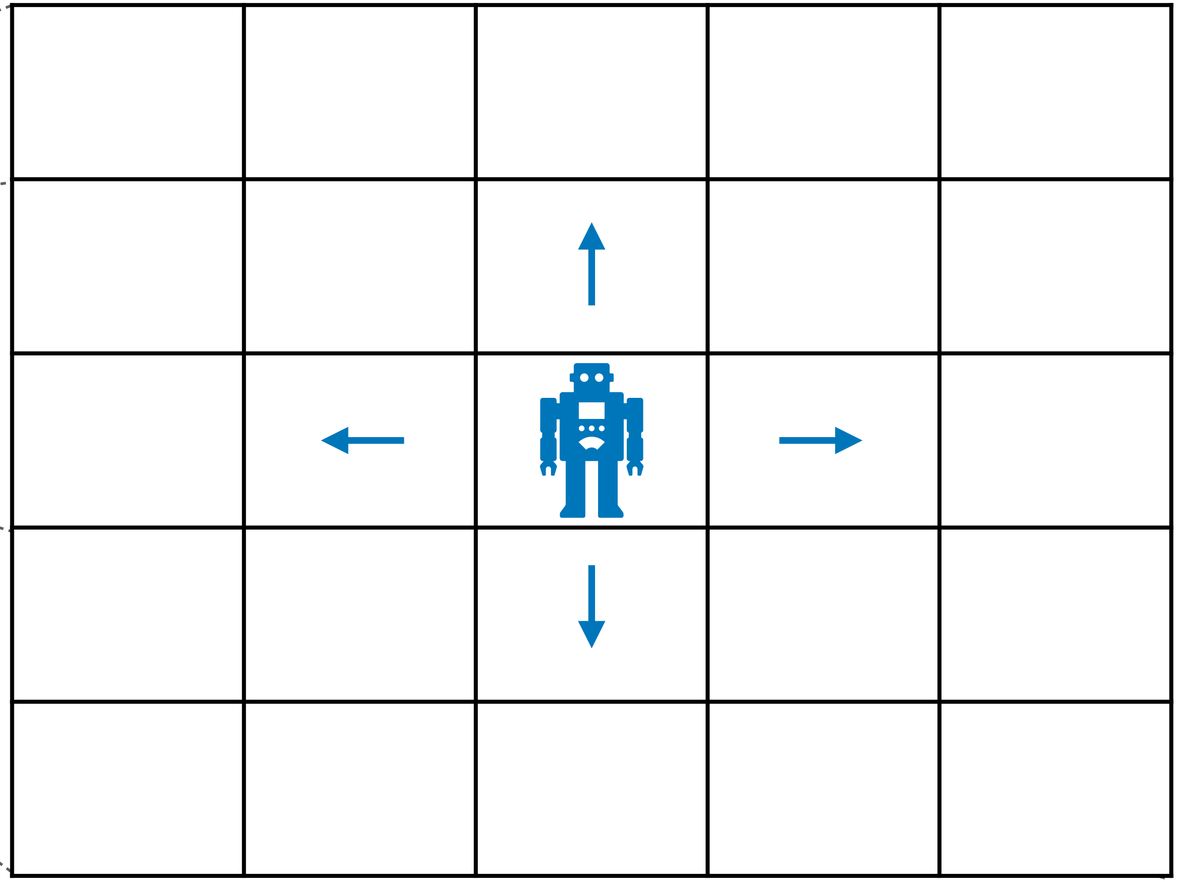




The “accumulator” agent

Moving to a grid world





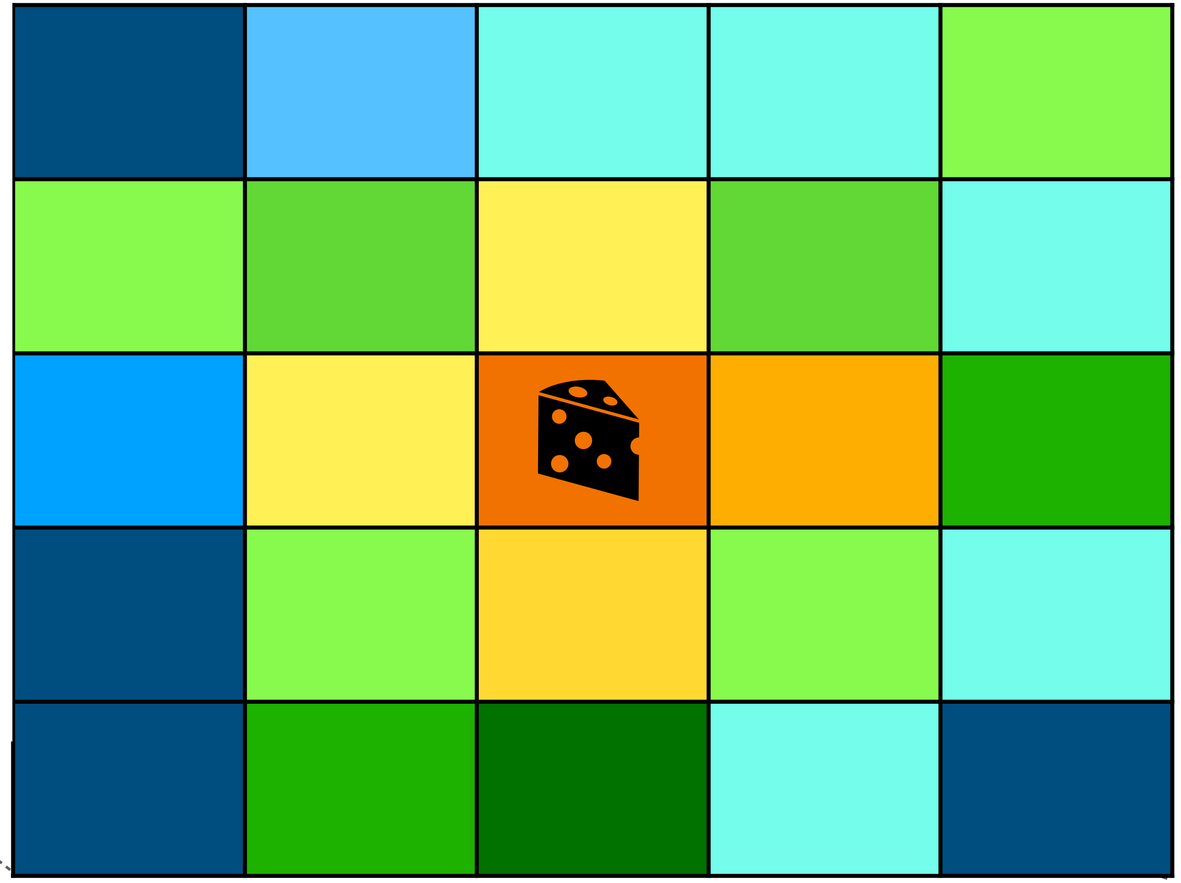
Four choices

up, $\theta_{\uparrow}$; down, $\theta_{\downarrow}$; left, $\theta_{\leftarrow}$; right, $\theta_{\rightarrow}$

The “accumulator” algorithm

Algorithm 3 Accumulator

Set n_{max} number steps
Set drift rate, ν
Set boundary height, a
Set accumulation noise, σ
Set initial direction: $\theta_0 \sim U(1, 4)$
for $step = 1, \ldots, n_{max}$ **do**
    Sample gradient: $\Delta o = o_s - o_{s-1}$
    Update evidence: $e_s = \nu \Delta o + \sigma \epsilon_s$
    if $e_t > a$ **then**
        Select new direction: $\theta_s \sim U(1, 4)$
    else
        Continue with old direction: $\theta_s = \theta_{s-1}$
    end if
    Move 1 step in θ_s direction
end for



Goal

Accumulate evidence to “decide” whether to tumble in a new direction

Take home message

- At any given moment, the ratio of likelihoods is the optimal method for evaluating two hypotheses (when probabilities are known).
- Accumulation of evidence optimally occurs via a sequential probability ratio test (SPRT), which follows an information diffusion process.
- Models like the drift diffusion model reflect operationalizations of the SPRT that can be fit to empirical data.

Lab 4: Evidence accumulation

URL: https://coaxlab.github.io/BIX-book/notebooks/lab4-evidence_accumulation.html