What is the nature of information?

Readings for today

- Timme, N. M., & Lapish, C. (2018). A tutorial for information theory in neuroscience. eneuro, 5(3).

Topics

- Entropy
- Mutual information
- Transfer entropy

Entropy

Information theory

<u>Goal</u>: A formal theory for the transmission, processing, extraction, and utilization of information.

<u>Approach</u>: Quantify the *amount* of information a channel, c, can convey about a signal, s, to a receiver, r.

signal

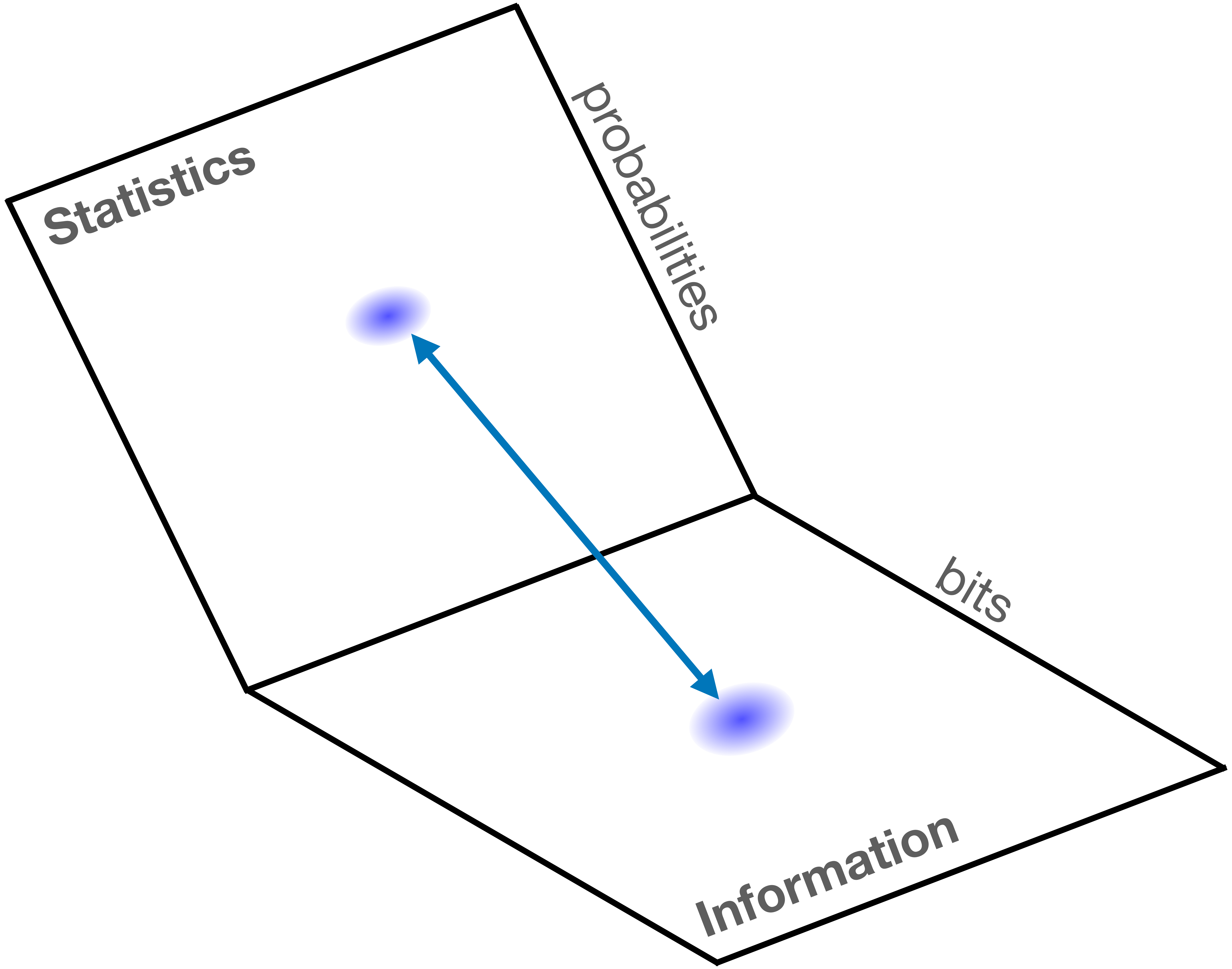
channel (c)

receiver

$$s \longrightarrow r$$

(Shannon & Weaver, 1949)

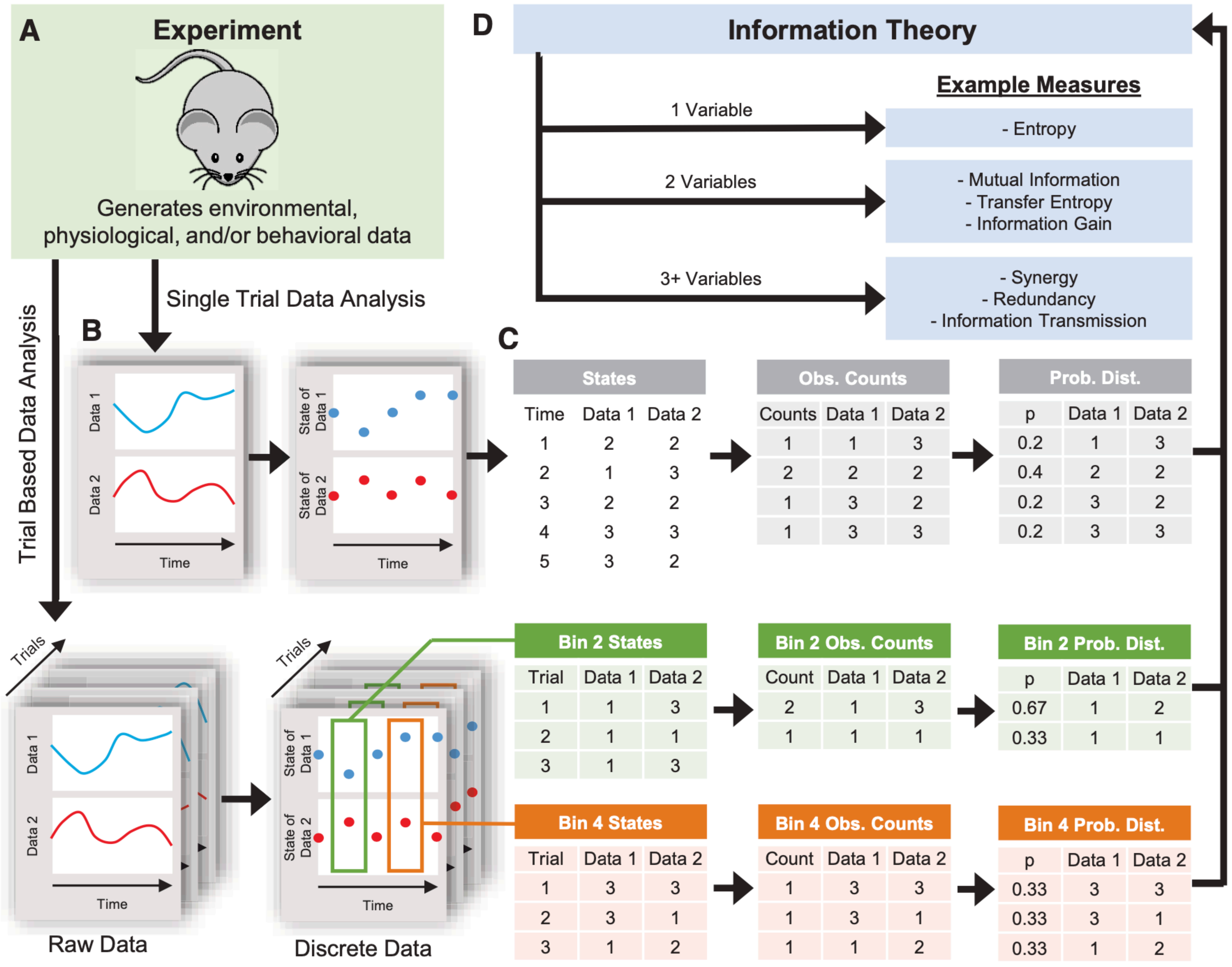
Information theory



Information theory is largely a projection of statistics, converting probabilities to bits (~$\log$ probabilities)

(Shannon & Weaver, 1949; Kullback 1959)

Information theory in neuroscience



Role in neuroscience & psychology:

- Data analysis
- Generative modeling

(Timme & Lapish, 2018)

Amount of information in s

Question: What is the *average* amount of information conveyed by s?

$$H(S) = \sum_{s \in S} p(s_i) I(s_i)$$

$$H(s) = -\sum_{s \in S} p(s) \log_2 p(s)$$

Entropy of signal s

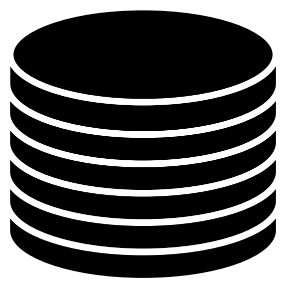
probability that i^{th} state is observed

information available from i^{th} state

$$I(s_i) = log_2(\frac{1}{p(s_i)}) \approx 0$$

$$p(s_i) = 0$$

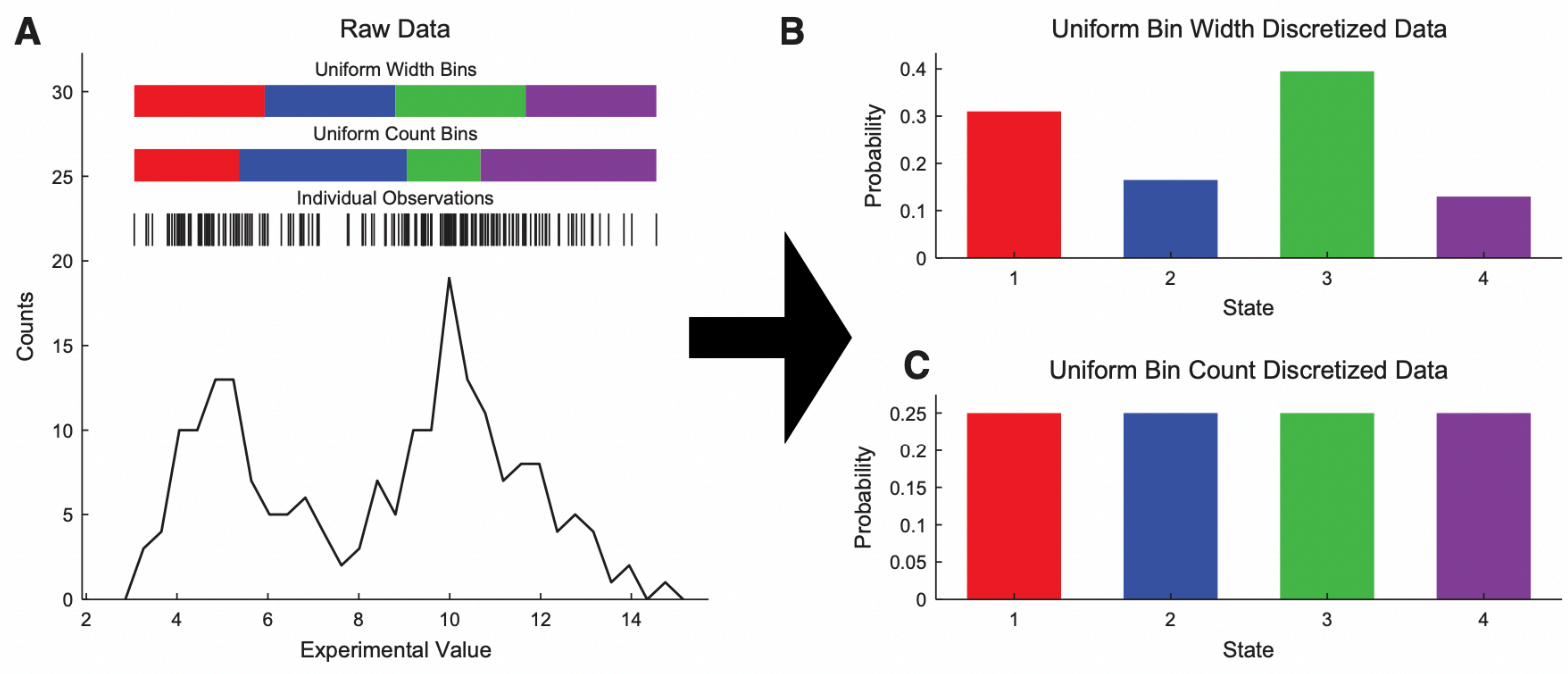
Example:



$$H(s) = -\frac{1}{2}log_2(\frac{1}{2}) - \frac{1}{2}log(\frac{1}{2})$$

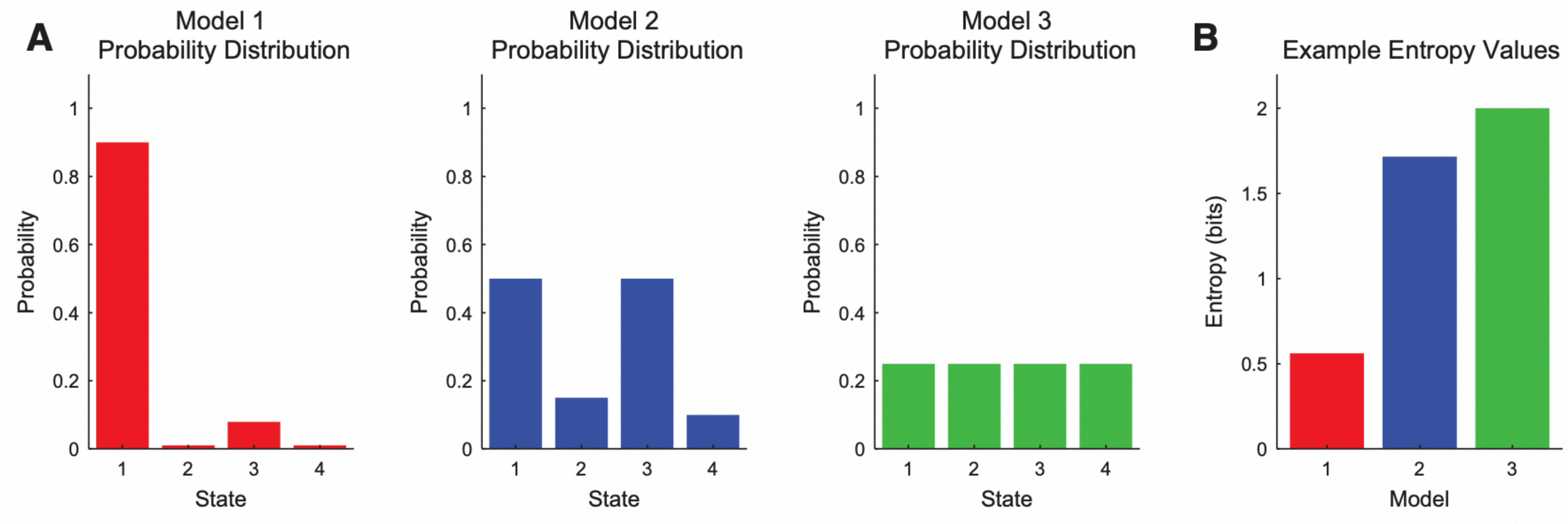
(Shannon & Weaver, 1949)

Discretizing your signal



(Timme & Lapish, 2018)

Entropy and structure



(Timme & Lapish, 2018)

Mutual information

Joint entropy

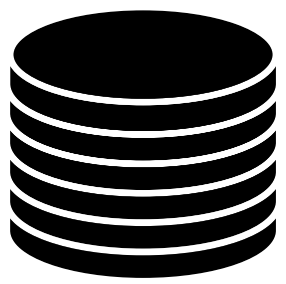
Question: What is the *average* amount of uncertainty in s and r?

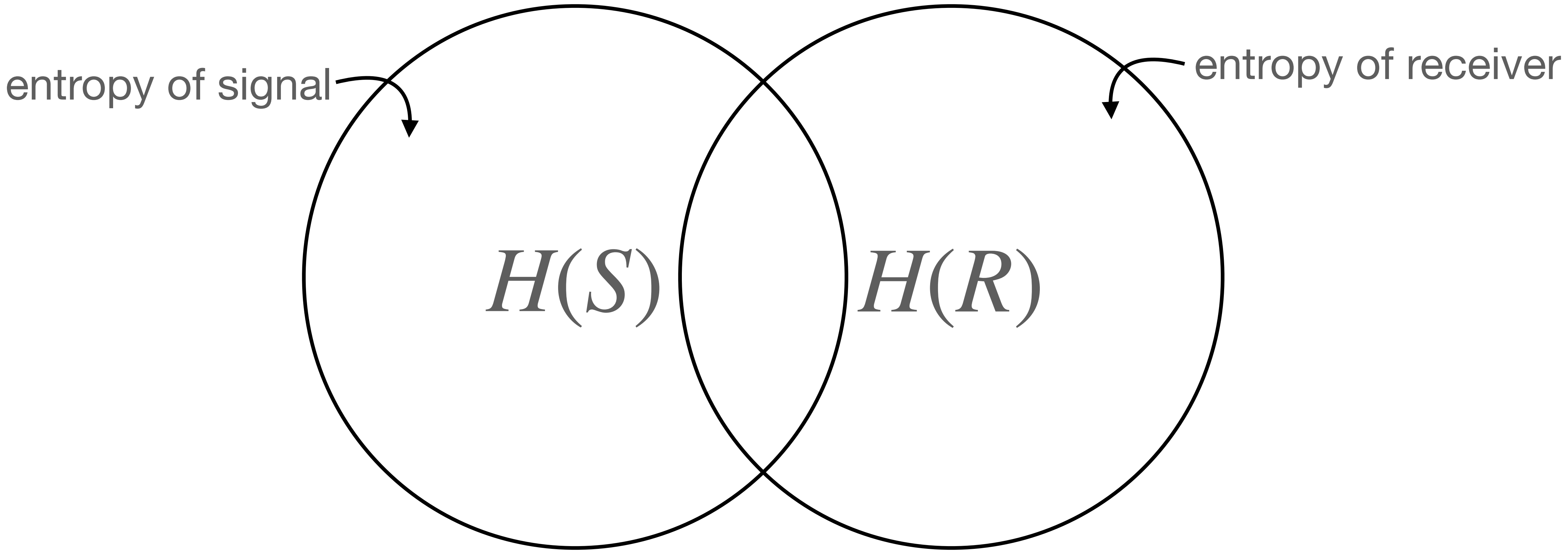
$$H(S,R) = -\sum_{s \in S, r \in R} p(s,r) \log_2 p(s,r)$$

Joint entropy of signal S & receiver R

Joint probability distribution

Example: $H(S,R) = 4[\frac{1}{4}\log_2(\frac{1}{4})]$





(Shannon & Weaver, 1949)

Conditional entropy

Question: What is the *average* amount of uncertainty in r after seeing s?

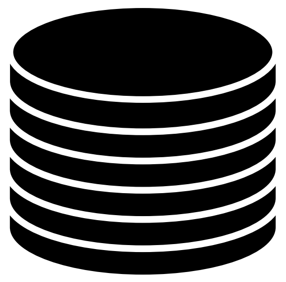
$$H(R\,|\,S) = -\sum_{s \in S, r \in R} p(s,r) \log_2 p(r\,|\,s)$$

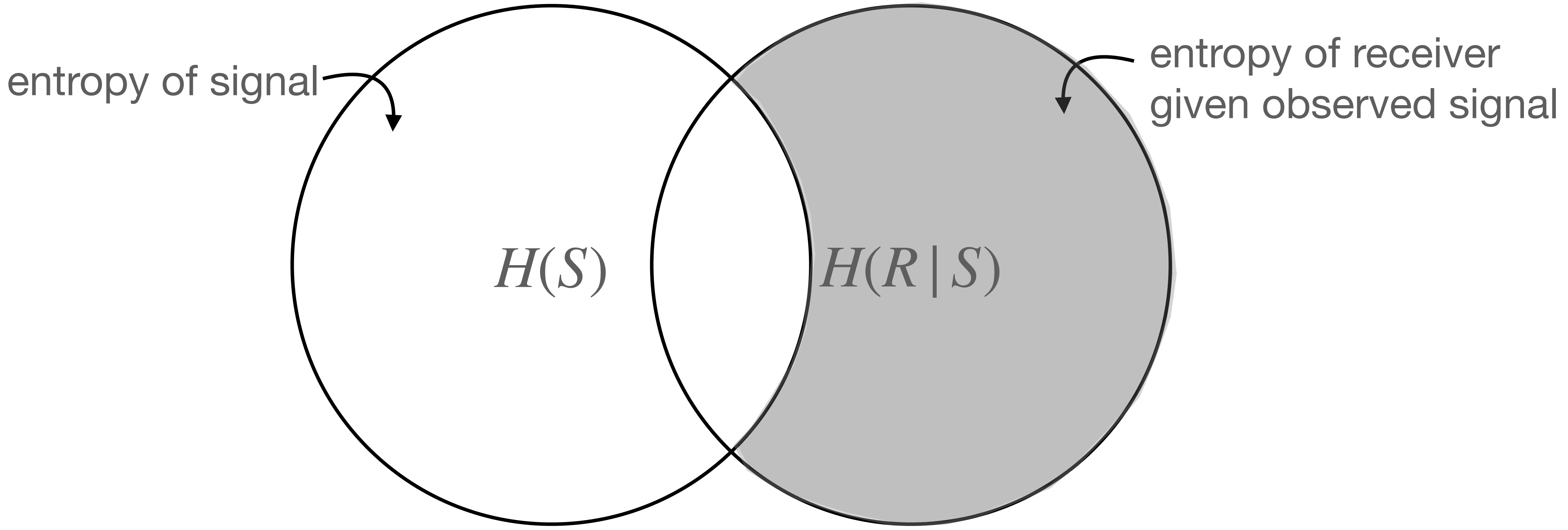
Joint entropy of signal S & receiver R

Joint probability distribution

Conditional probability distribution

Example: $H(R\,|\,S) = 2[0.4(\frac{1}{0.8})] + 2[0.1\log_2(\frac{1}{0.2})]$





$$H(S,R) = H(S) + H(R\,|\,S)$$

(Shannon & Weaver, 1949)

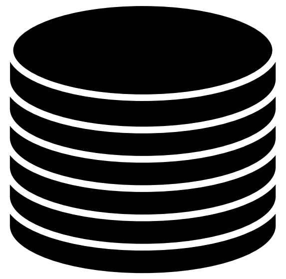
Conditional entropy

Example:

Table 1. Marginal and joint probability distributions for an example system of two dependent coins.

	$c_1 = heads$	$c_1 = tails$	Marginal Distributions for Coin 2
$c_2 = heads$	$p(c_1 = heads, c_2 = heads) = 0.4$	$p(c_1 = tails, c_2 = heads) = 0.1$	$p(c_2 = heads) = 0.5$
$c_2 = tails$	$p(c_1 = heads, c_2 = tails) = 0.1$	$p(c_1 = tails, c_2 = tails) = 0.4$	$p(c_2 = tails) = 0.5$
Marginal distributions for coin 1	$p(c_1 = heads) = 0.5$	$p(c_1 = tails) = 0.5$	

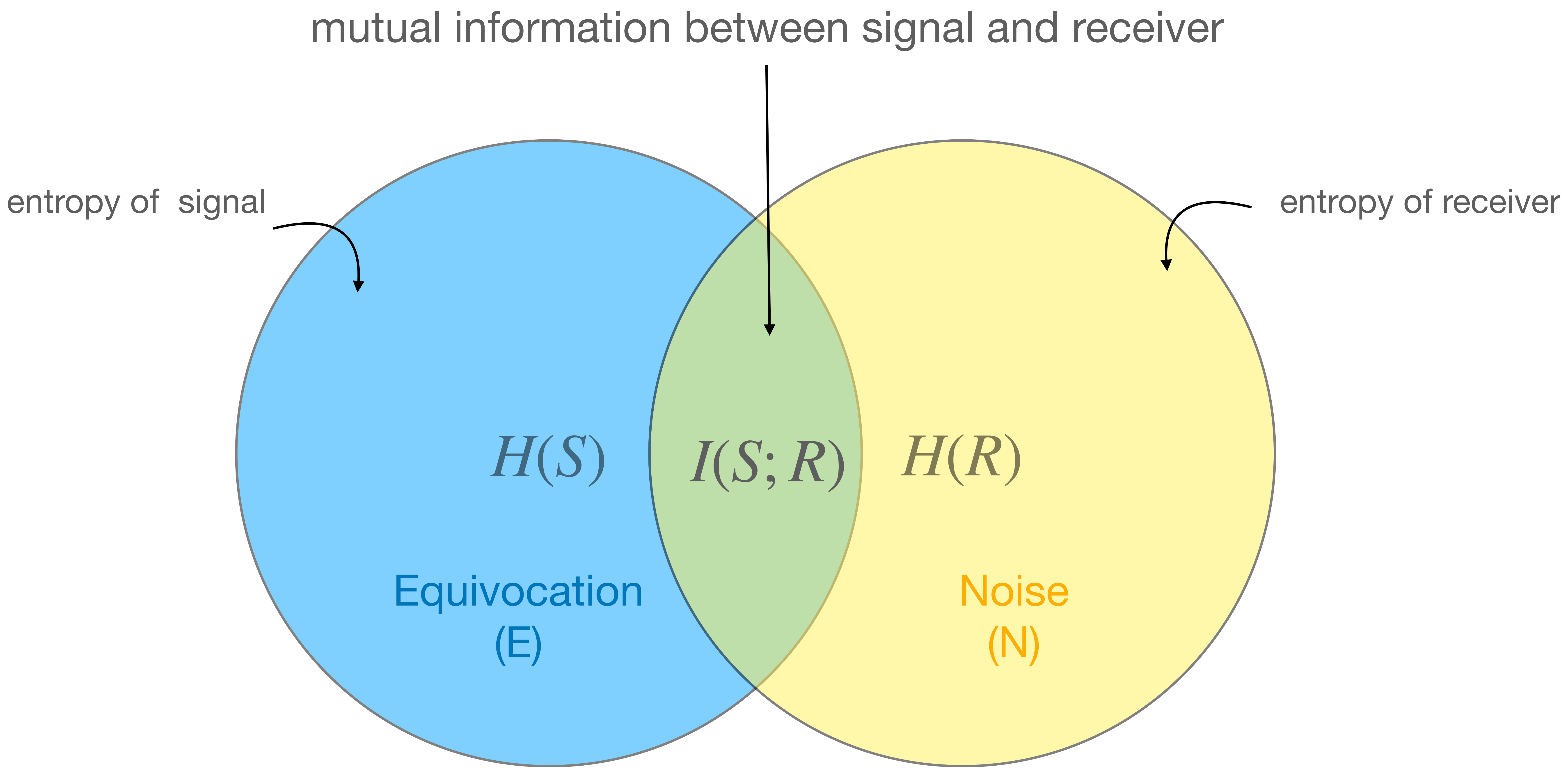
The joint distribution describe the likelihood for each possible combination of the two coins. The marginal distributions describe the likelihood for each coin alone. Marginal distributions can be found by summing across rows or columns of the joint distribution (Eqn. 1).



$$H(R \mid S) = 2[0.4(\frac{1}{0.8})] + 2[0.1 \log_2(\frac{1}{0.2})]$$

(Timme & Lapish, 2018)

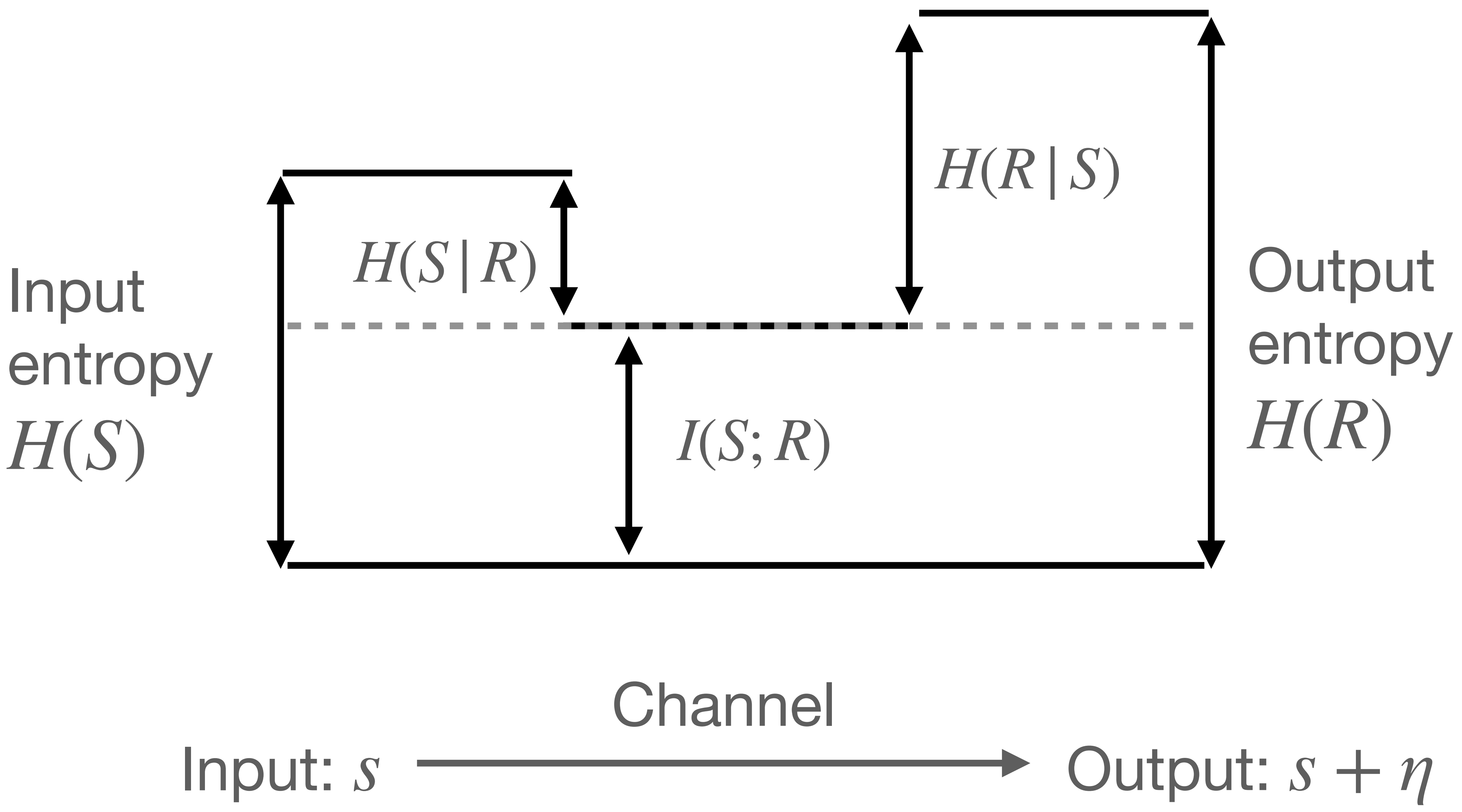
$I(S; R)$ mutual information



$I(S; R)$: The information transmitted from s to r is the total amount of information available at r, $H(R)$, minus noise.

(Timme & Lapish, 2018)

$I(S;R)$ mutual information

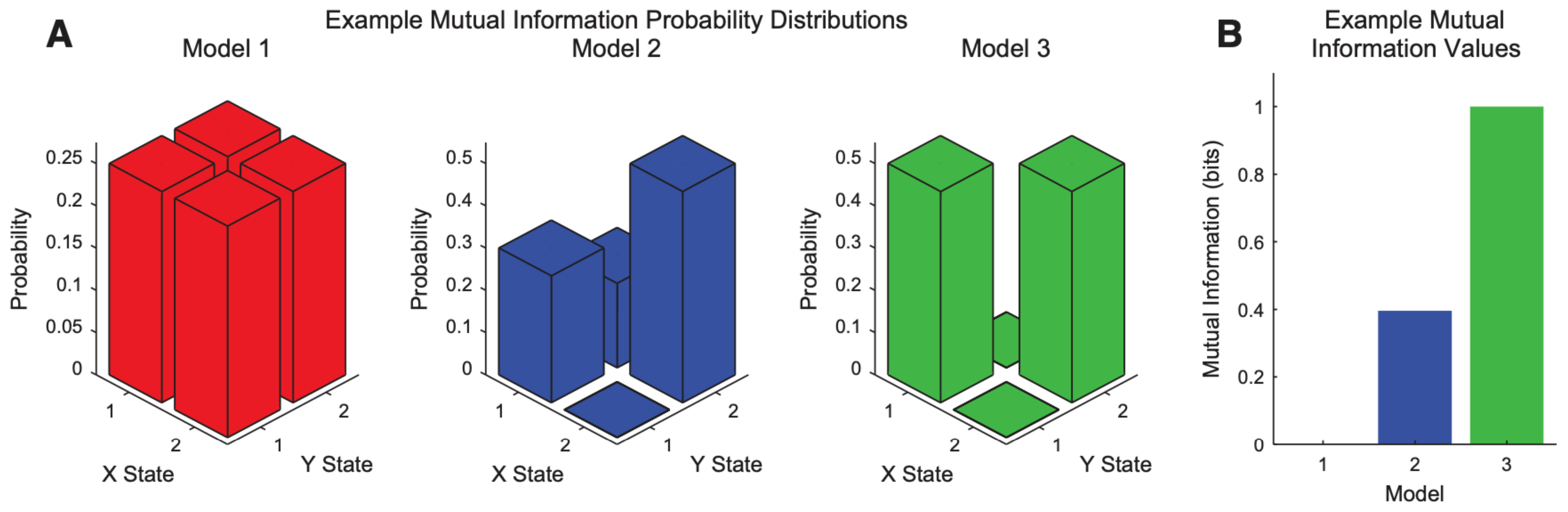


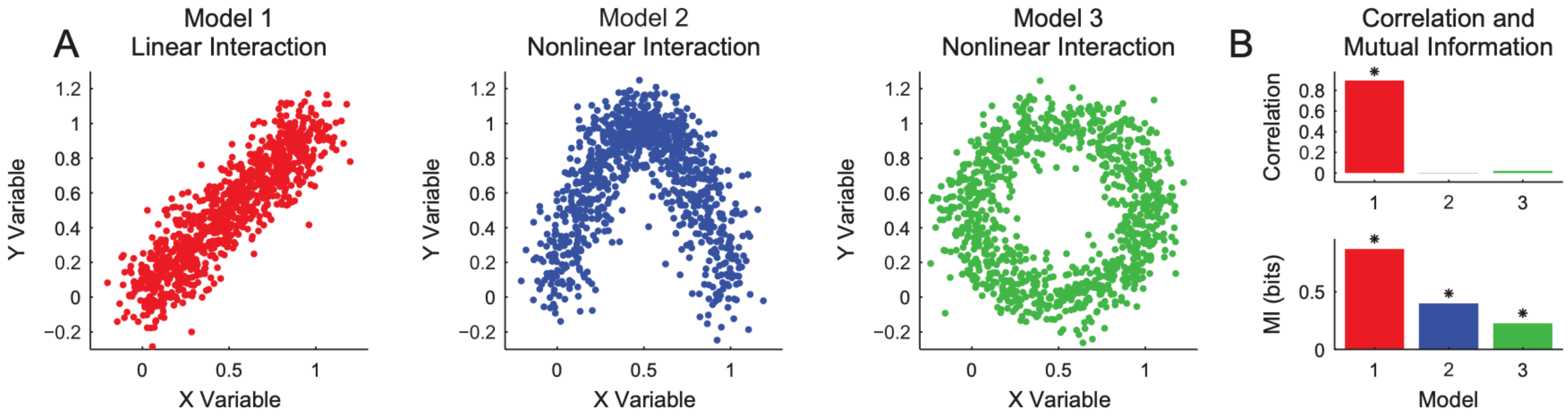
<u>Mutual information</u>:

$$I(S;R) = H(S) - H(S|R)$$
$$= H(R) - H(R|S)$$
$$= \sum_{s \in S, r \in R} p(s,r)\log_2\left(\frac{p(s,r)}{p(s)p(r)}\right)$$

(Stone, 2022; Timme & Lapish, 2018)

Examples mutual information





(Timme & Lapish, 2018)

Transfer entropy

$T(S \to R)$ Transfer entropy

<u>Question</u>: How much information is transferred $s \to r$?

$$T(S \to R) = I(R_f; S_p \mid S_f)$$

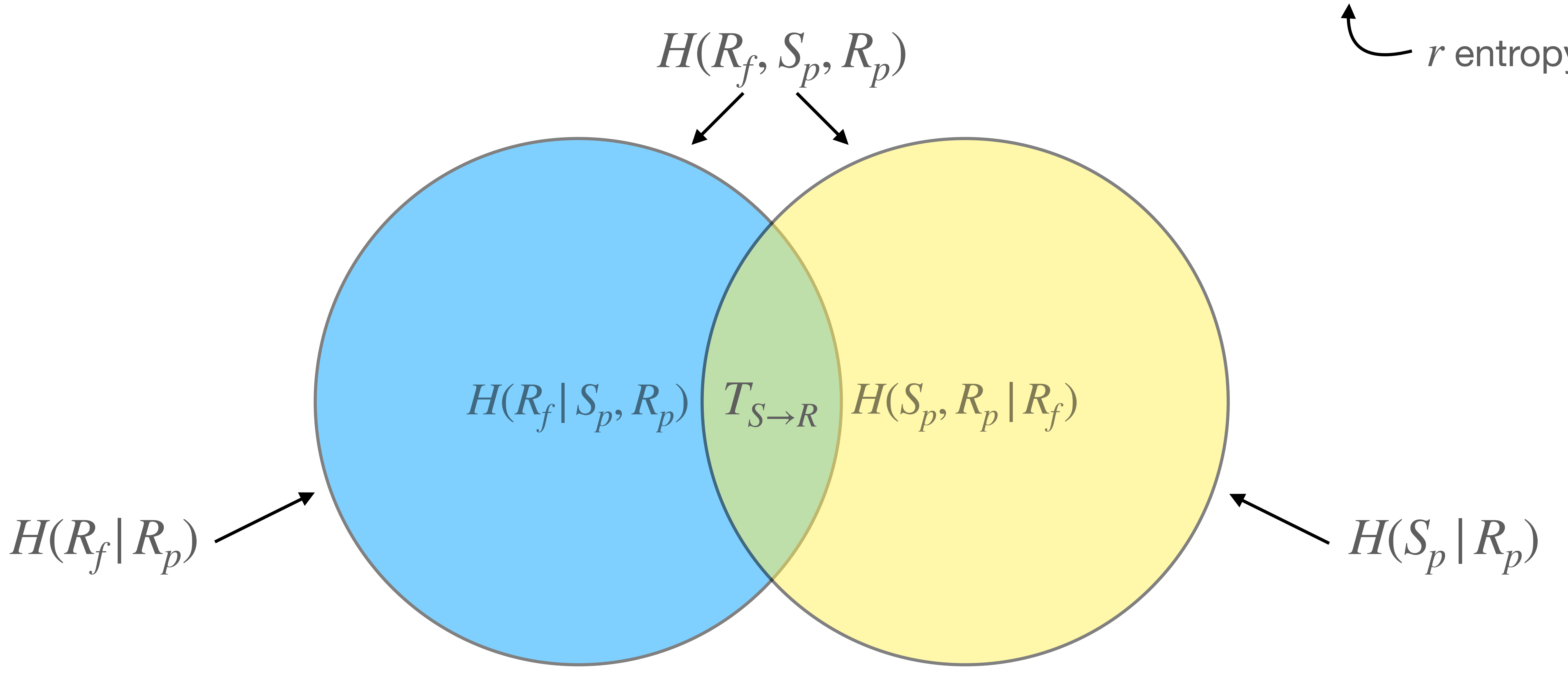
$$= H(R_f \mid R_p) - H(R_f \mid S_p, R_p)$$

Transfer entropy from S to R

r entropy in the past

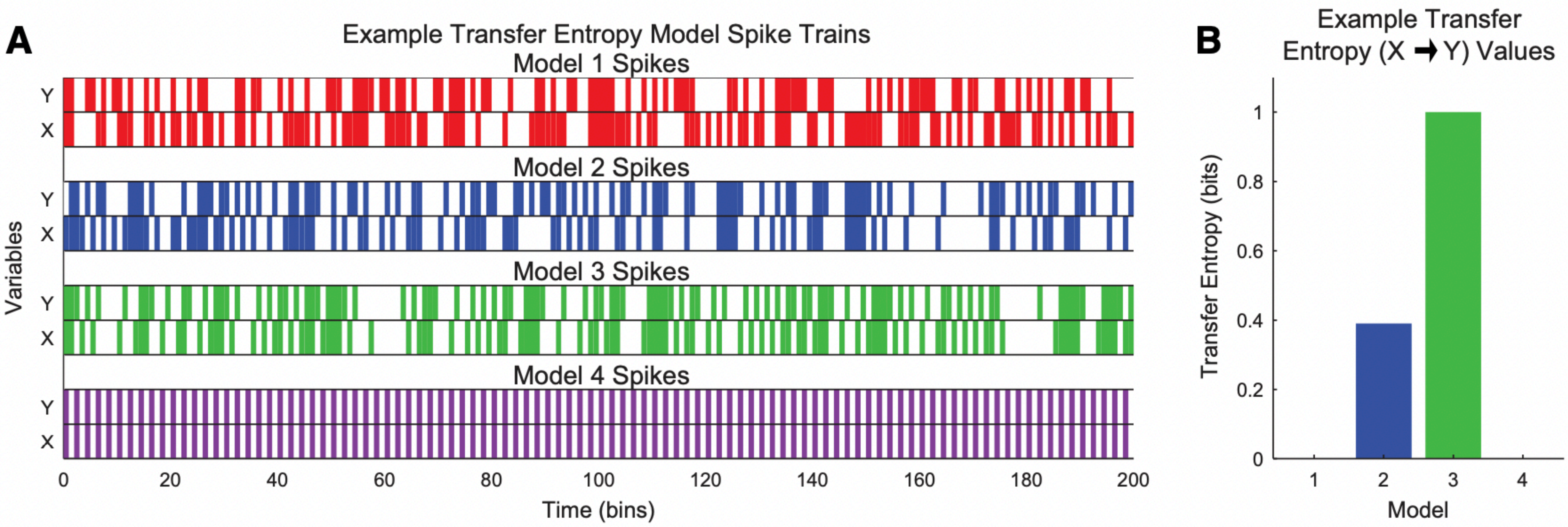
r entropy in the future

s entropy in the past



(Shannon & Weaver, 1949)

Measuring the flow of information



(Timme & Lapish, 2018)

Take home message

- Information is defined by its uncertainty. We call this entropy.
- Mutual information reflects the degree of association between signals.
- Transfer entropy quantifies how much information transfers from one signal to another.

Lab time!

https://coaxlab.github.io/BIX-book/notebooks/lab1-information.html