Homework 1

Tarek Abdel Sater – tga15

Problem 1: Fixing the mutex liveness model.

The objective is to fix the model in such a way that it satisfies both **liveness**, and **safety**.

Safety means that no two processes are in the critical section simultaneously.
This is represented by the formula: **AG(!(C1 & C2))**.

Liveness means that, at any time a process tries to go into the critical section, it will eventually enter it.
This is represented by the formula: **(AG(T1 => (AF(C1)))) & (AG(T2 => (AF(C2))))**.

To satisfy safety, we simply make sure that there exists no state where C1 and C2 both hold.

To satisfy liveness, we need to repair the model in such a way that C1 will eventually occur in all paths from states where T1 holds. The same goes for C2 and T2.

Problems arise whenever we have a state where only both T1 and T2 hold. In that case, it is difficult to satisfy liveness without the use of "black" transitions, i.e. illegal transitions where both processes transition to a single state at the same time.

To solve this, we use the F1 and F2 values to control the transitions in such a way that we 'synchronize' the two processes.

Problem 2: fO sequence.

Assuming O can be F or G.

We show that two sequential **F**s collapse to a single **F** i.e. **FFp** ⇔ **Fp**.

Suppose that **p** holds somewhere along the subsequent path, this means **Fp**. Since **Fp** holds along that path, then **F(Fp)** definitely holds.

By induction, any sequence **FFFFF…Fp** will collapse to a single **Fp**.

We will make the same argument for two sequential **G**s i.e. **GGp** ⇔ **Gp**.

Gp holds when **p** holds along an entire subsequent path. Since **p** holds along the entire subsequent path, **Gp** will hold at any point along that. This statement is equivalent to **GGp**.

By induction, any sequence **GGGG…Gp** will collapse to a single **Gp**.

For now, we know that any sequence of multiple **F**s and **G**s can be collapsed to a sequence of the form

({**F** | **G**} {**G** | **F**})* i.e. alternating **F**s and **G**s.

We now show that **FGp** ⇔ **Gp**.

If **p** will hold on the entire subsequent paths, then **Gp** holds. Since **Gp** holds in some future, then **FGp** holds.

We now show that **GFp** ⇔ **Fp**.

If **p** holds eventually in some path, then **Fp** holds along all that path. Since **Fp** holds along that entire path, then **GFp** holds.

By induction, a sequence of alternating **F**s and **G**s collapses to the last occurring temporal operator.

e.g. **FGFGFp** collapses to **Fp**. **FGFGFGp** collapses to **Gp**. The only thing that matters is the last occuring temporal operator.