Formal Methods Assignment 1

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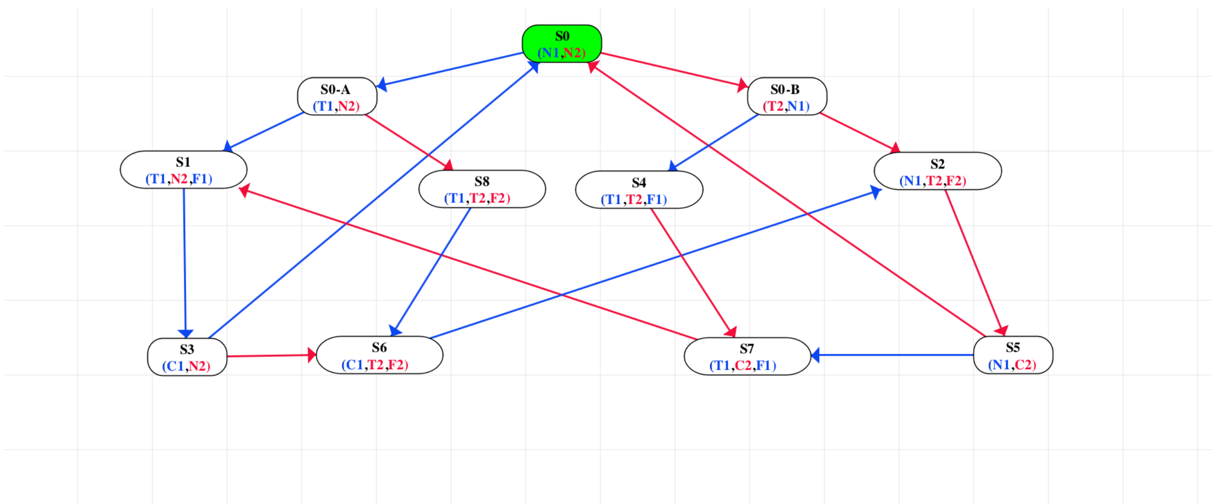
Question 1



State S0: initial state, both processes are in the neutral state.

States S0-A and S0-B: One of the processes is trying to get in the critical section while the other is still neutral. This leads to two options for each case: (S1,S8) from S0-A and (S2,S4) from S0-B.

S1 P1 is trying and it is its turn (F1) while P2 is still in the neutral state, thus leading to state S3 where P1 is in the critical section and P2 is still neutral. Since P1 is now in the critical section it is P2's turn now, and P2 might start trying to enter the critical section, while P1 is still in the critical section, leading to state S6. After P1 is done with the critical section it goes back to neutral, P2 is now trying and it is its turn (state S2) so now P2 enters in the critical section, hence leading to state S5. If P1 now tries again to enter the critical section (and it is its turn since P2 has entered and is now in the critical section) it can enter after P2 finishes and is in the neutral state (hence leading back to S2)

S2 Same path goes as in (S1) under the same circumstances but for P1 instead of P2 and P2 instead of P1.

It is safe because: No two processes are in the critical section at the same time.
It is live because: When one process enters the critical section it is the other one's turn.

Question 2

Given: $f = O_1O_2O_3...O_np$ where each O_i can be either F or G and p belongs to AP.

Question: Find a procedure to simplify f.

Answer: We can simplify each sequence of 3 or more consecutive Fs into FF.
and each sequence of 2 or more consecutive Gs into one G.

Proof for F: Fp means that p will be true eventually.
FFp means that Fp will be true eventually and p will be in the last state.
Similarly, $FFFp$ means that FFp will be true eventually, which means that Fp will be true eventually and p will be in the last state. Thus, $FFFp$ also means that p will be eventually true and at the last state, and so on.
Therefore, any sequence of 3 or more Fs then p is stating that p will eventually be true and will be encountered at the last state in our branching. This allows us to simplify such a sequence into FFp since they are equivalent.

Proof for G: if Gp means p is globally true, then GGp means Gp is globally true. Thus, GGp means that p is globally true. Therefore the sequence of $G...Gp$ can be reduced to Gp.

Thus: f can be one of the following permutations:

$$f = \begin{cases} F & \text{for } n = 1 \\ G & \text{where all } O_i = G \\ FF & \text{where all } O_i = F \\ (FG)^i & \text{for some } i \in N \\ (GF)^i & \text{for some } i \in N \\ (FGF)^i & \text{for some } i \in N \\ (GFG)^i & \text{for some } i \in N \\ (FFG)^i & \text{for some } i \in N \\ (GFF)^i & \text{for some } i \in N \end{cases}$$