1 $\mathsf{U}, \mathsf{V}, \mathbf{W}, \mathsf{U_w}$ in CTL

The purpose of this note is to clear up the relationship between the modalities $\mathsf{U}, \mathsf{V}, \mathsf{W}, \mathsf{U_w}$ in CTL.

We start by considering these as path modalities. Let $\pi = s_0, s_1, \ldots$ be a fullpath.

- $p\mathsf{U}q$ is strong until:
  p strong-until q means that q eventually holds and p holds until then
  $M, \pi \models p\mathsf{U}q$ iff
  $\exists k \geq 0 : M, s_k \models q$ and $(\forall j : 0 \leq j < k : M, s_j \models p)$

  So, there is one scenario for $M, \pi \models p\mathsf{U}q$: q holds at some state s_k along π, and p holds in all states up to, but not necessariy including, s_k.

- $p\mathsf{W}q$ is the dual of U [EC82]:
  $[p\mathsf{W}q] \equiv \neg[\neg p\mathsf{U}\neg q]$
  hence W is the same modality as V, see below

- $p\mathsf{V}q$ is release [GL94]:
  p releases q means that as long as p does not hold, q must hold.
  In particular, if p never holds, then q must hold forever.
  $M, \pi \models p\mathsf{V}q$ iff
  $\forall k \geq 0 : (\forall j : 0 \leq j < k : (M, s_j \not\models p)$ implies $M, s_k \models q)$

  So, there are two scenarios for $M, \pi \models p\mathsf{V}q$:

  1. p holds at some state s_k along π, and q holds in all states up to and including s_k
  2. p never holds along π, and so q holds in all states of π

- $p\mathsf{U_w}q$ is weak until:
  p weak-until q means that q eventually holds and p holds until then, or q never holds and so p always holds
  $M, \pi \models p\mathsf{U_w}q$ iff
  $\exists k \geq 0 : M, s_k \models q$ and $(\forall j : 0 \leq j < k : M, s_j \models p)$ or
  $\forall k \geq 0 : M, s_k \models p$

  So, there are two scenarios for $M, \pi \models p\mathsf{U_w}q$:

  1. q holds at some state s_k along π, and p holds in all states up to, but not necessariy including, s_k.
  2. q never holds along π, and so p holds in all states of π

2 Equivalences between the modalities

- $[p\mathsf{V}q] \equiv \neg[\neg p\mathsf{U}\neg q]$, from [GL94], page 847
- $[p\mathsf{W}q] \equiv \neg[\neg p\mathsf{U}\neg q]$, from [EC82], page 245

References

[EC82] E. A. Emerson and E. M. Clarke. Using branching time temporal logic to synthesize synchronization skeletons. *Science of Computer Programming*, 2(3):241–266, 1982.

[GL94] O Grumberg and D.E. Long. Model checking and modular verification. *TOPLAS*, 16(3):843–871, 1994.