

## STAT2110 Engineering Statistics

### Assignment 1

**Due** Canvas submit by 11:59pm Sunday March 31.

**Format & requirements** The solution report must contain definitions, justifications, equations – all workings necessary to obtain the final answer. The report can be either handwritten and scanned / photographed or typefaced as a pdf or a Word document.

In case you submit a handwritten document, it must be legible.

Each question must start from a new page.

**Integrity** Assignment must be performed individually. All students are required to meet the academic integrity standards of the University.

#### Question 1. [7 marks]

In a Legislative Assembly Election, nine candidates compete for four places in the Parliament. Each voter, including yourself, selects four candidates, ranking them from 1 to 4. After counting all votes, the candidates are ranked from 1 to 9, according to the received votes (if two candidates receive the same number of votes, the older one is ranked higher), and the best four candidates become MPs. Polls show that all candidates have the same popularity, so the outcome of the election is essentially a gamble.

(a) [1 mark]

What is the sample space of the election outcomes?

(b) [1marks]

How many different election outcomes are possible?

(c) [2 mark]

How many outcomes are possible in which your best candidate is actually ranked as  $X$ ?

(d) [1 mark]

How many outcomes are possible in which your worst candidate is actually ranked as  $Y$ ?

(e) [2 mark]

How many outcomes are possible in which your best candidate is actually ranked as  $X$  and your worst candidate is actually ranked as  $Y$ ?

#### Question 2. [8 marks]

Students who take STAT2110 pursue different engineering programs. The chances that a STAT2110 student pursues Chemical Engineering, Electrical & Electronic Engineering, Computer Systems Engineering are  $x_H$ ,  $x_E$ , and  $x_C$ , respectively. The remaining students pursue other engineering degrees. The probability of failing STAT2110, depending on the program, are  $y_H$ ,  $y_E$ ,  $y_C$ , and  $y_O$  respectively.

(a) [1 mark]

Introduce an appropriate notation and identify the sample space of possible events.

(b) [1 mark]

Using the introduced notation, identify the events, the probabilities of which are stated in the text, in the following form:

a.  $P(?) = x_H$

b.  $P(?) = x_E$

c.  $P(?) = x_C$

d.  $P(?) = y_H$

e.  $P(?) = y_E$

f.  $P(?) = y_C$

g.  $P(?) = y_O$

(c) [1 mark]

What is the probability that a student fails STAT2110?

(d) [1 mark]

What is the probability that a student pursues Chemical Engineering and fails STAT2110?

(e) [2 marks]

What is the probability that a student fails STAT2110, if they do not pursue Electrical & Electronic Engineering?

(f) [2 marks]

What is the probability that a student pursues Computer Systems Engineering, if they failed STAT2110?

### Question 3. [10 marks]

Suppose that the joint probability mass function for the random variables  $X$  and  $Y$  is

$$f(x, y) = \frac{x+y}{32}; \quad x \in \{1, 2\}, y \in \{1, 2, 3, 4\}$$

(a) [1 mark] Construct the contingency table of joint and marginal probabilities (their values).

(b) [1 mark] Verify that  $f(x, y)$  is a valid probability mass function.

(c) [2 marks] Derive the algebraic expressions for the marginal distributions  $g(x)$  and  $h(y)$ .

(d) [1 mark] Derive the algebraic expression for the conditional distribution of  $f(x|y)$ .

(e) [1 mark] Are the random variables  $X$  and  $Y$  independent?

(f) [2 marks] Derive the mean  $\mu_X$  and the variance  $\sigma_X^2$  of  $X$ .

(g) [2 marks] Given the mean and variance of  $Y$  are mean  $\mu_Y = \frac{90}{32}$  and  $\sigma_Y^2 = \frac{295}{256}$ , derive the covariance  $\sigma_{XY}$  and correlation coefficient  $\rho_{XY}$ .