STAT2110 PASS Worksheet 2

Jacob Mahony

1 Key Formulas

- Discrete cdf:

  $$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \; -\infty < x < \infty$$

- Continuous cdf:

  $$f(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt, \; -\infty < x < \infty$$

- Marginal distributions of X and Y alone (discrete):

  $$g(x) = \sum_{y} f(x,y) \text{ and } h(y) = \sum_{x} f(x,y)$$

- The marginal distributions of X and Y alone (continuous):

  $$g(x) = \int_{-\infty}^{\infty} f(x,y)dy \text{ and } h(y) = \int_{-\infty}^{\infty} f(x,y)dx$$

- Conditional distribution of Y given that $X = x$:

  $$f(y|x) = \frac{f(x,y)}{g(x)}, \; g(x) > 0$$

2 Questions

1. The probability that a vehicle entering the Luray Caverns has Canadian license plates is 0.12; the probability that it is a camper is 0.28; and the probability that it is a camper with Canadian license plates is 0.09. What is the probability that: **(2.83)**
   (a) a camper entering the Luray Caverns has Canadian license plates?
   (b) a vehicle with Canadian license plates entering the Luray Caverns is a camper?
   (c) a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper?

2. From a box containing 6 black balls and 4 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. What is the probability that all 3 are the same colour? **(3.26)**

3. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. **(3.11)**

4. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function: **(3.7)**

   $$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find the probability that over a period of one year, a family runs their vacuum cleaner:
   (a) less than 120 hours;
   (b) between 50 and 100 hours.

5. A continuous random variable X that can assume values between $x = 1$ and $x = 3$ has a density function given by $f(x) = 1/2$. **(3.17)**
   (a) Show that the area under the curve is equal to 1.
   (b) Find $P(2 < X < 2.5)$.
   (c) Find $P(X \leq 1.6)$.
6. Given the joint density function: **(3.53)**
$$f(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2,\ 2 < y < 4, \\ 0, & \text{elsewhere.} \end{cases}$$
   Find $P(1 < Y < 3 | X = 1)$.
7. Consider the following joint probability density function of the random variables X and Y: **(3.68)**
$$f(x,y) = \begin{cases} \frac{3x-y}{9}, & 1 < x < 3,\ 1 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$
   (a) Find the marginal density functions of X and Y.
   (b) Are X and Y independent?
   (c) Find $P(X > 2)$.