AERO3600 — Embedded Control Systems

Lab 02 - State-feedback Control Design[1]

Learning Outcomes

This lab will assess your ability to:

1. Design state-feedback controllers for physical systems using Matlab.
2. Build Simulink models of the closed-loop system and run numerical simulations.
3. Analyses and evaluate the state trajectories of the closed loop obtained via numerical simulations.

1 Physical systems

In this lab we continue the process to design Embedded Control Systems. We consider two benchmark systems, namely the rotary pendulum and 2-DOF aero systems shown in Figure 1.

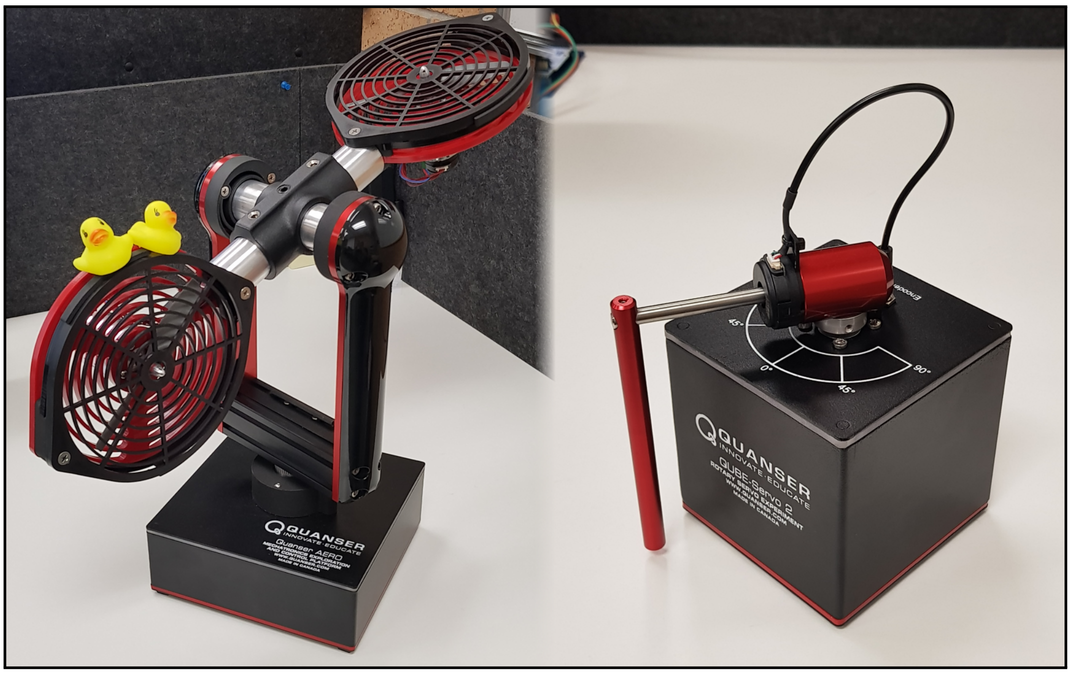


Figure 1: Rotary pendulum and aero systems.

[1]Updated: 12 March 2024.

2 Rotary pendulum system

In this section, we describe the control design task for the rotary pendulum system.

2.1 Problem formulation

We consider the nonlinear model of the rotary pendulum presented in the document *Lab 01 - Modelling and Simulation*, and the equilibrium points $\bar{x}_a$ and $\bar{x}_b$ defined as

$$\bar{x}_a = \begin{bmatrix} \bar{x}_{1a} \\ \bar{x}_{2a} \\ \bar{x}_{3a} \\ \bar{x}_{4a} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ; \qquad \bar{x}_b = \begin{bmatrix} \bar{x}_{1b} \\ \bar{x}_{2b} \\ \bar{x}_{3b} \\ \bar{x}_{4b} \end{bmatrix} = \begin{bmatrix} 0 \\ \pi \\ 0 \\ 0 \end{bmatrix}, \tag{1}$$

with input $\bar{V}_m = 0$ for both equilibria.

We also consider the linearised models about the equilibria. The linearised model about the equilibrium point $\bar{x}_a$ is

$$\dot{x}_a = A_a\, x_a + B_a\, u_a \tag{2}$$

where $x_a \triangleq x - \bar{x}_a$, $u_a \triangleq u - \bar{u}_a$, $u = V_m$, and the matrices A_a and B_a correspond to Jacobian matrices respect to states and input evaluated at the equilibrium $\bar{x}_a$ with $\bar{u}_a = \bar{V}_m = 0$.

The linearised model about the equilibrium point $\bar{x}_b$ is

$$\dot{x}_b = A_b\, x_b + B_b\, u_b \tag{3}$$

where $x_b \triangleq x - \bar{x}_b$, $u_b \triangleq u - \bar{u}_b$, $u = V_m$, and the matrices A_b and B_b correspond to Jacobian matrices respect to states and input evaluated at the equilibrium $\bar{x}_b$ with $\bar{u}_b = \bar{V}_m = 0$.

The first problem is to design a controller using the linearised model to stabilise the system about the equilibrium points $\bar{x}_a$. The second problem is to design another controller using the linearised model to stabilise the system about the equilibrium points $\bar{x}_b$. Notice that all the equations and matrices that characterise the nonlinear and linearised models are given in the document Lab 01 - Modelling and Simulation.

2.2 State-feedback control system design using linearised models.

In this section, you have to build and simulate the control system of the rotary pendulum system in closed loop with a state-feedback controller that stabilised the desired equilibrium points $\bar{x}_a$ and $\bar{x}_b$. We design the controller and compare the state histories of the control system using the nonlinear and its linear approximation. The task is to write a MATLAB script that performs the following actions:

1. Call the function `rp_parameters.m` that defines the parameters and matrices needed for the nonlinear and linearised models, as well as the equilibrium point $\bar{x}_a$ and the input $\bar{V}_m$. Store all these values in the structure `rp_p`.
2. Create the function `rp_sfc_design.m` that accepts the matrices A and B of the linearised model, and the vector E_c that contains the desired eigenvalues of the closed loop, and returns

   i) The variable `rp_p.COcheck=1` if the linearised model is completely controllable, otherwise `rp_p.COcheck=0`.

   ii) The gain of the state-feedback controller `rp_p.K` such that the eigenvalues of the closed loop are the entries of the vector `rp_p.Ec`.

   The function syntax should be

   ```
   [rp_p.COcheck,rp_p.K] = rp_sfc_design(rp_p.A,rp_p.B,rp_p.Ec);
   ```
3. Define the vector of desired eigenvalues of the closed loop in the main script and use the function `rp_sfc_design.m` to compute the gain of the state-feedback controller. Use first the eigenvalues selected for the semple solution. Once your are confident that your code is correct, change the desired eigenvalues and use simulations to select them

such that the input does not exceed its maximum value, that is max $|V_m| \leq 15$v, and simultaneously maximise the *region of attraction* with zero initial velocities, that is $x_3(0) = 0$ and $x_4(0) = 0$.

4. Simulate the control systems using the linearised and nonlinear models of the rotary pendulum. Use the initial conditions for both models that correspond to the same scenario.

5. Plots the time histories of the states from both the nonlinear and linearised control system against each other. Plot the states $x_1(t)$ and its linear approximation $x_{1a}(t) + \bar{x}_{1a}$ on top of each other in one graph, then plot $x_2(t)$ and $x_{2a}(t) + \bar{x}_{2a}$ together on another graph, and so on. Sample results for the initial conditions $x_1(0) = 60$ deg, $x_2(0) = 60$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$ deg/s are shown in Figure 2, where we selected the following eigenvalues $\{-5, -6, -7, -8\}$.

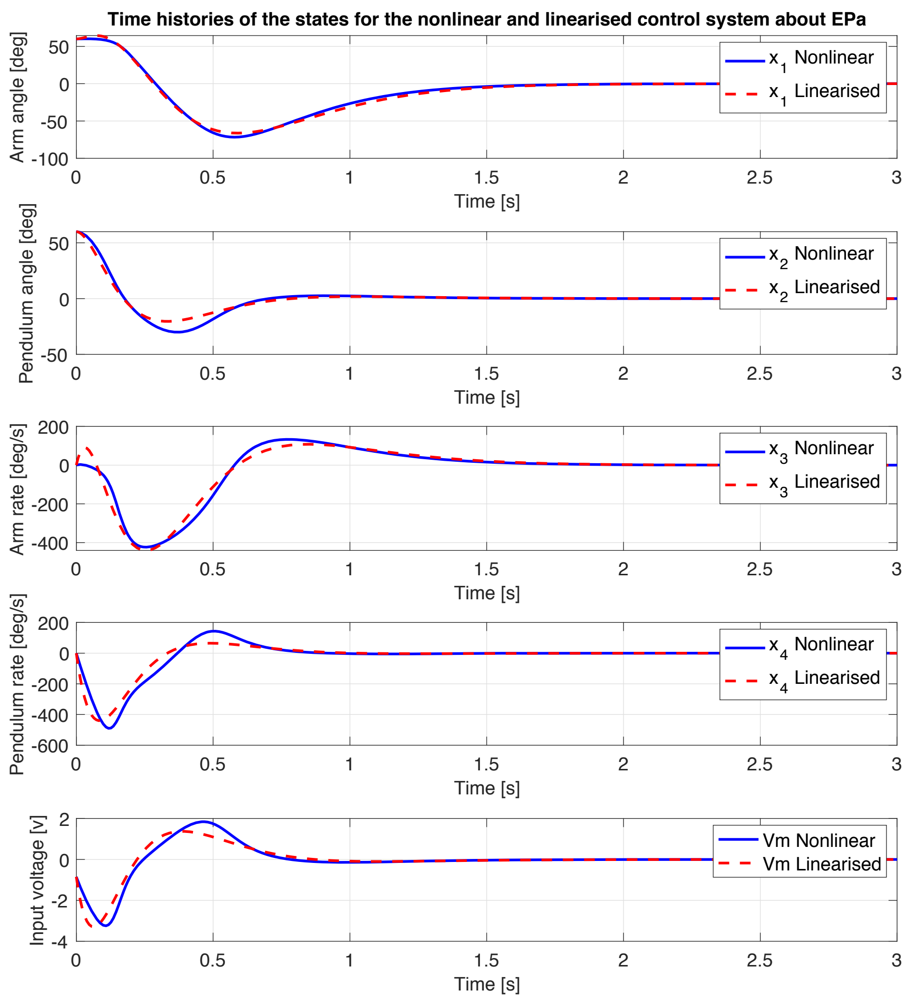


Figure 2: Time histories of the states and input voltage.

6. Save your script as `rp_mainfile_sfc_comparison_a.m`.

7. Repeat this process for initial conditions about the equilibrium point $\bar{x}_b$. Save your script as `rp_mainfile_sfc_comparison_b.m`. Sample results for the initial conditions $x_1(0) = 60$ deg, $x_2(0) = 160$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$deg/s are shown in Figure 3, where we selected the following eigenvalues $\{-5, -6, -7, -8\}$.

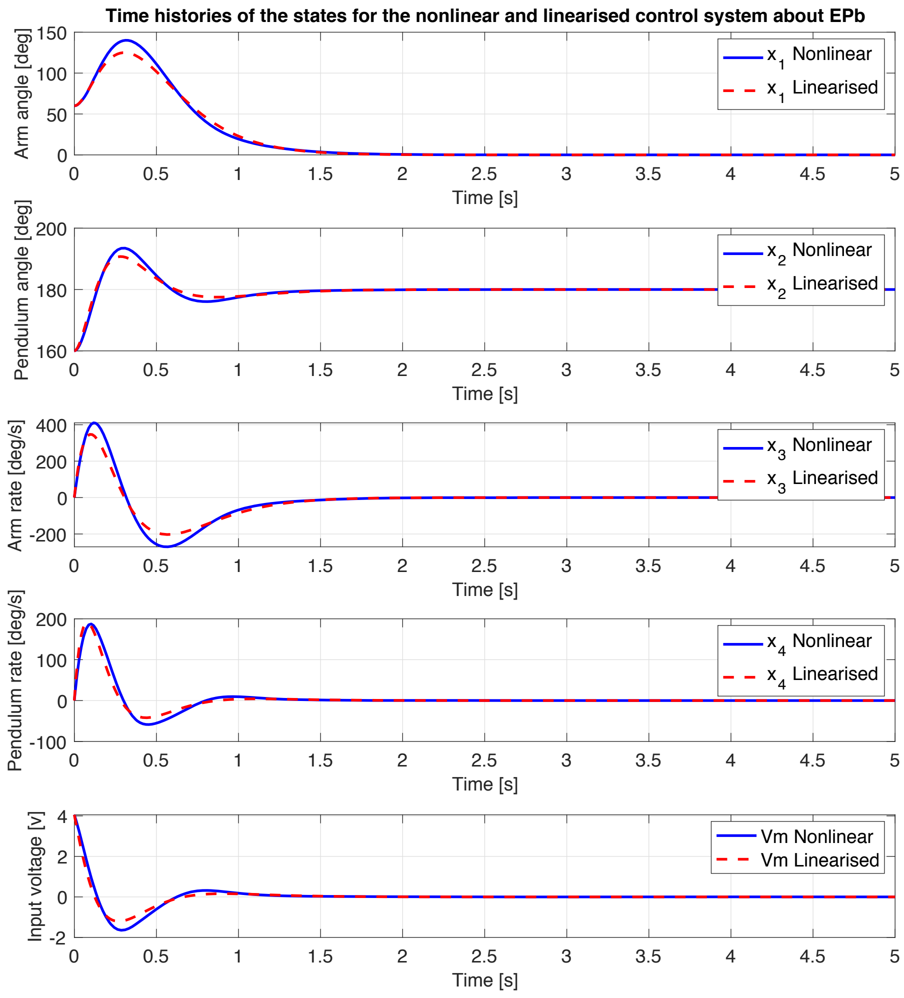


Figure 3: Time histories of the states and input voltage.

2.3 Marking Rubric

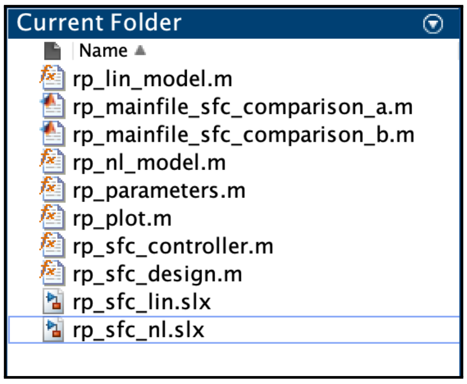
The following items will be verified in the assessment:

1. Correct output figure upon execution of the script `rp_mainfile_sfc_comparison_a.m` and `rp_mainfile_sfc_comparison_b.m`
2. Whether your functions `rp_nl_model.m`, `rp_lin_model.m`, `rp_parameters.m`, `rp_plot.m`, `rp_sfc_design.m`, `rp_sfc_controller.m` returns the correct expected values.
3. Whether your Simulink models `rp_sfc_nl.slx` and `rp_sfc_lin.slx` are correct and the simulation are correctly performed.

Please add your name and student number to all scripts and functions, for example see the following script

2.4 Code submission

We will need to submit your files for the lab assessments (see Canvas for the submission date). Please keep your files organised. Make sure that your lab folder has the following structure:



```
rp_lin_model.m
rp_mainfile_sfc_comparison_a.m
rp_mainfile_sfc_comparison_b.m
rp_nl_model.m
rp_parameters.m
rp_plot.m
rp_sfc_controller.m
rp_sfc_design.m
rp_sfc_lin.slx
rp_sfc_nl.slx
```

3 Aero system

In this section, we describe the control design task for the aero system.

3.1 Problem formulation

We consider the nonlinear model of the aero system presented in the document *Lab 01 - Modelling and Simulation*, and the equilibrium points $\bar{x}_a$ and $\bar{x}_b$ defined as

$$\bar{x}_a = \begin{bmatrix} \bar{x}_{1a} \\ \bar{x}_{2a} \\ \bar{x}_{3a} \\ \bar{x}_{4a} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; ; \qquad \bar{V}_{pa} = 0 \; ; \qquad \bar{V}_{ya} = 0, \tag{4}$$

and

$$\bar{x}_b = \begin{bmatrix} \bar{x}_{1b} \\ \bar{x}_{2b} \\ \bar{x}_{3b} \\ \bar{x}_{4b} \end{bmatrix} = \begin{bmatrix} \frac{\pi}{9} \\ 0 \\ 0 \\ 0 \end{bmatrix} \; ; \qquad \bar{V}_{pb} = \frac{K_{yy}}{K_T}\,\bar{\tau}_{pb} \; ; \qquad \bar{V}_{yb} = -\frac{K_{yp}}{K_T}\,\bar{\tau}_{pb}, \tag{5}$$

with $K_T = K_{pp}K_{yy} - K_{py}K_{yp}$ and $\bar{\tau}_{pb} = K_{sp}\sin(\bar{x}_{1b})$.

We also consider the linearised models about the equilibria. The linearised model about the equilibrium point $\bar{x}_a$ is

$$\dot{x}_a = A_a\, x_a + B_a\, u_a \tag{6}$$

where $x_a \triangleq x - \bar{x}_a$, $u_a \triangleq u - \bar{u}_a$, $u = \begin{bmatrix} V_p & V_y \end{bmatrix}^\top$, and the matrices A_a and B_a correspond to Jacobian matrices respect to x and u evaluated at the equilibrium $\bar{x}_a$ with $\bar{u}_a = \begin{bmatrix} \bar{V}_{pa} & \bar{V}_{ya} \end{bmatrix}^\top$.

The linearised model about the equilibrium point $\bar{x}_b$ is

$$\dot{x}_b = A_b\, x_b + B_b\, u_b \tag{7}$$

where $x_b \triangleq x - \bar{x}_b$, $u_b \triangleq u - \bar{u}_b$, $u = \begin{bmatrix} V_p & V_y \end{bmatrix}^\top$, and the matrices A_b and B_b correspond to Jacobian matrices respect to states and inputs evaluated at the equilibrium $\bar{x}_b$ with $\bar{u}_b = \begin{bmatrix} \bar{V}_{pb} & \bar{V}_{yb} \end{bmatrix}^\top$.

The first problem is to design a controller using the linearised model to stabilise the system about the equilibrium points $\bar{x}_a$. The second problem is to design another controller using the linearised model to stabilise the system about the equilibrium points $\bar{x}_b$. Notice that all the equations and matrices that characterise the nonlinear and linearised models are given in the document Lab 01 - Modelling and Simulation.

3.2 State-feedback control system design using linearised models.

In this section, you have to build and simulate the control system of the aero system in closed loop with a state-feedback controller that stabilised the desired equilibrium points $\bar{x}_a$ and $\bar{x}_b$. We design the controller and compare the state histories of the control system using the nonlinear and its linear approximation. The task is to write a MATLAB script that performs the following actions:

1. Call the function `aero_parameters.m` that defines the parameters and matrices needed for the nonlinear and linearised models, as well as the equilibrium point $\bar{x}_a$ and the inputs $\bar{V}_{pa}$ and $\bar{V}_{ya}$. Store all these values in the structure `aero_p`.

2. Create the function `aero_sfc_design.m` that accepts the matrices A and B of the linearised model, and the vector E_c that contains the desired eigenvalues of the closed loop, and returns

   i) The variable `aero_p.COcheck=1` if the linearised model is completely controllable, otherwise `aero_p.COcheck=0`.

   ii) The gain of the state-feedback controller `aero_p.K` such that the eigenvalues of the closed loop are the entries of the vector `aero_p.Ec`.

The function syntax should be

```
[aero_p.COcheck,aero_p.K] = aero_sfc_design(aero_p.A,aero_p.B,aero_p.Ec);
```

3. Define the vector of desired eigenvalues of the closed loop in the main script and use the function `aero_sfc_design.m` to compute the gain of the state-feedback controller. Use first the eigenvalues selected for the semple solution. Once your are confident that your code is correct, change the desired eigenvalues and use simulations to select them such that the inputs does not exceed their maximum values, that is $\max |V_p| \leq 15$v and $\max |V_y| \leq 15$v, and simultaneously maximise the *region of attraction* with zero initial velocities, that is $x_3(0) = 0$ and $x_4(0) = 0$.

4. Simulate the control systems using the linearised and nonlinear models of the aero system. Use the initial conditions for both models that correspond to the same scenario.

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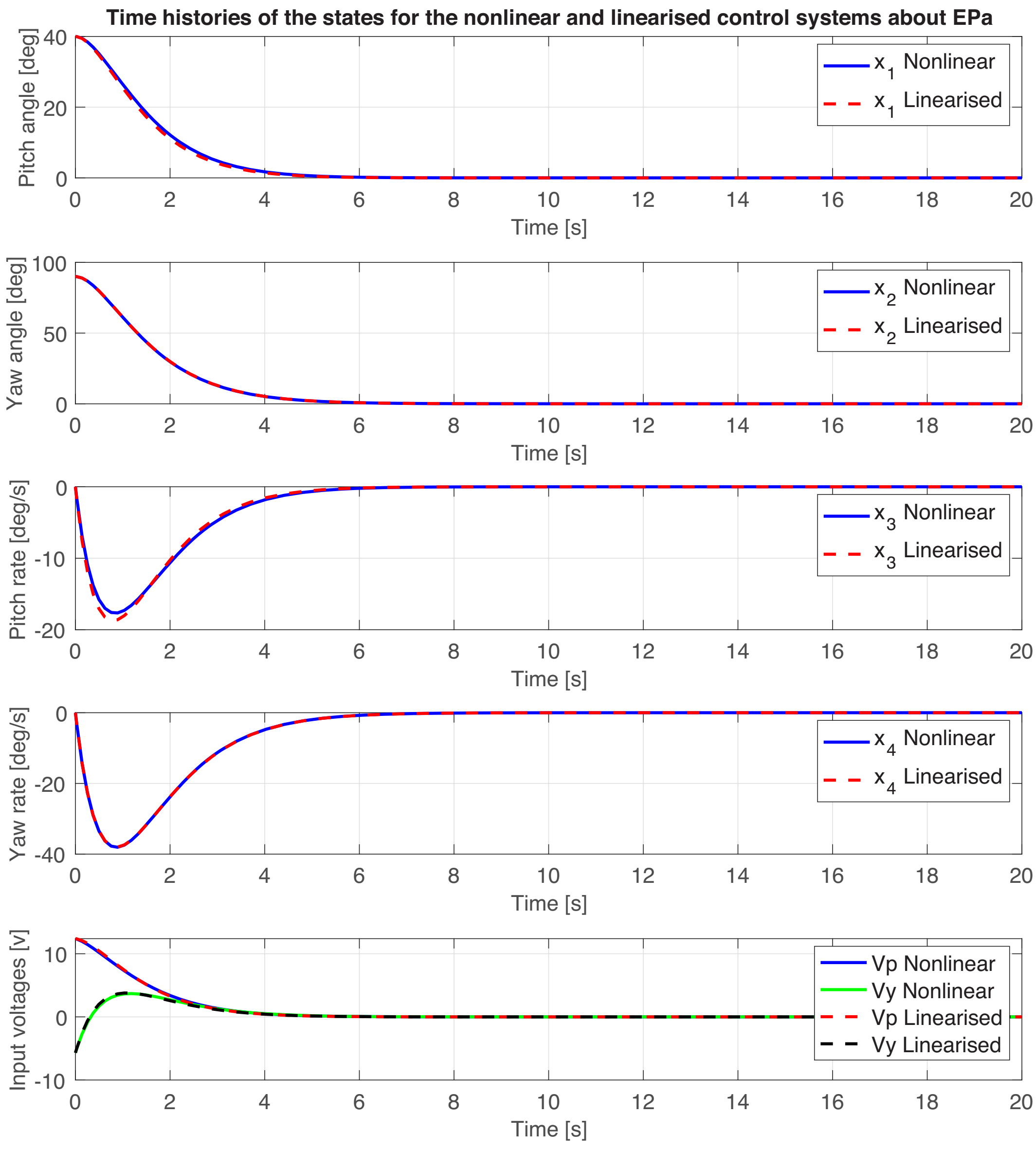

Figure 4: Time histories of the states and input voltage.

6. Save your script as `aero_mainfile_sfc_comparison_a.m`.

7. Repeat this process for initial conditions about the equilibrium point $\bar{x}_b$. Save your script as `aero_mainfile_sfc_comparison_b.m`. Sample results for the initial conditions $x_1(0) = 0$ deg, $x_2(0) = 90$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$deg/s are shown in Figure 5, where we selected the following eigenvalues $\{-1, -1.1, -1.2, -1.3\}$.

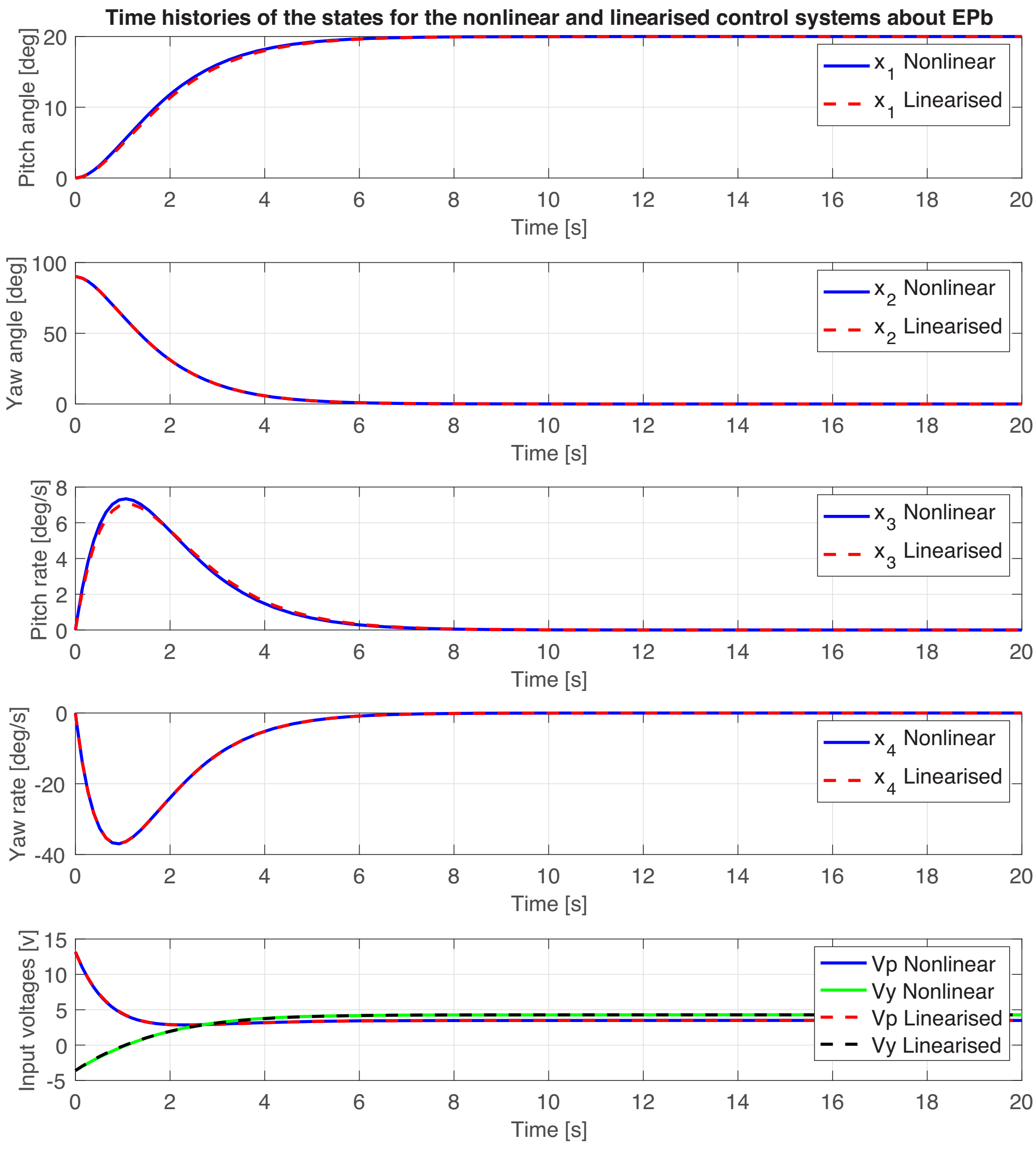

Figure 5: Time histories of the states and input voltage.

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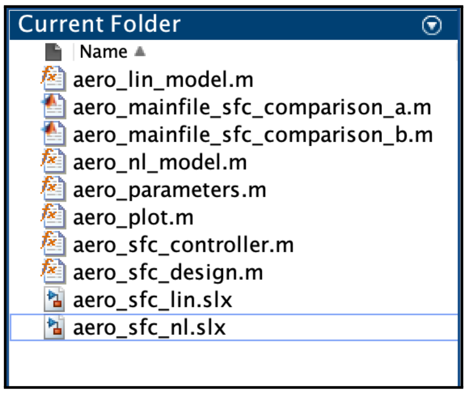

Current Folder

Name

aero_lin_model.m
aero_mainfile_sfc_comparison_a.m
aero_mainfile_sfc_comparison_b.m
aero_nl_model.m
aero_parameters.m
aero_plot.m
aero_sfc_controller.m
aero_sfc_design.m
aero_sfc_lin.slx
aero_sfc_nl.slx