AERO3600 — Embedded Control Systems

Lab 04 - Optimal Control Design[1]

Learning Outcomes

This lab will assess your ability to:

1. Use Matlab to design optimal controllers.
2. Build Simulink models of the closed-loop system and run numerical simulations.
3. Analyses and evaluate the state trajectories of the closed loop obtained via numerical simulations.

1 Optimal control design

In this lab, we redesign the control systems proposed in Lab 02 and Lab 03 using Optimal Control approach. The controllers are designed for the rotary pendulum and 2-DOF aero systems shown in Figure 1.

Figure 1: Rotary pendulum and aero systems.

The task in this lab is to replace the gain K of the controllers $u = -K\,x$ or $u = -K\,\hat{x}$ computed using eigenvalue assignment by a gain K computed by the Linear Quadratic Regulator approach.

[1]Updated: 17 Mar 2024.

2 Rotary pendulum system

Consider the scripts

- `rp_mainfile_ofc_comparison_a_studentnumber.m`
- `rp_mainfile_ofc_comparison_b_studentnumber.m`

and recompute the grain of the controller using LQR approach.

Create the function `rp_lqr_design.m` that accepts the matrices A and B of the linearised model and the matrices Q and R, and returns:

i) The variable `rp_p.COcheck=1` if the linearised model is completely controllable, otherwise `rp_p.COcheck=0`.

ii) The gain of the state-feedback controller `rp_p.K` obtained using the linear quadratic regulator method with weighting matrices `rp_p.Q` and `rp_p.R`. Hint: use the command `lqr` instead of `place`.

The function syntax should be

```
[rp_p.COcheck,rp_p.K] = rp_lqr_design(rp_p.A,rp_p.B,rp_p.Q,rp_p.R);
```

Select the matrices Q and R such that the closed loop achieve a satisfactory performance. Plots the time histories of the states for both the nonlinear and linearised control system against each other. Plot the states $x_1(t)$ and its linear approximation $x_{1a}(t) + \bar{x}_{1a}$ together with their estimations an on top of each other in one graph, then plot $x_2(t)$, $x_{2a}(t) + \bar{x}_{2a}$ and their estimations together on another graph, and so on. Sample results for the initial conditions $x_1(0) = 60$ deg, $x_2(0) = 60$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$ deg/s are shown in Figure 2, where we selected the following matrices

$$Q = \begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \qquad R = 1, \tag{1}$$

and the eigenvalues for the observer error dynamics are $\{-25, -26, -27, -28\}$. Note that the initial condition of the observer is set to zero.

Sample results for the initial conditions $x_1(0) = 10$ deg, $x_2(0) = 160$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$ deg/s are shown in Figure 3. The matrices Q and R are the same, but the eigenvalues of the observer error dynamics are $\{-30, -31, -32, -33\}$.

Compute the eigenvalues of the closed-loop using the command $\texttt{eig}(A - BK)$ and compare the closed loop eigenvalues with the desired eigenvalues proposed in Lab 03.

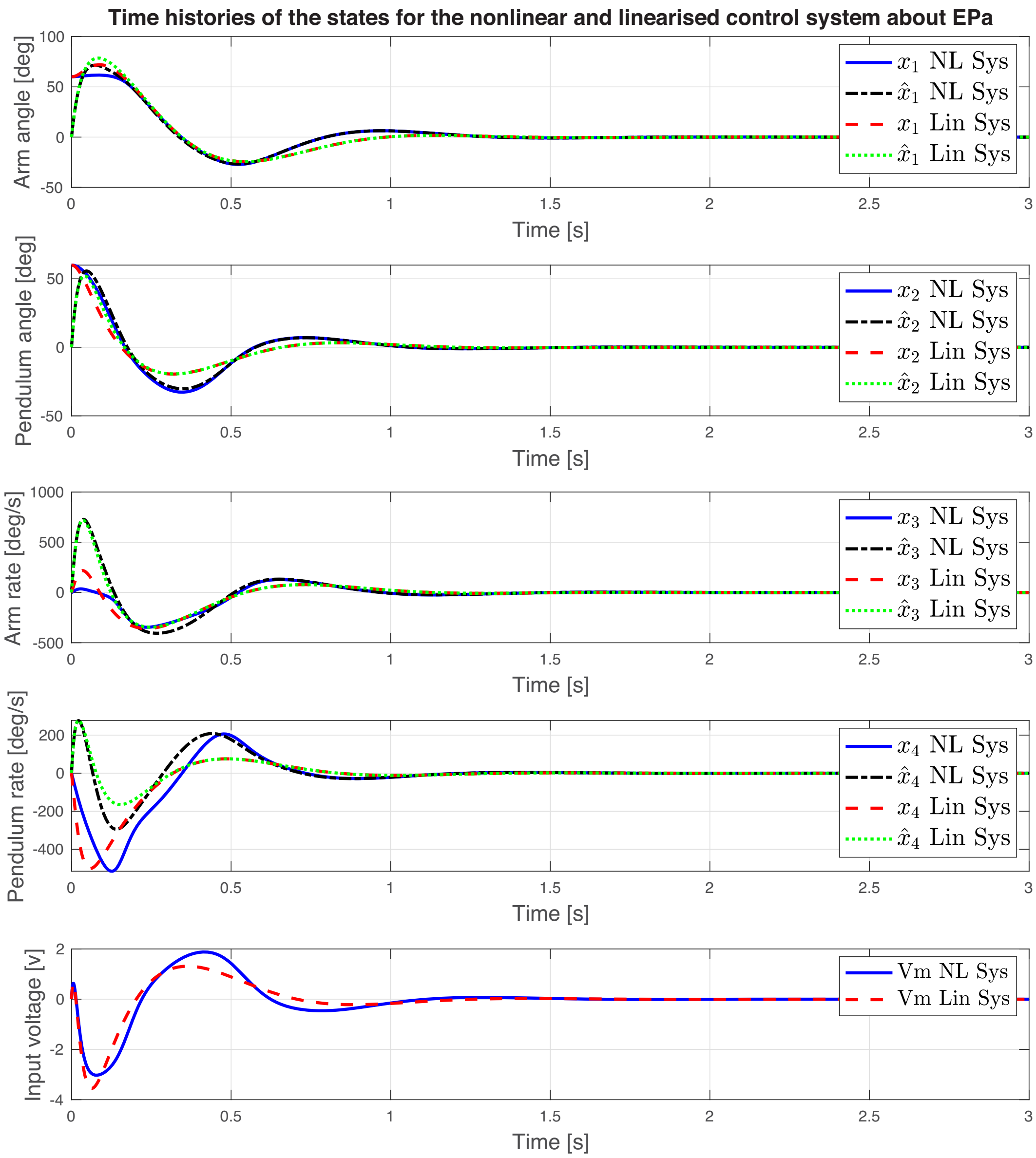

Figure 2: Time histories of the states, their estimations and input voltage.

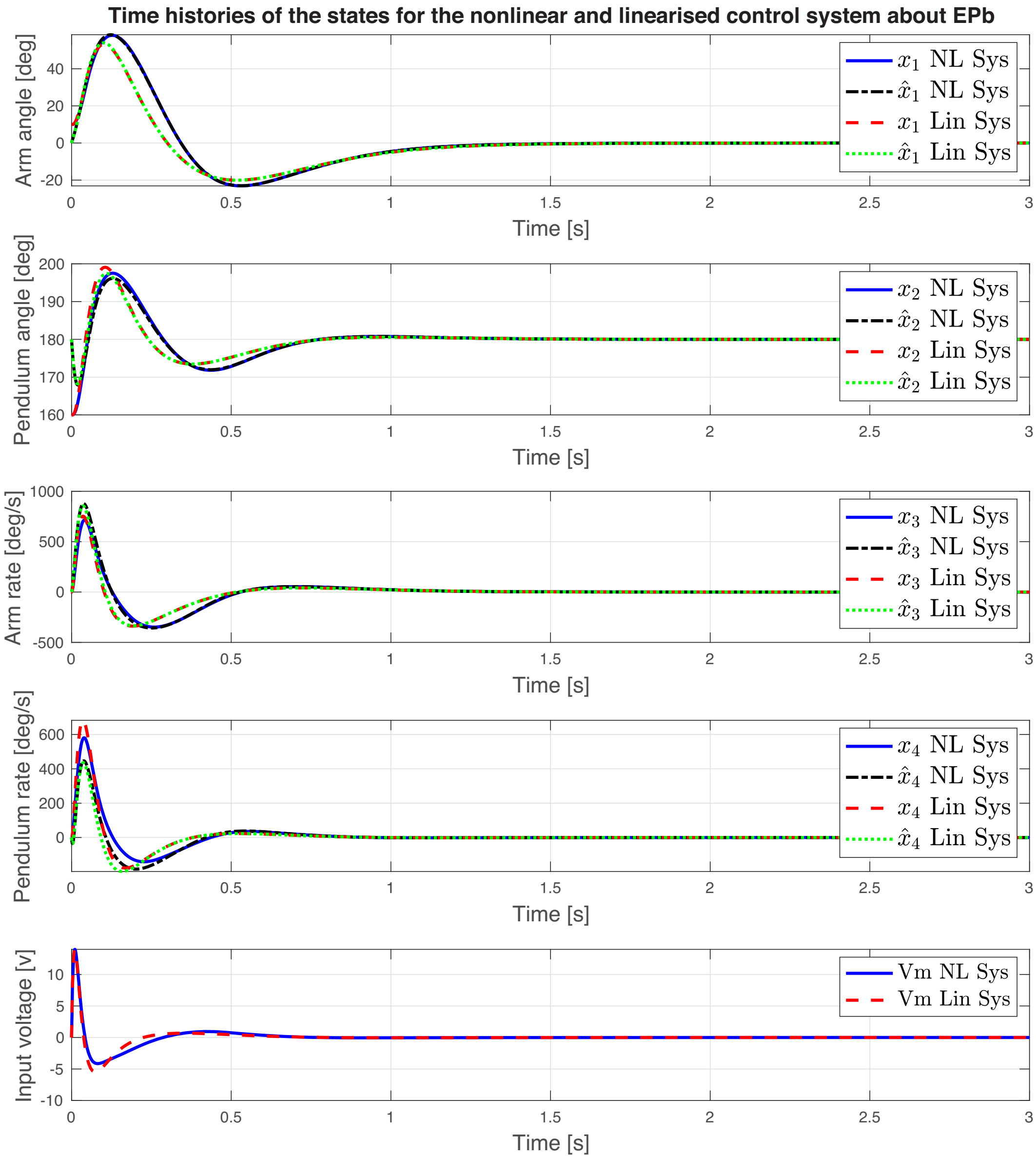

Figure 3: Time histories of the states, their estimations and input voltage.

3 Aero system

Consider the scripts

- `aero_mainfile_ofc_comparison_a_studentnumber.m`
- `aero_mainfile_ofc_comparison_b_studentnumber.m`

and recompute the grain of the controller using LQR approach.

Create the function `aero_lqr_design.m` that accepts the matrices A and B of the linearised model and the matrices Q and R, and returns:

i) The variable `aero_p.COcheck=1` if the linearised model is completely controllable, otherwise `aero_p.COcheck=0`.

ii) The gain of the state-feedback controller `aero_p.K` obtained using the linear quadratic regulator method with weighting matrices `aero_p.Q` and `aero_p.R`. Hint: use the command `lqr` instead of `place`.

The function syntax should be

```
[aero_p.COcheck,aero_p.K] = aero_lqr_design(aero_p.A,aero_p.B,aero_p.Q,aero_p.R);
```

Select the matrices Q and R such that the closed loop achieve a satisfactory performance. Plots the time histories of the states for both the nonlinear and linearised control system against each other. Plot the states $x_1(t)$ and its linear approximation $x_{1a}(t) + \bar{x}_{1a}$ together with their estimations an on top of each other in one graph, then plot $x_2(t)$, $x_{2a}(t) + \bar{x}_{2a}$ and their estimations together on another graph, and so on. Sample results for the initial conditions $x_1(0) = 40$ deg, $x_2(0) = 90$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$ deg/s are shown in Figure 4, where we selected the following matrices:

$$Q = \begin{bmatrix} 500 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix}, \qquad R = 1, \tag{2}$$

and the eigenvalues for the observer error dynamics are $\{-10, -11, -12, -13\}$. Note that the initial condition of the observer is set to zero.

Sample results for the initial conditions $x_1(0) = 40$ deg, $x_2(0) = 90$ deg, $x_3(0) = 0$ deg/s and $x_4(0) = 0$ deg/s are shown in Figure 5. The matrices Q and R and the eigenvalues of the observer error dynamics are the same.

Compute the eigenvalues of the closed-loop using the command `eig`$(A - BK)$ and compare the closed loop eigenvalues with the desired eigenvalues proposed in Lab 03.

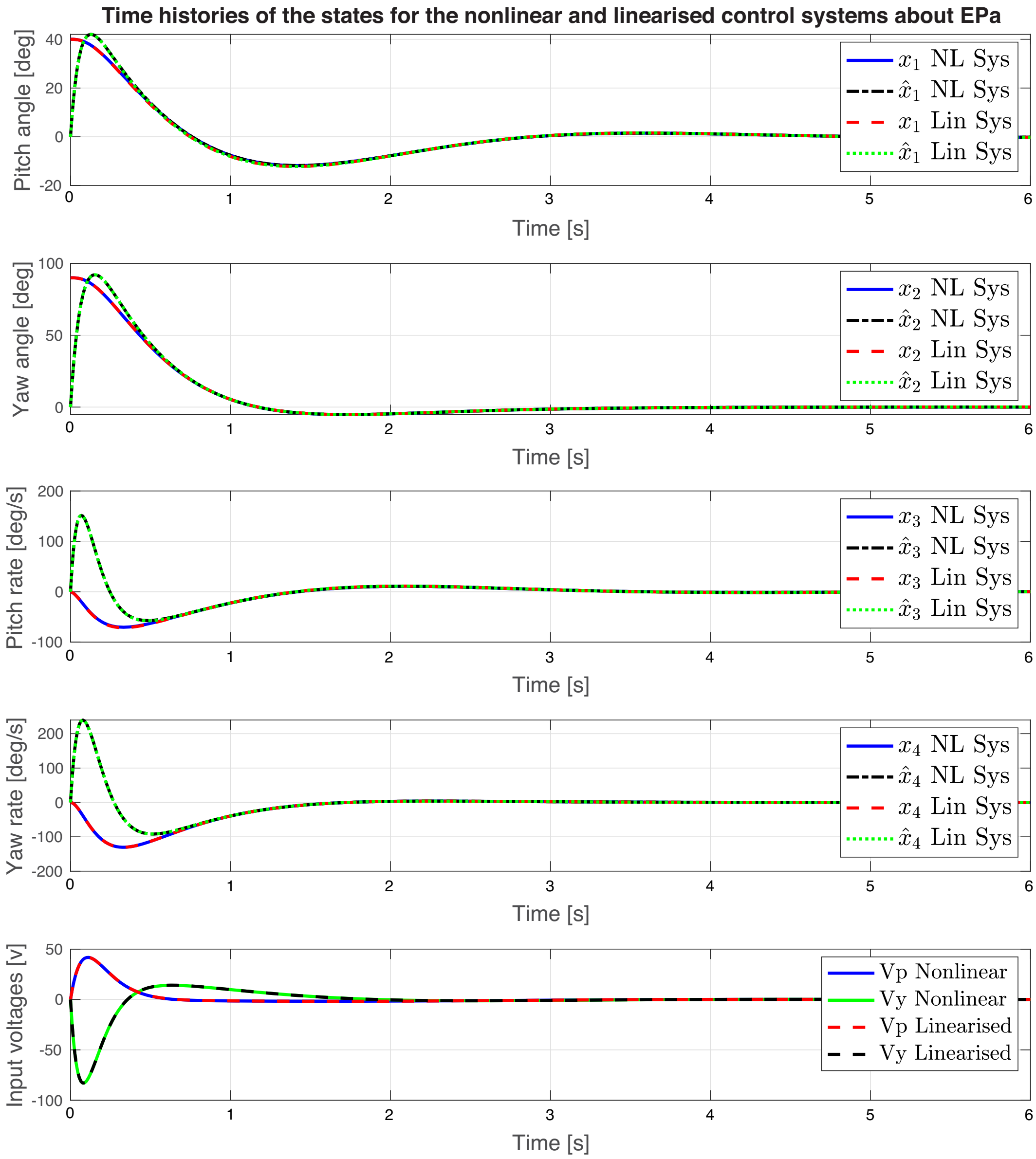

Figure 4: Time histories of the states, their estimations and input voltage.

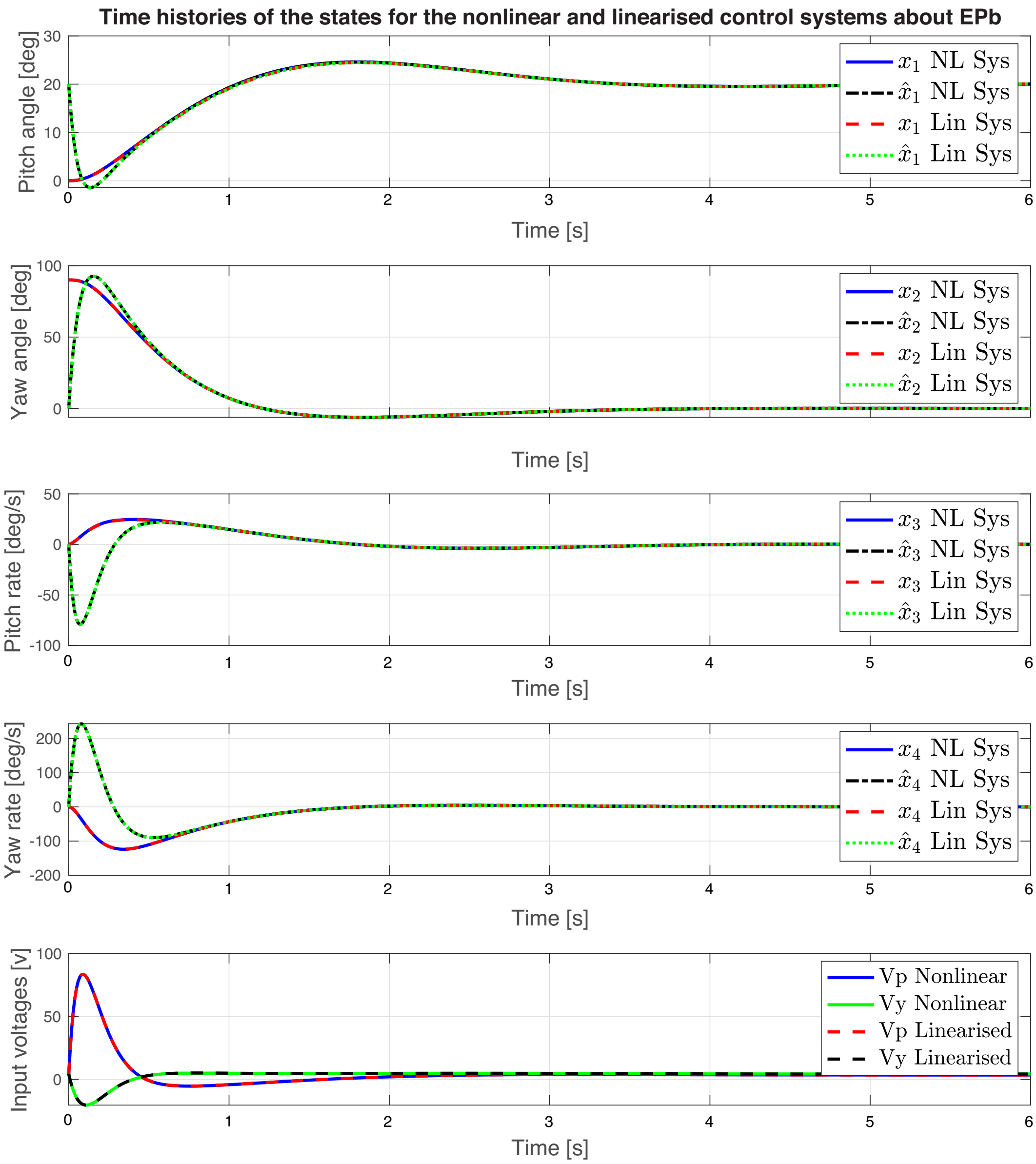

Figure 5: Time histories of the states, their estimations and input voltage.