Assignment 1: Bond graph modelling and simulation

Due 11:59pm, Friday 5th April

MCHA3400

Semester 1 2024

Background

Bond graphs are a graphical representation of physical system. The formalism allows for detailed analysis of causality and overall interconnection without having to initially derive a computational model of a system. Once a bond graph is constructed, it can be used to construct state-space models or simulations. In this assignment, you will be required to use the bond graph formalism to analyse and simulate a variety of systems.

Submission instructions

Assignment 1 is due at 11:59pm, Friday 5^{th} April via Canvas. Submission of this assignment will consist of both written solutions and Matlab scripts.

Software submission: Problems 2 and 4 required you to create simulations in Matlab. Your simulation files should be labeled as per the file names indicated throughout this document. Files with names different from those specified will be ignored.

Written submission: Your written solutions should be collected into a single `.pdf` file with the name `assignment1_written.pdf`. Please ensure that your solutions are neatly presented and organised in the correct order within your document. Examples of acceptable submissions include:

- Neatly handwritten on paper and scanned to a pdf,
- Neatly handwritten on a tablet (e.g., Surface, iPad) and saved to a pdf,
- Typed in a word processor (e.g., MS Word with equation editor) and exported to a pdf,
- Typeset in LaTeX and compiled to a pdf.

Warning

If you written submission cannot be easily read, it will receive a grade of 0.

Hint

Don't submit your initial working-out. Rewrite your solutions neatly in a logical order!

Submission format: Add both your written and software solutions into a single `.zip` file called *`studentNumber`*`_`*`studentName`*`_assignment1.zip`[1] and submit this file to Canvas for assessment. Your final `.zip` folder should include the following files:

- `assignment1_written.pdf`
- `Problem_2.m`
- `Problem_4.m`

Any files included in your submission not matching these filenames will be ignored.

[1]The expressions *`studentNumber`* and *`studentName`* should be replaced with your student number and name, respectively.

Problem 1 (30%)

A lumped-model representation of a drive system containing electrical, fluid power and mechanical elements is shown in Figure 1. In input voltage of V_{in} is applied to a series resistor (R_1), inductor (L_1) and ideal DC motor (K_m). The mechanical output of the motor is connected to a rotational inertia (J_1) and hydraulic pump (K_p). The pump draws fluid from a reservoir with constant pressure P_0 and feeds into a fluid accumulator. The accumulator is connected to a fluid resistor which feeds a fluid motor (K_h). The output of the fluid motor is connected to a spring (c) which is then connected to a rotational damper (b) and rotational inertia (J_2).

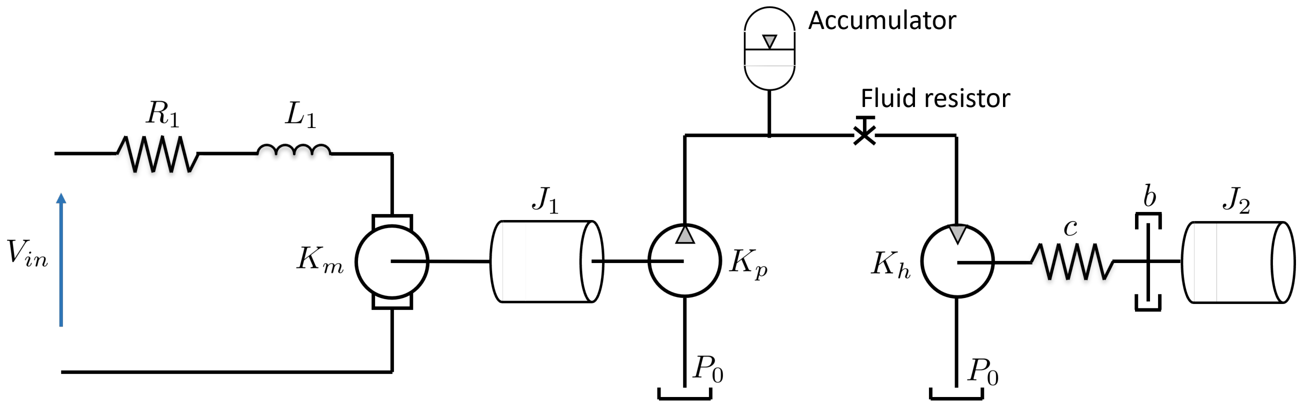


Figure 1: A system containing electrical, mechanical and fluid-power elements.

The CCRs of all components except the fluid accumulator and fluid resistor are assumed to be linear. The CCR of the ideal DC motor is given by

$$\begin{aligned} T_m &= K_m I_m \\ e = V_m &= K_m \omega_m, \end{aligned} \tag{1}$$

where T_m is the torque exerted by the motor, I_m is the current through the motor, $e = V_m$ is the back-emf and ω_m is the motor angular velocity. The fluid pump has CCRs

$$Q_p = K_p \omega_p, \ P_p = \frac{1}{K_p} T_p, \tag{2}$$

where Q_p is the volumetric flow rate through the pump, P_p is the pressure difference across the pump and T_p is the torque applied to the pump. Similarly, the hydraulic motor has CCRs

$$T_h = K_h P_h, \ \omega_h = \frac{1}{K_h} Q_h \tag{3}$$

with similar definitions as above. The fluid accumulator is assumed to have a nonlinear CCR given by

$$P_a = \Phi_a(V_a), \tag{4}$$

where V_a is the volume of the accumulator and P_a is the pressure of the accumulator. The fluid resistor is similarly assumed to have a nonlinear CCR given by

$$Q_r = \Omega_r(P_r), \tag{5}$$

where Q_r is the volumetric flow rate through the resistor and P_r is the change in pressure across the resistor.

a) **(10%)** Construct a bond graph for the system shown in Figure 1 and assign causality and causal strokes to the graph. Note that the graph should be causal.

b) **(10%)** Using your bond graph, determine a set of differential equations that describe the dynamic behaviour of the system. All components can be considered linear, except for the accumulator and fluid resistor.

c) **(10%)** Find a purely electrical circuit that has the same bond graph as the system shown in Figure 1. Submit a drawing of the electrical circuit and the corresponding bond graph. You can use the symbol shown in Figure 2 to represent an ideal DC-DC converter, which acts as a transformer element.

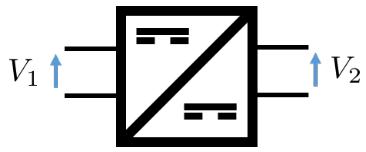


Figure 2: An ideal DC-DC converter (transformer) element.

To represent a purely electrical gyrator, use the symbol show in Figure 3.

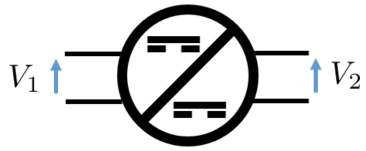


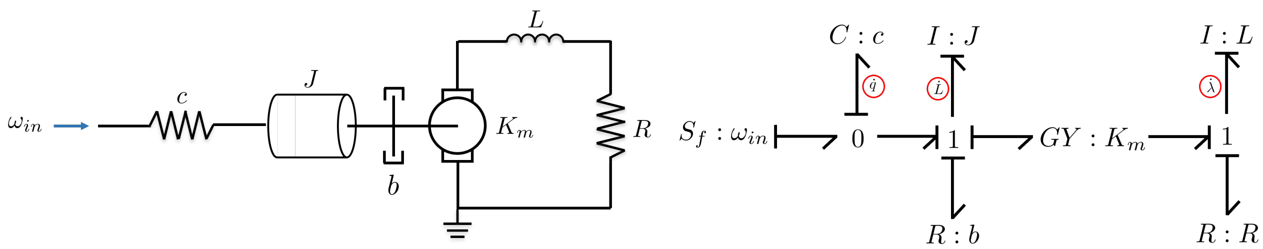
Figure 3: An ideal gyrator element.

Problem 2 (30%)

A simple electro-mechanical network is shown in Figure 4a. An angular velocity input ω_{in} is applied to a spring with compliance c, which is then connected to a rotational inertia with moment of inertia J and a rotational damper with damping coefficient b. The rotational inertia is coupled with an ideal DC motor which has CCRs

$$\begin{aligned} V_m &= K_m \omega_m \\ I_m &= \frac{1}{K_m} T_m, \end{aligned} \tag{6}$$

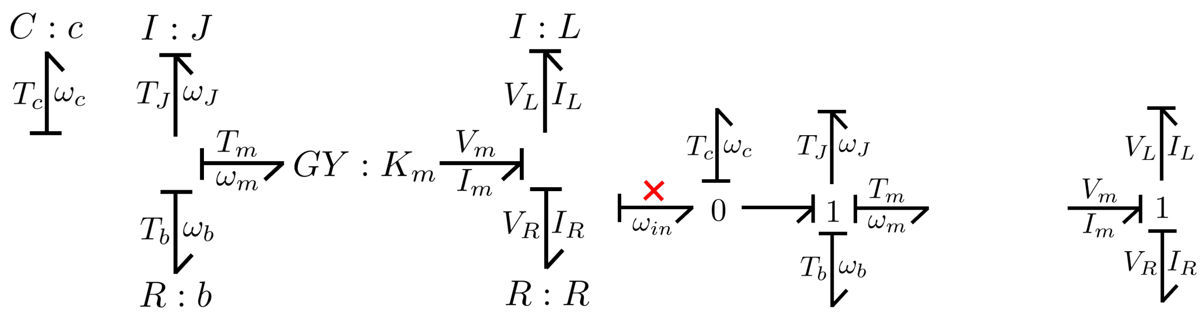
where K_m is the modulus of the motor, V_m is the voltage across the motor, ω_m is the angular velocity of the motor, I_m is the current through the motor and T_m is the torque exerted by the motor. The electrical side of the ideal DC motor is connected to series inductor and resistor with inductance L and resistance R, respectively. The bond graph of the network is given in Figure 4b.



(a) An electrical network with DC-DC transformer. (b) Bond graph of the electrical network.

Figure 4: A simple electrical network.

The bond graph of the network can be decomposed into the system CCRs and SSRs, which are shown in Figures 5a and 5b, respectively.



(a) System CCRs. (b) System SSRs.

Figure 5: Bond graph decomposition of the electrical network.

a) **(4%)** Create a new file in Matlab called `Problem_2.m` and include the commands `clear` and `clc` at the top of your script. Create a parameters structure called `params` that will store all of the system parameters. Add the following parameters to the structure.

$$\begin{aligned} c &= 0.1 \ \frac{rad}{Nm} & b &= 1.5 \ \frac{Nm \ s}{rad} \\ J &= 2 \ kg \ m^2 & K_m &= 8.2 \ \frac{V \ s}{rad} \\ L &= 1 \times 10^{-3} \ F & R &= 10 \ \Omega \end{aligned} \tag{7}$$

b) **(10%)** For each of the static CCRs associated with the components shown in Figure 5a, create an anonymous function describing the component behaviour with the appropriate inputs and outputs. For each of the SSRs described in Figure 5b, create an anonymous function describing the power variable interconnections with the appropriate inputs and outputs.

c) **(8%)** For each of the dynamic CCRs associated with the components shown in Figure 5a, create an anonymous function that describes the individual state equations of each state. Make use of the previously defined CCRs and SSRs to describe each state's behaviour as a function of states and inputs. Once completed, combine all states into a single state equation of the form

$$\dot{x} = f(x, u), \tag{8}$$

where $x = [q, L, \lambda]$ is the state, q is the displacement of the spring, L is the angular momentum of the rotational inertia, λ is the magnetic flux of the inductor and $u = \omega_{in}$ is the input.

d) **(8%)** Making use of the function defined in the previous steps, create a simulation of the electrical network using the `ode23s` solver with a relative error tolerance of 1×10^{-6}. Simulate the system for 5.0 seconds with the input

$$\omega_{in}(t) = \begin{cases} 1 + \cos(5t), & t < 2.5s \\ \frac{t \sin(2t)}{5}, & t \geq 2.5s. \end{cases} \tag{9}$$

and initial conditions $q(0) = 0$, $L(0) = 0$ and $\lambda(0) = 0$. Plot the displacement of the spring, velocity of the rotational inertia and current through the inductor on separate plots. Include the resulting plots in your `.pdf` submission.

Problem 3 (20%)

A simple mechanical system is shown in Figure 6. The system consisting of a torque input T_{in} which is applied to a rotational inertial with moment of inertia J_1. The inertia is coupled to a spring with compliance c_1, which is also coupled to a rotational damper, with damping coefficient b_1, and gear set. The gears have number of teeth N_1, N_2 on the low and high sides, respectively. The high side of the gears is connected to a rotational damper with damping coefficient b_2 and spring with compliance c_2. The second terminal of the spring is rigidly fixed to a wall.

Warning

Note that you should not make any simplifying modifications to the diagram and resolve any a-causality using the techniques studied in this class. Submissions using alternate methods will receive a grade of 0.

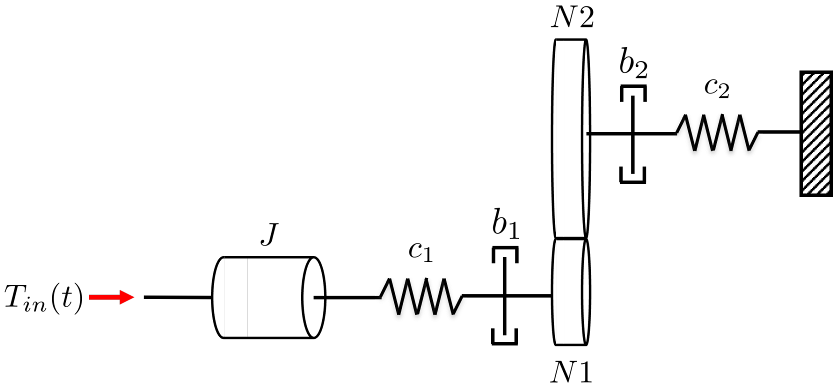


Figure 6: A simplified model of an electric vehicle drive system.

a) **(10%)** Construct a bond graph for the system shown in Figure 6, assigning a power convention and causal strokes. Indicate any causality issues with the graph. Note that your graph should be a-causal.

b) **(10%)** Taking L to be the angular momentum of the rotational inertia, q_1 the displacement of the spring with compliance c_1 and q_2 the displacement of the spring with compliance c_2 as the model states, construct a state-space model of the system.

Problem 4 (20%)

A cam-follower system is shown in Figure 7. The relationship between the cam angle and follower angle is described by the generic function

$$\phi_f = f(\phi_c). \tag{10}$$

The cam is has moment of inertia J_c about the pivot point of the motor whereas the follower has moment of inertia J_f about the follower's pivot-point. The spring and damper on the follower support shaft have compliance c_f and damping coefficient b_f, respectively. The effects of the motor on the cam can be modelled as a source of torque.

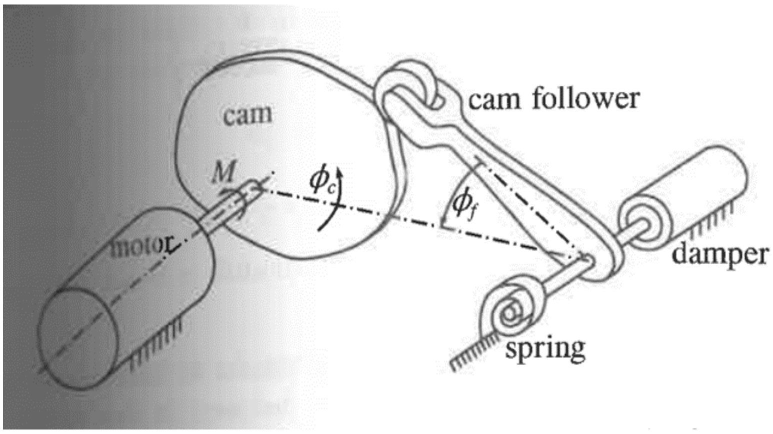


Figure 7: A simplified cam-follower system.

The bond graph of the cam-follower system is given in Figure 8, where the cam inertia element has been put into integral causality and causality fully propagated. As the follower inertia element has been forced into derivative causality, it is clear that the system is over-causal.

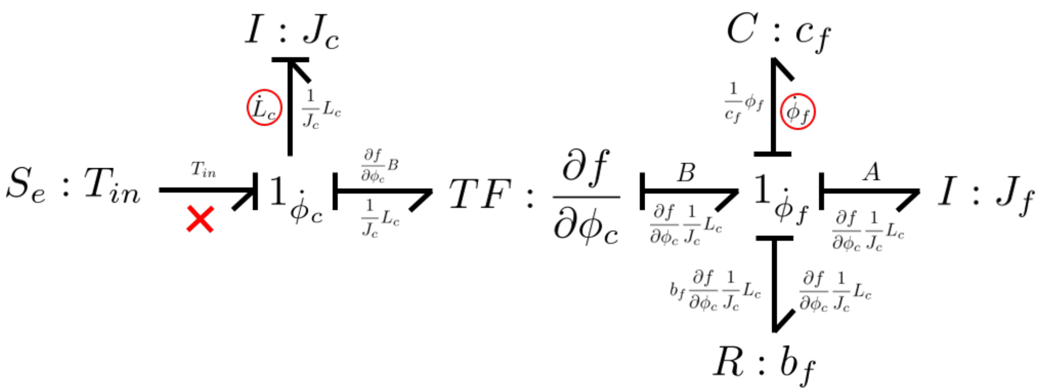


Figure 8: A simplified cam-follower system.

By assigning states to all components in integral causality, the power variables can be propagated as shown in Figure 8 where

$$\begin{aligned} A &= J_f \frac{d}{dt}\left[\frac{\partial f}{\partial \phi_c}\frac{1}{J_c}L_c\right] \\ &= J_f\left[\frac{\partial^2 f}{\partial \phi_c^2}\dot{\phi}_c\frac{1}{J_c}L_c + \frac{\partial f}{\partial \phi_c}\frac{1}{J_c}\dot{L}_c\right] \\ &= J_f\frac{\partial^2 f}{\partial \phi_c^2}\left(\frac{1}{J_c}L_c\right)^2 + \frac{\partial f}{\partial \phi_c}\frac{J_f}{J_c}\dot{L}_c \end{aligned} \tag{11}$$

and

$$B = A + \frac{1}{c_f}\phi_f + b_f\frac{\partial f}{\partial \phi_c}\frac{1}{J_c}L_c. \tag{12}$$

In order to describe the angle of the cam, an additional state ϕ_c is added to the model. The resulting state equations are given implicitly by

$$\begin{aligned} \dot{L}_c &= T_{in} - \frac{\partial f}{\partial \phi_c}\left[J_f\frac{\partial^2 f}{\partial \phi_c^2}\left(\frac{1}{J_c}L_c\right)^2 + \frac{\partial f}{\partial \phi_c}\frac{J_f}{J_c}\dot{L}_c + \frac{1}{c_f}\phi_f + b_f\frac{\partial f}{\partial \phi_c}\frac{1}{J_c}L_c\right] \\ \dot{\phi}_f &= \frac{\partial f}{\partial \phi_c}\frac{1}{J_c}L_c \\ \dot{\phi}_c &= \frac{1}{J_c}L_c. \end{aligned} \tag{13}$$

a) **(15%)** Create a Matlab script called `Problem_4.m` and include the commands `clear` and `clc` at the top of your script. Using the `ode45` solver with a relative error tolerance of 1×10^{-6}, construct a numerical simulation of the cam mechanism in Matlab. The CCRs of the cam inertia, spring and damper can be considered linear. The cam mechanism has geometry given by

$$\phi_f = f(\phi_c) = 2 - 0.5\cos(\phi_c), \tag{14}$$

where ϕ_c and ϕ_c are in radians. Use the parameters and inputs

Parameter	Value
J_c	6×10^{-4} $[kg\ m^2]$
J_f	3×10^{-4} $[kg\ m^2]$
c_f	5×10^{-1} $[\frac{rad}{Nm}]$
b_f	3×10^{-2} $[\frac{Nm\ s}{rad}]$

Input	Value
$T_{in}(\omega_c)$	$-k_p(\omega_c - \omega_c^\star)$ [Nm]

where ω_c is the angular velocity of the cam, $k_p = 5.0$ and $\omega_c^\star = 2$. Use the initial conditions $L_c(0) = 0, \phi_f(0) = 1.5, \phi_c(0) = 0$ and run the simulation for 15 seconds. Using the `subplot` command, plot the cam angular velocity and follower angle vs time on two separate sub-plots. Include your output plot as part of your written submission.

b) **(5%)** Modify your simulation to compute the value of the states at a fixed step size of $0.1s$. Configure an output function that prints the step time and state ϕ_f after each step of the ODE solver.