STAT2110 PASS Worksheet 3

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1 Key Formulas

- Expected value of X (continuous):
$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$
- Expected value of $g(X)$ (continuous):
$$\mu_{g(X)} = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$
- Expected value of $g(X, Y)$ (continuous):
$$\begin{aligned}\mu_{g(X,Y)} &= E[g(X,Y)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y) dx dy\end{aligned}$$
- Variance of X (continuous):
$$\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$
- Covariance of X and Y:
$$\begin{aligned}\sigma_{XY} &= E[(X-\mu_X)(Y-\mu_Y)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y) f(x,y) dx dy\end{aligned}$$
- If a and b are constants, then
$$E(aX+b) = aE(X)+b$$
- The correlation coefficient of X and Y is
$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

2 Questions

1. By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probability 0.7. What is this person's expected gain? **(4.7)**
2. The density function of coded measurements of the pitch diameter of threads of a fitting is:
$$f(x) = \begin{cases} \frac{4}{\pi(1+x^2)}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$
Find the expected value of X. **(4.11)**
3. Suppose that X and Y have the following joint probability function: **(4.23)**
Note that X and Y are discrete.

$f(x,y)$		$x=2$	$x=4$
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

   (a) Find the expected value of $g(X,Y) = XY^2$.
   (b) Find μ_X and μ_Y.

4. The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable $Y = 3X - 2$, where X has the density function
$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$
Find the mean and variance of the random variable Y. **(4.43)**

5. Let X be a random variable with the following probability distribution:

x	-2	3	5
$f(x)$	0.3	0.2	0.5

Find the standard deviation of X. **(4.34)**

6. Let X be a random variable with the following probability distribution:

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ and then, using these values, evaluate $E[(2X+1)^2]$. **(4.57)**