Exercises

4.1 The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given in Exercise 3.13 on page 112 as

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

4.2 The probability distribution of the discrete random variable, X, is

$$f(x) = \binom{5}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{5-x}, \quad x = 0, 1, 2, 3, 4, 5.$$

Find the mean of X.

4.3 Find the mean of the random variable T representing the total of the three coins in Exercise 3.25 on page 113.

4.4 A coin is biased such that a head is twice as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice.

4.5 In a gambling game, a woman is paid \$2 if she draws a jack or a queen and \$4 if she draws a king or an ace from an ordinary deck of 52 playing cards. If she draws any other card, she loses. How much should she pay to play if the game is fair?

4.6 An attendant at a car wash is paid according to the number of cars that pass through. Suppose the probabilities are 1/12, 1/12, 1/4, 1/4, 1/6, and 1/6, respectively, that the attendant receives \$7, \$9, \$11, \$13, \$15, or \$17 between 4:00 P.M. and 5:00 P.M. on any sunny Friday. Find the attendant's expected earnings for this particular period.

4.7 By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probability 0.7. What is this person's expected gain?

4.8 Suppose that an antique jewelry dealer is interested in purchasing a gold necklace for which the probabilities are 0.2, 0.3, 0.4, and 0.1, respectively, that she will be able to sell it for a profit of \$250, sell it for a profit of \$150, break even, or sell it for a loss of \$150. What is her expected profit?

4.9 A private pilot wishes to insure an airplane for \$200,000. The insurance company estimates that a total loss will occur with probability 0.001, a 50% loss with probability 0.01, and a 25% loss with probability 0.2. Ignoring all other partial losses, what premium should the insurance company charge each year to realize an average profit of \$500?

4.10 Two tire-quality experts examine stacks of tires and assign a quality rating to each tire on a 3-point scale. Let X denote the rating given by expert A and Y denote the rating given by B. The following table gives the joint distribution for X and Y.

$f(x,y)$		$y=1$	$y=2$	$y=3$
x	1	0.10	0.05	0.02
	2	0.10	0.35	0.05
	3	0.03	0.10	0.20

Find μ_X and μ_Y.

4.11 The density function of coded measurements of the pitch diameter of threads of a fitting is

$$f(x) = \begin{cases} \frac{1}{\ln(2)(1+x)}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of X.

4.12 If a dealer's profit, in units of \$5000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

find the average profit per automobile.

4.13 The density function of the continuous random variable X, the total number of hours, in units of 100 hours, that a family runs a vacuum cleaner over a period of one year, is given in Exercise 3.7 on page 112 as

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the average number of hours per year that families run their vacuum cleaners.

4.14 Find the proportion X of individuals who can be expected to respond to a certain mail-order solicitation if X has the density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

4.15 Assume that two random variables (X, Y) are uniformly distributed on a circle with radius a. Then the joint probability density function is

$$f(x, y) = \begin{cases} \frac{1}{\pi a^2}, & x^2 + y^2 \leq a^2, \\ 0, & \text{otherwise.} \end{cases}$$

Find μ_X, the expected value of X.

4.16 Suppose that you are inspecting a lot of 1000 light bulbs, among which 30 are defectives. You choose two light bulbs randomly from the lot without replacement. Let

$$X_1 = \begin{cases} 1, & \text{if the 1st light bulb is defective.} \\ 0, & \text{otherwise.} \end{cases}$$

$$X_2 = \begin{cases} 1, & \text{if the 2nd light bulb is defective.} \\ 0, & \text{otherwise.} \end{cases}$$

Find the probability that at least one light bulb chosen is defective. [*Hint*: Compute $P(X_1 + X_2 \geq 1)$.]

4.17 Let X be a random variable with the following probability distribution:

x	-3	6	9
$f(x)$	$1/6$	$1/2$	$1/3$

Find $\mu_{g(X)}$, where $g(X) = (2X + 1)^2$.

4.18 Find the expected value of the random variable $g(X) = X^2$, where X has the probability distribution of Exercise 4.2.

4.19 A large industrial firm purchases several new computers at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of computers, X, purchased each year has the following probability distribution:

x	0	1	2	3
$f(x)$	$1/10$	$3/10$	$2/5$	$1/5$

If the cost of the desired model is \$1200 per unit and at the end of the year a refund of $50X^2$ dollars will be issued, how much can this firm expect to spend on new computers during this year?

4.20 A continuous random variable X has the density function

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of $g(X) = e^{X/4}$.

4.21 What is the dealer's average profit per automobile if the profit on each automobile is given by $g(X) = X^2$, where X is a random variable having the density function of Exercise 4.12?

4.22 The hospitalization period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 5$, where X has the density function

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the average number of days that a person is hospitalized following treatment for this disorder.

4.23 Suppose that X and Y have the following joint probability function $f(x, y)$:

	x	
y	2	4
1	0.15	0.10
3	0.25	0.25
5	0.15	0.10

(a) Find the expected value of $g(X, Y) = XY^2$.

(b) Find μ_X and μ_Y.

4.24 Referring to the random variables whose joint probability distribution is given in Exercise 3.39 on page 125,

(a) find $E(X^2Y - 2XY)$;

(b) find $\mu_X - \mu_Y$.

4.25 Referring to the random variables whose joint probability distribution is given in Exercise 3.51 on page 126, find the mean for the total number of jacks and kings when 3 cards are drawn without replacement from the 12 face cards of an ordinary deck of 52 playing cards.

4.26 Let X and Y be random variables with joint density function

$$f(x, y) = \begin{cases} 4xy, & 0 < x,\ y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of $Z = \sqrt{X^2 + Y^2}$.

4.27 In Exercise 3.27 on page 113, a density function is given for the time to failure of an important component of a DVD player. Find the mean number of hours to failure of the component and thus the DVD player.

4.28 Consider the information in Exercise 3.28 on page 113. The problem deals with the weight in ounces of the product in a cereal box, with

$$f(x) = \begin{cases} \frac{2}{5}, & 23.75 \leq x \leq 26.25, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Plot the density function.

(b) Compute the expected value, or mean weight, in ounces.

(c) Are you surprised at your answer in (b)? Explain why or why not.

4.29 Exercise 3.29 on page 113 dealt with an important particle size distribution characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Plot the density function.

(b) Give the mean particle size.

4.30 In Exercise 3.31 on page 114, the distribution of times before a major repair of a washing machine was given as

$$f(y) = \begin{cases} \frac{1}{4}e^{-y/4}, & y \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

What is the population mean of the times to repair?

4.31 Consider Exercise 3.32 on page 114.

(a) What is the mean proportion of the budget allocated to environmental and pollution control?

(b) What is the probability that a company selected at random will have allocated to environmental and pollution control a proportion that exceeds the population mean given in (a)?

4.32 In Exercise 3.13 on page 112, the distribution of the number of imperfections per 10 meters of synthetic fabric is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

(a) Plot the probability function.

(b) Find the expected number of imperfections, $E(X) = \mu$.

(c) Find $E(X^2)$.

Exercises

4.33 Use Definition 4.3 on page 140 to find the variance of the random variable X of Exercise 4.7 on page 137.

4.34 Let X be a random variable with the following probability distribution:

x	-2	3	5
$f(x)$	0.3	0.3	0.4

Find the standard deviation of X.

4.35 The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution:

x	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	0.3	0.04

Using Theorem 4.2 on page 141, find the variance of X.

4.36 Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1, 2, or 3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of power failures striking this subdivision.

4.37 A dealer's profit, in units of \$5000, on a new automobile is a random variable X having the density function given in Exercise 4.12 on page 137. Find the variance of X.

4.38 The proportion of people who respond to a certain mail-order solicitation is a random variable X having the density function given in Exercise 4.14 on page 137. Find the variance of X.

4.39 The total number of hours, in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a random variable X having the density function given in Exercise 4.13 on page 137. Find the variance of X.

4.40 Referring to Exercise 4.14 on page 137, find $\sigma^2_{g(X)}$ for the function $g(X) = 3X + 4$.

4.41 Find the standard deviation of the random variable $h(X) = (3X + 1)^2$ in Exercise 4.17 on page 138.

4.42 Using the results of Exercise 4.21 on page 138, find the variance of $g(X) = X^2$, where X is a random variable having the density function given in Exercise 4.12 on page 137.

4.43 The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable $Y = 3X - 2$, where X has the density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the mean and variance of the random variable Y.

4.44 Find the covariance of the random variables X and Y of Exercise 3.39 on page 125.

4.45 Find the covariance of the random variables X and Y of Exercise 3.49 on page 126.

4.46 Find the covariance of the random variables X and Y of Exercise 3.44 on page 125.

4.47 For the random variables X and Y whose joint density function is given in Exercise 3.40 on page 125, find the covariance.

4.48 Given a random variable X, with standard deviation σ_X, and a random variable $Y = a + bX$, show that if $b < 0$, the correlation coefficient $\rho_{XY} = -1$, and if $b > 0$, $\rho_{XY} = 1$.

4.49 Consider the situation in Exercise 4.32 on page 139. The distribution of the number of imperfections per 10 meters of synthetic failure is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Find the variance and standard deviation of the number of imperfections.

4.50 For a laboratory assignment, if the equipment is working, the density function of the observed outcome X is

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the variance and standard deviation of X.

4.51 For the random variables X and Y in Exercise 3.39 on page 125, determine the correlation coefficient between X and Y.

4.52 Random variables X and Y follow a joint distribution

$$f(x, y) = \begin{cases} 2, & 0 < x \le y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the correlation coefficient between X and Y.

Exercises

4.53 Referring to Exercise 4.35 on page 147, find the mean and variance of the discrete random variable $Z = 3X - 2$, when X represents the number of errors per 100 lines of code.

4.54 Using Theorem 4.5 and Corollary 4.6, find the mean and variance of the random variable $Z = 5X + 3$, where X has the probability distribution of Exercise 4.36 on page 147.

4.55 Suppose that a grocery store purchases 5 cartons of skim milk at the wholesale price of \$1.20 per carton and retails the milk at \$1.65 per carton. After the expiration date, the unsold milk is removed from the shelf and the grocer receives a credit from the distributor equal to three-fourths of the wholesale price. If the probability distribution of the random variable X, the number of cartons that are sold from this lot, is

x	0	1	2	3	4	5
$f(x)$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{2}{15}$

Find the expected profit.

4.56 Repeat Exercise 4.43 on page 147 by applying Theorem 4.5 and Corollary 4.6.

4.57 Let X be a random variable with the following probability distribution:

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

Find $E(X)$ and $E(X^2)$ and then, using these values, evaluate $E[(2X+1)^2]$.

4.58 The total time, measured in units of 100 hours, that a teenager runs their hair dryer over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2-x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Use Theorem 4.6 to evaluate the mean of the random variable $Y = 60X^2 + 48X$, where Y is equal to the number of kilowatt hours expended annually.

4.59 If a random variable X is defined such that $E[(X-1)^2] = 10$ and $E[(X-2)^2] = 5$, find μ and σ^2.

4.60 Suppose that X and Y are independent random variables having the joint probability distribution

y \ x	2	4
1	0.15	0.10
3	0.25	0.25
5	0.15	0.10

Find
(a) $E(2X - 3Y)$;
(b) $E(XY)$.

4.61 Use Theorem 4.7 to evaluate $E(2XY^2 - X^2Y)$ for the joint probability distribution shown in Table 3.1 on page 116.

4.62 If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$, find the variance of the random variable $Z = -2X + 4Y - 3$.

4.63 Repeat Exercise 4.62 if X and Y are not independent and $\sigma_{XY} = 1$.

4.64 Suppose that X and Y are independent random variables with probability densities and

$$g(x) = \begin{cases} \frac{24}{x^4}, & x > 2, \\ 0, & \text{elsewhere,} \end{cases}$$

and

$$h(y) = \begin{cases} 2y, & 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of $Z = XY$.

4.65 Let X represent the number that occurs when a red die is tossed and Y the number that occurs when a green die is tossed. Find
(a) $E(2X + Y)$;
(b) $E(X - 2Y)$;
(c) $E(2XY)$.

4.66 Let X represent the number that occurs when a green die is tossed and Y the number that occurs when a red die is tossed. Find the variance of the random variable
(a) $3X - Y$;
(b) $X + 5Y - 5$.

4.67 If the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{2}{7}(x+2y), & 0 < x < 1,\ 1 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the expected value of $g(X,Y) = \frac{X}{Y^4} + X^2Y$.

4.68 The power P, in watts, which is dissipated in an electric circuit with a resistance of 50 ohms is known to be given by $P = I^2R$, where I is current in amperes and R is the resistance. However, I is a random variable with $\mu_I = 10$ amperes and $\sigma_I^2 = 0.02$ amperes^2. Give numerical approximations to the mean and variance of the power P.

4.69 Consider Review Exercise 3.77 on page 128. The random variables X and Y represent the number of vehicles that arrive at two separate street corners during a certain 2-minute period in the day. The joint distribution is

$$f(x,y) = \left(\frac{1}{4^{(x+y)}}\right)\left(\frac{9}{16}\right),$$

for $x = 0, 1, 2, \ldots$ and $y = 0, 1, 2, \ldots$.
(a) Give $E(X)$, $E(Y)$, $\mathrm{Var}(X)$, and $\mathrm{Var}(Y)$.
(b) Consider $Z = X + Y$, the sum of the two. Find $E(Z)$ and $\mathrm{Var}(Z)$.

4.70 Consider Review Exercise 3.64 on page 127. There are two service lines. The random variables X and Y are the proportions of time that line 1 and line 2 are in use, respectively. The joint probability density function for (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), & 0 \le x,\ y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine whether or not X and Y are independent.

(b) It is of interest to know something about the proportion of $Z = X + Y$, the sum of the two proportions. Find $E(X + Y)$. Also find $E(XY)$.

(c) Find $\text{Var}(X)$, $\text{Var}(Y)$, and $\text{Cov}(X, Y)$.

(d) Find $\text{Var}(X + Y)$.

4.71 The length of time Y, in minutes, required to generate a human reflex to tear gas has the density function

$$f(y) = \begin{cases} \frac{1}{2}e^{-y/2}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) What is the mean time to reflex?

(b) Find $E(Y^2)$ and $Var(Y)$.

4.72 A manufacturing company has developed a machine for cleaning carpet that is fuel-efficient because it delivers carpet cleaner so rapidly. Of interest is a random variable Y, the amount in gallons per minute delivered. It is known that the density function is given by

$$f(y) = \begin{cases} 1, & 7 \le y \le 8, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Sketch the density function.

(b) Give $E(Y)$, $E(Y^2)$, and $\text{Var}(Y)$.

4.73 For the situation in Exercise 4.72, compute $E(e^Y)$ using Theorem 4.1, that is, by using

$$E(e^Y) = \int_7^8 e^y f(y)\, dy.$$

Then compute $E(e^Y)$ not by using $f(y)$, but rather by using the second-order adjustment to the first-order approximation of $E(e^Y)$. Comment.

4.74 Consider again the situation of Exercise 4.72. It is required to find $\text{Var}(e^Y)$. Use Theorems 4.2 and 4.3 and define $Z = e^Y$. Thus, use the conditions of Exercise 4.73 to find

$$\text{Var}(Z) = E(Z^2) - [E(Z)]^2.$$

Then do it not by using $f(y)$, but rather by using the first-order Taylor series approximation to $\text{Var}(e^Y)$. Comment!

4.75 An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package, has a mean life of 800 hours with a standard deviation of 50 hours. At most, what percentage of the bulbs fail to last even 700 hours? Assume that the distribution is symmetric about the mean.

4.76 Suppose 120 new jobs are opening up at an automobile manufacturing plant, and 1000 applicants show up for the 70 positions. To select the best 120 from among the applicants, the company gives a test that covers mechanical skill, manual dexterity, and mathematical ability. The mean grade on this test turns out to be 60, and the scores have a standard deviation of 6. Can a person who scores 78 count on getting one of the jobs? [*Hint*: Use Chebyshev's theorem.] Assume that the distribution is symmetric about the mean.

4.77 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev's theorem, find

(a) $P(|X - 10| \ge 4)$;

(b) $P(|X - 10| < 4)$;

(c) $P(4 < X < 16)$;

(d) the value of the constant c such that $P(|X - 10| \ge c) \le 0.05$.

4.78 Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$, where X has the density function

$$f(x) = \begin{cases} 30x^2(1-x)^2, & 0 < x < 1, \\ 0, & \text{elsewhere} \end{cases}$$

and compare with the result given in Chebyshev's theorem.

Review Exercises

4.79 Prove Chebyshev's theorem.

4.80 Find the covariance of random variables X and Y having the joint probability density function

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1,\ 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

4.81 Referring to the random variables whose joint probability density function is given in Exercise 3.47 on page 125, find the average amount of kerosene left in the tank at the end of the day.

4.82 Assume the length X, in minutes, of a particular type of telephone conversation is a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{3}e^{-x/3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine the mean length $E(X)$ of this type of telephone conversation.

(b) Find the variance and standard deviation of X.

(c) Find $E[(X+5)^2]$.

4.83 Referring to the random variables whose joint density function is given in Exercise 3.41 on page 125, find the covariance between the weight of the creams and the weight of the toffees in these boxes of chocolates.

4.84 Referring to the random variables whose joint probability density function is given in Exercise 3.41 on page 125, find the expected weight for the sum of the creams and toffees if one purchased a box of these chocolates.

4.85 Suppose it is known that the life X of a particular compressor, in hours, has the density function

$$f(x) = \begin{cases} \frac{1}{900}e^{-x/900}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the mean life of the compressor.

(b) Find $E(X^2)$.

(c) Find the variance and standard deviation of the random variable X.

4.86 Referring to the random variables whose joint density function is given in Exercise 3.40 on page 125,

(a) find μ_X and μ_Y;

(b) find $E[(X+Y)/2]$.

4.87 Show that $\text{Cov}(aX, bY) = ab\,\text{Cov}(X, Y)$.

4.88 Consider the density function of Review Exercise 4.85. Demonstrate that Chebyshev's theorem holds for $k = 2$ and $k = 3$.

4.89 Consider the joint density function

$$f(x, y) = \begin{cases} \frac{16y}{x^3}, & x > 2,\ 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Compute the correlation coefficient ρ_{XY}.

4.90 Consider random variables X and Y of Exercise 4.63 on page 158. Compute ρ_{XY}.

4.91 A dealer's profit, in units of \$5000, on a new automobile is a random variable X having density function

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the variance of the dealer's profit.

(b) Demonstrate that Chebyshev's theorem holds for $k = 2$ with the density function above.

(c) What is the probability that the profit exceeds \$500?

4.92 Consider Exercise 4.10 on page 137. Can it be said that the ratings given by the two experts are independent? Explain why or why not.

4.93 A company's marketing and accounting departments have determined that if the company markets its newly developed product, the contribution of the product to the firm's profit during the next 6 months will be described by the following:

Profit Contribution	Probability
−\$5,000	0.2
\$10,000	0.5
\$30,000	0.3

What is the company's expected profit?

4.94 In a support system in the U.S. space program, a single crucial component works only 85% of the time. In order to enhance the reliability of the system, it is decided that 3 components will be installed in parallel such that the system fails only if they all fail. Assume the components act independently and that they are equivalent in the sense that all 3 of them have an 85% success rate. Consider the random variable X as the number of components out of 3 that fail.

(a) Write out a probability function for the random variable X.

(b) What is $E(X)$ (i.e., the mean number of components out of 3 that fail)?

(c) What is $\text{Var}(X)$?

(d) What is the probability that the entire system is successful?

(e) What is the probability that the system fails?

(f) If the desire is to have the system be successful with probability 0.99, are three components sufficient? If not, how many are required?

4.95 In business, it is important to plan and carry out research in order to anticipate what will occur at the end of the year. Research suggests that the profit (loss) spectrum for a certain company, with corresponding probabilities, is as follows:

Profit	Probability
$-\$15{,}000$	0.05
\$0	0.15
\$15,000	0.15
\$25,000	0.30
\$40,000	0.15
\$50,000	0.10
\$100,000	0.05
\$150,000	0.03
\$200,000	0.02

(a) What is the expected profit?

(b) Give the standard deviation of the profit.

4.96 It is known through data collection and considerable research that the amount of time in seconds that a certain employee of a company is late for work is a random variable X with density function

$$f(x) = \begin{cases} \frac{3}{(4)(50^3)}(50^2 - x^2), & -50 \le x \le 50, \\ 0, & \text{elsewhere.} \end{cases}$$

In other words, he not only is slightly late at times, but also can be early to work.

(a) Find the expected value of the time in seconds that he is late.

(b) Find $E(X^2)$.

(c) What is the standard deviation of the amount of time he is late?

4.97 A delivery truck travels from point A to point B and back using the same route each day. There are four traffic lights on the route. Let X_1 denote the number of red lights the truck encounters going from A to B and X_2 denote the number encountered on the return trip. Data collected over a long period suggest that the joint probability distribution for (X_1, X_2) is given by

	x_2				
x_1	0	1	2	3	4
0	0.01	0.01	0.03	0.07	0.01
1	0.03	0.05	0.08	0.03	0.02
2	0.03	0.11	0.15	0.01	0.01
3	0.02	0.07	0.10	0.03	0.01
4	0.01	0.06	0.03	0.01	0.01

(a) Give the marginal density of X_1.

(b) Give the marginal density of X_2.

(c) Give the conditional density distribution of X_1 given $X_2 = 3$.

(d) Give $E(X_1)$.

(e) Give $E(X_2)$.

(f) Give $E(X_1 \mid X_2 = 3)$.

(g) Give the standard deviation of X_1.

4.98 A convenience store has two separate locations where customers can be checked out as they leave. These locations each have two cash registers and two employees who check out customers. Let X be the number of cash registers being used at a particular time for location 1 and Y the number being used at the same time for location 2. The joint probability function is given by

	y		
x	0	1	2
0	0.12	0.04	0.04
1	0.08	0.19	0.05
2	0.06	0.12	0.30

(a) Give the marginal density of both X and Y as well as the probability distribution of X given $Y = 2$.

(b) Give $E(X)$ and $\text{Var}(X)$.

(c) Give $E(X \mid Y = 2)$ and $\text{Var}(X \mid Y = 2)$.

4.99 Consider a ferry that can carry both buses and cars across a waterway. Each trip costs the owner approximately \$10. The fee for cars is \$3 and the fee for buses is \$8. Let X and Y denote the number of buses and cars, respectively, carried on a given trip. The joint distribution of X and Y is given by

	x		
y	0	1	2
0	0.01	0.01	0.03
1	0.03	0.08	0.07
2	0.03	0.06	0.06
3	0.07	0.07	0.13
4	0.12	0.04	0.03
5	0.08	0.06	0.02

Compute the expected profit for the ferry trip.

4.100 As we shall illustrate in Chapter 12, statistical methods associated with linear and nonlinear models are very important. In fact, exponential functions are often used in a wide variety of scientific and engineering problems. Consider a model that is fit to a set of data involving measured values k_1 and k_2 and a certain response Y to the measurements. The model postulated is

$$\hat{Y} = e^{b_0 + b_1 k_1 + b_2 k_2},$$

where $\hat{Y}$ denotes the **estimated value of Y**, k_1 and k_2 are fixed values, and b_0, b_1, and b_2 are **estimates** of constants and hence are random variables. Assume that these random variables are independent and use the approximate formula for the variance of a nonlinear function of more than one variable. Give an expression for $\text{Var}(\hat{Y})$. Assume that the means of b_0, b_1, and b_2 are known and are β_0, β_1, and β_2, and assume that the variances of b_0, b_1, and b_2 are known and are σ_0^2, σ_1^2, and σ_2^2.

4.101 Consider Review Exercise 3.73 on page 128. It involved Y, the proportion of impurities in a batch, and the density function is given by

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the expected percentage of impurities.

(b) Find the expected value of the proportion of quality material (i.e., find $E(1-Y)$).

(c) Find the variance of the random variable $Z = 1 - Y$.

4.102 **Project**: Let X = number of hours each student in the class slept the night before. Create a discrete variable by using the following arbitrary intervals: $X < 3$, $3 \le X < 6$, $6 \le X < 9$, and $X \ge 9$.

(a) Estimate the probability distribution for X.

(b) Calculate the estimated mean and variance for X.