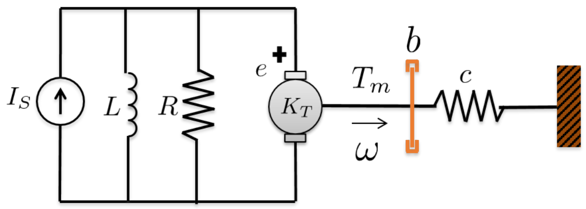
Lab task 2: Simulation of an under-causal system

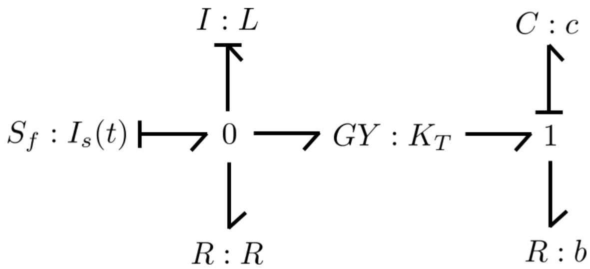
In this task, you will construct a numerical simulation of an under-causal system. In your lab A session, you have considered the simple electro-mechanical system shown in Figure 5a. All components are assumed to have linear CCRs and the motor behaves according to

$$T_m = K_T I_m$$
$$e = K_T \omega.$$

From the bond graph in Figure 5b it is clear that the system is under-causal due to the inability to fully assign causality to the graph.



(a) Electro-mechanical network.

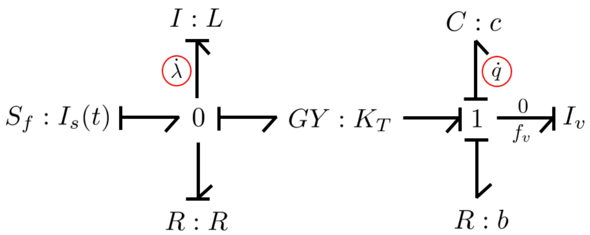


(b) Under-causal bond graph.

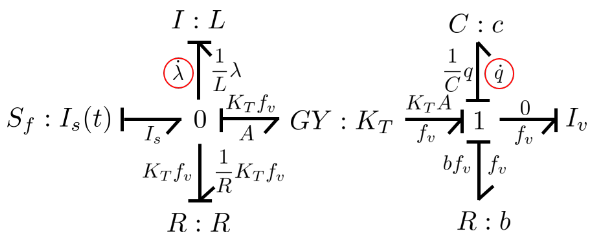
Figure 5: A simple electro-mechanical system.

In order to resolve the under-causality, a virtual inertia is added to the right-hand 1-junction as shown in Figure 6a. The states are then propagated through the graph as shown in Figure 6b. The term A is described by

$$A = I_s - \frac{1}{L}\lambda - \frac{1}{R}K_T f_v. \tag{5}$$



(a) Under-causal network with virtual inertia.



(b) Under-causal network with propagated power variables.

Figure 6: Resolution to the under-causal network.

Once all power bonds have been propagated, the dynamic equations describing the system can be resolved. First the virtual flow f_v is evaluated by considering the sum of efforts about the right-hand

1-junction. This resolves to

$$
\begin{aligned}
0 &= K_T A - \frac{1}{c} q - b f_v \\
0 &= K_T \left[I_s - \frac{1}{L}\lambda - \frac{1}{R} K_T f_v \right] - \frac{1}{c} q - b f_v \\
\left[\frac{1}{R} K_T^2 + b \right] f_v &= K_T \left[I_s - \frac{1}{L}\lambda \right] - \frac{1}{c} q \\
f_v &= \frac{R}{K_T^2 + bR} \left[K_T I_s - K_T \frac{1}{L}\lambda - \frac{1}{c} q \right].
\end{aligned}
\tag{6}
$$

Considering the bond graph in Figure 6b, the sate equations can be described in terms of the flow f_v by

$$
\begin{aligned}
\dot{\lambda} &= K_T f_v \\
\dot{q} &= f_v.
\end{aligned}
\tag{7}
$$

Tasks:

a) Create a new file in Matlab and include the commands `clear` and `clc` at the top of your script. Create a parameters structure called `params` that will store all of the system parameters. Add the following parameters to the structure.

$$
\begin{aligned}
L &= 10.5 \times 10^{-3}\ H & c &= 0.227\ [\mathrm{Nm/rad}] \\
R &= 12.8 \times 10^{3}\ \Omega & K_T &= 2.64\ [\mathrm{Nm/A}]. \\
b &= 0.042\ [\mathrm{Nms/rad}]
\end{aligned}
\tag{8}
$$

b) Create an anonymous function that describes the flow f_v, defined in (6), as a function of states and inputs.

c) For each of the dynamic CCRs shown in (7), create an anonymous function that describes the individual state equation as a function of f_v. Make use of the function describing f_v to describe each state's behaviour as a function of states and inputs. Once completed, combine all states into a single state equation of the form

$$
\dot{x} = f(x, u),
\tag{9}
$$

where $x = [\lambda, q]$ is the state of the system and $u = I_s$ is the input.

d) Using the `ode45` solver with a relative error tolerance of 1×10^{-6}, run the simulation for 0.01 seconds from the initial conditions $\lambda(0) = 0, q(0) = 1$ and input

$$
I_s(t) = 0.1 \sin(1000t).
\tag{10}
$$

Plot the current through the inductor and the spring displacement vs time. Your results should agree with Figure 7.

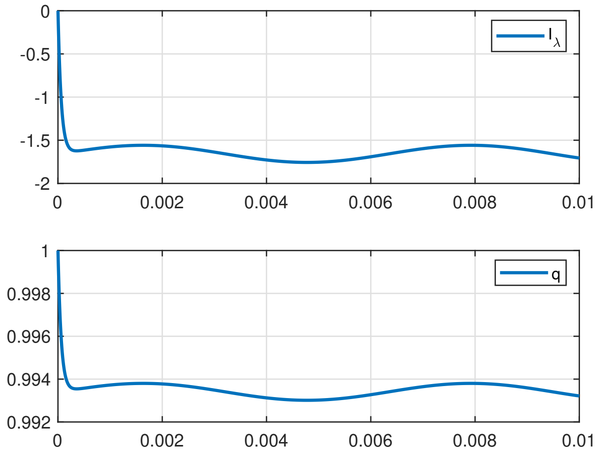


Figure 7: Example output from problem 2.e.