School of Engineering, Discipline of Mechanical Engineering and Mechatronics, pp. 1–8

THE UNIVERSITY OF NEWCASTLE AUSTRALIA

Lab 3: Simulation causality-constrained systems

MCHA3400

Semester 1 2024

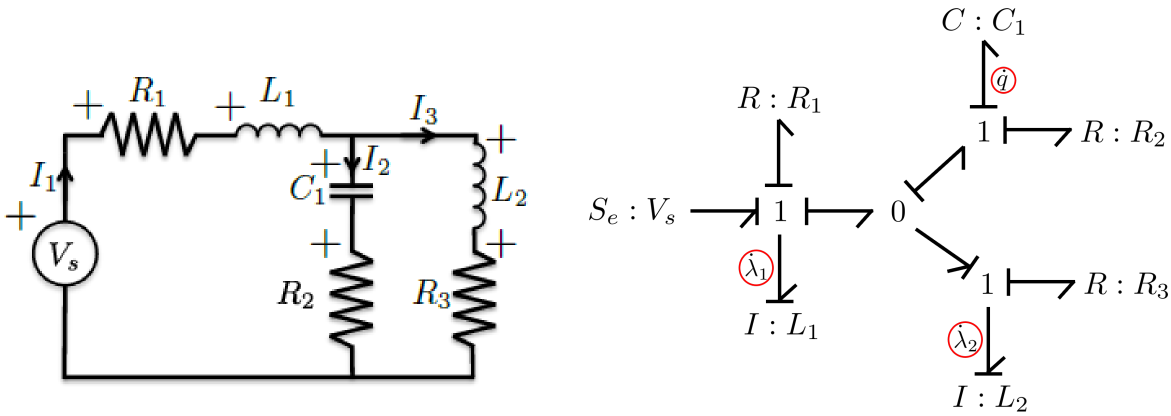
Introduction

In lab 2, we practised modelling physical systems and simulating them in Matlab using the ode45 command. In this lab, you will continue to develop these skills by constructing simulations for two separate systems. The first system is a simple electrical system which can be modelled and simulated without consideration of constraints. The second system is a simple electro-mechanical system which is under-causal.

This lab is worth 1.5% of your overall grade. Show your results to your tutor by the end of your lab session to be awarded marks. Labs will also be accepted at the beginning of your lab 4 session. After this point in time, no marks will be awarded.

Lab task 1: Simulation of an electrical network

This this task you will construct a numerical simulation of the electrical network shown in Figure 1a. In your lab A session, you will have constructed the bond graph shown in Figure 1b which fully describes the electrical network.

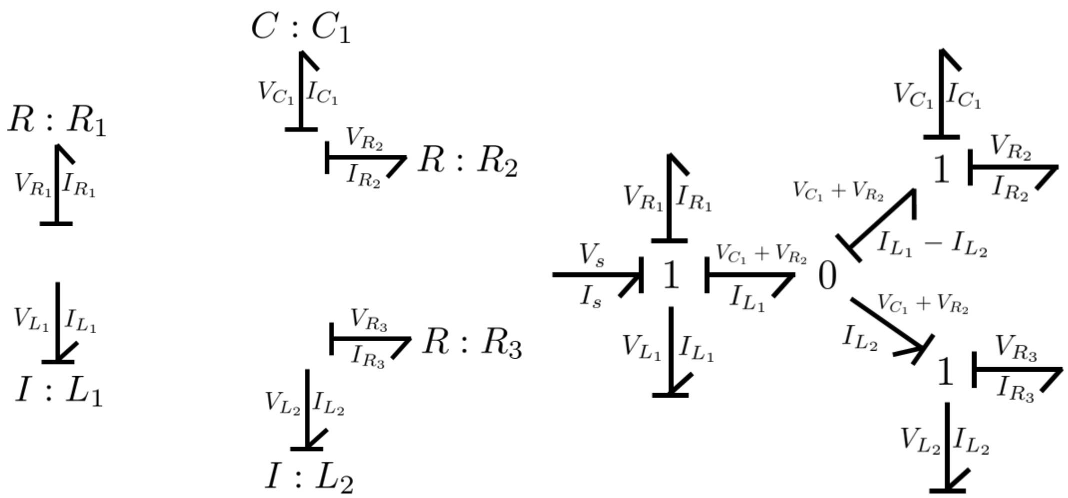


(a) An electrical network.

(b) Bond graph of the electrical network.

Figure 1: A simple electrical network.

The bond graph of the electrical network can be decomposed into the component CCRs, shown in Figure 2a, and SSRs, shown in Figure 2b.



(a) Electrical network CCRs.

(b) Electrical network SSRs.

Figure 2: CCR and SSR de-constructions of the electrical network bond graph.

The CCRs of all electrical components are assumed to be linear.

Tasks:

a) Create a new file in Matlab and include the commands `clear` and `clc` at the top of your script. Create a parameters structure called `params` that will store all of the system parameters. Add the following parameters to the structure.

$$
\begin{aligned}
R_1 &= 120\ \Omega & R_2 &= 100\ \Omega \\
L_1 &= 100 \times 10^{-6}\ H & L_2 &= 10 \times 10^{-6}\ H \\
C_1 &= 47 \times 10^{-6}\ F & R_3 &= 330\ \Omega
\end{aligned} \tag{1}
$$

b) For each of the static CCRs associated with the components shown in Figure 2a, create an anonymous function describing the component behaviour with the appropriate inputs and outputs.

c) For each of the SSRs described in Figure 2b, create an anonymous function describing the power variable interconnections with the appropriate inputs and outputs.

d) For each of the dynamic CCRs associated with the components shown in Figure 2a, create an anonymous function that describes the individual state equations of each state. Make use of the previously defined CCRs and SSRs to describe each state's behaviour as a function of states and inputs. Once completed, combine all states into a single state equation of the form

$$\dot{x} = f(x, u), \tag{2}$$

where $x = [\lambda_1, \lambda_2, q]$ is the state of the system and $u = V_s$ is the input.

e) Making use of the function defined in the previous steps, create a simulation of the electrical network using the `ode23s` solver with a relative error tolerance of 1×10^{-6}. Simulate the system for 0.5 seconds with the input

$$V_s = 10 \tag{3}$$

and initial conditions $\lambda_1(0) = 0$, $\lambda_2(0) = 0$ and $q_1(0) = 0$. Plot the current through each of the inductors and the voltage across the capacitor. Your results should agree with Figure 3.

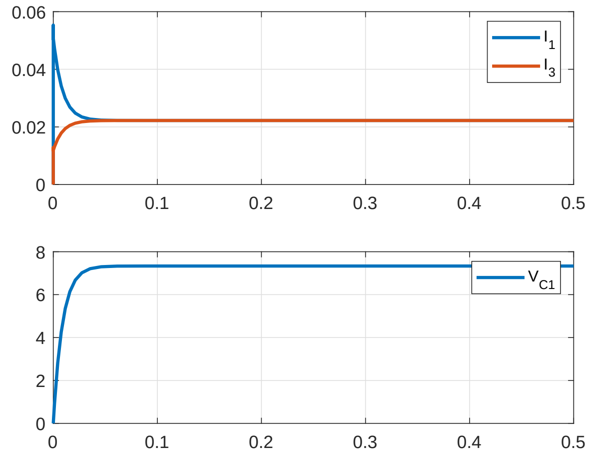


Figure 3: Example output from problem 1.c.

Hint

You are required to plot the current and voltage which are not the states of the system.

> Consider how the CCRs of the components shown in Figure 2a can be used to relate the states to the variables to be plotted.

f) Re-run the simulation for 0.5 seconds using the new input

$$V_s(t) = \max\left\{\min\left\{10sin(100t), 8\right\}, -8\right\}. \tag{4}$$

Your figure should agree with Figure 4.

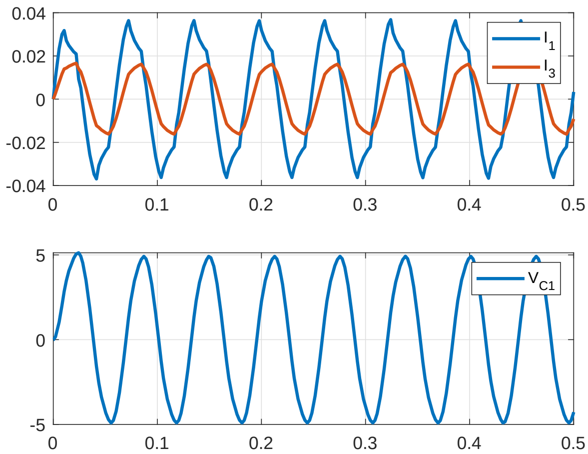


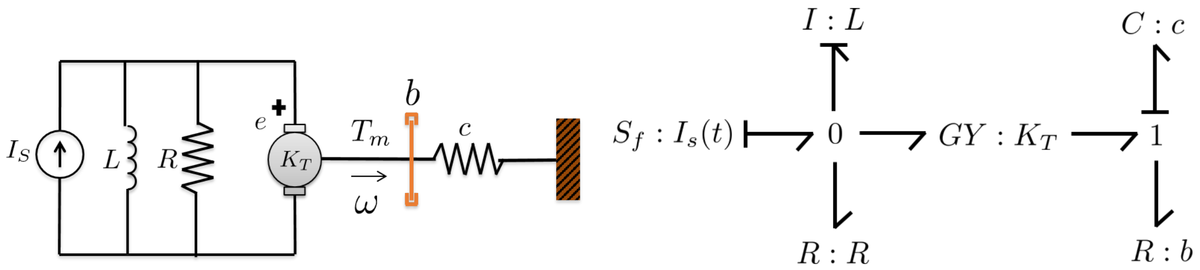
Figure 4: Example output from problem 1.e.

Lab task 2: Simulation of an under-causal system

In this task, you will construct a numerical simulation of an under-causal system. In your lab A session, you have considered the simple electro-mechanical system shown in Figure 5a. All components are assumed to have linear CCRs and the motor behaves according to

$$
\begin{aligned}
T_m &= K_T I_m \\
e &= K_T \omega.
\end{aligned}
$$

From the bond graph in Figure 5b it is clear that the system is under-causal due to the inability to fully assign causality to the graph.

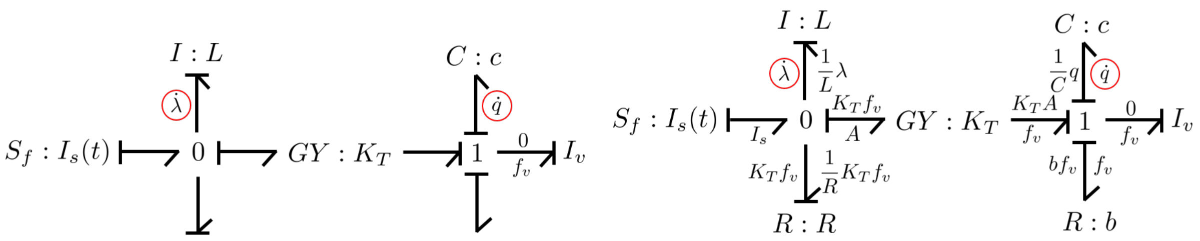


(a) Electro-mechanical network. (b) Under-causal bond graph.

Figure 5: A simple electro-mechanical system.

In order to resolve the under-causality, a virtual inertia is added to the right-hand 1-junction as shown in Figure 6a. The states are then propagated through the graph as shown in Figure 6b. The term A is described by

$$
A = I_s - \frac{1}{L}\lambda - \frac{1}{R}K_T f_v. \tag{5}
$$



(a) Under-causal network with virtual inertia. (b) Under-causal network with propagated power variables.

Figure 6: Resolution to the under-causal network.

Once all power bonds have been propagated, the dynamic equations describing the system can be resolved. First the virtual flow f_v is evaluated by considering the sum of efforts about the right-hand

1-junction. This resolves to

$$
\begin{aligned}
0 &= K_T A - \frac{1}{c}q - bf_v \\
0 &= K_T\left[I_s - \frac{1}{L}\lambda - \frac{1}{R}K_T f_v\right] - \frac{1}{c}q - bf_v \\
\left[\frac{1}{R}K_T^2 + b\right] f_v &= K_T\left[I_s - \frac{1}{L}\lambda\right] - \frac{1}{c}q \\
f_v &= \frac{R}{K_T^2 + bR}\left[K_T I_s - K_T\frac{1}{L}\lambda - \frac{1}{c}q\right].
\end{aligned} \tag{6}
$$

Considering the bond graph in Figure 6b, the sate equations can be described in terms of the flow f_v by

$$
\begin{aligned}
\dot{\lambda} &= K_T f_v \\
\dot{q} &= f_v.
\end{aligned} \tag{7}
$$

Tasks:

a) Create a new file in Matlab and include the commands `clear` and `clc` at the top of your script. Create a parameters structure called `params` that will store all of the system parameters. Add the following parameters to the structure.

$$
\begin{aligned}
L &= 10.5 \times 10^{-3}\ H & c &= 0.227\ [\mathrm{Nm/rad}] \\
R &= 12.8 \times 10^{3}\ \Omega & K_T &= 2.64\ [\mathrm{Nm/A}]. \\
b &= 0.042\ [\mathrm{Nms/rad}]
\end{aligned} \tag{8}
$$

b) Create an anonymous function that describes the flow f_v, defined in (6), as a function of states and inputs.

c) For each of the dynamic CCRs shown in (7), create an anonymous function that describes the individual state equation as a function of f_v. Make use of the function describing f_v to describe each state's behaviour as a function of states and inputs. Once completed, combine all states into a single state equation of the form

$$\dot{x} = f(x, u), \tag{9}$$

where $x = [\lambda, q]$ is the state of the system and $u = I_s$ is the input.

d) Using the `ode45` solver with a relative error tolerance of 1×10^{-6}, run the simulation for 0.01 seconds from the initial conditions $\lambda(0) = 0, q(0) = 1$ and input

$$I_s(t) = 0.1\sin(1000t). \tag{10}$$

Plot the current through the inductor and the spring displacement vs time. Your results should agree with Figure 7.

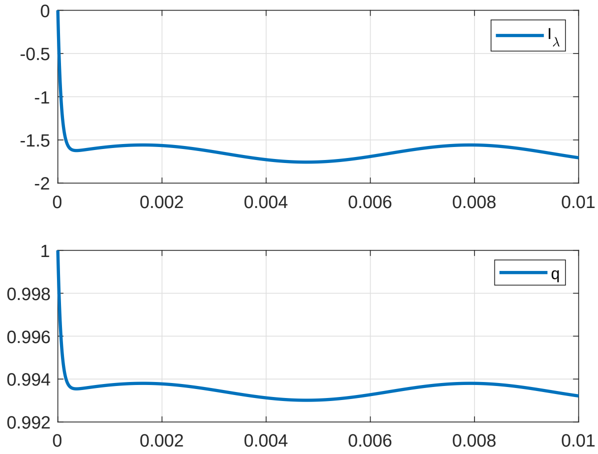


Figure 7: Example output from problem 2.e.