# Analysing Spatial Data in R
# Worked examples: Small Area Estimation

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## Small Area Estimation

- Small Area Estimation provides a general framework for investigating the spatial distribution of variables at different administrative levels
- Disease Mapping is a particular case of Small Area Estimation
- Very important for governemt agencies and statistical bureaus
- Lehtonen and Pahkinen describe different direct and regression-based estimators and provide trainning materials on-line
- Rao (2003) provides a complete summary of different methods for SAE.

## How do we get the data?

### Statistical offices

- Different types of small area data
- Public release as yearly reports, books, atlas, etc.
- Aggregated data (usualy)
- Individual data might be available (on request)

### Survey data

- Provide accurate information at individual level (person, houshold, ...)
- Difficult to obtain from public sources
- *Ad-hoc* surveys can be carried and linked to aggregated public data
- Some way of combining individual and aggregated data

## Overview of R packages for SAE

- `sampling`: Sampling methods for complex surveys
- `survey`: Analysis of data from complex surveys
- `glm`: Generalised Linear Models
- `nlme`: Mixed-effect models
- `SAE`: Some EBLUP estimators for Small Area Estimation
- `spsurvey`: Spatial survey design and analysis

## The MSU284 Population

The MSU284 Population (Särndal et al., 2003) describes the 284 municipalities of Sweden. It is included in package `sampling`.

- LABEL. Identifier.
- P85. Population in 1985
- RMT85. Revenues from the 1985 municipal taxation
- ME84. Number of Municipal Employees in 1984
- REG. Geographic region indicator (8 regions)
- CL. *Cluster* indicator (50 clusters)

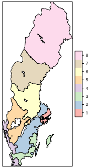
```
> library(sampling)
> data(MU284)
> MU284 <- MU284[order(MU284$REG), ]
> MU284$LABEL <- 1:284
> summary(MU284)
```

## Basics of Survey Design

- Surveys are used to obtain representative data on all the population in the study region
- Ideally, the survey data would contain a small sample for each area
- In practice, surveys are clustered to reduce costs (for example, *two-stage* sampling)
- Define *sampling frame*
- Example: General Houshold Survey 2000 (ONS)
  - Primary Sampling Units (PSUs): Postcode
  - Secondary Sampling Units (SSUs): Household
- Outcome is $\{(x_{ij}, y_{ij}), j \in s_i; i = 1, \ldots, K\}$
  - $y_{ij}$ target variable
  - $x_{ij}$ covariates

## Regions in Sweden

- Municipalities in Sweden can be grouped into 8 regions
- We will treat the municipalities as the *units*
- To estimate the regional mean we will sample from the municipalities




## Survey sampling with R

### Simple Random Sampling Without Replacement

- Sample is made of 32 municipalities (∼11% sample)
- Equal probabilities for all municipalities

```
> N <- 284
> n <- 32
> nreg <- length(unique(MU284$REG))
> set.seed(1)
> smp <- srswor(n, N)
> dsmp <- MU284[smp == 1, ]
> table(dsmp$REG)
```

```
1 2 3 4 5 6 7 8
2 5 6 3 7 3 2 4
```

## Survey sampling with R

### Stratified SRS Without Replacement

- Sample is made of 32 municipalities ($\sim$11% sample)
- 4 municipalities sampled per region
- Equal probabilities for all municipalities **within** strata

```
> set.seed(1)
> smpcl <- mstage(MU284, stage = list("cluster", "cluster"),
+     varnames = list("REG", "LABEL"), size = list(8, rep(4,
+         8)), method = "srswor")
> dmpcl <- MU284[smpcl[[2]]$LABEL, ]
> table(dsmpcl$REG)

1 2 3 4 5 6 7 8
4 4 4 4 4 4 4 4
```

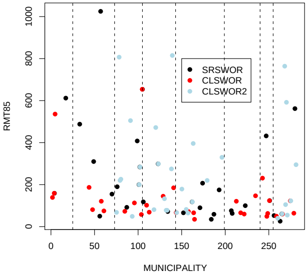
## Survey sampling with R

### Stratified SRS Without Replacement (Two-Stage Sampling)

- Sample is made of 32 municipalities ($\sim$11% sample)
- 8 municipalities sampled per region
- Equal probabilities for all municipalities **within** strata
- Some regions do not contribute to the survey sample

```
> set.seed(1)
> smpcl2 <- mstage(MU284, stage = list("cluster", "cluster"),
+     varnames = list("REG", "LABEL"), size = list(4, rep(8,
+         8)), method = "srswor")
> dsmpcl2 <- MU284[smpcl2[[2]]$LABEL, ]
> table(dsmpcl2$REG)

3 4 5 8
8 8 8 8
```

## Survey sampling with R




## Small Area Estimators

### Sample-based Estimators

Based on the survey data

- *Direct Estimator*
- *GREG Estimator*

### Indirect Estimators

Based on survey data and some appropriate model

- (Generalised) *Linear Regression*
- Mixed-Effects Models
- EBLUP Estimation
- Models with Spatially Correlated Effects

## Direct Estimation

- Direct estimators rely on the survey sample to provide small area estimates
- Not appropriate if there are out-of-sample areas

Horvitz-Thomson estimator:

$$\hat{Y}_{direct} = \sum_{i \in s} \frac{1}{\pi_i} y_i \qquad \hat{\bar{Y}}_{direct} = \sum_{i \in s} \frac{\frac{1}{\pi_i} y_i}{\sum_{i \in s} \frac{1}{\pi_i}}$$

For SRS without replacement: $\pi_i = \frac{n}{N}$

```
> library(survey)
> RMT85 <- sum(MU284$RMT85)
> RMT85REG <- as.numeric(by(MU284$RMT85, MU284$REG, sum))
```

## Direct Estimation

- Direct estimators rely on the survey sample to provide small area estimates
- Not appropriate if there are out-of-sample areas

$$Y_{direct} = \sum_{i \in s} \frac{1}{\pi_i} y_i$$

For SRS without replacement: $\pi_{ij} = \frac{n_i}{N_i}$

```
> library(survey)
> svy <- svydesign(~1, data = dsmp, fpc = rep(284, n))
> dest <- svytotal(~RMT85, svy)
```

## Direct Estimation

A domain refers to a subpopulation of the area of interest
In the example, we may estimate the revenues for each region

$$Y_{direct,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij}$$

```
> fpc <- lreg[dsmpcl$REG]
> svycl <- svydesign(id = ~1, strata = ~REG, data = dsmpcl,
+     fpc = fpc)
> destcl <- svytotal(~RMT85, svycl)
```

## Direct Estimation

A domain refers to a subpopulation of the area of interest
In the example, we may estimate the revenues for each region

$$Y_{direct,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij}$$

```
> fpc2 <- lreg[dsmpcl2$REG]
> svycl2 <- svydesign(id = ~1, strata = ~REG, data = dsmpcl2,
+     fpc = fpc2)
> destcl2 <- svytotal(~RMT85, svycl2)
```

## Direct Estimation of Domains

A domain refers to a subpopulation of the area of interest
In the example, we may estimate the revenues for each region

$$Y_{direct,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij}$$

```
> svyby(~RMT85, ~REG, svy, svytotal)

  REG statistics.RMT85 se.RMT85
1   1         6842.625 5244.545
2   2        17998.500 9620.438
3   3        16223.500 6874.105
4   4         4339.875 2699.869
5   5         6656.250 2505.059
6   6         2121.125 1138.299
7   7         4934.500 3725.099
8   8         6230.250 4711.205
```

## Direct Estimation of Domains

A domain refers to a subpopulation of the area of interest
In the example, we may estimate the revenues for each region

$$Y_{direct,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij}$$

```
> svyby(~RMT85, ~REG, svycl, svytotal)

  REG statistics.RMT85    se.RMT85
1   1         44356.25 34347.1708
2   2          5568.00  1184.5134
3   3          7184.00  4299.5057
4   4          4759.50   908.4262
5   5          3360.00   455.2333
6   6          4038.50   825.9968
7   7          1751.25   532.0153
8   8          2153.25   444.6669
```

## Direct Estimation of Domains

A domain refers to a subpopulation of the area of interest
In the example, we may estimate the revenues for each region

$$Y_{direct,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij}$$

```
> svyby(~RMT85, ~REG, svycl2, svytotal)

  REG statistics.RMT85 se.RMT85
3   3         9436.000 2450.388
4   4        10597.250 3080.939
5   5        10199.000 2299.526
8   8         7376.875 2418.904
```

## Generalised Regression Estimator

### Definition

- Model-assisted estimator
- Relies on survey design and (linear) regression
- It can be expressed as a direct estimator plus some correction term based on additional information (covariates)

$$\hat{Y}_{GREG} = \sum_{j \in s} \frac{1}{\pi_j} y_j + \sum_k \beta_k \left( \sum_{p=1}^{N} x_p - \sum_{j \in s} \frac{1}{\pi_j} x_j \right)$$

$$\hat{Y}_{GREG,i} = \sum_{j \in s_i} \frac{1}{\pi_{ij}} y_{ij} + \sum_k \beta_k \left( \sum_{p=1}^{N_i} x_p - \sum_{j \in s_i} \frac{1}{\pi_{ij}} x_{ij} \right)$$

Coefficients $\beta_k$ are estimated using weigthed linear regression.

## GREG Estimation with R

```
> pop.totals = c("(Intercept)" = N, ME84 = sum(MU284$ME84))
> svygreg <- calibrate(svy, ~ME84, calfun = "linear", population = pop.
> svytotal(~RMT85, svygreg)

      total     SE
RMT85 67473 1217.2

> svygregcl <- calibrate(svycl, ~ME84, calfun = "linear",
+     population = pop.totals)
> svytotal(~RMT85, svygregcl)

      total     SE
RMT85 68170 873.04

> svygregcl2 <- calibrate(svycl2, ~ME84, calfun = "linear",
+     population = pop.totals)
> svytotal(~RMT85, svygregcl2)

      total     SE
RMT85 68387 914.81
```

## Linear Regression

- `lm` assumes that the sample comes from an *infinite* population
- `svyglm` accounts for the survey design and provides a correction for *finite population* in the estimation of the standard errors

We are trying to model the total tax revenues according to the number of municipal employees

```
> plot(MU284$ME84, MU284$RMT85)
> plot(MU284$ME84, MU284$RMT85, xlim = c(0, 10000))
> survlm <- lm(RMT85 ~ ME84, dsmp)
> survglm <- svyglm(RMT85 ~ ME84, svy)
> summary(survlm)
> summary(survglm)
```

## Mixed-effects models and EBLUP estimators

- Mixed-effects models can be used to improve estimation
- Random Effects measure variation due to unmesared factors
- Spatial patterns can be accounted for by means of random effects

### Fay-Herriot Area Level Model

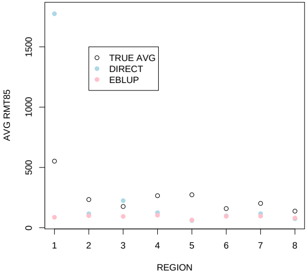
$$\begin{array}{lcllcl} \hat{\bar{Y}}_i & = & \mu_i + e_i & e_i & \sim & N(0, \hat{\sigma}_i^2) \\ \mu_i & = & \beta X_i + u_i & u_i & \sim & N(0, \sigma_u^2) \end{array}$$

- $\hat{\bar{Y}}_i$ is often a direct estimator
- $\hat{\sigma}_i^2$ is the variance of the direct estimator
- $\hat{\mu}_i$ is a new (improved) small area estimator
- $\hat{u}_i$ are estimated using EBLUP estimators

## EBLUP estimators with R

```
> library(SAE)
> destmean <- svyby(~RMT85, ~REG, svycl, svymean)
> Y <- matrix(destmean[, 2], ncol = 1)
> sigma2i <- matrix(destmean[, 3], ncol = 1)^2
> X <- matrix(as.numeric(by(MU284$ME84, MU284$REG, mean)),
+     ncol = 1)
> ebluparea <- EBLUP.area(Y, cbind(1, X), sigma2i, 8)
> print(sum((destmean[, 2] - (RMT85REG/lreg))^2))

[1] 1590108

> print(sum((ebluparea$EBLUP - (RMT85REG/lreg))^2))

[1] 329263.7

> print(ebluparea$randeff[, 1])

[1]   0.3319200   9.6791711   2.6907938  13.8812442 -25.4537694
[6]   3.4234902   5.9494749 -10.5023248
```

## EBLUP estimators with R




## Spatial EBLUP estimators

- The random effects can be used to model spatial dependence
- There are different approaches to model spatial dependence
- Petrucci and Salvati (2006) propose a Spatial EBLUP estimator based in a SAR specification

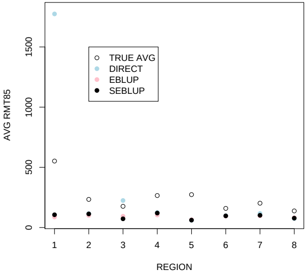
$$\begin{array}{rclcrcl} \hat{\bar{Y}}_i & = & \mu_i + e_i & & e_i & \sim & N(0, \hat{\sigma}_i^2) \\ \mu_i & = & \beta X_i + v_i & & v & \sim & N(0, \sigma_u^2[(I-\rho W)(I-\rho W^T)]^{-1}) \end{array}$$

- $\rho$ measures spatial correlation
- $W$ is a *proximity* matrix which can be defined in different ways

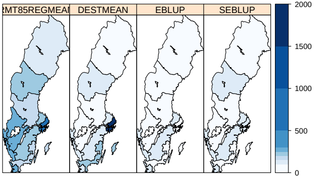
## Spatial EBLUP estimators with R

```
> moran.test(Y, nb2listw(nb), alternative = "two.sided")

Moran's I test under randomisation

data:  Y
weights: nb2listw(nb)

Moran I statistic standard deviate = 1.1501, p-value = 0.2501
alternative hypothesis: two.sided
sample estimates:
Moran I statistic       Expectation          Variance
      -0.02635814       -0.14285714        0.01026137

> sebluparea <- SEBLUP.area(Y, matrix(cbind(1, X), ncol = 2),
+     sigma2i, 8, W, init = c(0, ebluparea$sigma2u))
> print(paste("Rho:", sebluparea$rho, "s.d.", sqrt(sebluparea$varsigmar
+     2]), sep = " ", collapse = " "))

[1] "Rho: -0.402461548158343 s.d. 0.120181628230132"

> print(sebluparea$randeff[, 1])

[1]  -9.097686  18.450828 -19.126460  23.199879 -35.424211   6.951748
[7]   8.234322 -11.566655
```

## EBLUP estimators with R



## Mapping the results




## References and other sources


- Additional documentation for `survey` package: `http://faculty.washington.edu/tlumley/survey/`
- Practical Exemplars and Survey Analysis (ESRC/NCRM): `http://www.napier.ac.uk/depts/fhls/peas/`
- A. Petrucci and N. Salvati (2006). Small Area Estimation for Spatial Correlation in Watershed Erosion Assessment. *Journal of Agricultural, Biological & Environmental Statistics* **11** (2): 169-182.
- J.N.K. Rao (2003). *Small Area Estimation*. John Wiley & Sons, Inc.
- C.E. Särndall, B. Swensson and J. Wretman (2003). *Model Assisted Survey Sampling*. Springer-Verlag.


## Assessment of the Estimators

$$AEMSE = \frac{1}{K}\sum_{i=1}^{K}(\hat{Y}_i - Y_i)^2$$

### Estimation of the National Mean

| Estimator | sqrt(AEMSE) |
|---|---|
| Direct (SRS) | 4258.4 |
| Direct (CL) | 3565.8 |
| Direct (CL2) | 31996 |

### Estimation in Domains

| Estimator | sqrt(AEMSE) |
|---|---|
| Direct (CL) | 157.62 |
| EBLUP | 71.727 |
| SEBLUP | 69.355 |