Analysing Spatial Data in R: Worked examples: (Bayesian) disease mapping II

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Bayesian Disease mapping

- Bayesian Estimation in Disease Mapping has been one of the leading topics in spatial statistics in the last 20 years
- Bayesian Hierarchical Models can be used to model complex data structures
- The Bayesian approach offers an easy approach to the estimation of complex models via Markov Chain Monte Carlo
- Spatial analysis of routinoulsy collected health data is standard practise nowadays
- Spatio-temporal models can be used
- Waller & Gotway (2004) and Banerjee et al. (2003) account for a comprehensive summary on spatial models

Bayesian Inference

- Bayesian Inference is based on estimating the probability density of the parameters θ in the model *after* observing the data, i.e., their posterior distributions: $p(\theta|y)$
- $p(\theta|y)$ is usually difficult to derive:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{\int_\theta p(y|\theta)p(\theta)} \propto p(y|\theta)p(\theta)$$

- $p(y|\theta)$ is the likelihood of the model, which reflects the relationship between the data and the parameters
- $p(\theta)$ is the *prior* distribution of the parameters, which reflects the initial information on the parameters
- Usually, $p(\theta|y)$ is computed by simulation using *Markov Chain Monte Carlo* techniques
- WinBUGS in a generic software to fit a wide range of models. It uses the Gibbs sampler for that.

Benefits of Bayesian Inference

- Suitable framework to deal with a large number of problems
- Priors can be used to account for initial information (for example, spatial dependence)
- If no prior information is available, vague (or non-informative) priors can be used so that the posterior distribution *will only depend* on the data and the model.
- Multilevel models can be used: Bayesian Hierarchical Models
- Complex effects, such as spatial and/or temporal dependence, can be modeled easily
- When the posterior distribution is not in a closed form, different simulation techniques can be used to approximate them.
- Missing values are treated similarly as the parameters in the model

Markov Chain Monte Carlo/Gibbs sampler

MCMC aims at simulating a series of values for the parameters in the model, so that, in the end, these values will be draws from the posterior distribution.

- Assign initial values to every parameter in the model (and missing values)
- At every step, Gibbs sampler simulates from the full conditional distribution:

$$p(\theta_i|\theta_{-i}, y)$$

- After a *burn in* period, the simulated values are draws from the posterior $p(\theta|y)$
- Convergence of the simulated values should be assessed

WinBUGS

- BUGS stands for *Bayesian inference Using Gibbs Sampler*
- Developed at the MRC and Imperial College London
- Provides a generic language to Bayesian Hierarchical models
- Models can be specified graphically as well
- Several utilities to assess the convergence of the chain and display results
- GeoBUGS is an extension to deal with spatial models and maps
- PkBUGS is another extension to deal with Pharmacokinetics models
- A developer interface has been included so that the user can extend the range of functions available
- OpenBUGS is the *open source* version of WinBUGS

Calling WinBUGS from `R`

- Packages `R2WinBUGS` and `BRUGS` can call WinBUGS and OpenBUGS from `R`
- `R2WinBUGS` calls WinBUGS using the scripting language and then reads the output log file
- `BRUGS` is an interface to the actual OpenBUGS (NOT WinBUGS) routines
- `R2WinBUGS` can run on several platforms (Windows, Linux/Unix, Mac)
- Other alternatives to call WinBUGS externally in different ways are available at `http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/remote14.shtml`

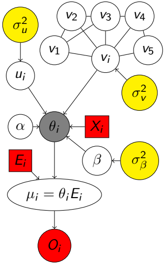
Leukemia Cancer Data revisited

We need...

- Model specification (using the BUGS language)
- Mortality Data (in a `list`)
- Spatial data describing the neighbourhood structure, in a specific format
- Initial values of the parameters
- Optionally, we may want to export the map information to be used within WinBUGS

Bayesian Spatial Modelling

$$
\begin{aligned}
O_i &\sim Poisson(\mu_i)\\
\mu_i &= \theta_i E_i\\
\\
\log(\theta_i) &= \alpha + \beta X_i + u_i + v_i\\
u_i &\sim Normal(0, \sigma_u^2)\\
v_i | v_{-i} &\sim Normal(\textstyle\sum_{j\sim i} v_j/n_i, \sigma_v^2/n_i)\\
\\
f(\alpha) &\propto 1\\
f(\beta) &\propto 1\\
\sigma_u^2 &\sim Gamma^{-1}(.001, .001)\\
\sigma_v^2 &\sim Gamma^{-1}(.001, .001)
\end{aligned}
$$



J. Besag, J. York, A. Mollie (1991). Bayesian image restoration, with two applications in spatial statistics (with discussion). *Annals of the Institute of Statistical Mathematics* **43(1)**, 1-59

Model specification using the BUGS language

```
model{
        for(i in 1:N)
        {
                O[i] ~ dpois(mu[i])
                mu[i]<-theta[i] * E[i]
            log(theta[i]) <- alpha + beta[1]*PCTAGE65P[i]+
                beta[2]*PCTOWNHOME[i]+beta[3]*AVGIDIST[i]+u[i] + v[i]

                u[i] ~ dnorm(0, precu)

                SMR[i]<- O[i] / E[i]
                prob[i]<-step(theta[i]-1)
        }
        v[1:N]~car.normal(adj[], weights[], num[], precv)

        alpha~dflat()
        for(i in 1:3) {beta[i] ~dflat()}
        precu~dgamma(0.001, 0.001)
        precv~dgamma(0.001, 0.001)

        sigmau<-1/precu
        sigmav<-1/precv
}
```

Preparing data...

1.- Read maps

```
> library(maptools)
> nymap <- readShapePoly("NY8_utm18")
```

2.- Create list of observed, expected

```
> nymap$EXP <- nymap$POP8 * sum(nymap$Cases)/sum(nymap$POP8)
```

3.- Create adjacency matrix

```
> library(spdep)
> nynb <- poly2nb(nymap)
```

4.- Create weights

```
> nyWBweights <- nb2WB(nynb)
```

```
> d <- c(list(O = nymap$Cases, E = nymap$EXP), N = 281,
+     list(PCTAGE65P = nymap$PCTAGE65P, PCTOWNHOME = nymap$PCTOWNHOME,
+         AVGIDIST = nymap$AVGIDIST))
> inits1 <- list(alpha = 1, beta = c(0, 0, 0), u = rep(0,
+     281), v = rep(0, 281), precu = 1, precv = 1)
> inits2 <- list(alpha = 10, beta = c(1, 1, 1), u = rep(1,
+     281), v = rep(1, 281), precu = 0.1, precv = 0.1)
```

Calling WinBUGS using `R2WinBUGS`

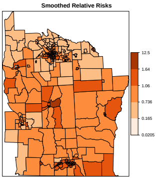
5.- Call WinBUGS

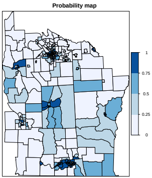
```
> library(R2WinBUGS)
> mfile <- paste(getwd(), "/model.txt", sep = "", collapse = "")
> tdir <- paste(getwd(), "/NYoutput", sep = "", collapse = "")
> dir.create(tdir)
> res <- bugs(data = c(d, nyWBweights), inits = list(inits1,
+     inits2), parameters.to.save = c("u", "v", "theta",
+     "prob", "sigmau", "sigmav"), model.file = mfile,
+     working.directory = tdir, n.thin = 3, n.chains = 2,
+     n.iter = 6000, n.burnin = 3000)
```

6.- Add results to map object

```
> nymap$prob <- res$mean$prob
> nymap$theta <- res$mean$theta
> nymap$u <- res$mean$u
> nymap$v <- res$mean$v
> logfile <- paste(getwd(), "/NYoutput/log.txt", sep = "",
+     collapse = "")
> reslog <- bugs.log(file = logfile)
```

Mapping the results





Exporting the data to work directly with WinBUGS

1. Export the maps with spdep

   > sp2WB(map = nymap, file = "NY_WB.txt")

2. Import map with WB first, and "reboot"
3. Use `bugs.data` (from R2WinBUGS) to create the files with data and initial values

   > bugs.data(d)
   > file.rename("data.txt", "dataNY.txt")
   > bugs.data(nyWBweights)
   > file.rename("data.txt", "data-spatialNY.txt")
   > bugs.data(inits1)
   > file.rename("data.txt", "inits1NY.txt")
   > bugs.data(inits2)
   > file.rename("data.txt", "inits2NY.txt")

Running WinBUGS directly

1. Open all needed files in WinBUGS
2. *Check* the model syntax
3. *Load* data (health and spatial)
4. *Compile* the model
5. *Load* inital values
6. *Run* the model (burn in period)
7. *Monitor* parameters of interest and DIC
8. *Rerun* the model
9. *Assess* convergence of the simulations
10. *Show* summary statistics of the parameters of the model
11. *Display* results on a map

Further references

- S. Banerjee, B.P. Carlin and A.E. Gelfand (2003). *Hierarchical Modeling and Analysis for Spatial Data*. Chapman & Hall.
- A.B. Lawson, W.J. Browne and C.L. Vidal Rodeiro (2003). *Disease Mapping with WinBUGS and MLwiN*. Wiley & Sons.
- OpenBUGS: `http://mathstat.helsinki.fi/openbugs/`
- R programming language: `http://www.r-project.org`
- D.J. Spiegelhalter, N.G. Best, B.P. Carlin and A. Van der Linde (2002). Bayesian Measures of Model Complexity and Fit (with Discussion), *Journal of the Royal Statistical Society, Series B* **64(4)**, 583-616.
- L.A. Waller and C.A. Gotway (2004). *Applied Spatial Statistics for Public Health Data*. Wiley & Sons.