

Lecture 13: Joins Part II

Today's Lecture

1. Sort-Merge Join (SMJ)
2. Hash Join (HJ)
3. The Cage Match: SMJ vs. HJ

1. Sort-Merge Join (SMJ)



What you will learn about in this section

1. Sort-Merge Join
2. “Backup” & Total Cost
3. Optimizations
4. ACTIVITY: Sequential Flooding

Sort Merge Join (SMJ): Basic Procedure

To compute $R \bowtie S$ on A :

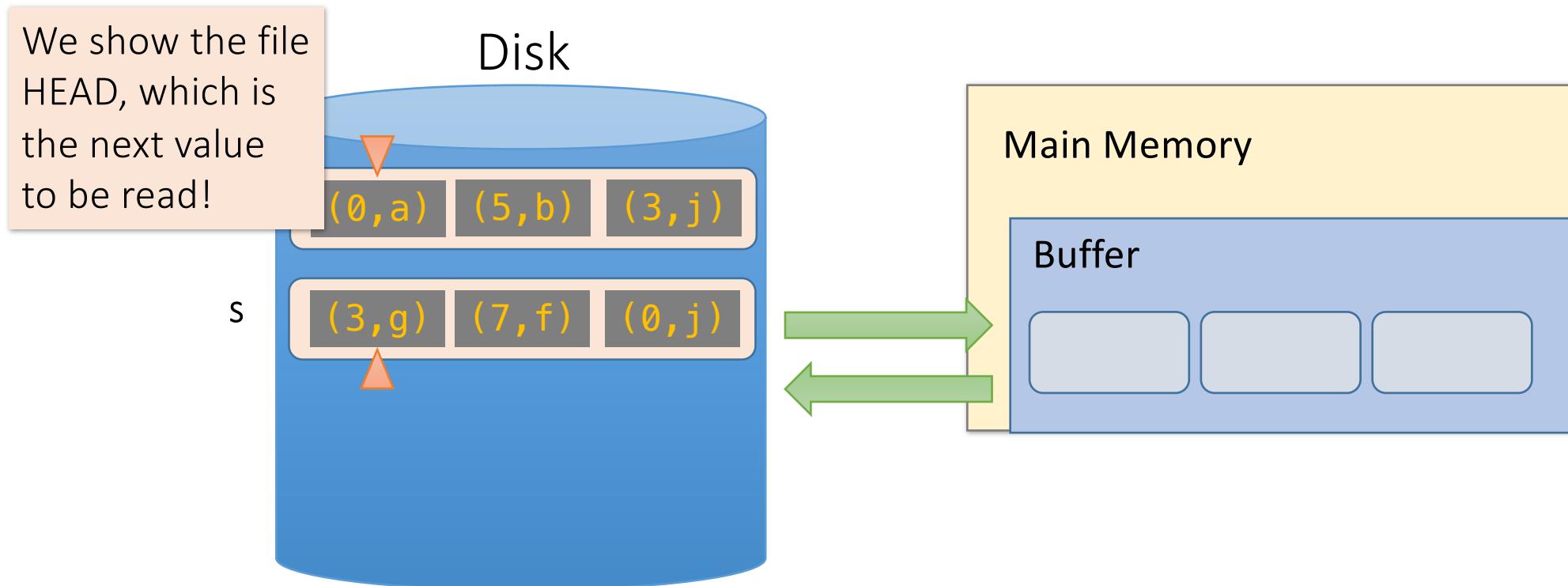
1. Sort R, S on A using ***external merge sort***
2. ***Scan*** sorted files and “merge”
3. [May need to “backup”- see next subsection]

Note that we are only considering equality join conditions here

Note that if R, S are already sorted on A ,
SMJ will be awesome!

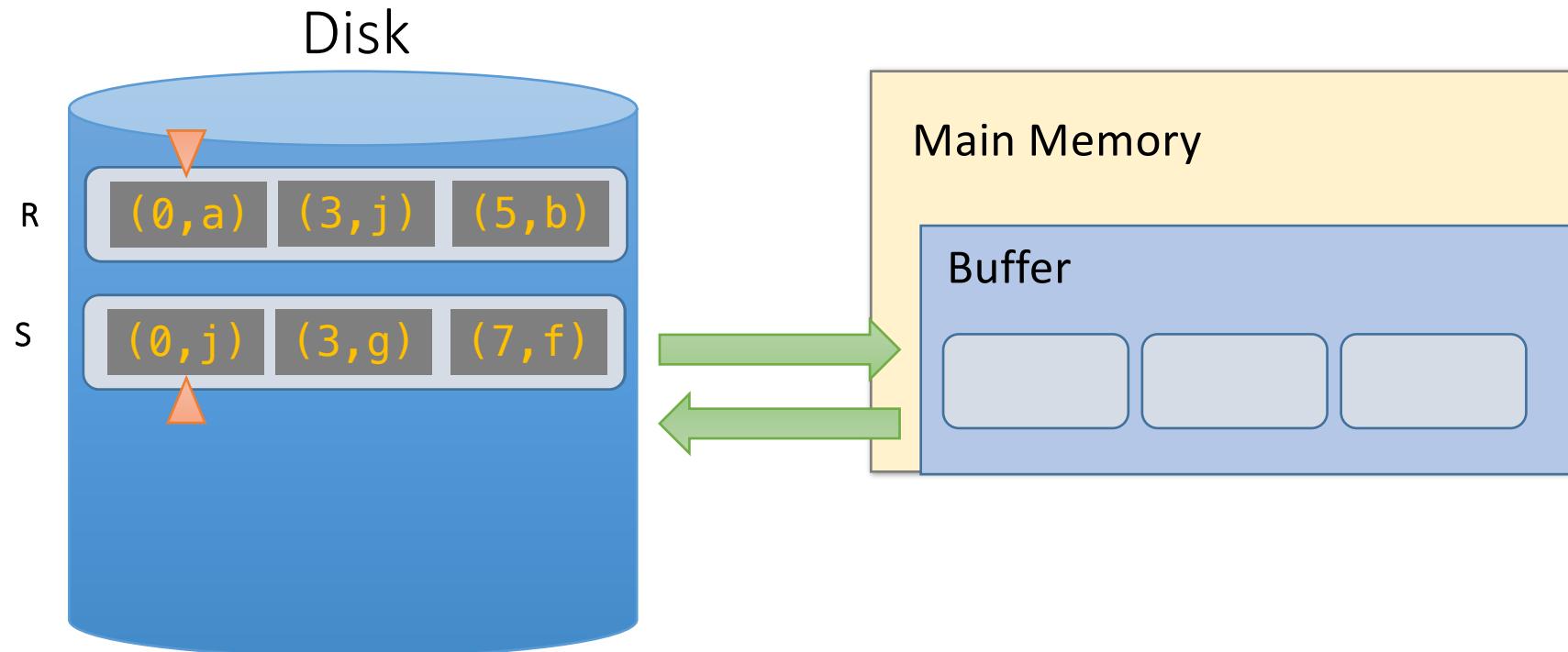
SMJ Example: $R \bowtie S$ on A with 3 page buffer

- For simplicity: Let each page be ***one tuple***, and let the first value be A



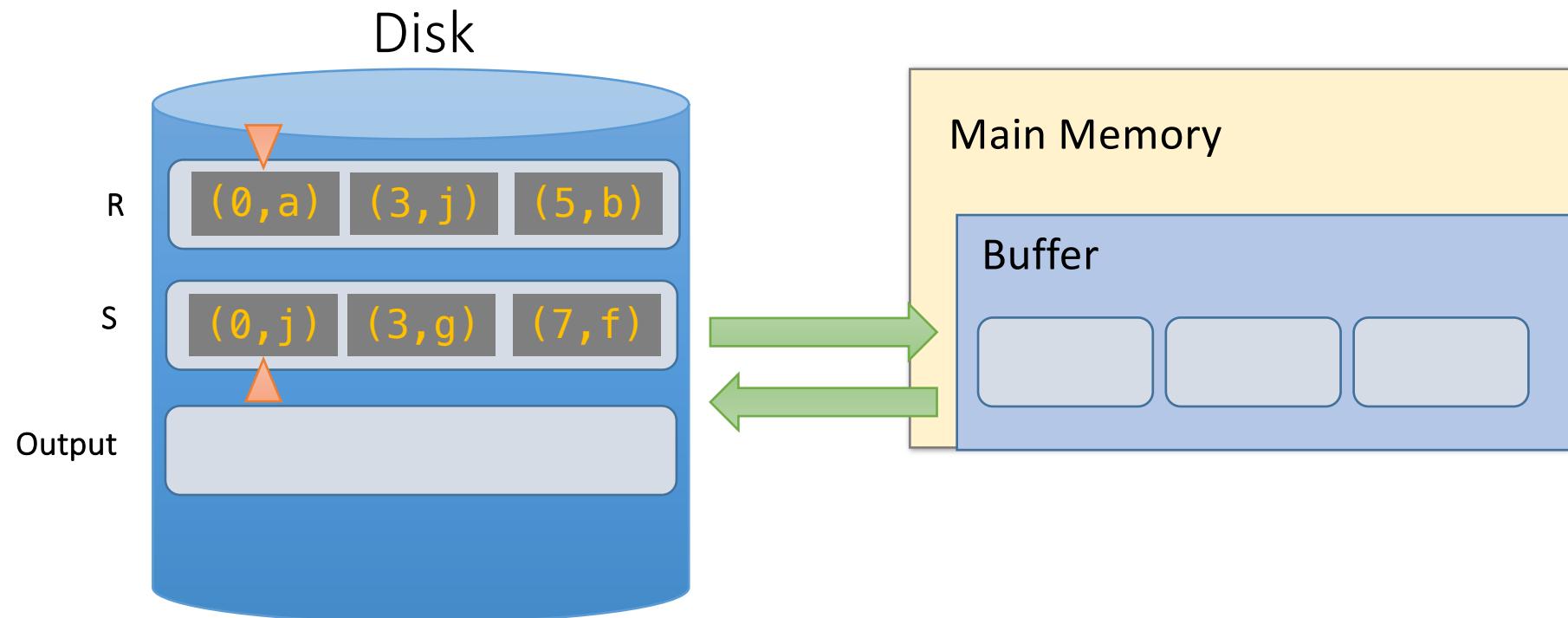
SMJ Example: $R \bowtie S$ on A with 3 page buffer

1. Sort the relations R, S on the join key (first value)



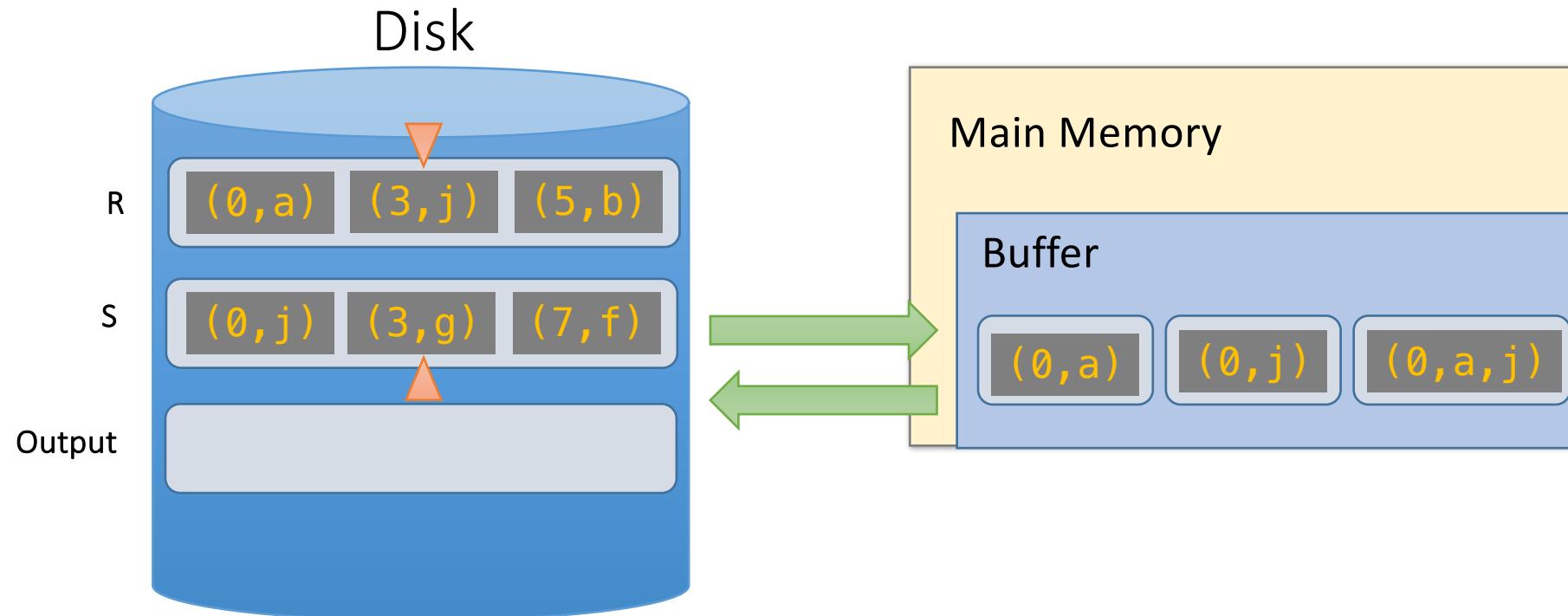
SMJ Example: $R \bowtie S$ on A with 3 page buffer

2. Scan and “merge” on join key!



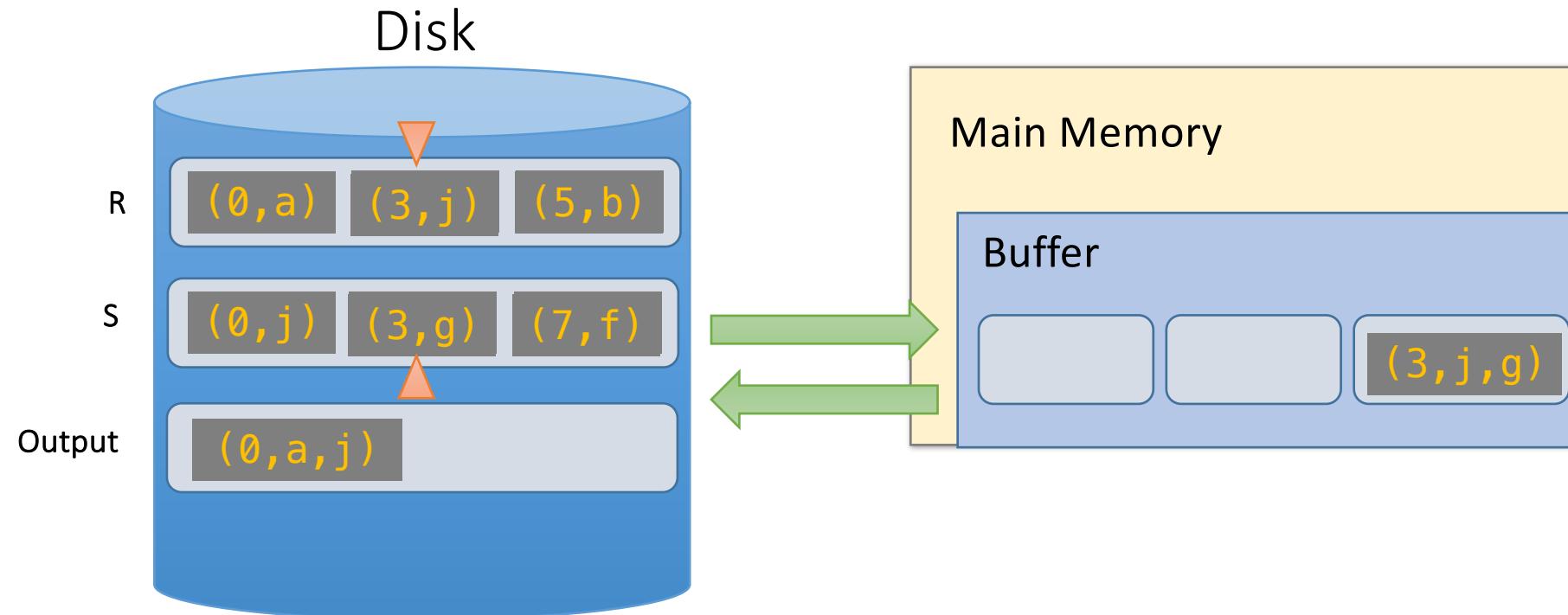
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2. Scan and “merge” on join key!



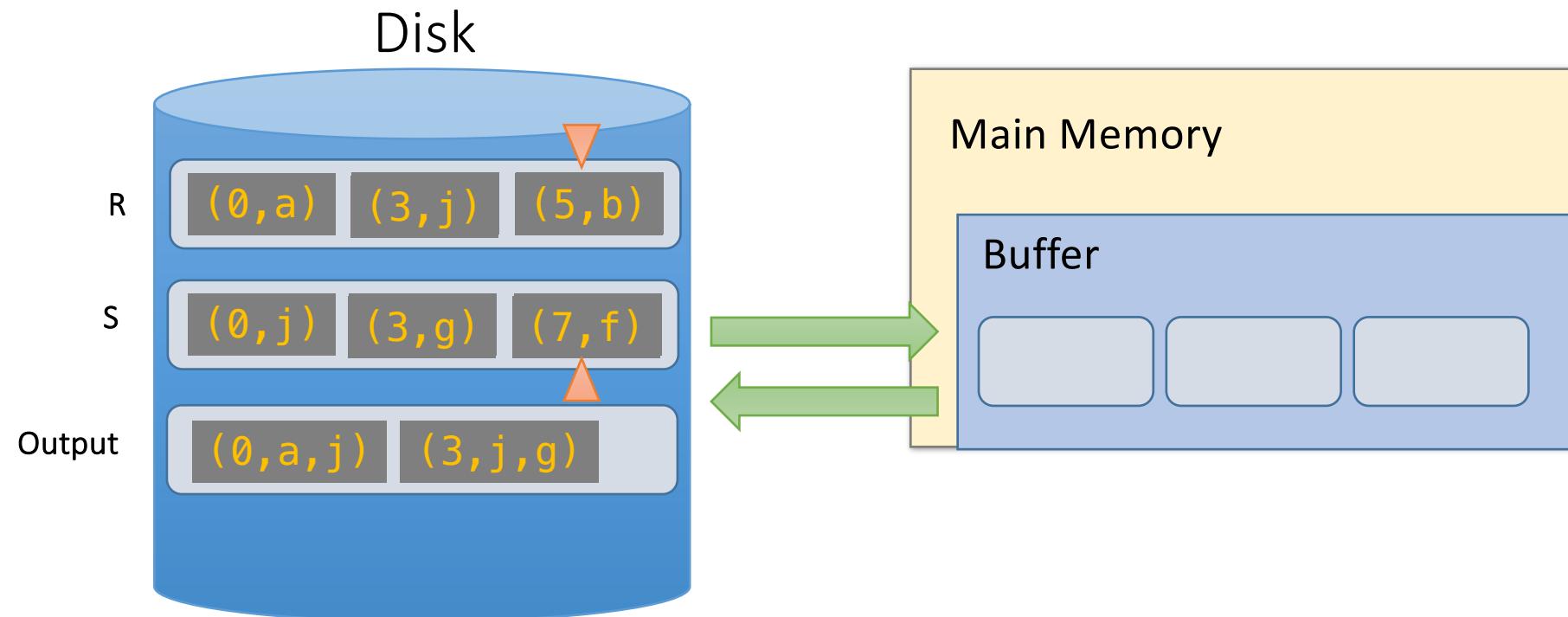
SMJ Example: $R \bowtie S$ on A with 3 page buffer

2. Scan and “merge” on join key!



SMJ Example: $R \bowtie S$ on A with 3 page buffer

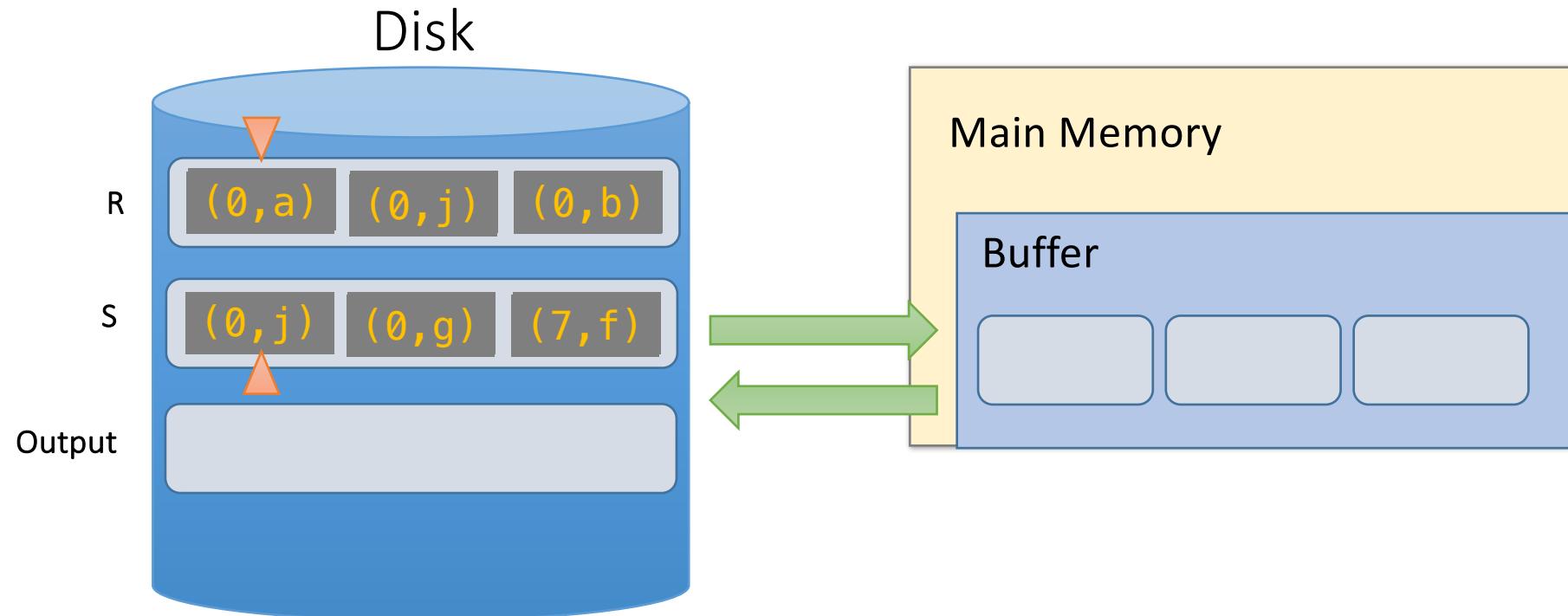
2. Done!



What happens with duplicate join keys?

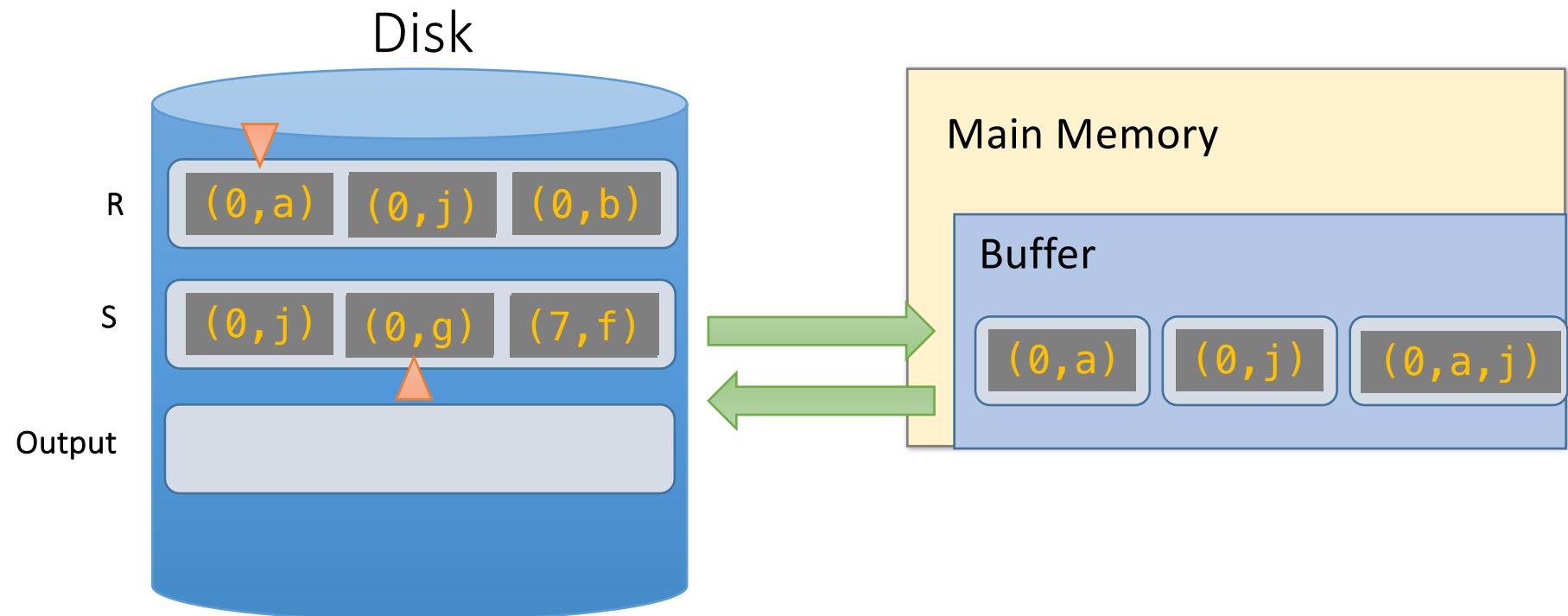
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...



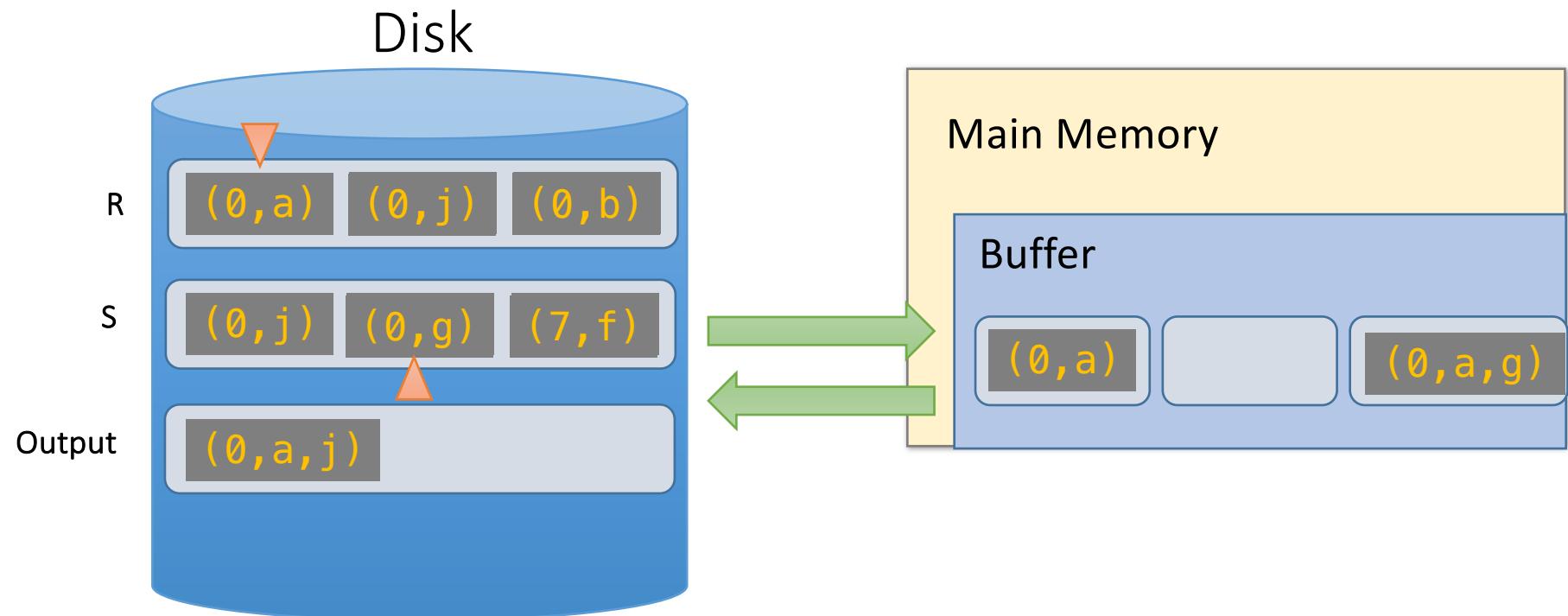
Multiple tuples with Same Join Key: “Backup”

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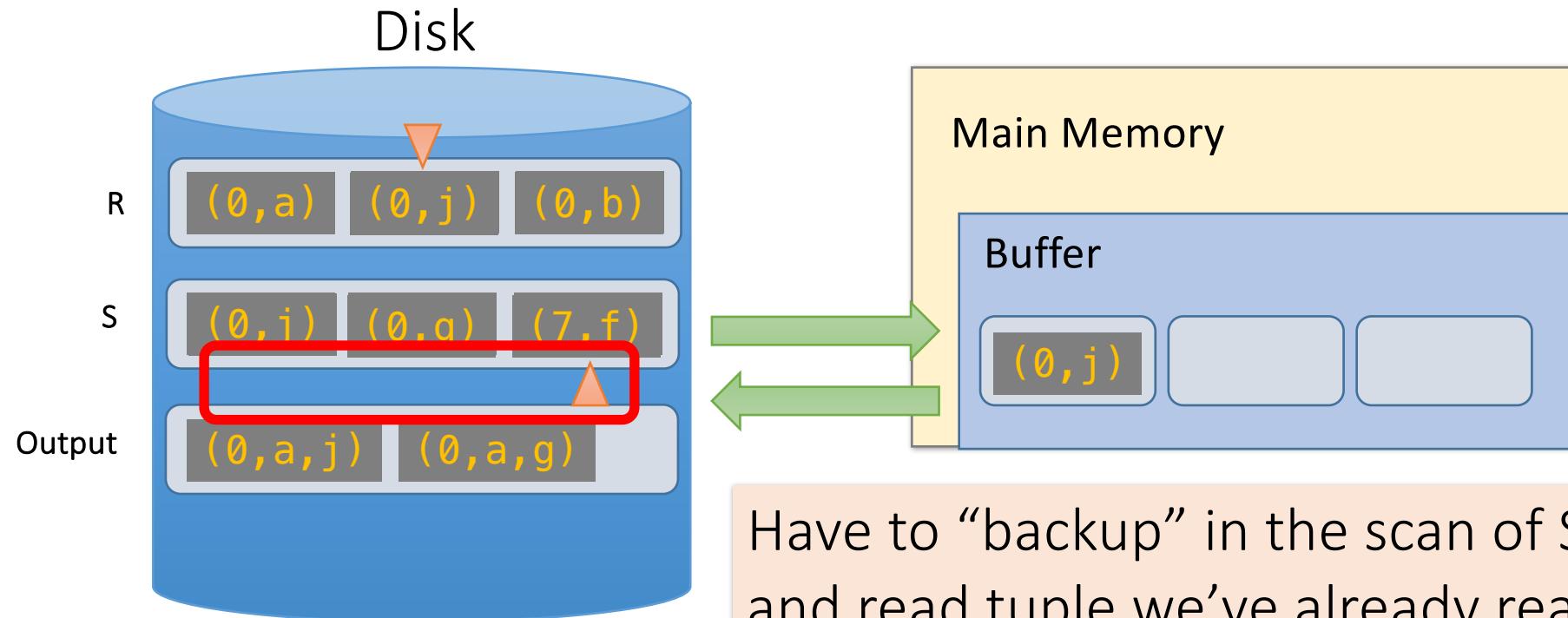
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...



Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...



Backup

- At best, no backup → scan takes $P(R) + P(S)$ reads
 - For ex: if no duplicate values in join attribute
- At worst (e.g. full backup each time), scan could take $P(R) * P(S)$ reads!
 - For ex: if *all* duplicate values in join attribute, i.e. all tuples in R and S have the same value for the join attribute
 - Roughly: For each page of R, we'll have to *back up* and read each page of S...
- Often not that bad however, plus we can:
 - Leave more data in buffer (for larger buffers)
 - Can “zig-zag” (see animation)

SMJ: Total cost

- Cost of SMJ is **cost of sorting** R and S...
- Plus the **cost of scanning**: $\sim P(R) + P(S)$
 - Because of *backup*: in worst case $P(R)*P(S)$; but this would be very unlikely
- Plus the **cost of writing out**: $\sim P(R) + P(S)$ but in worst case $T(R)*T(S)$

$\sim \text{Sort}(P(R)) + \text{Sort}(P(S))$
 $+ P(R) + P(S) + \text{OUT}$

Recall: $\text{Sort}(N) \approx 2N \left(\left\lceil \log_B \frac{N}{2(B+1)} \right\rceil + 1 \right)$

Note: this is using repacking, where we estimate that we can create initial runs of length $\sim 2(B+1)$

SMJ vs. BNLJ: Steel Cage Match

- If we have 100 buffer pages, $P(R) = 1000$ pages and $P(S) = 500$ pages:
 - Sort both in two passes: $2 * 2 * 1000 + 2 * 2 * 500 = 6,000 \text{ IOs}$
 - Merge phase $1000 + 500 = 1,500 \text{ IOs}$
 - **= 7,500 IOs + OUT**

What is BNLJ?

- $500 + 1000 * \left\lceil \frac{500}{98} \right\rceil = \underline{\b{6,500 IOs + OUT}}$
- But, if we have 35 buffer pages?
 - Sort Merge has same behavior (still 2 passes)
 - BNLJ? **15,500 IOs + OUT!**



SMJ is ~ linear vs. BNLJ is quadratic...
But it's all about the memory.

A Simple Optimization: Merges Merged!

Given $B+1$ buffer pages

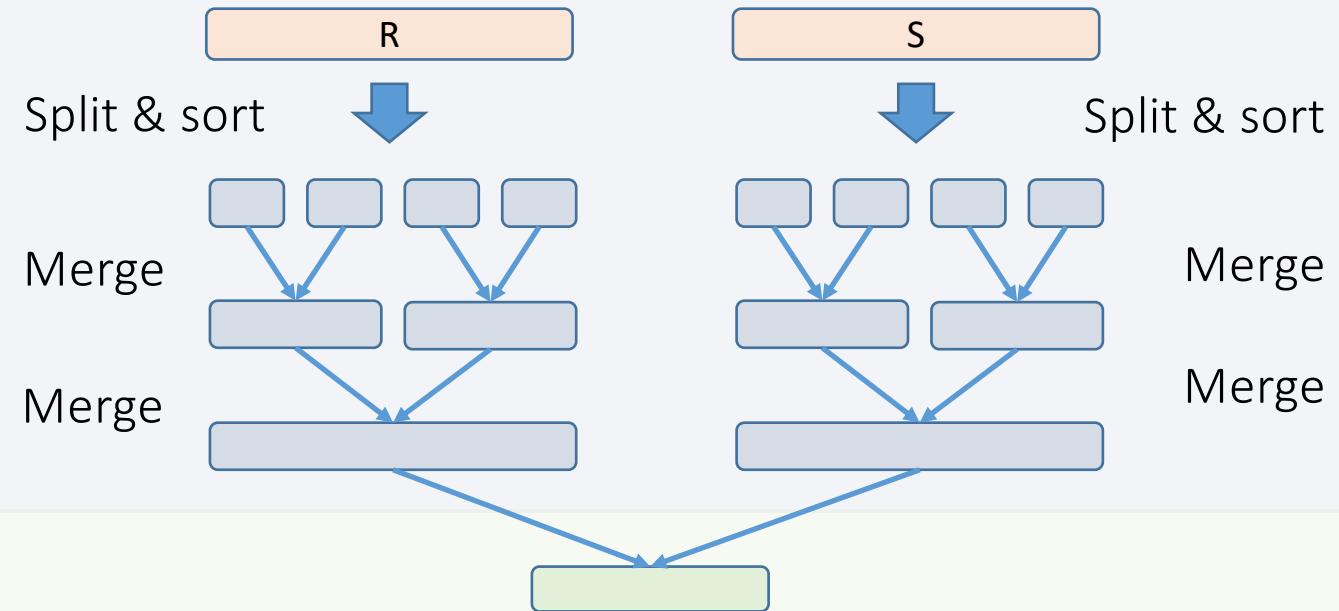
- SMJ is composed of a ***sort phase*** and a ***merge phase***
- During the ***sort phase***, run passes of external merge sort on R and S
 - Suppose at some point, R and S have $\leq B$ (sorted) runs in total
 - We could do two merges (for each of R & S) at this point, complete the sort phase, and start the merge phase...
 - OR, we could combine them: do **one** B-way merge and complete the join!

Un-Optimized SMJ

Given $B+1$ buffer pages

Unsorted input relations

Sort Phase
(Ext. Merge Sort)



Merge / Join Phase

Joined output
file created!

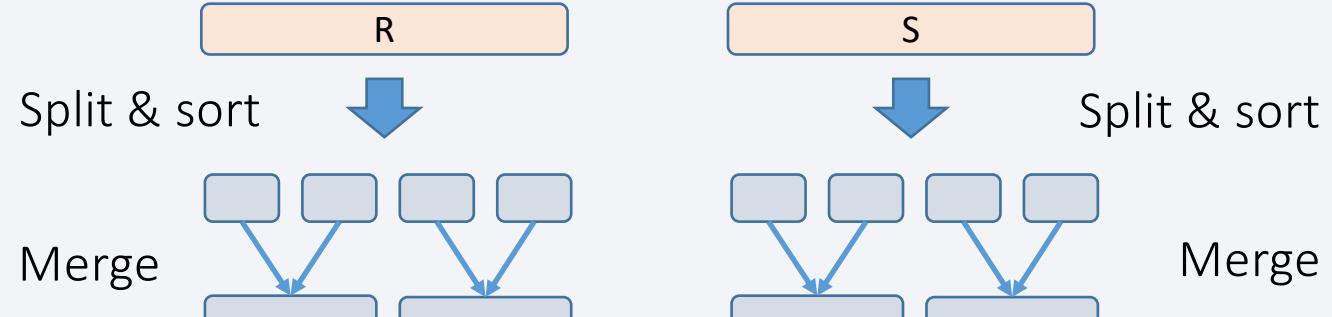
Simple SMJ Optimization

Given $B+1$ buffer pages

Unsorted input relations

Sort Phase
(Ext. Merge Sort)

$\leq B$ total runs



Merge / Join Phase

B-Way Merge / Join

Joined output
file created!

Simple SMJ Optimization

Given $B+1$ buffer pages

- Now, on this last pass, we only do $P(R) + P(S)$ IOs to complete the join!
- If we can initially split R and S into **B total runs each of length approx. $\leq 2(B+1)$** , *assuming repacking lets us create initial runs of $\sim 2(B+1)$* - then we only need **$3(P(R) + P(S)) + OUT$** for SMJ!
 - 2 R/W per page to sort runs in memory, 1 R per page to B-way merge / join!
- How much memory for this to happen?
 - $\frac{P(R)+P(S)}{B} \leq 2(B + 1) \Rightarrow \sim P(R) + P(S) \leq 2B^2$
 - Thus, $\max\{P(R), P(S)\} \leq B^2$ is an approximate sufficient condition**

See Lecture 13,
Slide 13-14 – to
clarify this slide.

If the larger of R,S has $\leq B^2$ pages, then SMJ costs
 $3(P(R)+P(S)) + OUT!$

Takeaway points from SMJ

If input already sorted on join key, skip the sorts.

- SMJ is basically linear.
- Nasty but unlikely case: Many duplicate join keys.

SMJ needs to sort **both** relations

- If $\max \{ P(R), P(S) \} < B^2$ then cost is $3(P(R)+P(S)) + OUT$



Bonus questions.



- Q1: Fast dog.
 - If $\max \{P(R), P(S)\} < B^2$ then SMJ takes $3(P(R) + P(S)) + OUT$
 - What is the similar condition to obtain $5(P(R) + P(S)) + OUT$?
 - What is the condition for $(2k+1)(P(R) + P(S)) + OUT$
- Q2: BNLJ V. SMJ
 - Under what conditions will BNLJ outperform SMJ?
 - Size of R, S and # of buffer pages
- Discuss! And We'll put up a google form.

2. Hash Join (HJ)



What you will learn about in this section

1. Hash Join
2. Memory requirements

Recall: Hashing

- **Magic of hashing:**
 - A hash function h_B maps into $[0, B-1]$
 - And maps nearly uniformly
- A hash **collision** is when $x \neq y$ but $h_B(x) = h_B(y)$
 - Note however that it will never occur that $x = y$ but $h_B(x) \neq h_B(y)$
- We hash on an attribute A , so our hash function is $h_B(t)$ has the form $h_B(t.A)$.
 - **Collisions** may be more frequent.

Recall: Mad Hash Collisions



Say something here to justify this slide's existence? [TODO]

Hash Join: High-level procedure

To compute $R \bowtie S$ on A :

Note again that we are only considering equality constraints here

1. **Partition Phase:** Using one (shared) hash function h_B , partition R and S into B buckets
2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for h , and join these
 1. Use BNLJ here; or hash again → either way, operating on small partitions so fast!

We *decompose* the problem using h_B , then complete the join

Hash Join: High-level procedure

To compute $R \bowtie S$ on A :

1. **Partition Phase:** Using one (shared) hash function h_B per pass partition R and S into B buckets.

- Each phase creates B more buckets that are a factor of B smaller.
- Repeatedly partition with a new hash function
- Stop when all buckets for one relation are smaller than $B-1$ (Why?)

Each pass takes $2(P(R) + P(S))$

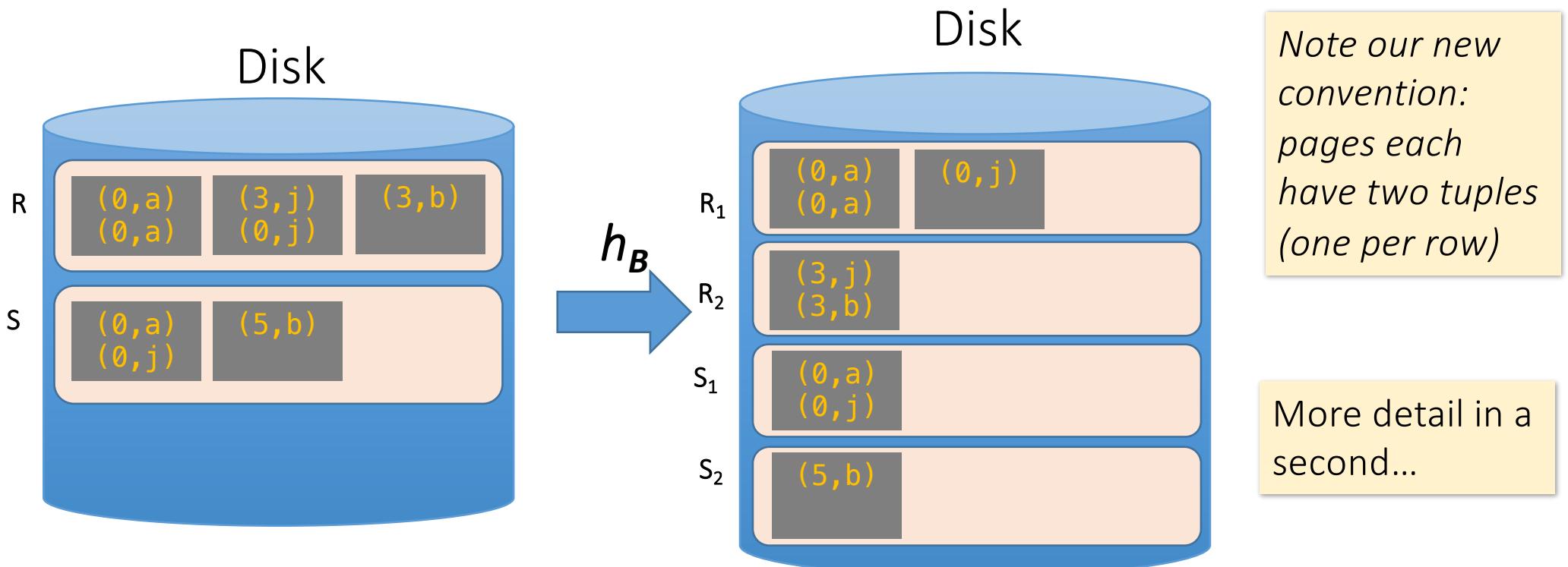
2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for h , and join these
 - Use BNLJ here for each matching pair.

$P(R) + P(S) + OUT$

We *decompose* the problem using h_B , then complete the join

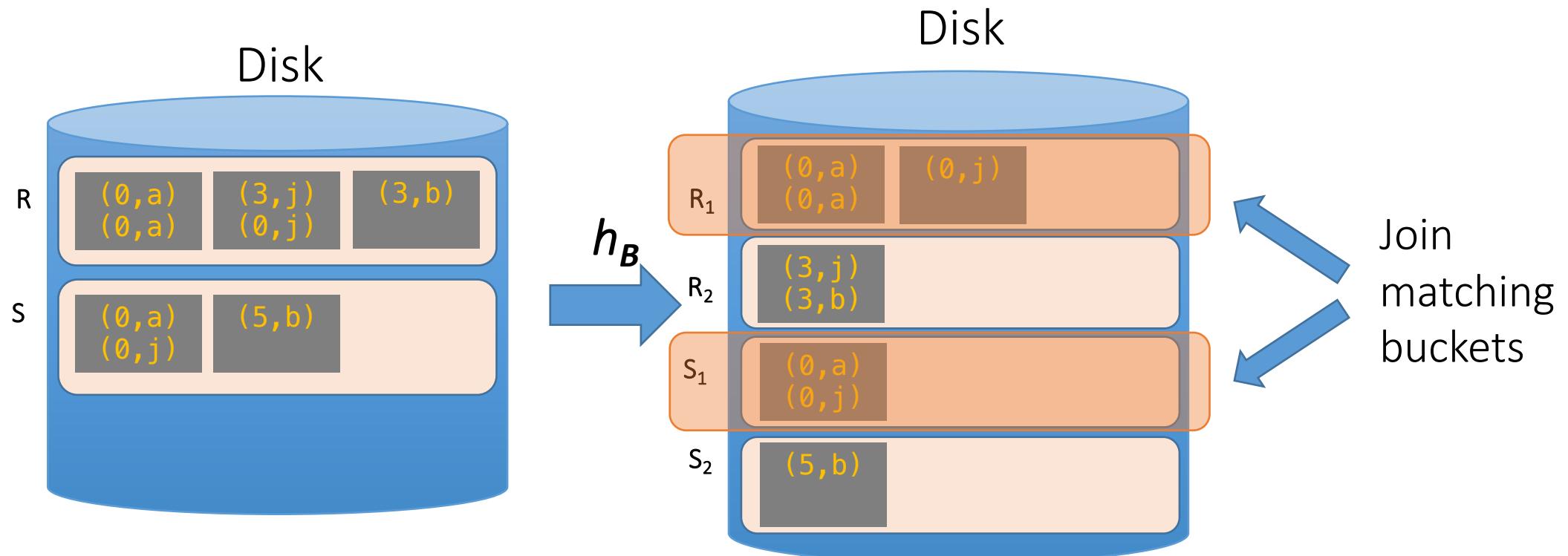
Hash Join: High-level procedure

1. Partition Phase: Using one (shared) hash function h_B , partition R and S into B buckets



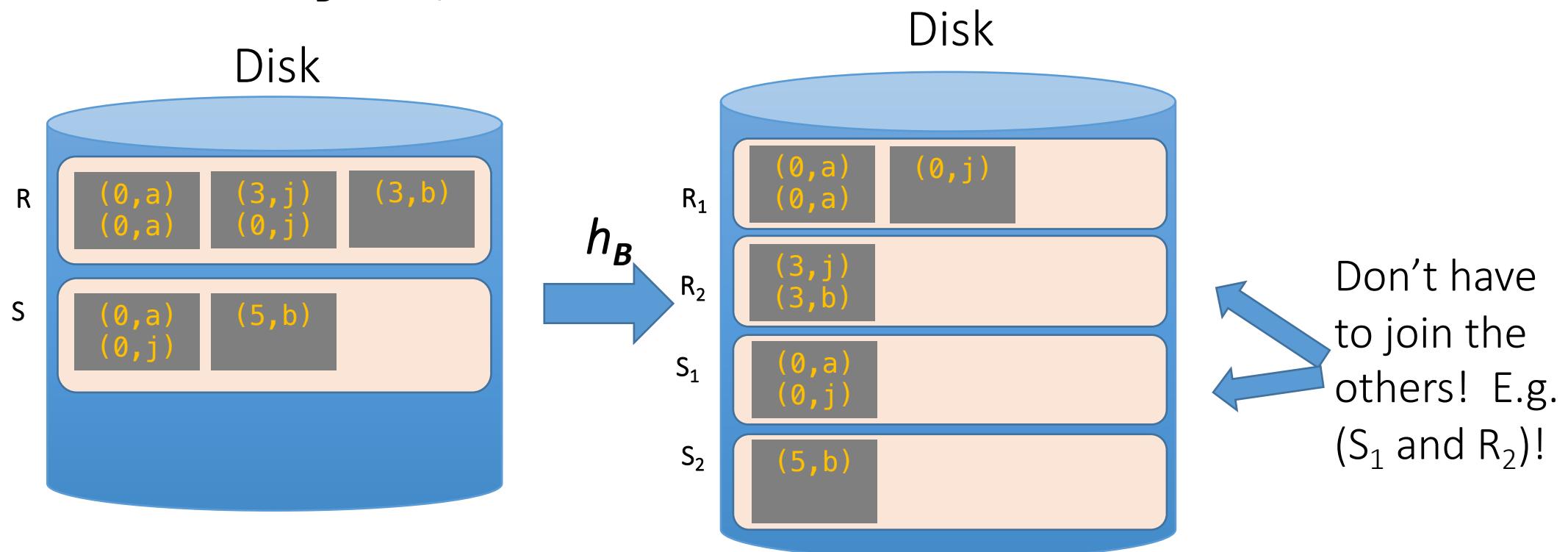
Hash Join: High-level procedure

2. Matching Phase: Take pairs of buckets whose tuples have the same values for h_B , and join these



Hash Join: High-level procedure

2. Matching Phase: Take pairs of buckets whose tuples have the same values for h_B , and join these



Hash Join Phase 1: Partitioning

Goal: For each relation, partition relation into **buckets** such that if $h_B(t.A) = h_B(t'.A)$ they are in the same bucket

Given $B+1$ buffer pages, we partition into B buckets:

- We use B buffer pages for output (one for each bucket), and 1 for input
 - The “dual” of sorting.
 - For each tuple t in input, copy to buffer page for $h_B(t.A)$
 - When page fills up, flush to disk.

How big are the resulting buckets?

Given $B+1$ buffer pages

- Given **N input pages, we partition into B buckets:**
 - → Ideally our buckets are each of size $\sim N/B$ pages
- What happens if there are **hash collisions?**
 - Buckets could be $> N/B$
 - **We'll do several passes...**
- What happens if there are **duplicate join keys?**
 - Nothing we can do here... could have some **skew** in size of the buckets

How big *do we want* the resulting buckets?

- Ideally, our buckets would be of size $\leq B - 1$ pages
 - 1 for input page, 1 for output page, $B-1$ for each bucket
- Recall: If we want to join a bucket from R and one from S, we can do BNLJ in linear time if for *one of them (wlog say R)*,
 $P(R) \leq B - 1$!
 - And more generally, being able to fit bucket in memory is advantageous
- We can keep partitioning buckets that are $> B-1$ pages, until they are $\leq B - 1$ pages
 - Using a new hash key which will split them...

Given $B+1$ buffer pages

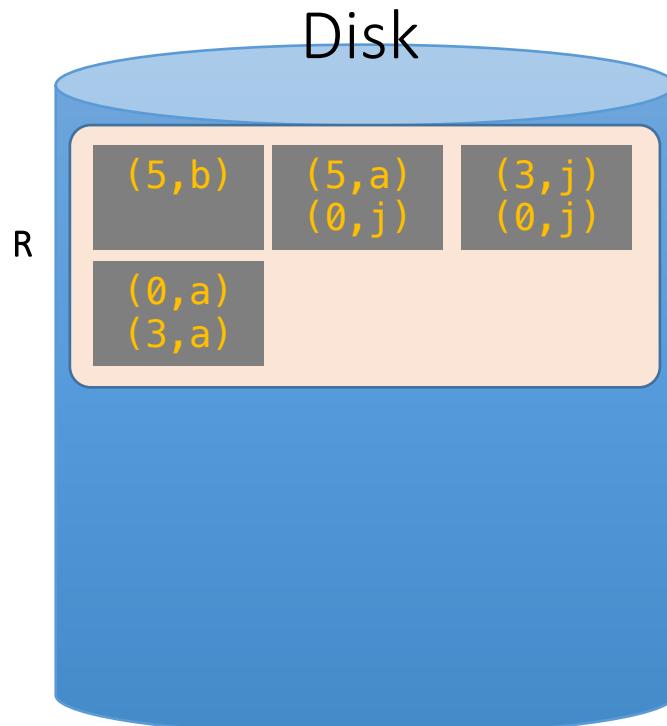
Recall for BNLJ:
$$P(R) + \frac{P(R)P(S)}{B - 1}$$

We'll call each of these
a "pass" again...

Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

We partition into $B = 2$ buckets using hash function h_2 so that we can have one buffer page for each partition (and one for input)



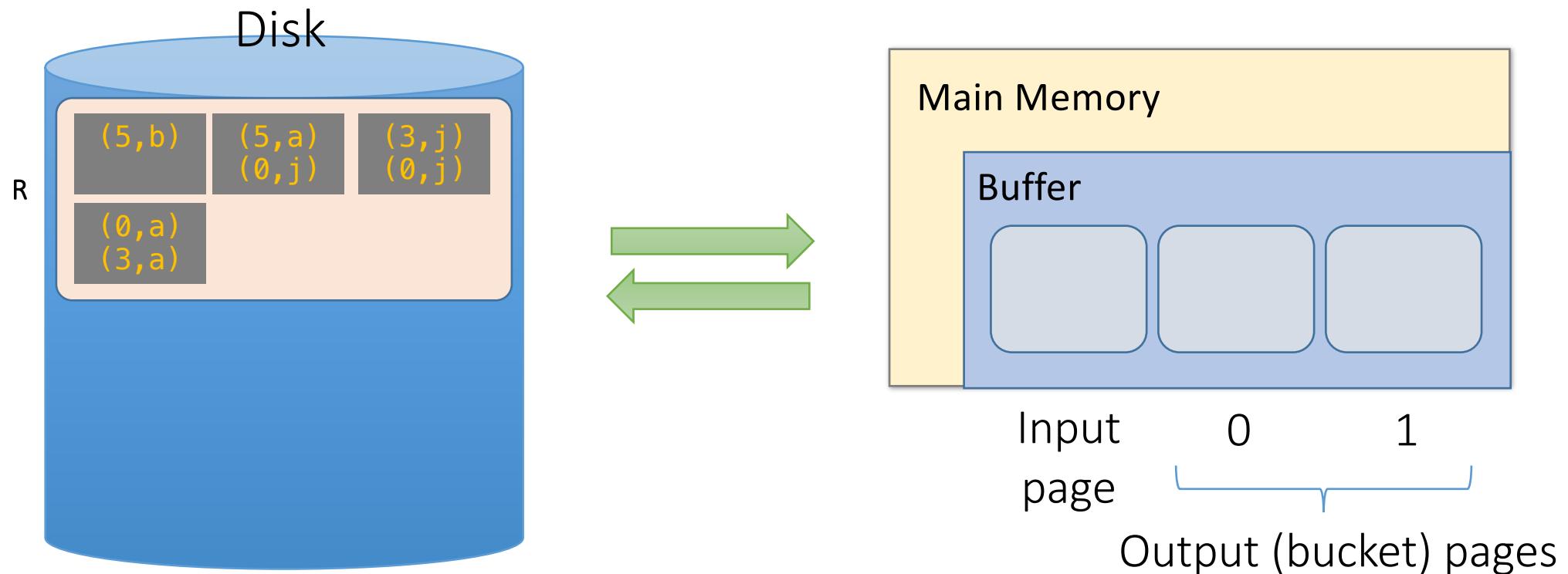
For simplicity, we'll look at partitioning one of the two relations- we just do the same for the other relation!

Recall: our goal will be to get $B = 2$ buckets of size $\leq B-1 \rightarrow 1$ page each

Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

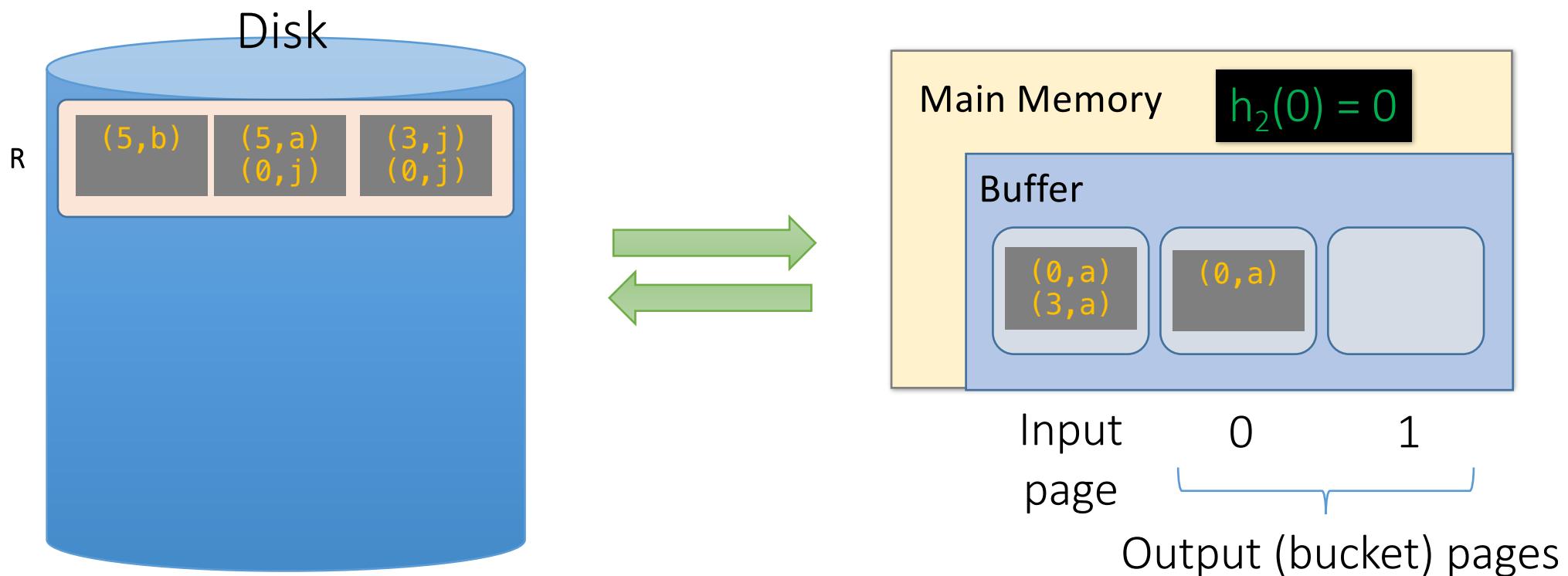
1. We read pages from R into the “input” page of the buffer...



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

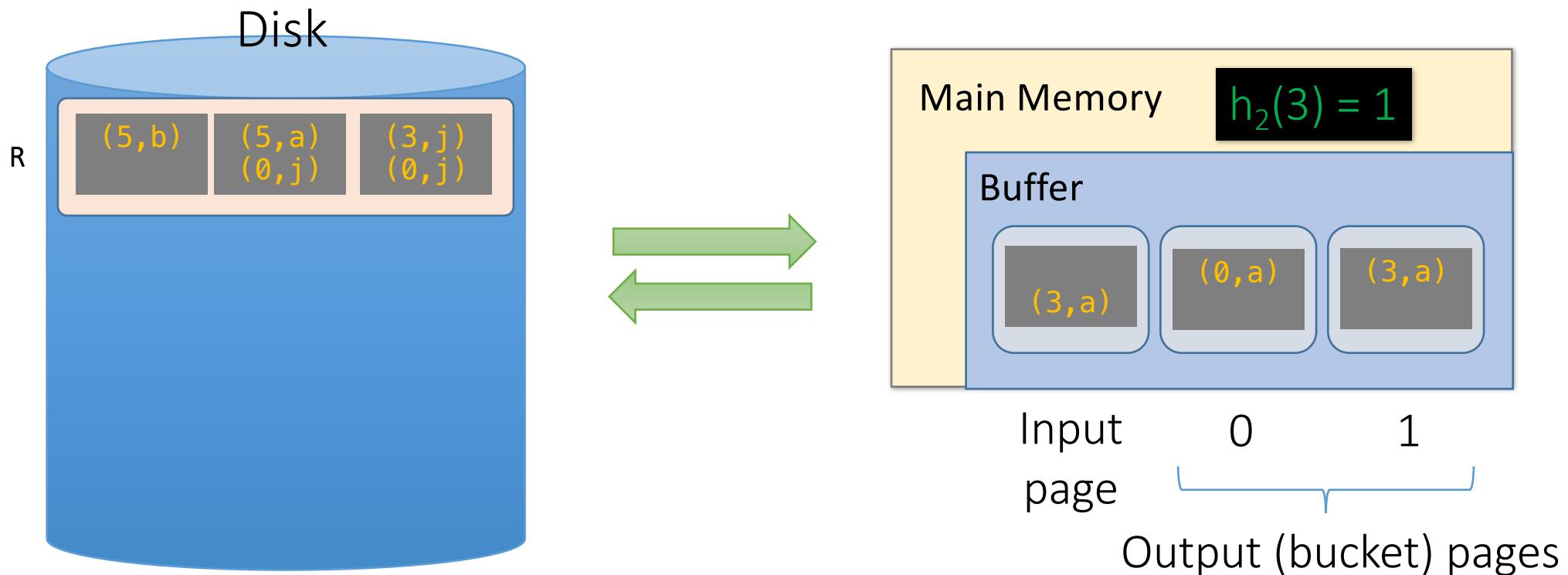
2. Then we use **hash function h_2** to sort into the buckets, which each have one page in the buffer



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

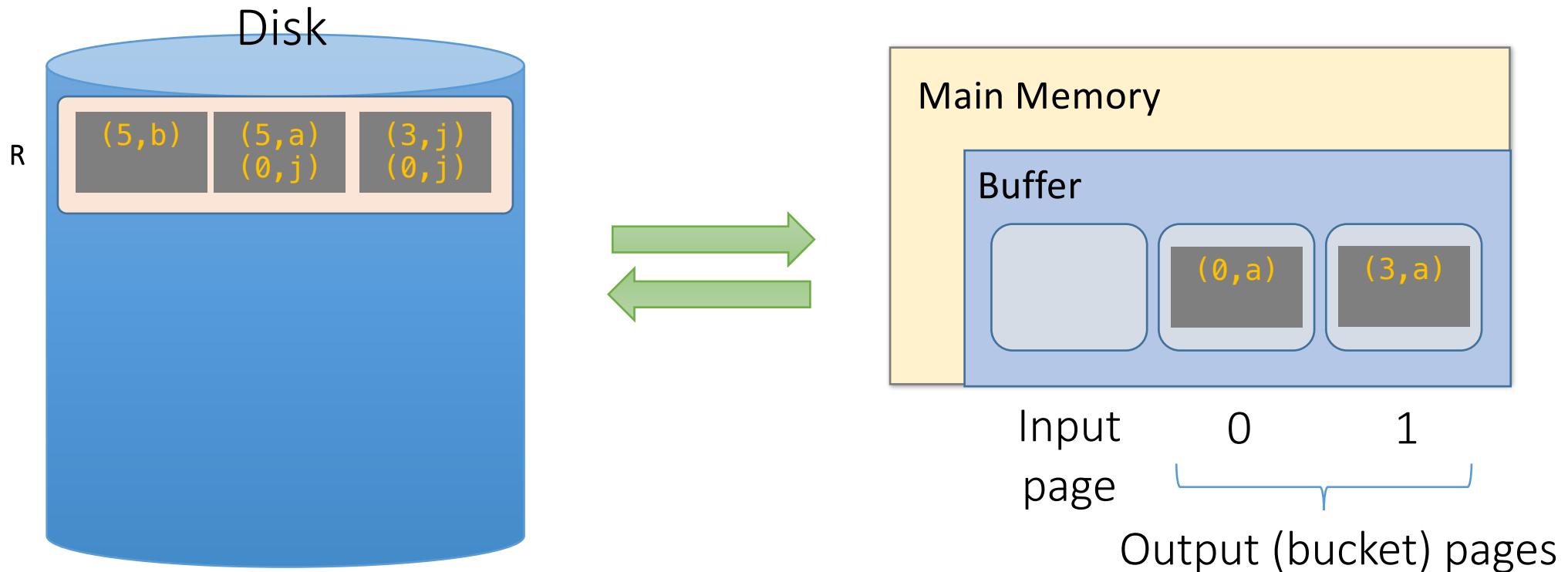
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

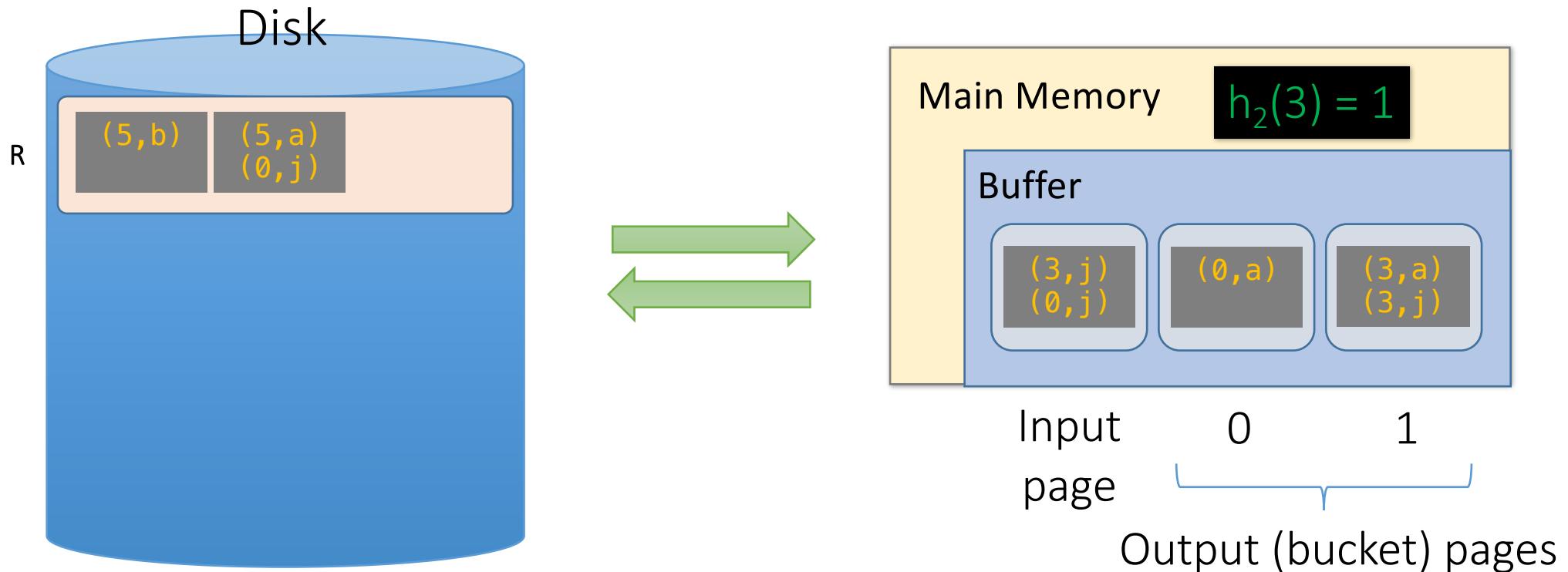
3. We repeat until the buffer bucket pages are full...



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

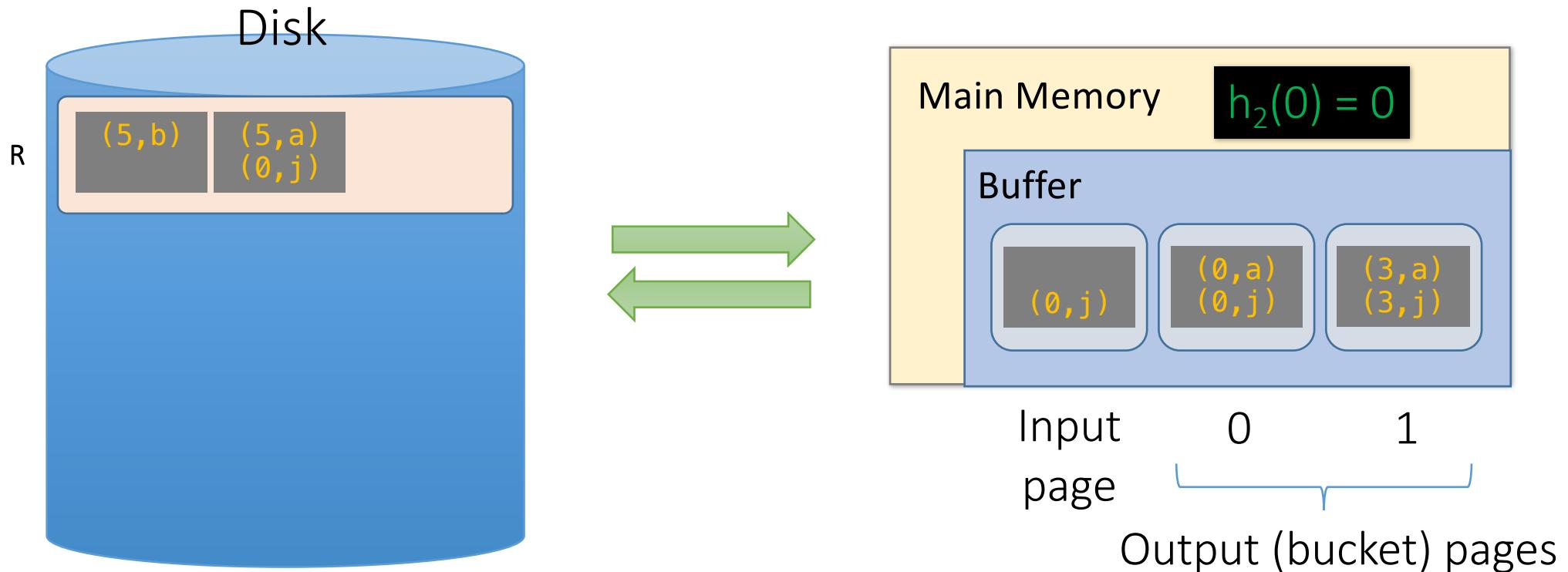
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

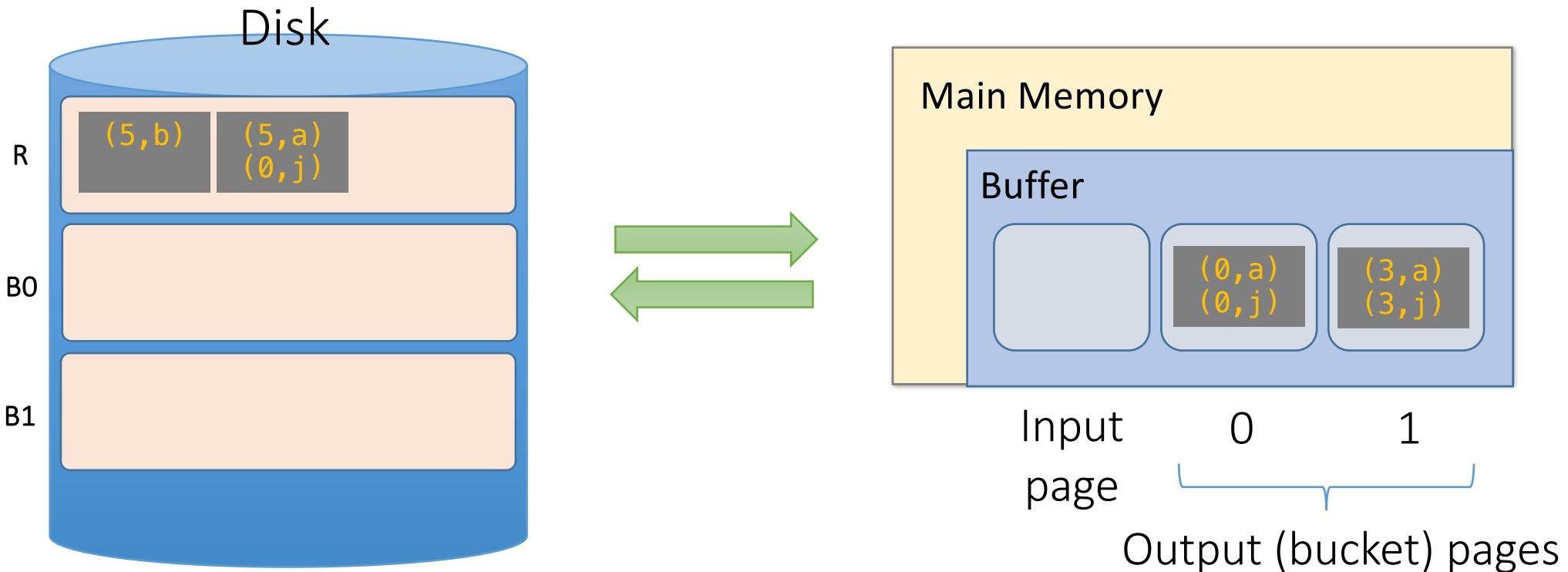
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Hash Join Phase 1: Partitioning

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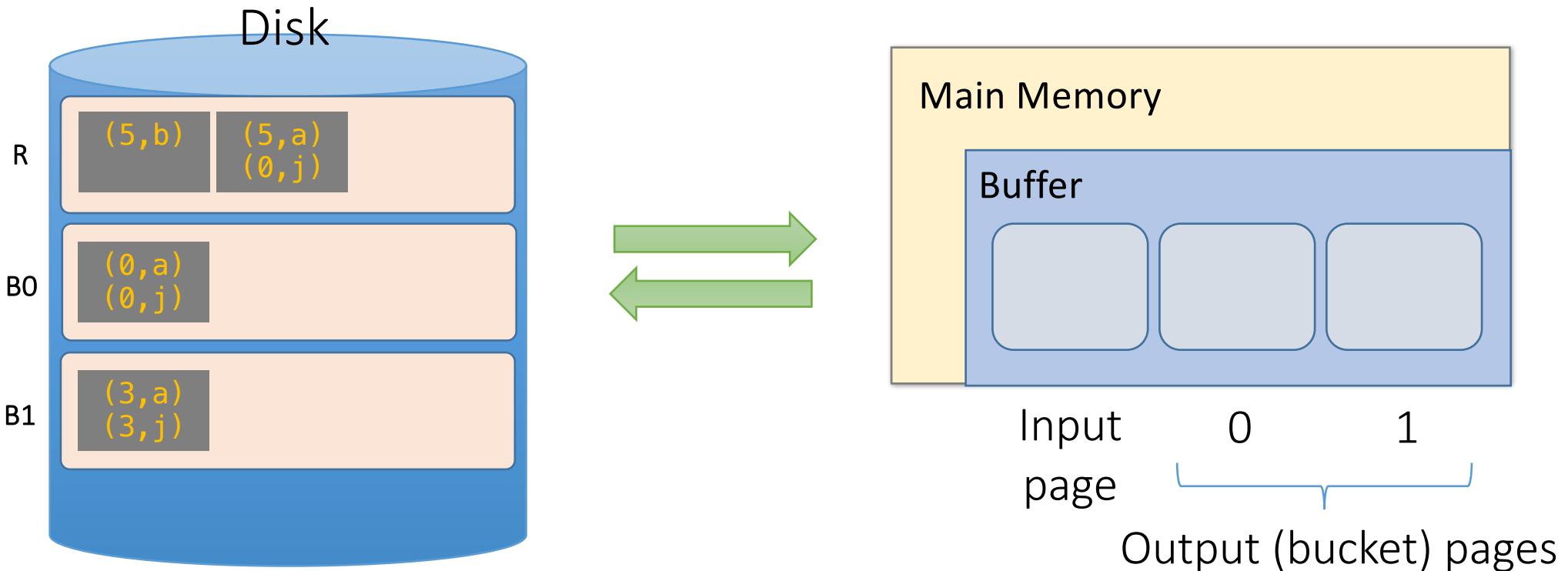
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

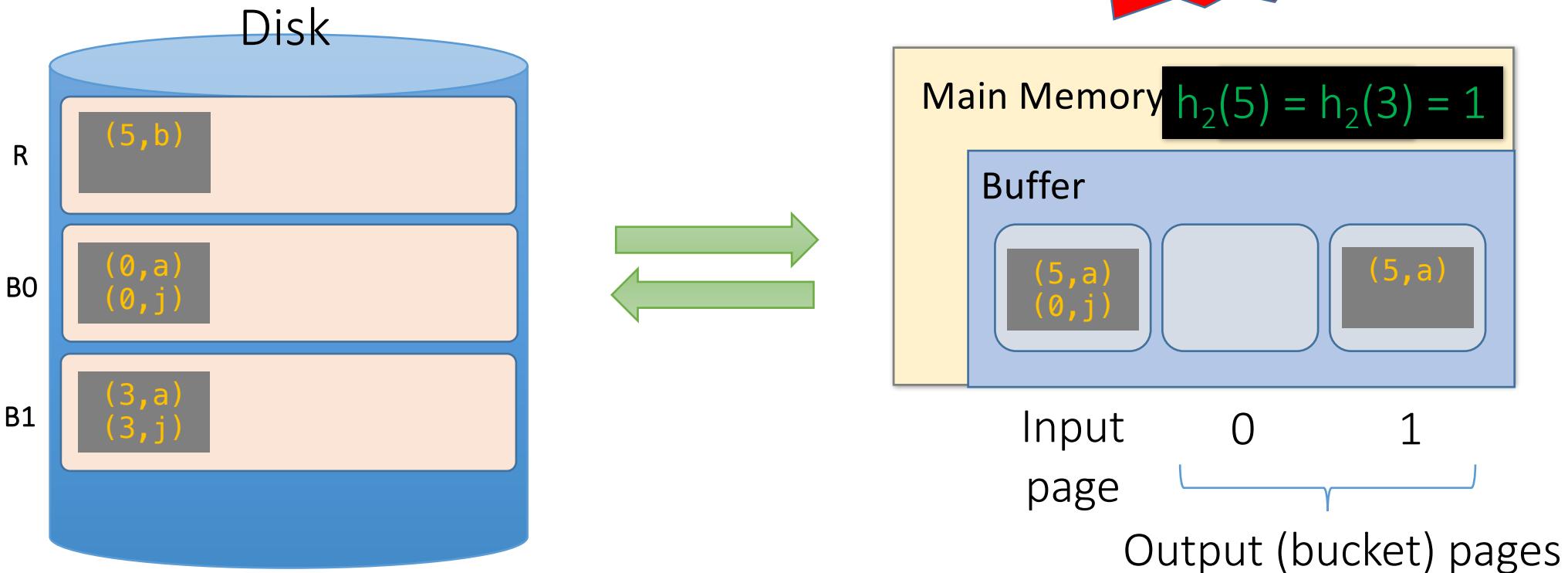
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

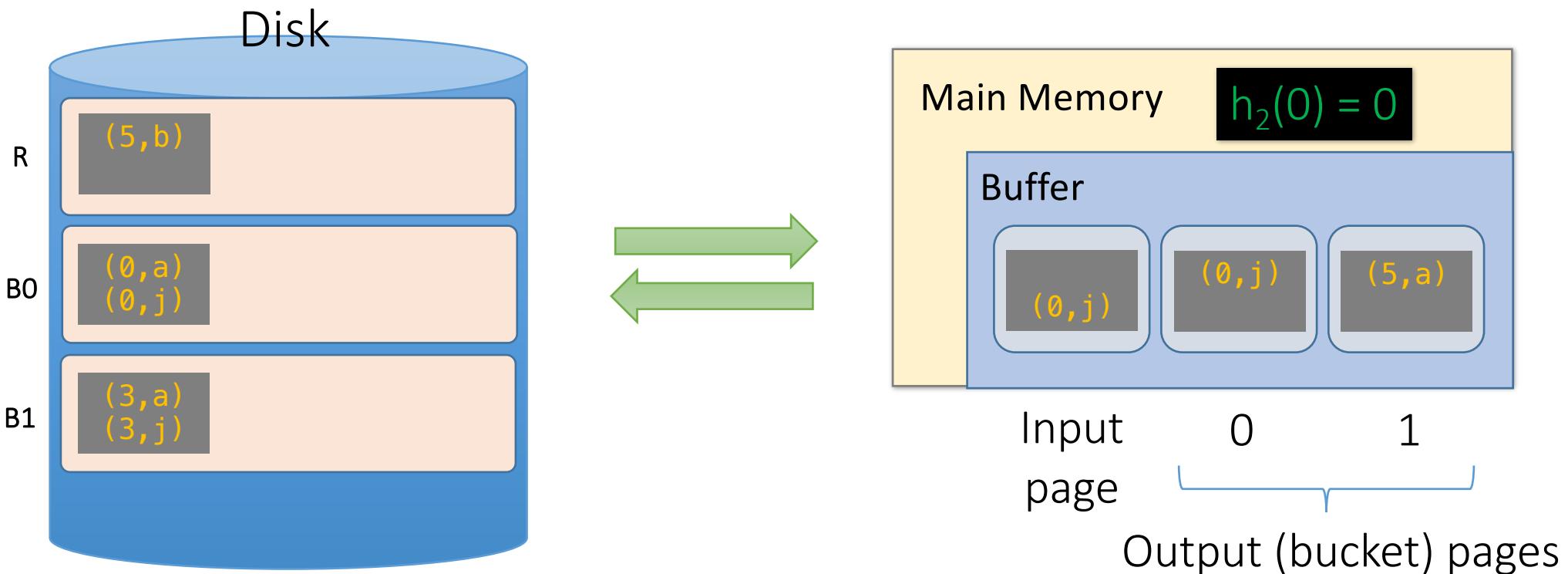
Note that collisions can occur!



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

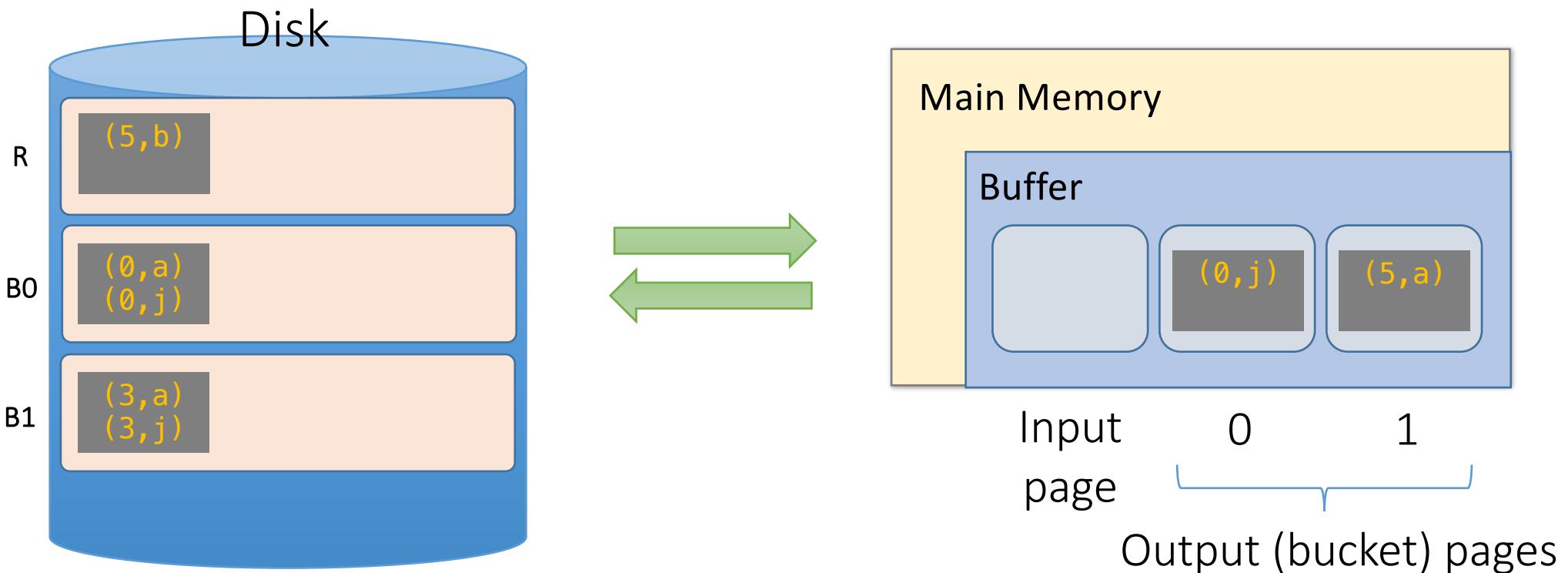
Finish this pass...



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

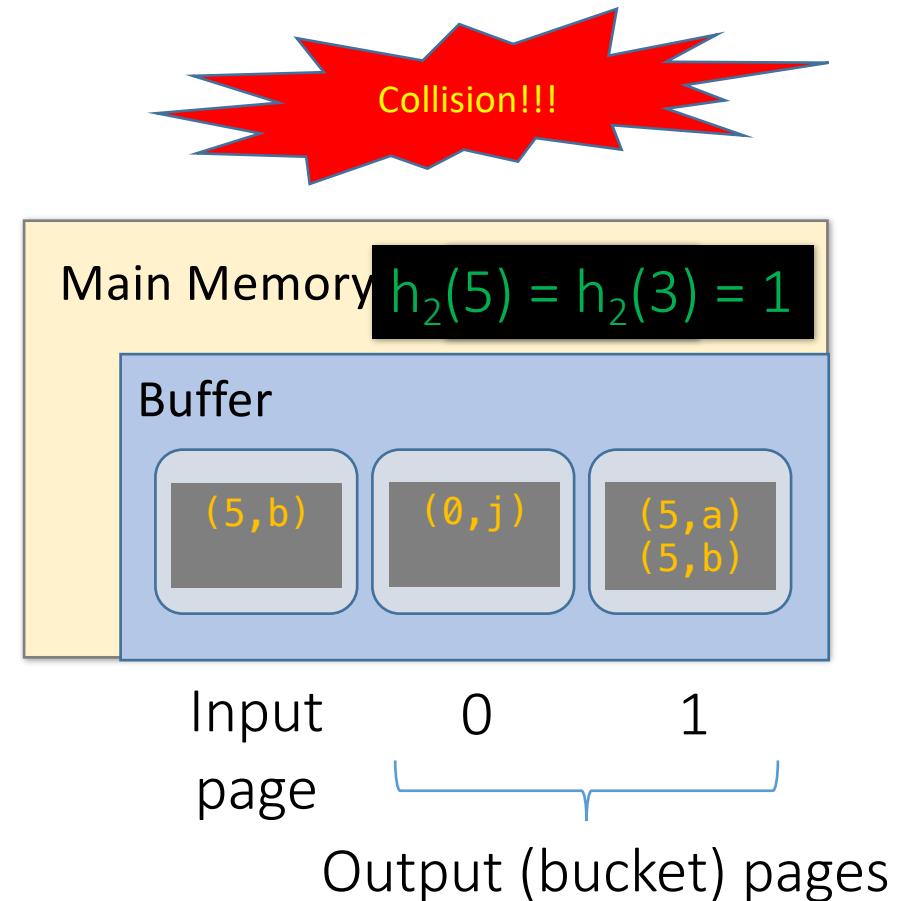
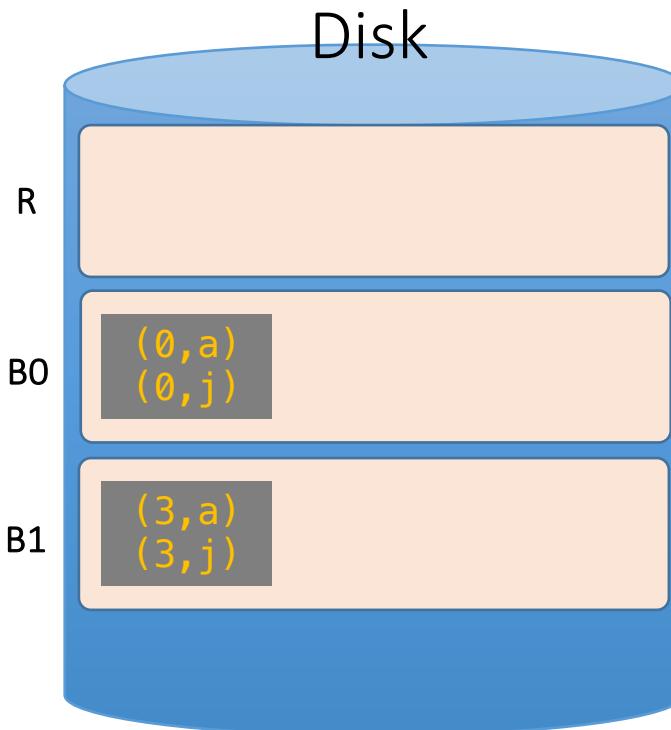
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

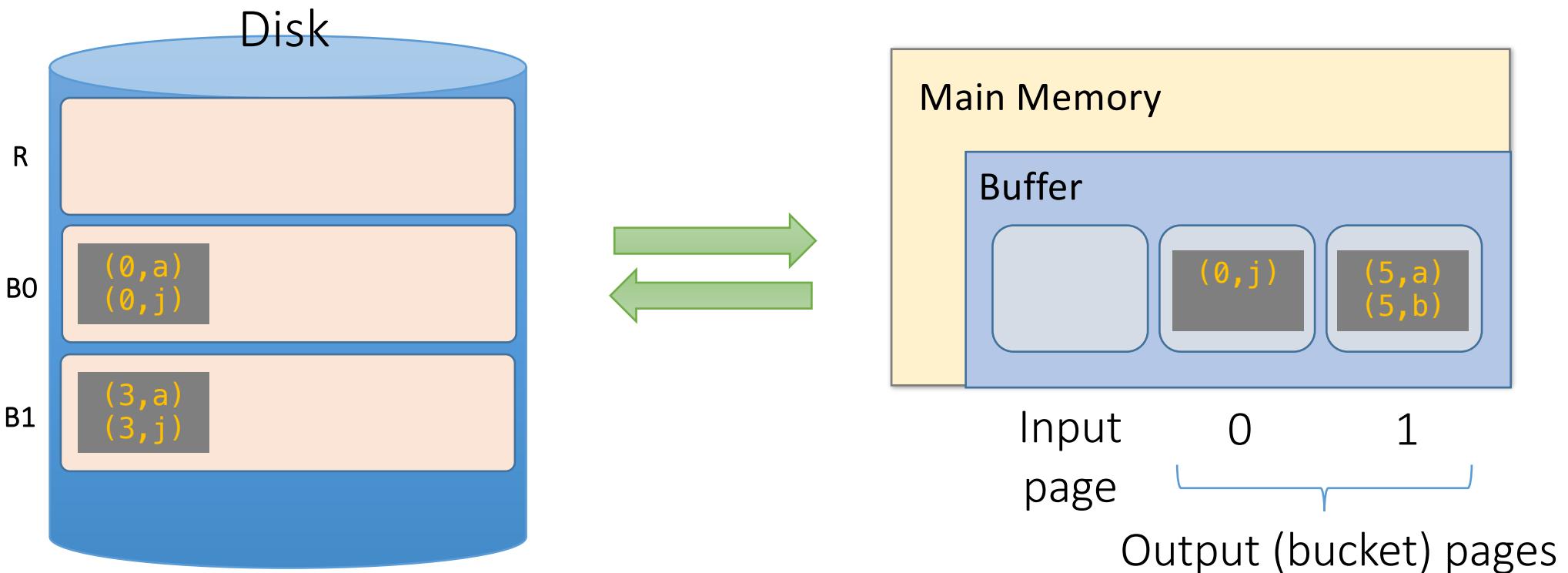
Finish this pass...



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

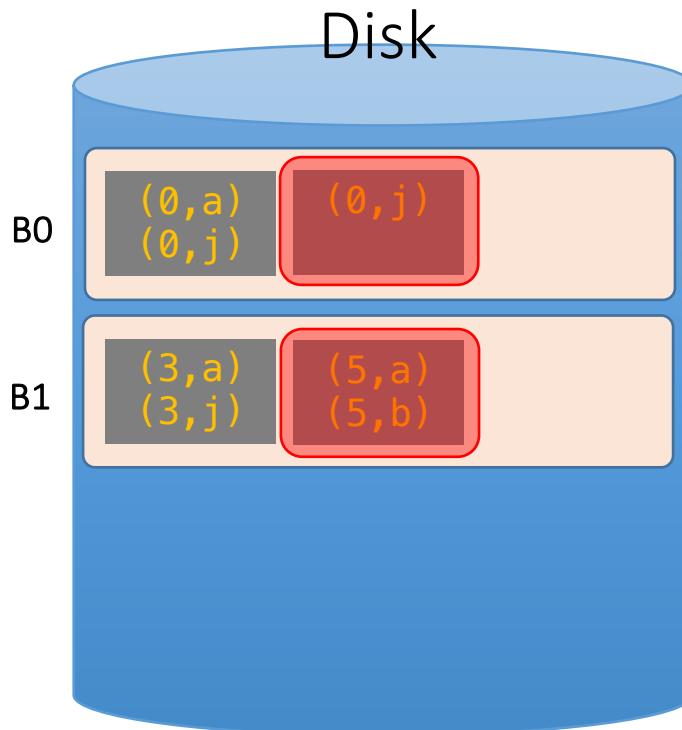
Finish this pass...



Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

We wanted buckets of size $B-1 = 1...$
however we got larger ones due to:

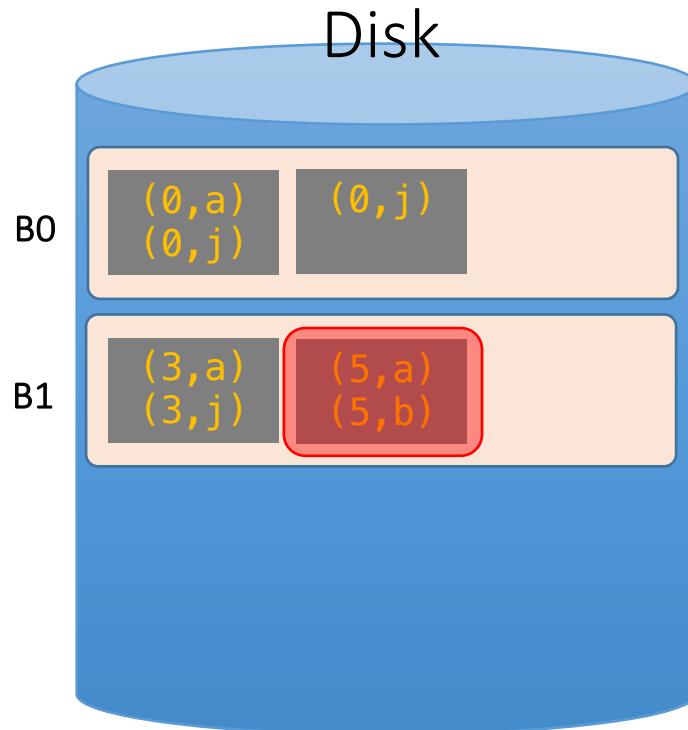


(1) Duplicate join keys

(2) Hash collisions

Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages



To take care of larger buckets caused by (2) hash collisions, we can just do another pass!

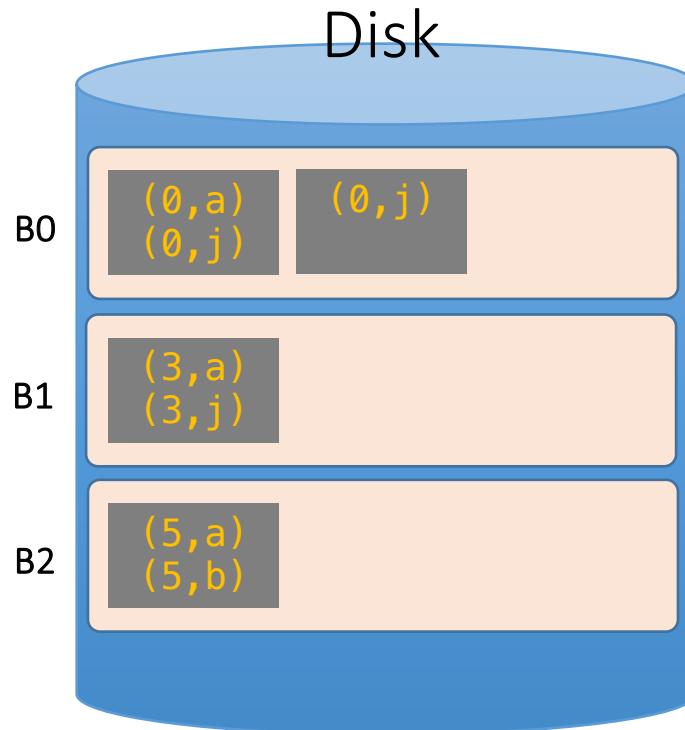
What hash function should we use?

Do another pass with a different hash function, h'_2 , ideally such that:

$$h'_2(3) \neq h'_2(5)$$

Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages



To take care of larger buckets caused by (2) hash collisions, we can just do another pass!

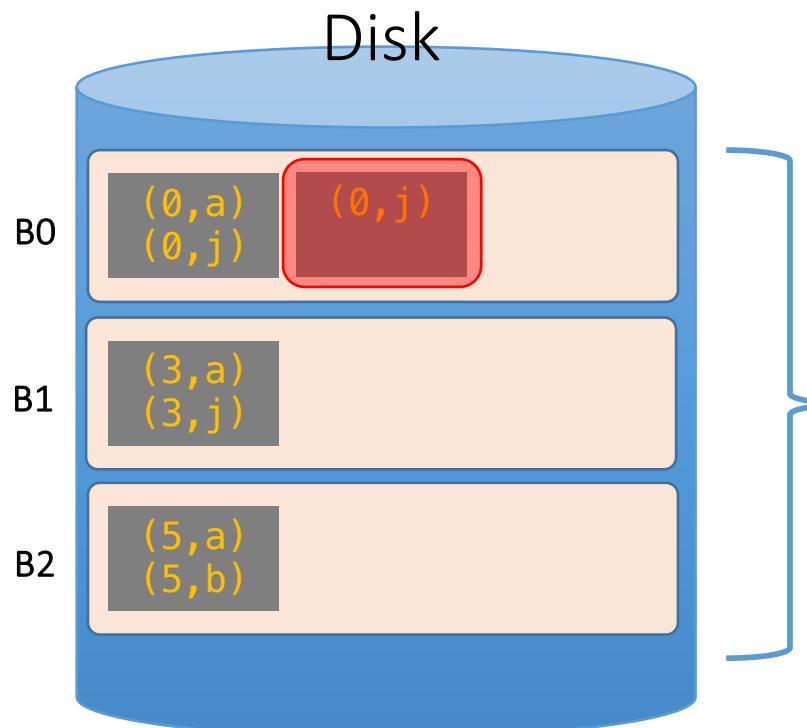
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Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages



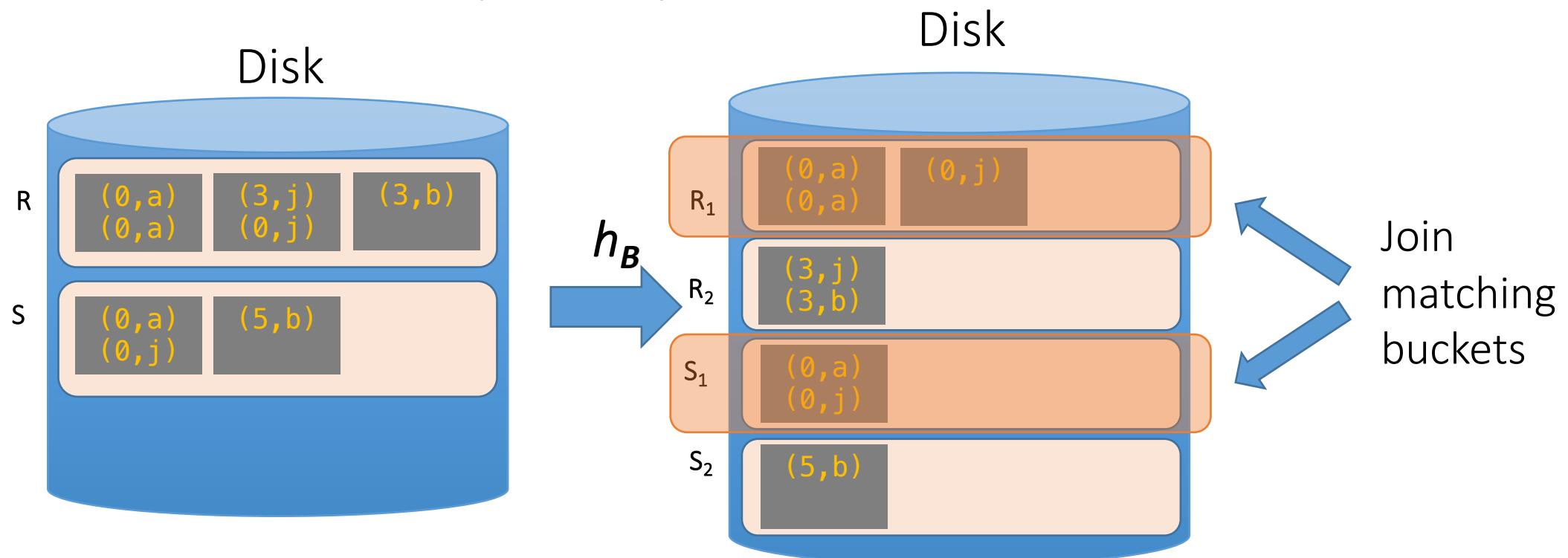
What about duplicate join keys?
Unfortunately this is a problem... but
usually not a huge one.

We call this unevenness
in the bucket size skew

Now that we have partitioned R and S...

Hash Join Phase 2: Matching

- Now, we just join pairs of buckets from R and S that have the same hash value to complete the join!



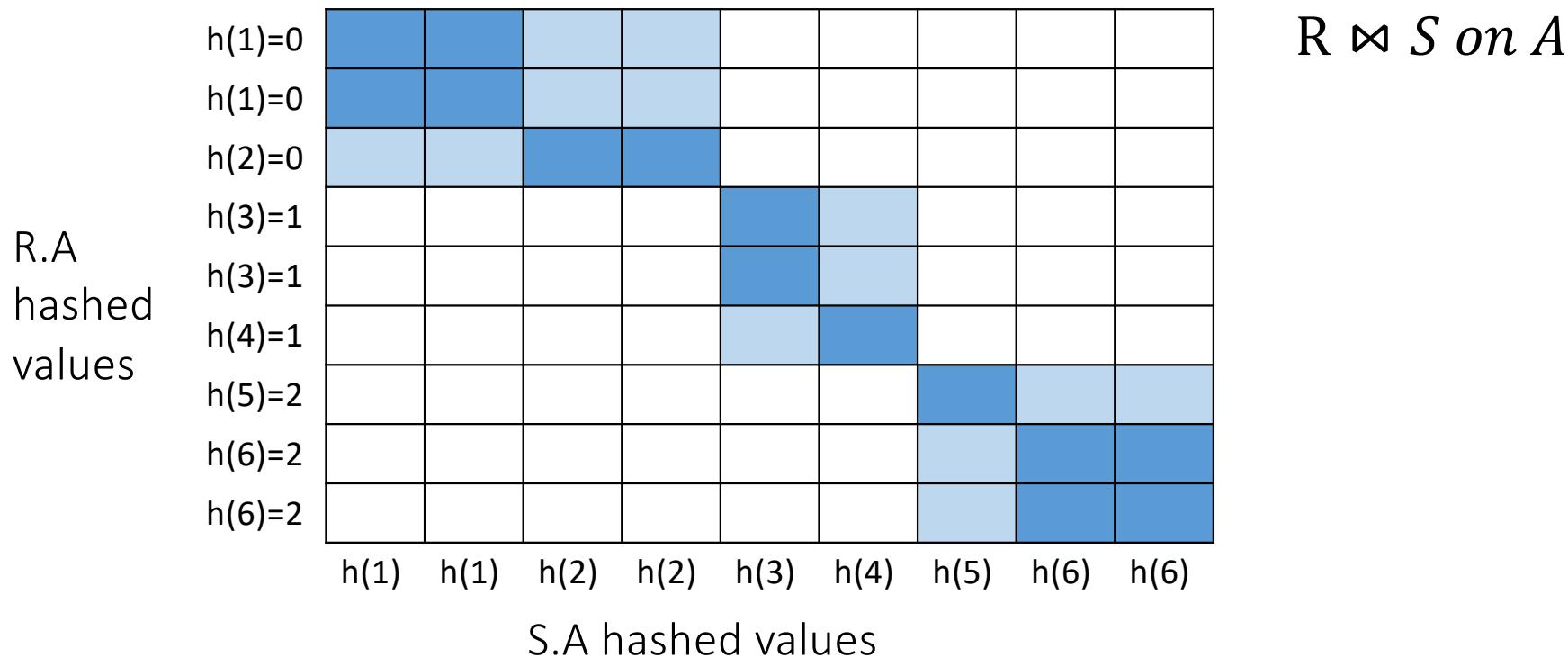
Hash Join Phase 2: Matching

- Note that since $x = y \rightarrow h(x) = h(y)$, we only need to consider pairs of buckets (one from R, one from S) that have the same hash function value
- If our buckets are $\sim B - 1$ pages, can join each such pair using BNLJ *in linear time*; recall (with $P(R) = B-1$):

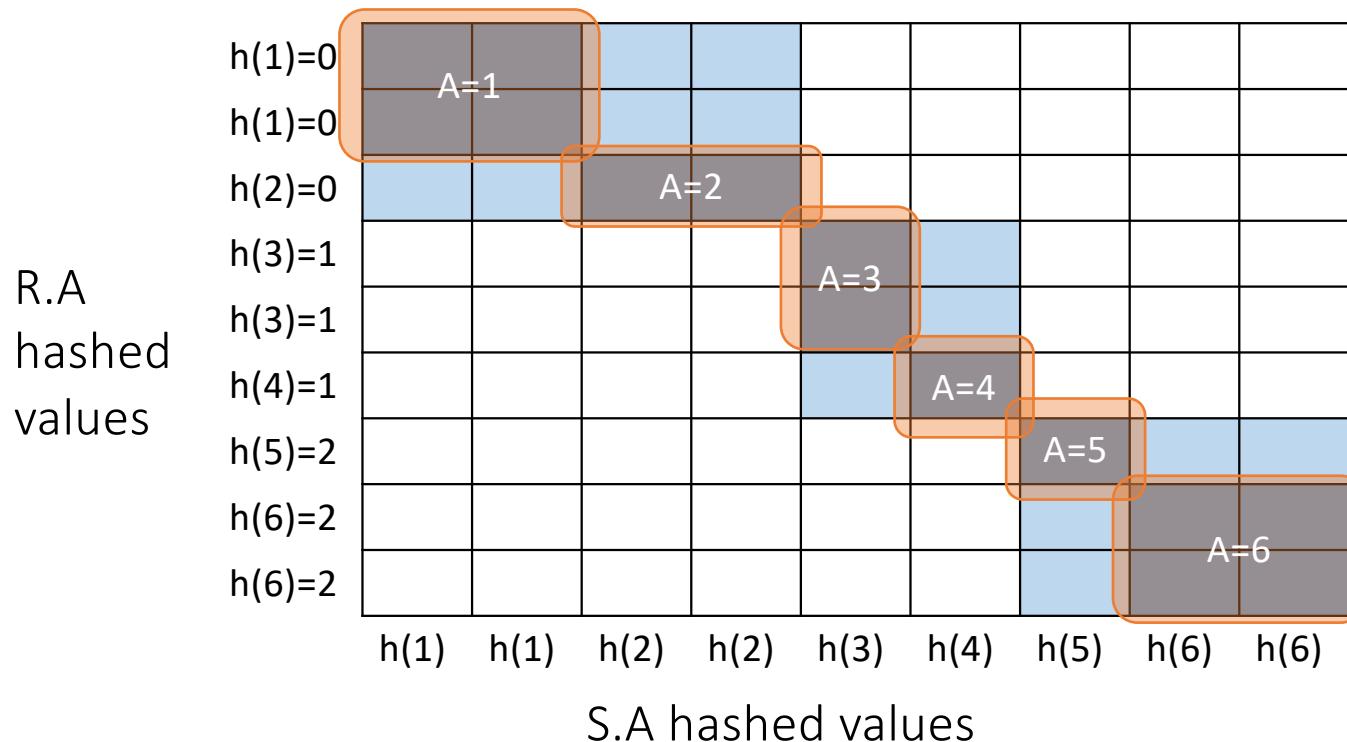
$$\text{BNLJ Cost: } P(R) + \frac{P(R)P(S)}{B-1} = P(R) + \frac{(B-1)P(S)}{B-1} = P(R) + P(S)$$

Joining the pairs of buckets is linear!
(As long as smaller bucket $\leq B-1$ pages)

Hash Join Phase 2: Matching



Hash Join Phase 2: Matching

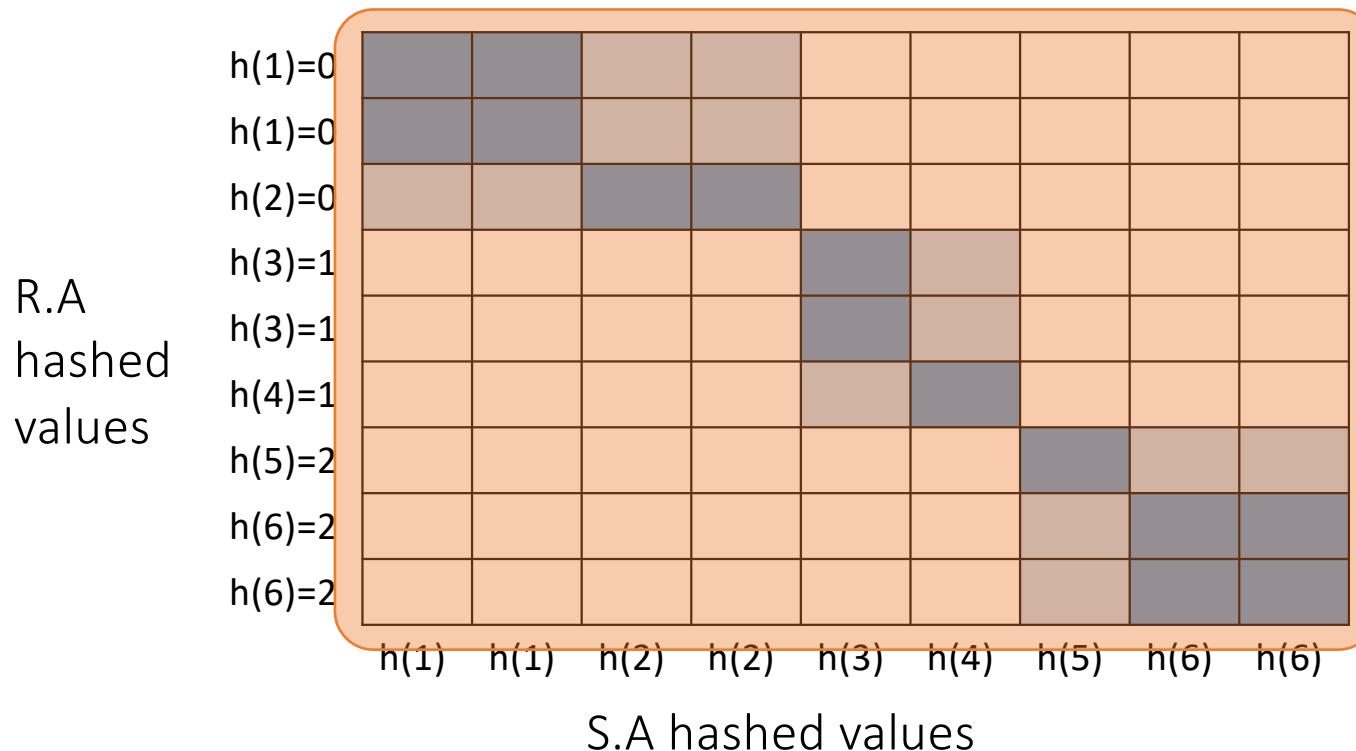


$R \bowtie S$ on A

To perform the join, we ideally just need to explore the dark blue regions

= the tuples with same values of the join key A

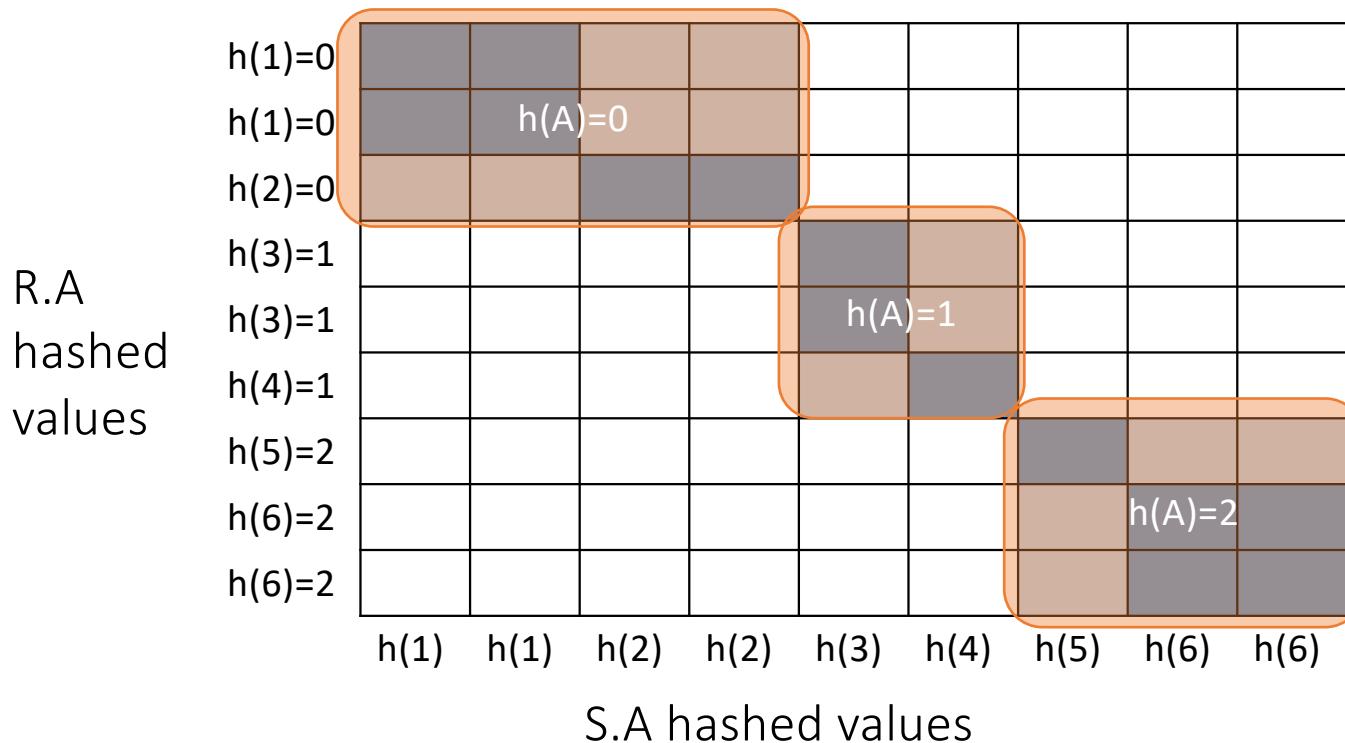
Hash Join Phase 2: Matching



$R \bowtie S \text{ on } A$

With a join algorithm like BNLJ that doesn't take advantage of equijoin structure, we'd have to explore this ***whole grid!***

Hash Join Phase 2: Matching



$R \bowtie S$ on A

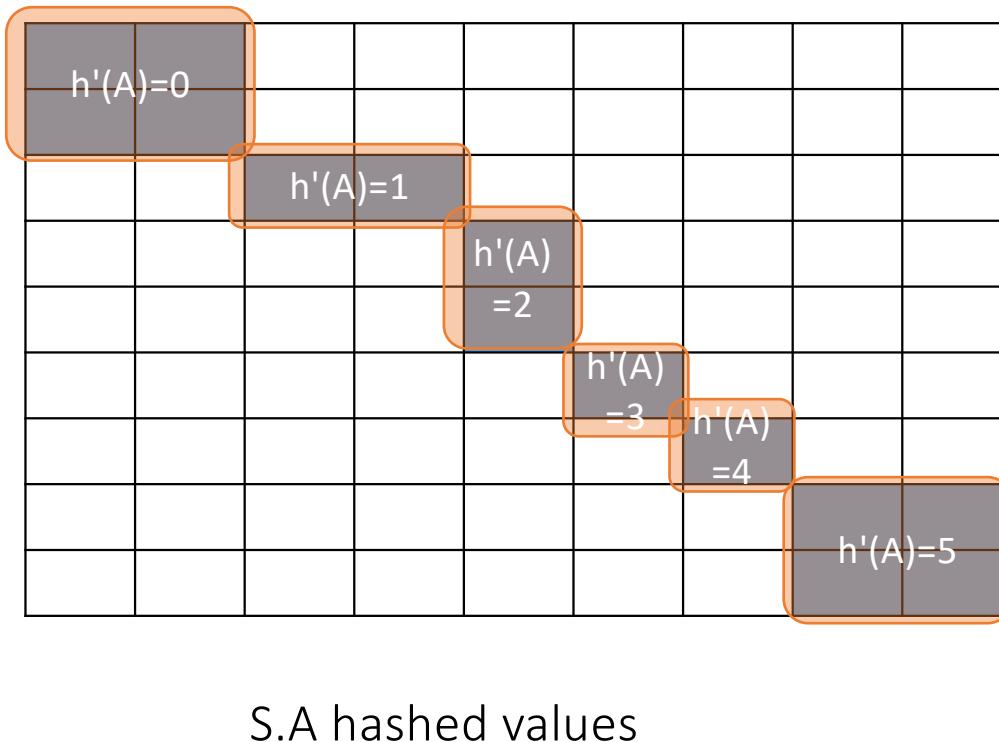
With HJ, we only explore the *blue* regions

= the tuples with same values of $h(A)$!

We can apply BNLJ to each of these regions

Hash Join Phase 2: Matching

R.A
hashed
values



$R \bowtie S \text{ on } A$

An alternative to
applying BNLJ:

We could also hash
again, and keep doing
passes in memory to
reduce further!

How much memory do we need for HJ?

- Given $B+1$ buffer pages + WLOG: Assume $P(R) \leq P(S)$
- Suppose (reasonably) that we can partition into B buckets in 2 passes:
 - For R , we get B buckets of size $\sim P(R)/B$
 - To join these buckets in linear time, we need these buckets to fit in $B-1$ pages, so we have:

$$B - 1 \geq \frac{P(R)}{B} \Rightarrow \sim B^2 \geq P(R)$$

Quadratic relationship
between *smaller*
relation's size & memory!



Hash Join Summary

- *Given enough buffer pages as on previous slide...*
- **Partitioning** requires reading + writing each page of R,S
 - $\rightarrow 2(P(R)+P(S))$ IOs
- **Matching** (with BNLJ) requires reading each page of R,S
 - $\rightarrow P(R) + P(S)$ IOs
- **Writing out results** could be as bad as $P(R)*P(S)$... but probably closer to $P(R)+P(S)$

HJ takes $\sim 3(P(R)+P(S)) + OUT$ IOs!



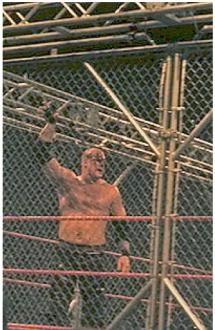
Bonus questions #2



- Q1: Fast little dog.
 - If $\min \{P(R), P(S)\} < B^2$ then HJ takes $3(P(R) + P(S)) + OUT$
 - What is the similar condition to obtain $5(P(R) + P(S)) + OUT$?
 - What is the condition for $(2k+1)(P(R) + P(S)) + OUT$
- Q2: SMJ V. HJ
 - Under what conditions will HJ outperform SMJ?
 - Under what conditions will SMJ outperform SMJ?
 - Size of R, S and # of buffer pages
- Discuss! And We'll put up a google form.

3. The Cage Match

Sort-Merge v. Hash Join



- ***Given enough memory***, both SMJ and HJ have performance:

$$\sim 3(P(R) + P(S)) + OUT$$



- ***“Enough” memory*** =

- SMJ: $B^2 > \max\{P(R), P(S)\}$

- HJ: $B^2 > \min\{P(R), P(S)\}$

Hash Join superior if relation sizes *differ greatly*. Why?

Further Comparisons of Hash and Sort Joins

- Hash Joins are highly parallelizable.



- Sort-Merge less sensitive to data skew and result is sorted



Summary

- Saw IO-aware join algorithms
 - Massive difference
- Memory sizes key in hash versus sort join
 - Hash Join = Little dog (depends on smaller relation)
- Skew is also a major factor
- Message: The database can compute IO costs, and these are different than a traditional system