One dimension quadrotor vertical flight

Yingying Li

January 2022

1 Problem setup

The dynamical model of 1D quadrotor with linear air drag force is approximated as follows.

$$x_{t+1} = \begin{bmatrix} x_{1,t+1} \\ x_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_c \\ 0 & 1-\beta_* \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_* \end{bmatrix} u_t + \begin{bmatrix} 0 \\ -g\Delta_c \end{bmatrix} + \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} \tag{1}$$

where g is the gravitational acceleration, x_1 is altitude, $x_2 = \dot{x}_1$ is velocity, u_t is the total thrust on the vertical direction, Δ_c is time discretization for control computation, $\alpha_* = \Delta_c/m$, where m is mass, $\beta_* = I^a \Delta_c/m$, I^a is the drag force coefficient of aerodynamics (air/wind). Here we approximate the aerodynamics by a linear drag force. The disturbances $w_t = (w_{1,t}, w_{2,t})^\top$ contains time discretization errors and other unmodeled forces, such as unmodeled aerodynamics and unmodeled thrusts.

We consider a dynamical system on the error state and error control inputs, so it has no drifting term from gravitation acceleration.

$$x^e_{t+1} = \begin{bmatrix} x^e_{1,t+1} \\ x^e_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_c \\ 0 & 1-\beta_* \end{bmatrix} \begin{bmatrix} x^e_{1,t} \\ x^e_{2,t} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_* \end{bmatrix} u^e_t + \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix}$$

where $x^e_t = x_t - x^{ref}$, $u^e_t = u_t - u^{ref}$, (x^{ref}, u^{ref}) are a reference equilibrium point.
Claim: The equilibrium to 1 is $x_2^{eq} = 0$, $u^{eq} = g\Delta_c/\alpha_*$, and x_1^{eq} is anything.
Proof: By a direct check.

We consider quadratic cost on the deviation from equilibrium reference point:

$$c(x^e_t, u^e_t) = (x^e_t)^\top Q x^e_t + (u^e_t)^\top R u^e_t$$

2 Parameter choices

Cost parameters $Q = I, R = 1$.

Dynamics parameters

- Mass $m = 1$kg.
- Time discretization for control computation: $\Delta_c = 1$s.
- Air force drag coefficient $I^a = 0.25$kg/s.
- gravitational acceleration $g = 9.8$m/s^2.
- As a result, $\alpha_* = 1$s/kg, $\beta_* = 0.25$ (no unit).
- Equilibrium reference point: $x_1^{ref} = 1$m, $x_2^{ref} = 0$m/s, $u^{ref} = 9.8$N.

Constraints parameters

- Altitude constraint: $0 \leq x_1 \leq 6$m. Hence, altitude error constraint: $-1 \leq x_1^e \leq 5$m.
- Velocity constraint: $-2 \leq x_2 \leq 2$m/s. Hence, velocity error constraint $-2 \leq x_2^e \leq 2$m/s.
- Thrust constraint: $0 \leq u_t \leq 9.8 + 2$N. Hence, thrust error constraint: $-9.8 \leq u_t^e \leq 5$N.

Uncertainty parameters

- Disturbance bound: $|w_{2,t}| \leq 0.2$m/s. $|w_{1,t}| \leq 0.2$m (this is only from time discretization, maybe I can make it smaller, but safe control reformulation can be more difficult).
- Disturbance distribution: each coordinate are i.i.d., and follows Bern with projection to on $[-0.2, 0.2]$
- Uncertainty on α_*: $0.5 \leq \hat{\alpha}^0 \leq 1.2$ (mass's uncertainty is 0.83kg to 2kg. This comes from uncertainty on mass (unknown load)
- Uncertainty on β_*: $0.2 \leq \hat{\beta}^0 \leq 0.4$. This is from aerodynamics unknown drag coefficient and from unknown mass.

3 Plots to make for the paper

I wrote down what our theoretical results imply, in other words, what we hope to observe. However, it is totally okay to observe something different. In that case, let's discuss and see what's going on.

1. System estimation error vs. the number of stages in a pure exploration setting. Change $\bar{\eta}$ to plot mutliple lines.
   Hope to observe: when $\bar{\eta}$ increases, estimation error decreases. When t increases, estimation error decreases.

2. State vs. the number of stages in safe adaptive control, control input vs. the number of stages in safe adaptive control. Plot multiple trajectories. Show all of them satisfy the constraints. **Additional hope:** plot one unconstrained learning trajectory that violate the constraints.

3. Regret vs. the number of stages. One line is with pure exploitation. Another line is without pure exploitation, but $\bar{\eta}^e = 1/(T^e)$ changes with the episode e's length T^e.
   Hope to observe: By letting T^e increases with episode e (your current T^e is the same, so please change it to be $T^e = 50 * 1.2^e$), and observe with pure exploitation's regret is smaller that without pure exploitation's regret.

4. In addition to item 3 above, change $\bar{\eta}$ for with pure exploitation, plot different curves for different $\bar{\eta}$.

   Hope to observe: regret first decrease then increase as $\bar{\eta}$ goes from 0 to very large.