Comilla University
Department of Computer Science and Engineering
2nd year 1st Semester Final Examination-2020
Session: 2018-19
Course Name (Course Code): Algebra, Trigonometry and Matrices (MATH 2108)

Full Marks : 60 Time Allowed: 3 Hours

[Answer any five (5) from the following questions. Figures to the right indicate full marks. Answer each part of the question consecutively. Writing anything in the question is strictly prohibited.]

1\. a. Define function and relation with example. Let the functions f and g on the real numbers R be defined by $f(x) = x^2 + 2x - 3, g(x) = 3x - 4$. Find the value of gof(2). [2+2+2=6]

b. If A=R \ $\{-\frac{1}{2}\}$ and B=R\ $\{\frac{1}{2}\}$ two non-empty sets. A function f: A→ B which is defined as $f(x) = \frac{x-3}{2x+1}$. Prove that f(x) is one-one, onto. Find also inverse of f(x). [4]

c. Show that $f(x) = \ln(x + \sqrt{1+x^2})$ is an odd function. [2]

2\. a. Prove that every equation of degree n has exactly n roots. [5]

b. If A and B are any two sets then prove that $(A \cup B)^c = A^c \cap B^c$ [4]

12 c. If a, b, c are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of (b + c) (c + a) (a + b) in p, q and r. [3]

3\. a. What do you mean by symmetric functions of the roots of f(x)= 0. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2, \sum \alpha^2 \beta$ [2+2+2=6]

12 b. Find the sum of the fourth powers and six powers of the roots of the equation [6]

$$x^4 - x^3 - 7x^2 + x + 6 = 0$$

4\. a. State and prove the Descartes's rule of sign theorem. [6]

b. Define complex number with graphical representation. Find the nature of the roots of the equation $6x^4 - 25x^3 + 81x^2 - 9x - 13 = 0$ [3+3=6]

5\. a. State and prove the De Moivre's Theorem [7]

12 b. Show that in the Argand's plane a variable Point P ($\equiv$ z) subject to the condition |z+1| +|z-1|=3 describe an ellipse. [5]

6\. a. Define vector space. If S and T are subspace of a finite dimensional vector space V over the field F then prove that dim(S+T) =dim S+ dim T -dim (S∩T) [1+6=7]

b. Define hyperbolic function and Gregory's series. Show that [1+2+2=5]

$$\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \text{------------}$$

7. a. Define line integral where $\bar{r}$ is position vector of a point. If $\bar{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} - 20xz^2\hat{k}$ [3+3

Evaluate $\int_c \bar{A}.d\bar{r}$ from (0,0,0) to (1,1,1) along the following paths c

i) $x=t,\ y=t^2,\ z=t^3$

ii) The straight line joining (0,0,0) to (1,0,0) then (1,1,0) and then to (1,1,1).

b. Find the angle between the surface $z=x^2 + y^2$ and $z = (x - \frac{\sqrt{6}}{6})^2 - (y - \frac{\sqrt{6}}{6})^2$ at the point [

$P = (\frac{\sqrt{6}}{12}, \frac{\sqrt{6}}{12}, \frac{1}{12})$

c. Prove that $\nabla^2 \left(\frac{1}{r}\right) = 0$

8. a. State and Prove Green's theorem in the plane.

b. Use Stokes theorem to evaluate $\oint_c \bar{F}.d\bar{r}$, if $\bar{F} = xz\hat{\imath} + xy\hat{\jmath} + 3xz\hat{k}$ and c in the boundary of the portion of the plane 2x+y+z=2 in the first octant, traversed counterclockwise as viewed from above.