

COMILLA UNIVERSITY

Faculty of Engineering

Department of Computer Science & Engineering

2<sup>nd</sup> Year 1<sup>st</sup> Semester Final Examination '14, Session 2012-2013

Course Code: MATH-215

Course Title: Algebra, Trigonometry and Matrices

Full Marks: 60

Time: 3 Hours

Answer any five questions.

N. B. Figures in the right margin indicate marks

1. (a) Define with examples: union and intersection of two sets; difference of two sets and complement of a set. [4]  
(b) Prove that  $\overline{A \cap B} = \overline{A} \cup \overline{B}$  [4]  
(c) Draw the Venn diagram for each of the combinations of the sets A, B and C: [4]  
i)  $A \cap (B \cup C)$ , ii)  $\overline{A} \cap \overline{B} \cap \overline{C}$
2. (a) Define function. Using example show that every function is a relation but the converse is not true. [4]  
(b) Is  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = 1/x$  onto? [3]  
(c) Let  $f(x) = \frac{2}{\sqrt{x}}$ ,  $x, y > 0$ . Find the inverse of  $f$  and also state the domains and ranges of  $f$  and  $f^{-1}$ . [5]
3. (a) State and prove De-Moivre's theorem. [6]  
(b) Solve the equation  $x^7 + x^4 + x^3 + 1 = 0$  with the help of De-Moivre's theorem. [6]
4. (a) Represent graphically the set of values of  $z$  for which [6]  
a)  $\left| \frac{z-3}{z+3} \right| = 2$  b)  $\left| \frac{z-3}{z+3} \right| < 2$   
(b) Prove that if the ratio  $\frac{z-i}{z-1}$  is purely imaginary, then the point  $z$  lies on the circle whose centre is the point  $\frac{1}{2}(1+i)$  and radius  $\frac{1}{\sqrt{2}}$ . [6]
5. (a) Separate into real and imaginary parts  $\tan^{-1}(x+iy)$ . [6]  
(b) Express  $\cosh^5 \theta$  in terms of hyperbolic cosines of the multiple of  $\theta$ . [6]

6. (a) Describe the meaning of summation of series of n-terms. Also sum to n-terms of the series: [6]

$$\frac{\sin \theta}{\cos \theta + \cos 2\theta} + \frac{\sin \theta}{\cos \theta + \cos 4\theta} + \frac{\sin \theta}{\cos \theta + \cos 6\theta} + \dots$$

(b) Find the sum of the following series: [6]

$$\cos 2\theta + 2\cos 3\theta + 3\cos 4\theta + \dots + n\cos(n+1)\theta$$

7. (a) Let  $f(x) = -4x^7 + x^3 - x^2 + 2$ . Tell the maximum number of real zeroes. Use Descartes rule of signs to determine how many positive, negative and complex zeroes. [4]

(b) Find the multiple roots of  $x^4 - 2x^3 + 2x - 1 = 0$  and thus solve the equation. [4]

(c) Find the equation whose roots are those of  $x^4 + 10x^3 + 39x^2 + 76x + 65 = 0$  each increased by 4. [4]

8. (a) Find the inverse of the matrix: [6]

$$\bar{A} = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

(b) Define rank of a matrix. Find the rank of the matrix [6]

$$\bar{B} = \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$