Comilla University

Department of CSE

2^{nd} year 2^{nd} Semester Final Examination-2019

Course Code: CSE-2205

Course Title: Complex variable and Geometry

Session: 2017-18 Time: 03 Hours Full Marks: 60

[Answer any **five** from the following questions:]

1. a) Define complex number with example. Describe geometrically the region of the following function: $\left|\frac{z-i}{z+1}\right| = 2$. **4** 9

   b) For any complex number z_1, z_2 prove that $|z_1 + z_2| \le |z_1| + |z_2|$ **5**

   c) Find the modulus and general argument and principle argument of the following complex number: $\frac{-2}{1+i\sqrt{3}}$. **3**

2. a) State the Cauchy-Riemann equations. For the function, $f(z)$ define by **6**

$$f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & when\ z \neq 0 \\ 0, & when\ \ z = 0 \end{cases}$$ 10

   Show that the Cauchy Riemann equations are satisfied at (0, 0) but the function is not differentiable at the origin.

   b) Define harmonic function. Show that $U = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic function hence find its harmonic conjugate V such that, $f(z) = U + iV$ is analytic. **6**

3. a) State Cauchy's Residue theorem. Find the residues of the function **6** 9

$$f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$$

   b) Show that $\int_B \frac{\sin 3z}{z+\frac{\pi}{2}} dz = 2\pi i$, where B is the circle $|z| = 5$. **6**

4. a) Expand $f(z) = \sin z$ in a Taylor's series about $z = \pi/4$. **5**

   b) State Laurent series. Expand the function $f(z) = \frac{1}{(z^2+1)(z+2)}$ in a laurent series for the region $1 < |z| < 2$. **7**

5. Using the method of contour integration, evaluate

   a) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ $\frac{2\pi}{\sqrt{3}}$ b) $\int_0^{2\pi} \frac{d\theta}{3+2\sin\theta}$ $\frac{2\pi}{\sqrt{5}}$ **6** 10 **6**

6. a) Find the condition that the general equation of the second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represents a pair of straight line. **6**

b) Show that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines if $a:h = h:b = g:f$. Also, show that the distance between them is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$ 6

7. a) Reduce the equation $16x^2 - 24xy + 9y^2 - 104x - 172y - 44 = 0$ to the standard form. Also find its all properties. 6 2/1

b) Find the equation of the circle passing through the three points $(-3, 2), (1, 7)$ and $(5, -3)$. 6 4

$-62/61$

8. a) Transform the equation $17x^2 + 8xy - 7y^2 - 16x - 32y - 18 = 0$ to one in which there is no term involving x, y *and* xy, both sets of axes being rectangular. 6

b) Show that $25x^2 + 2xy + 25y^2 - 130x - 130y + 169 = 0$ represent an ellipse referred to rectangular Cartesian coordinates (x, y). 6

$\frac{x^3 - 3xy^2}{x^2+y^2}$ $\frac{y^3 - 3x^2y}{x^2+y^2}$