Introduction to Digital Image Processing

— Image Transform

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Image Translation

- The translation maps each pixel in an input image into a new position in the output image
  - The contents of the image remain the same.
- Given the displacements (Δx, Δy), the coordinates of pixel (x_0, y_0) is transformed to (x_1, y_1) as follows:

$$x_1 = x_0 + \Delta x \qquad\qquad \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$
$$y_1 = y_0 + \Delta y \quad OR$$

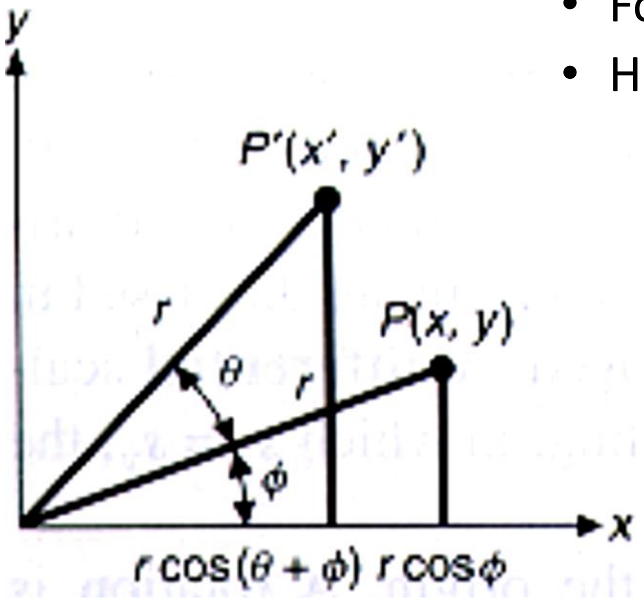
- Note that the pixel value is not changed in the transformation.
- All pixels in an image are translated in the same amount

Image Rotation

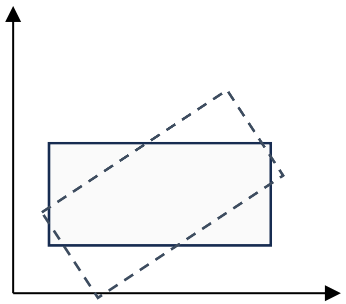
- Under rotation, pixel (x_0, y_0) in an image is relocated to a new position (x_1, y_1) in the output image by an angle θ about the origin as follows:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$



- For triangle ABP, we have $\cos(\phi) = x/r$ and $sin(\phi) = y/r$
- For triangle ABP’, we have $\cos(\theta + \phi) = x'/r$ and $sin(\theta + \phi) = y'/r$
- Hence, we have

$$x = r\cos(\phi)\,, y = rsin(\phi)$$
$$x' = r\cos(\theta + \phi) = rcos(\phi)cos(\theta) - rsin(\phi)\sin(\theta)$$
$$y' = rsin(\theta + \phi) = rcos(\phi)\sin(\theta) + rsin(\phi)\cos(\theta)$$



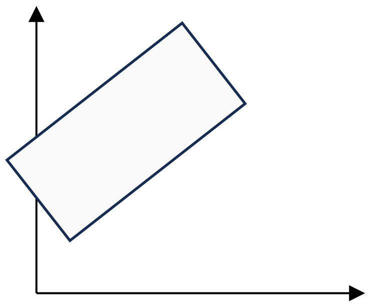


Image Scaling

- Image scaling changes the size of the object by multiplying the coordinates of the points by scaling factors.
- With scaling, a pixel (x0, y0) in an image is relocated to a new position (x1, y1) in the output image by displacing it through a user-specified scaling factor s as follows:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = s \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

- A non-uniform scaling can be achieved with

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

- The gaps (pixels without a value) between the pixels in the output image need to be filled in with interpolation.

homogeneous coordinates

- Add one additional coordinate w, i.e.,

$$(\mathrm{x}', \mathrm{y}') \rightarrow (x, y, w), \;\; w \neq 0$$

- Recover (x’, y’) by homogenizing (x, y, w) as follows:

$$x' = \frac{x}{w}, \qquad y' = \frac{y}{w}$$

- So, we have x=x’w, y=y’w,

$$(\mathrm{x}', \mathrm{y}') \rightarrow (x'w, y'w, w)$$

- Translation in homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x' + dx \\ y' + dy \\ 1 \end{bmatrix}$$

homogeneous coordinates

- Scaling

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_s & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x' \\ s_y y' \\ 1 \end{bmatrix}$$

- Rotation

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x' \cos(\theta) - y' sin(\theta) \\ x' \sin(\theta) + y' cos(\theta) \\ 1 \end{bmatrix}$$

Successive Transformation

- Successive translations

$$\begin{bmatrix}1 & 0 & dx\\0 & 1 & dy\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & dx'\\0 & 1 & dy'\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & dx+dx'\\0 & 1 & dy+dy'\\0 & 0 & 1\end{bmatrix}$$

- Successive scaling

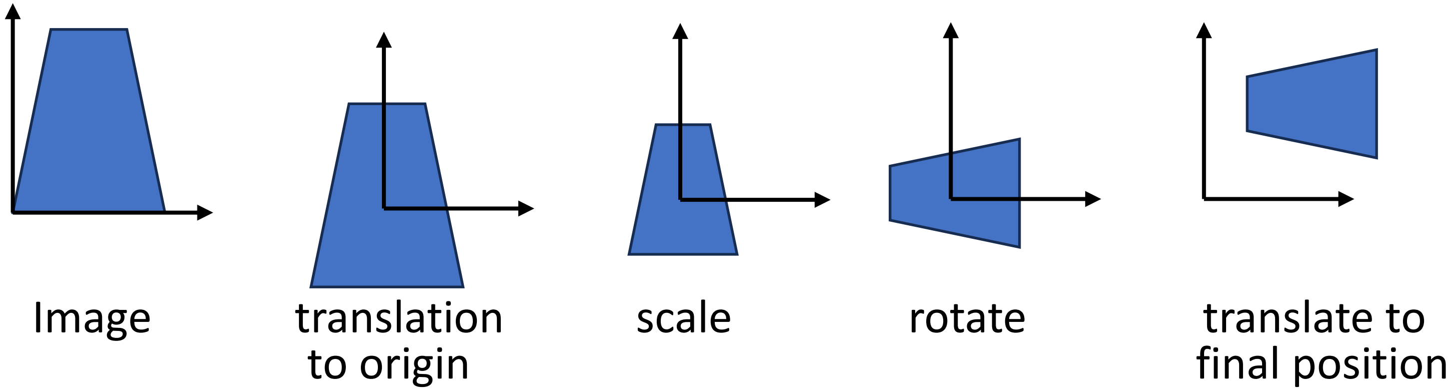
$$\begin{bmatrix}s_{x'} & 0 & 0\\0 & s_{y'} & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}s_x & 0 & 0\\0 & s_y & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}s_x s_{x'} & 0 & 0\\0 & s_y s_{y'} & 0\\0 & 0 & 1\end{bmatrix}$$

- Successive rotations

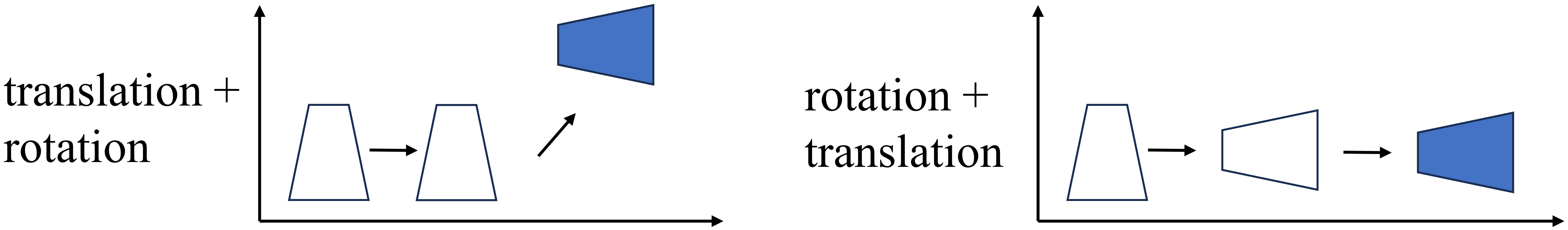
$$\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\\sin(\theta) & \cos(\theta) & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\phi) & -\sin(\phi) & 0\\\sin(\phi) & \cos(\phi) & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & 0\\\sin(\theta+\phi) & \cos(\theta+\phi) & 0\\0 & 0 & 1\end{bmatrix}$$

Composition of Transformations

- The transformation matrices of a series of transformations can be concatenated into a single transformation matrix.
- Example:
  - Translate to origin
  - Perform scaling and rotation
  - Translate to location P_2



- Important: preserve the order of transformations!



General form of transformation

- The general form of transformation

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \text{Rotation and scale} & \text{translation} \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

- Representing a sequence of transformations as a single transformation matrix

$$x = a_{11}x' + a_{12}y' + a_{13}$$
$$y = a_{21}x' + a_{22}y' + a_{23}$$

Special cases of transformations

- Rigid transformations: involve translation and rotation
  - Preserve angles and lengths
- Similarity transformations: involve rotation, translation, and <u>scaling</u>
  - Preserve angles but not lengths
- Affine transformations: involve translation, rotation, scaling, and <u>shear</u>
  - Preserve parallelism of lines but <u>not</u> lengths or angles.
  - Shearing along x-axis: $x = x' + ay, y = y'$ or $\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
  - Shearing along y-axis: $x = x', y = y' + by$ or $\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Image Registration

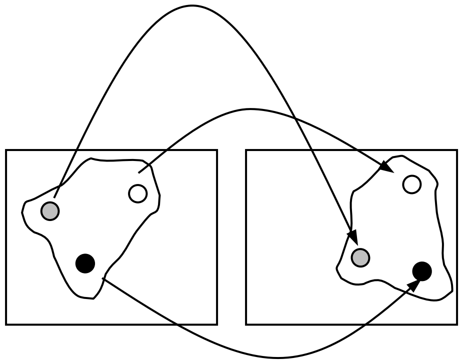
- Give two images I_1 and I_2, image registration finds transformation functions f and g such that after applying these functions to image I_1 the transformed image matches I_2 without error

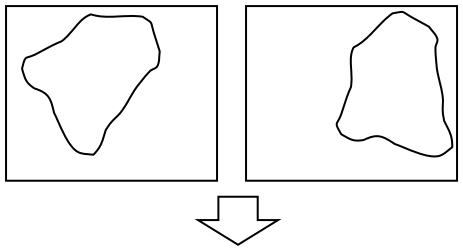
$$I_2(x, y) = g(I_1(f(x, y)))$$

where f is a 2D spatial transformation function and g is a 1D intensity transformation function

Registration Approaches

- Control point-based registration
  - Fast
  - Any registration problem
  - ➢User operation is needed
  - ➢Accuracy relies on experience
- Content-based registration
  - Automatic process
  - Performance is consistent
  - ➢High computational expense





Similarity measurements