Introduction to Digital Image Processing

— Morphological Operations

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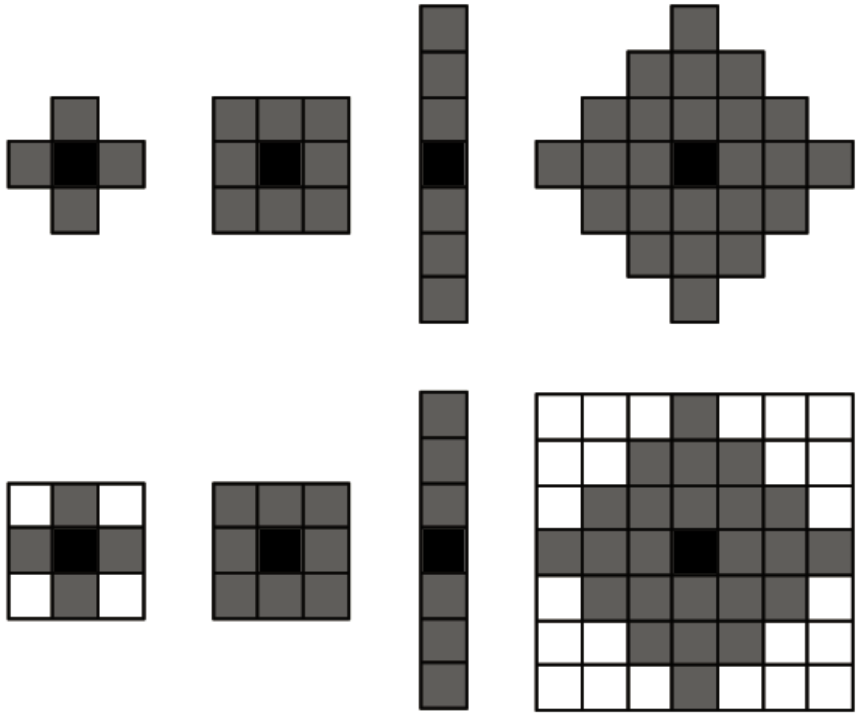
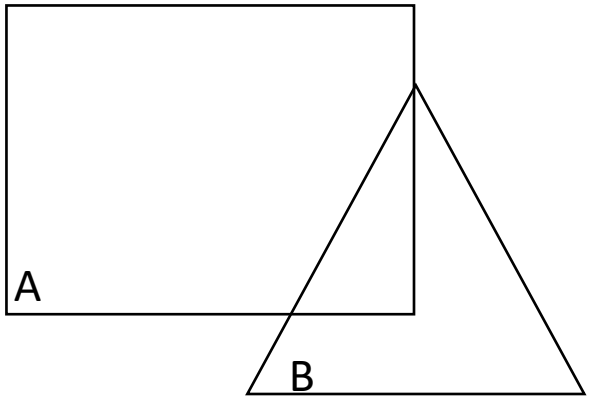
Morphological operations and Structuring Element

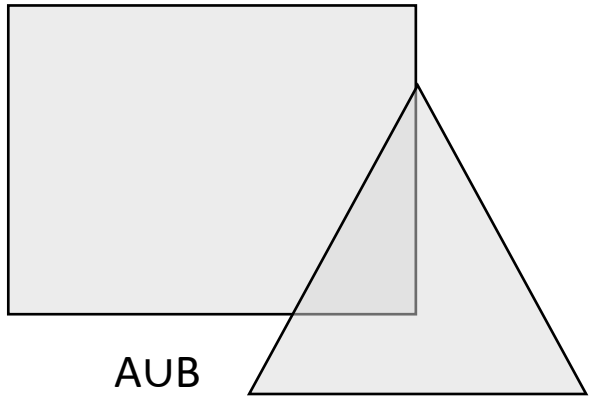
- Morphological operations are used to extract image components for the representation and description of image regions, such as boundaries and skeletons.
  - Typical operations include Erosion, Dilation, Opening, Closing, Hit-or-miss transformation, and Thinning, Thickening

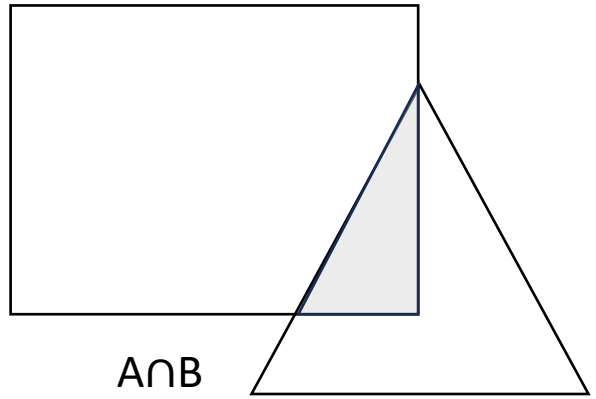
- **Structuring Elements (SE)** contain 0, 1 and (or) Not-a-Number (NaN)
  - NaNs in an SE that does not involve computing
- Every Structuring Element has an “origin”
  - The computing result is applied to the origin
- Shape and size can be adapted to geometric properties of the objects
- Examples of Structuring Elements

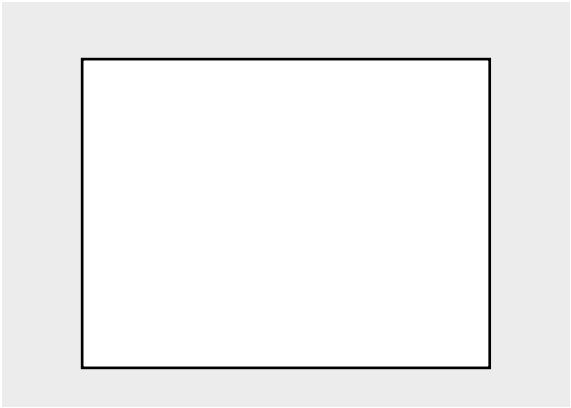


Basic Set Operations

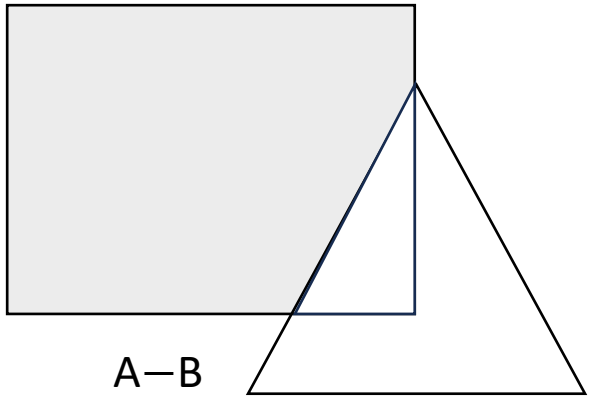








$(A)^c$

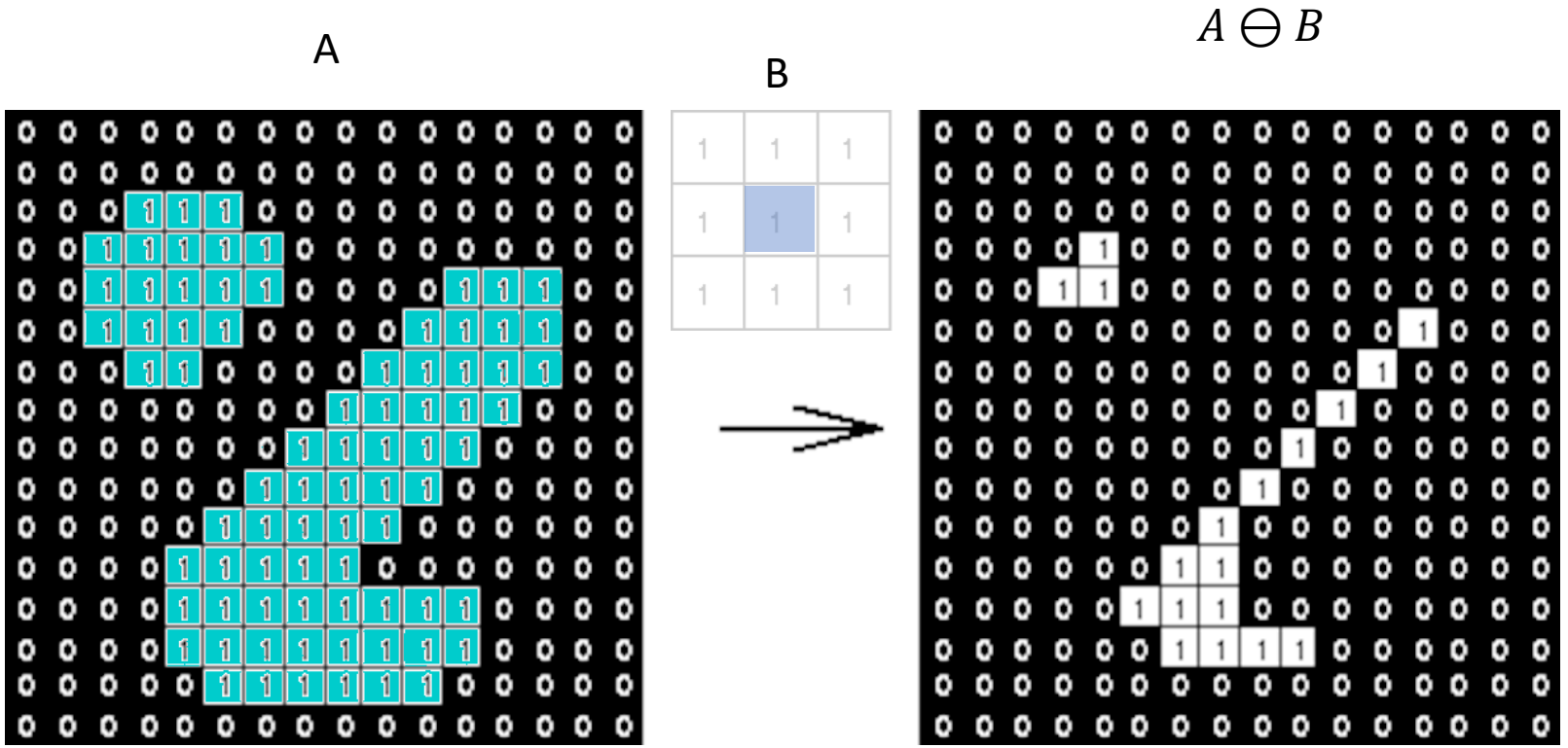


Erosion

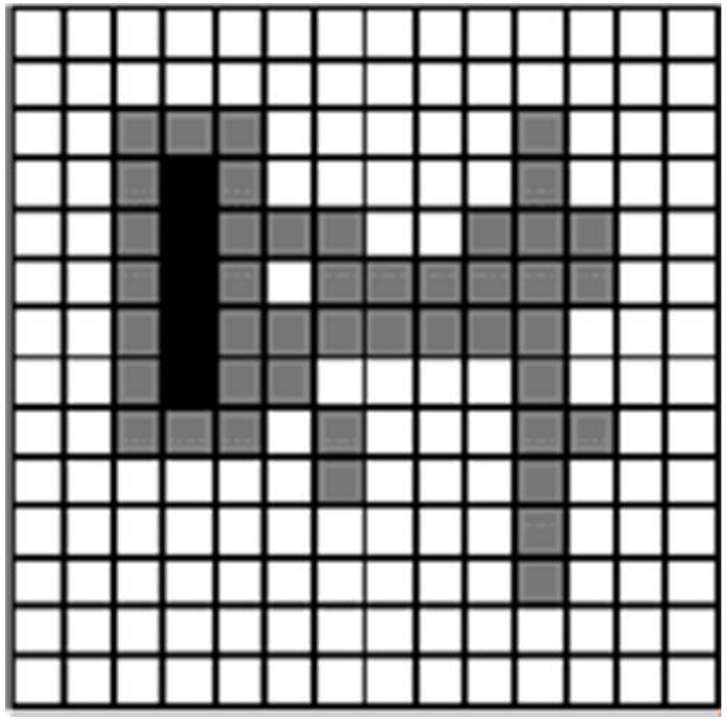
- With an image A and an SE B, the erosion of A by B, denoted by $A \ominus B$, is defined as

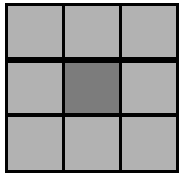
$$A \ominus B = \{z|(B)_z \subseteq A\}$$

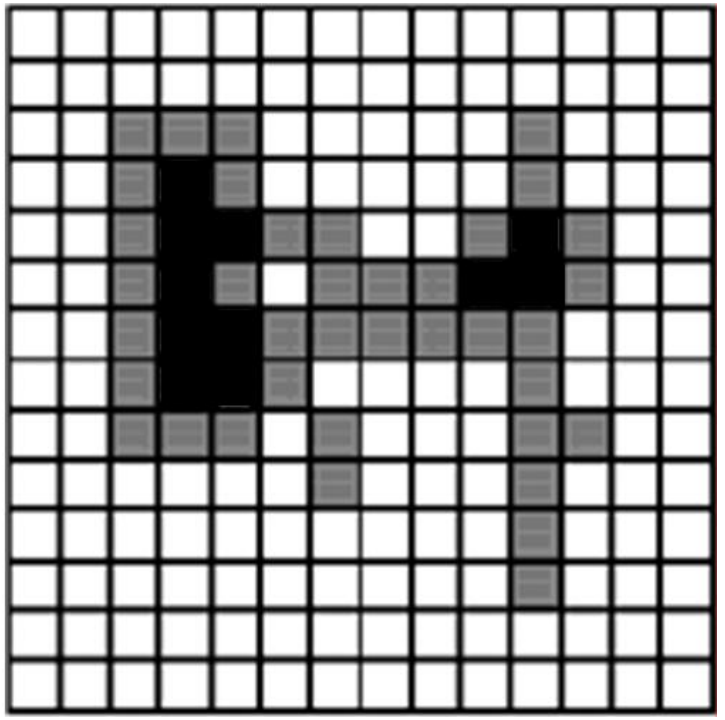
z is an element of A

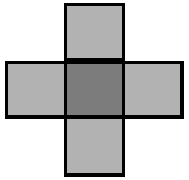


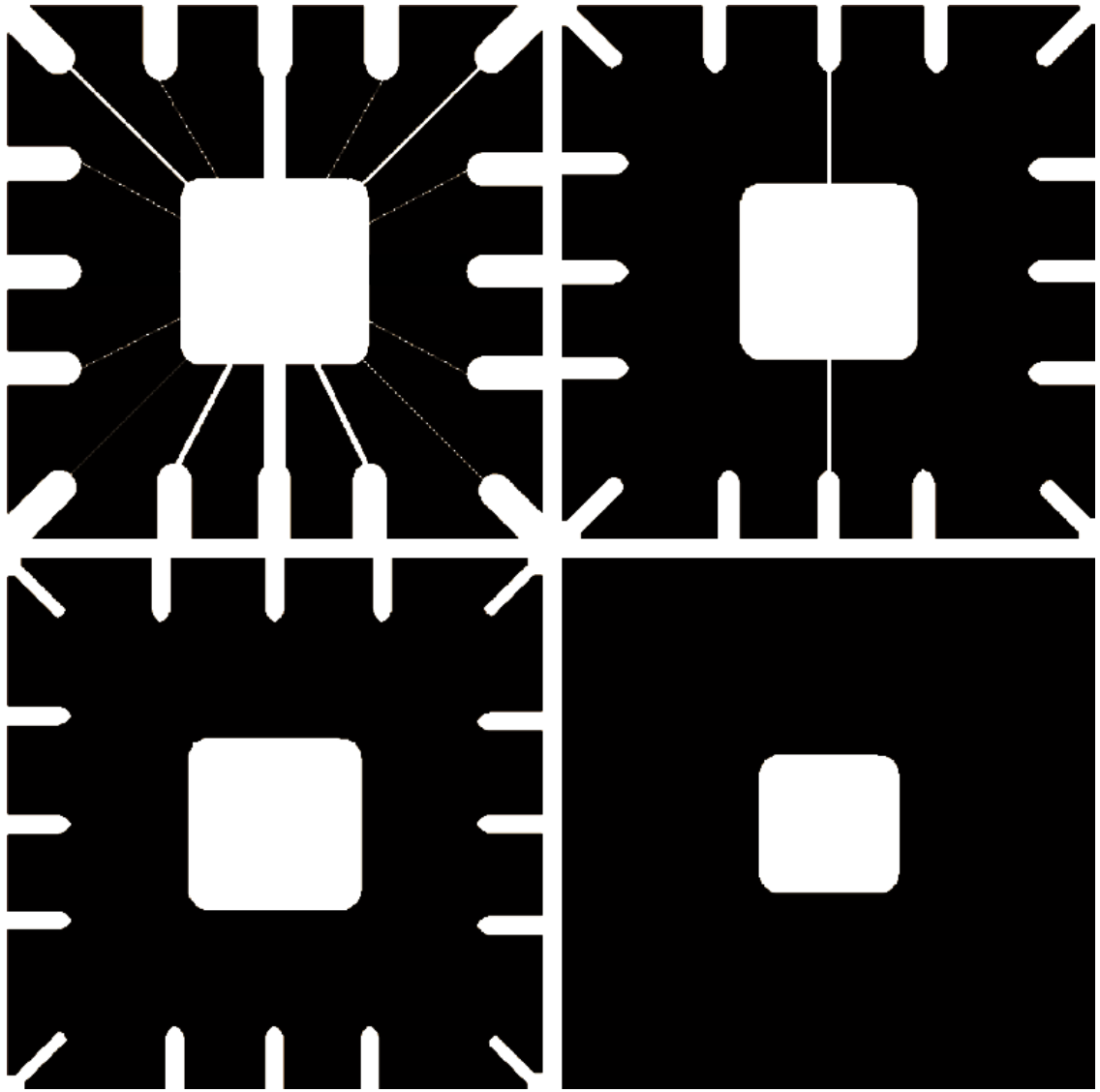
Examples of Erosion



SE= 



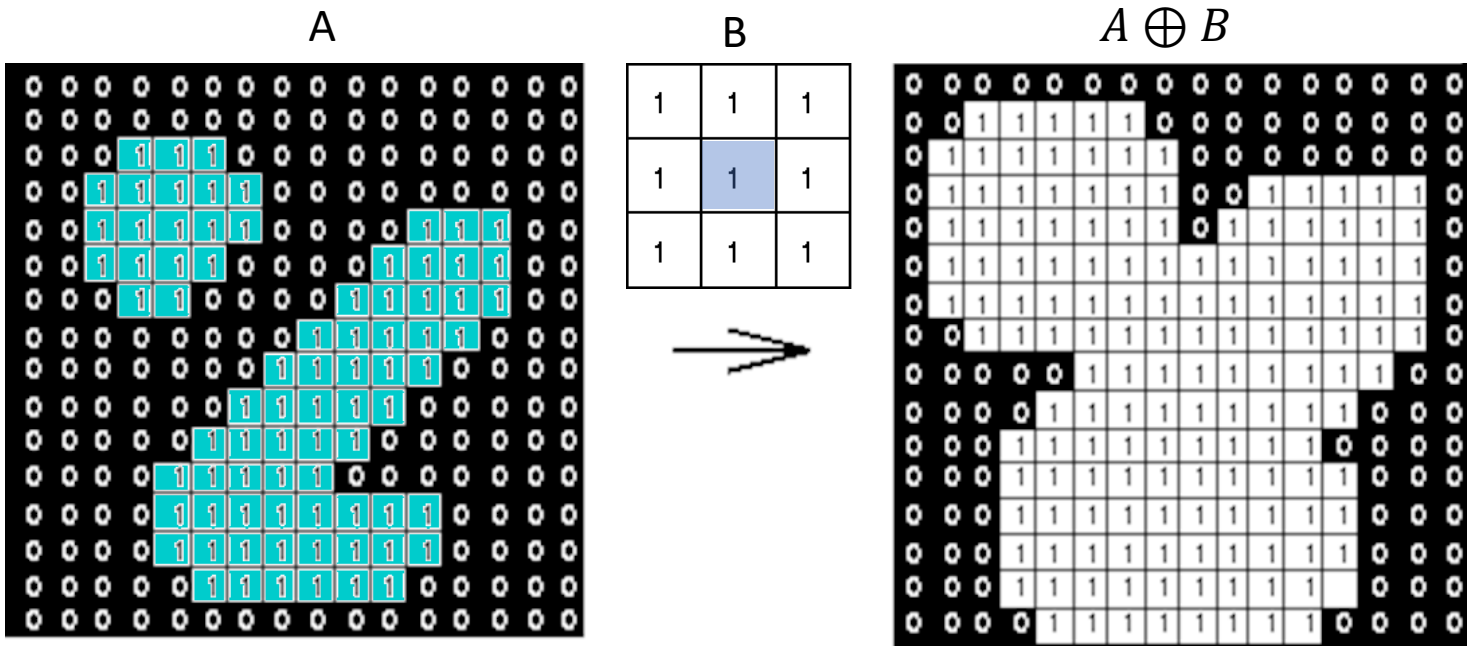
SE= 



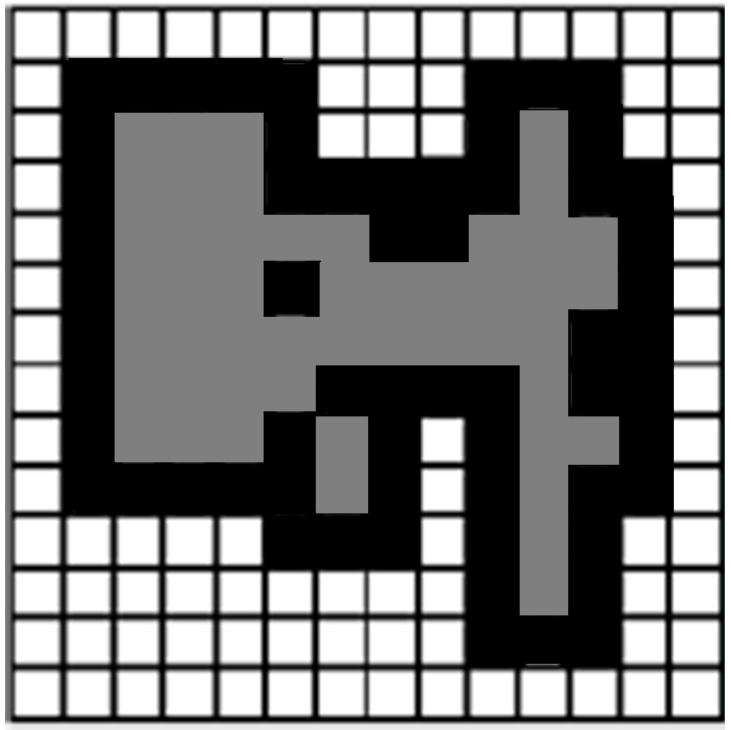
Dilation

- With an image A and a structuring element B, the dilation of A by B, denoted by $A \oplus B$, is defined as

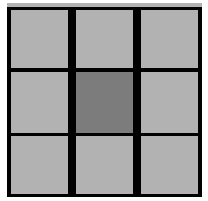
$$A \oplus B = \{z | (B)_z \cap A \neq \emptyset\}$$

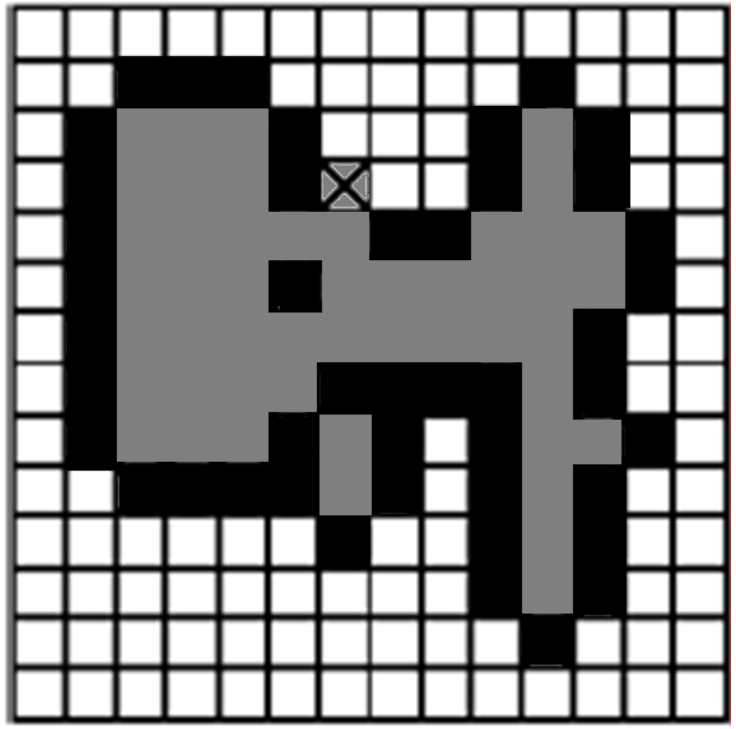


Dilation

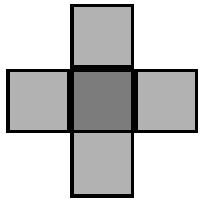


SE=

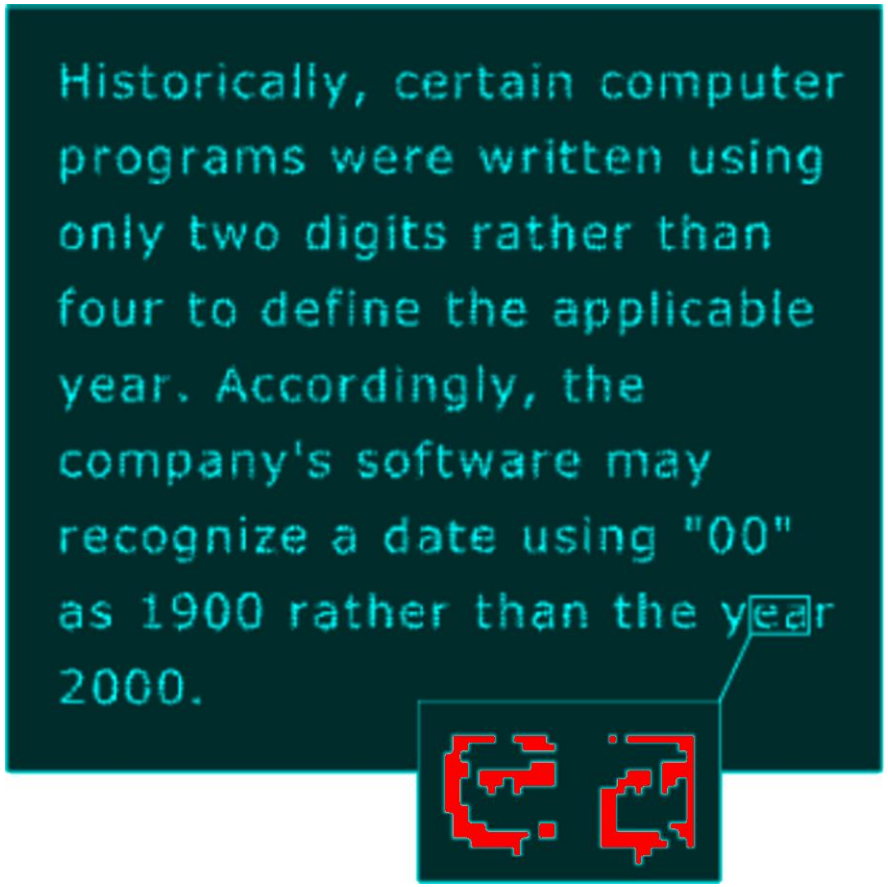


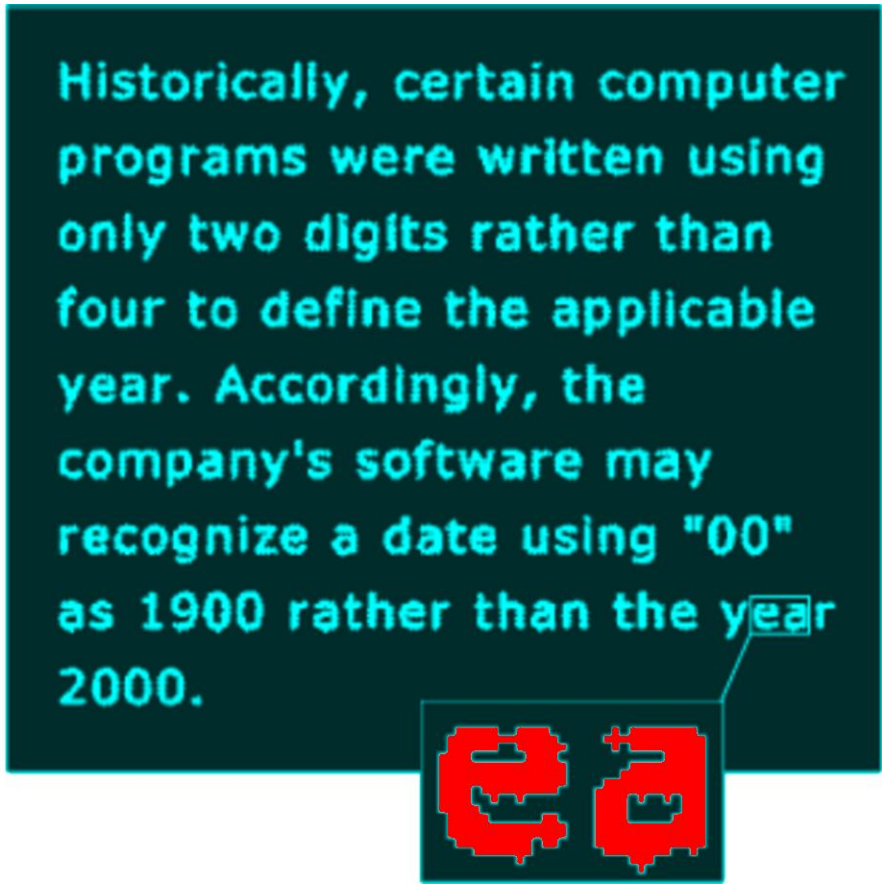


SE=



Using Dilation to Bridging Gaps



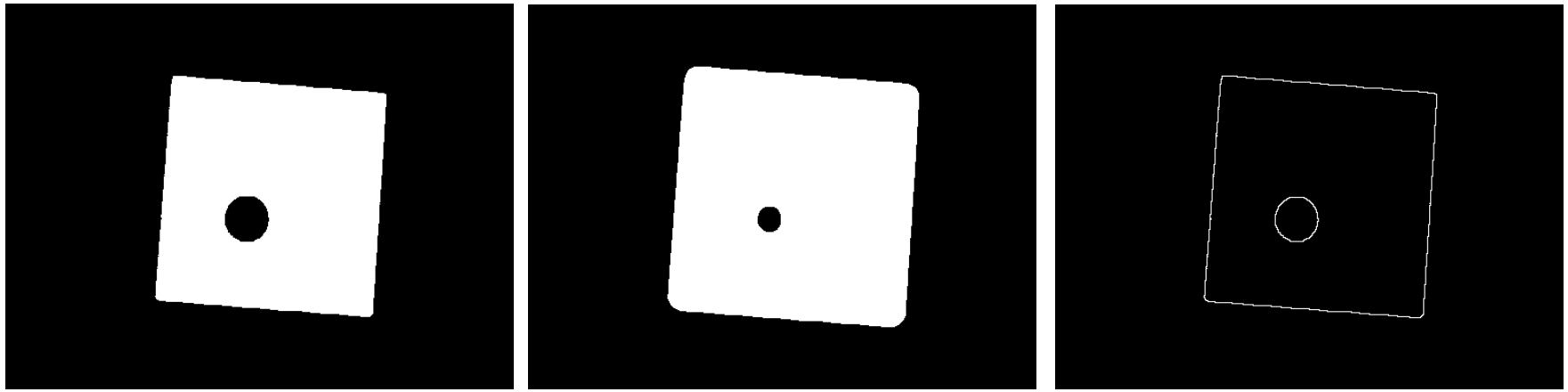


0	1	0
1	1	1
0	1	0

Edge Detection

Using dilation and image subtraction

1. Dilate the original image
2. Subtract the original image from the dilated version



Opening

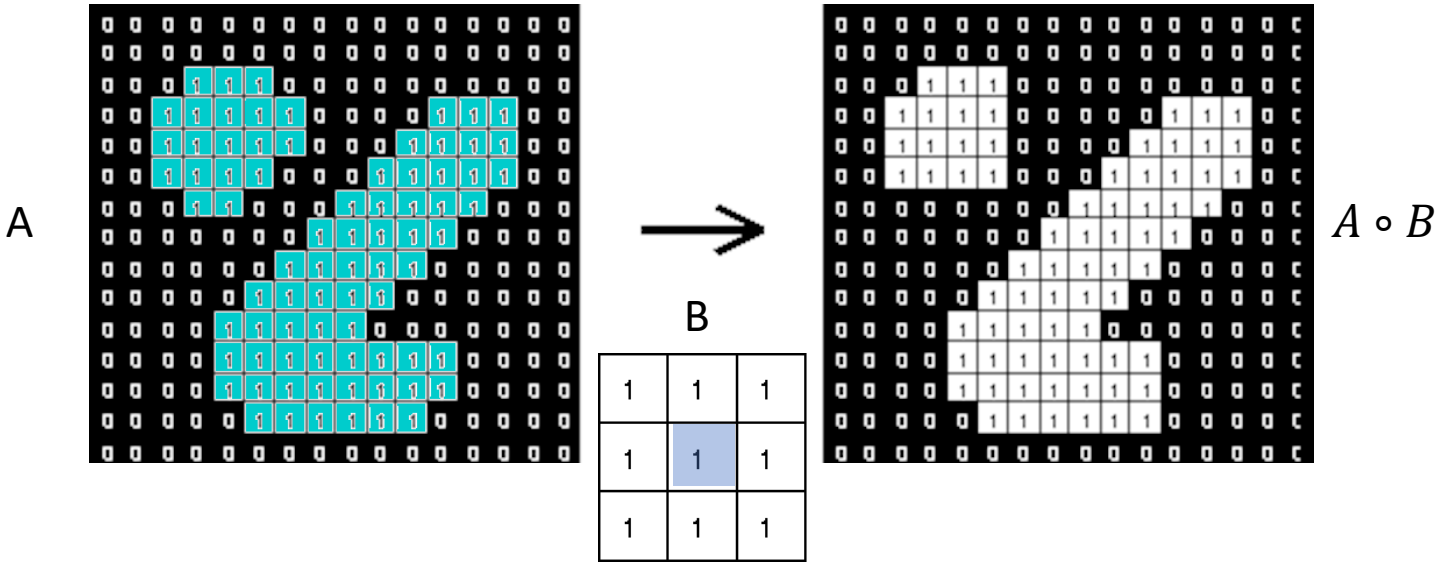
- Opening is achieved by erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

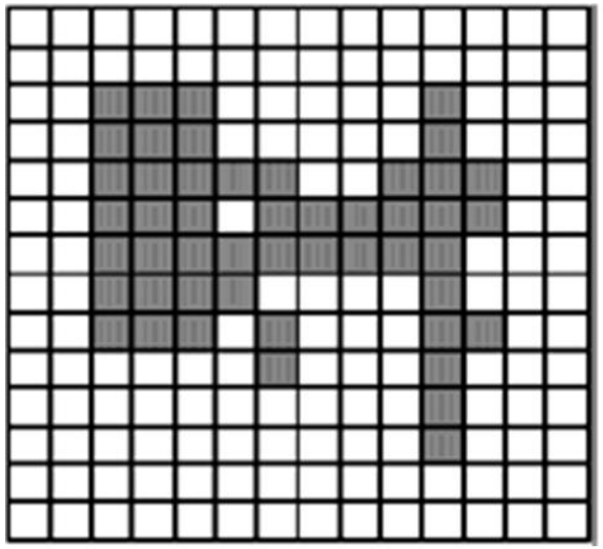
- Opening is idempotent, i.e., repeated application has no further effects:

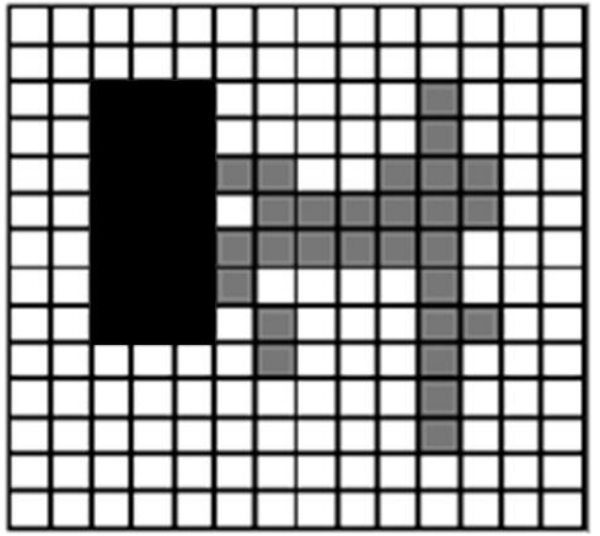
$$(A \circ B) \circ B = A \circ B$$

- It is used to eliminate protrusions and break connections.

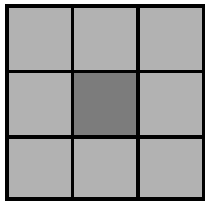


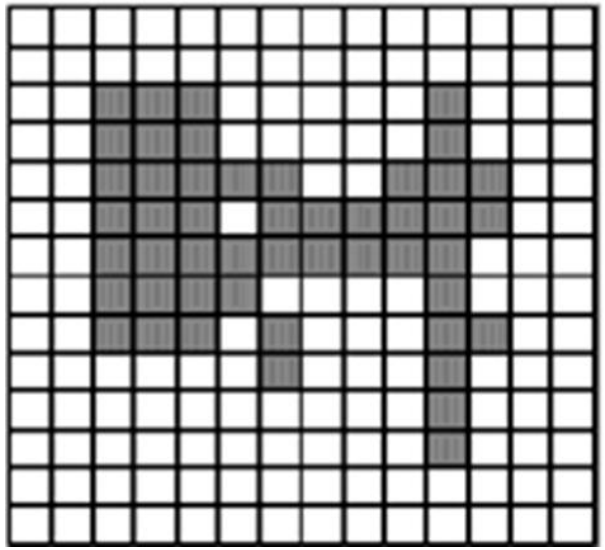
Opening

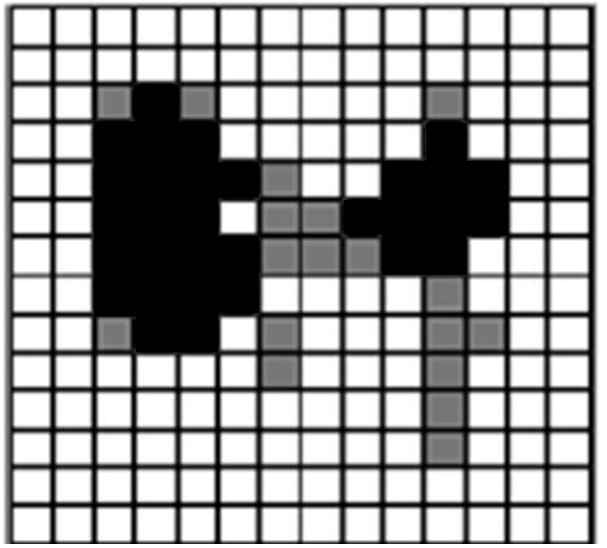




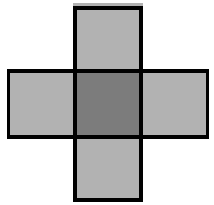
B=





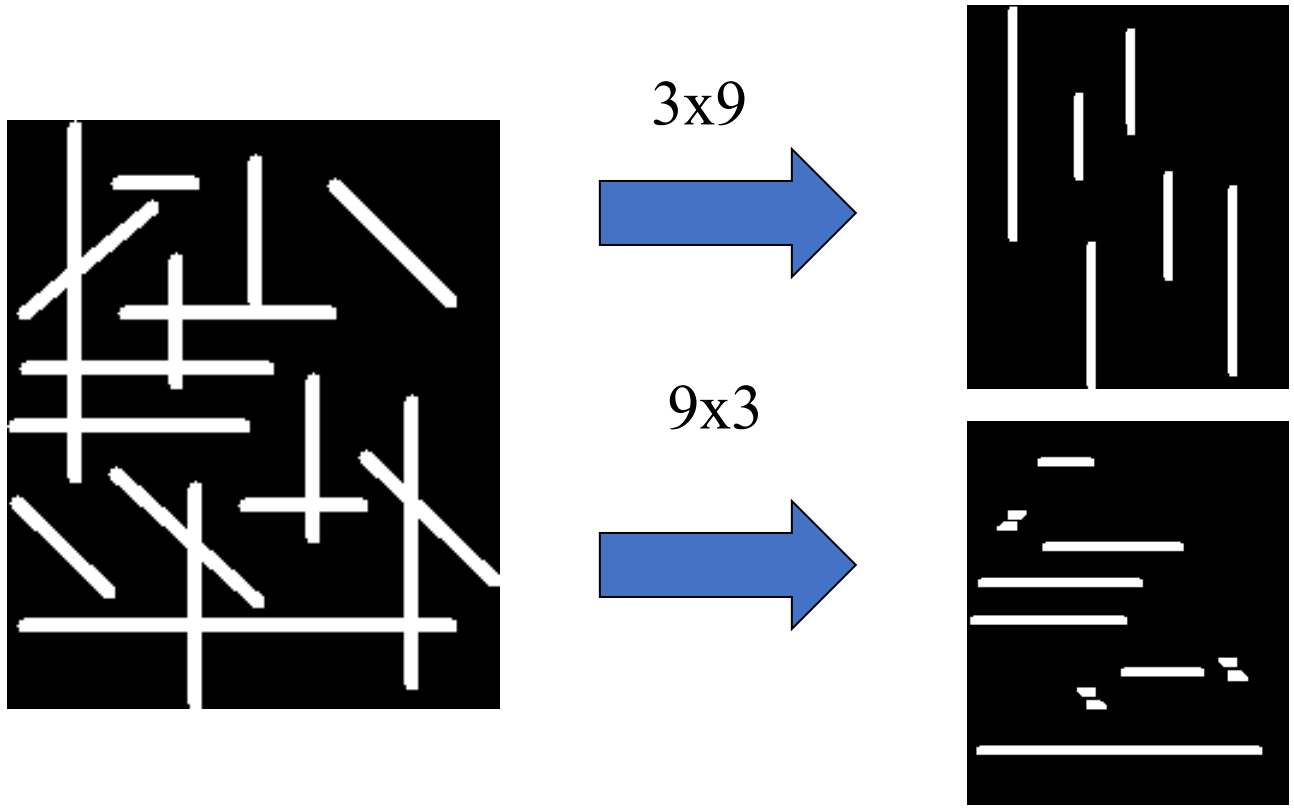


B=



Opening Example

- 3x9 and 9x3 Structuring Element



Closing

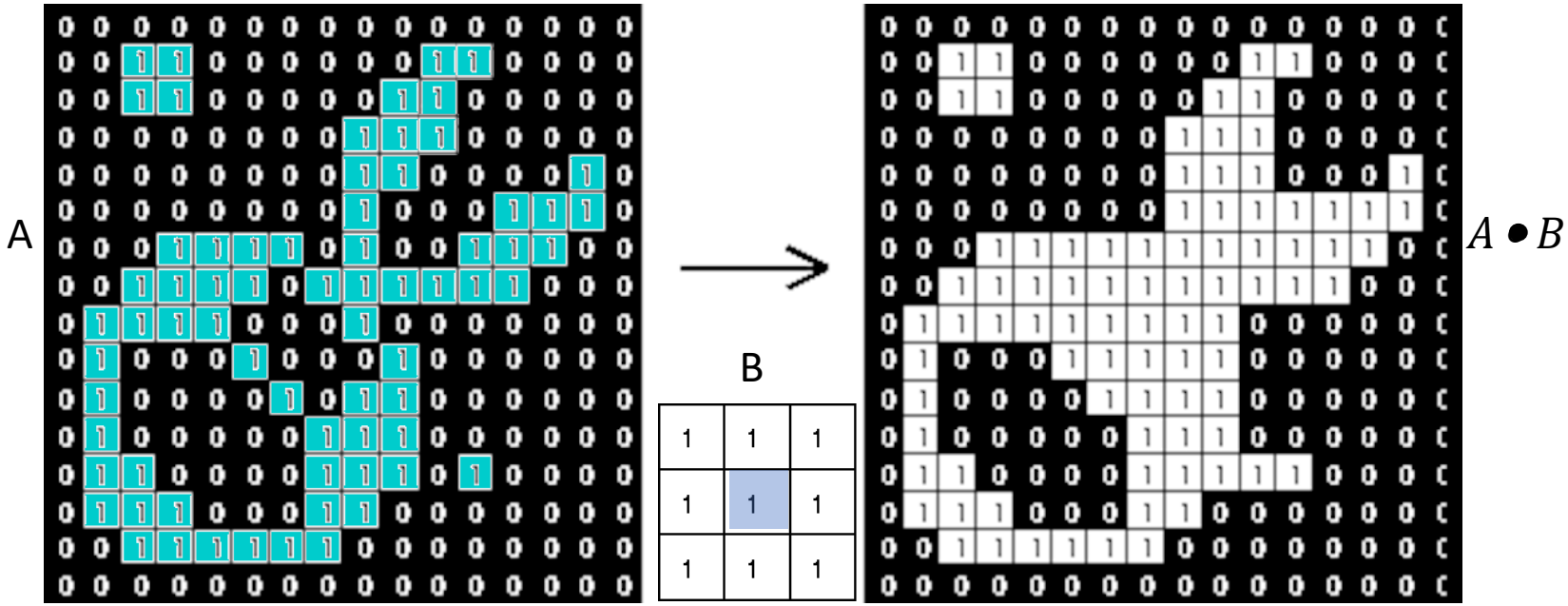
- Take the structuring element (SE) and slide it around *outside* each foreground region.
  - All background pixels that can be covered by the SE with the SE being entirely within the background region will be preserved.
  - All background pixels that cannot be reached by the structuring element without lapping over the edge of the foreground object will be turned into a foreground.
- Closing is **idempotent:** Repeated application has no further effects!

Closing

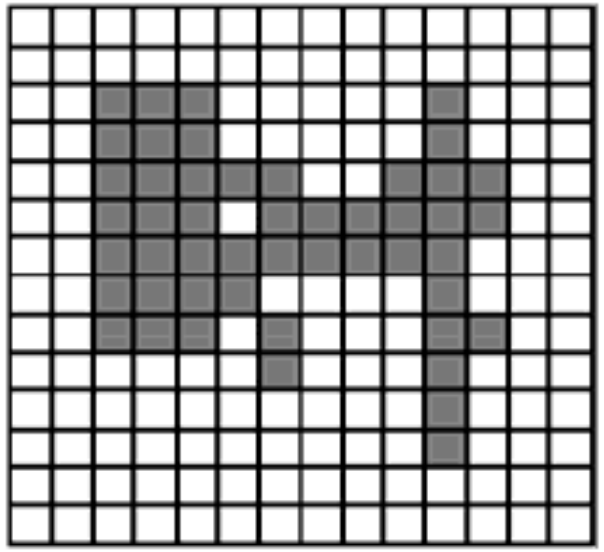
- Closing is achieved by dilation followed by erosion

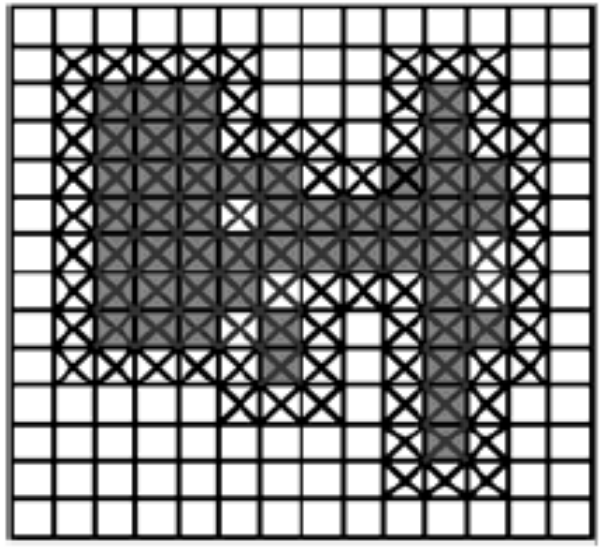
$$A \bullet B = (A \oplus B) \ominus B$$

- It is also idempotent: $(A \bullet B) \bullet B = A \bullet B$
- It is used to fuse narrow gaps and eliminate small holes

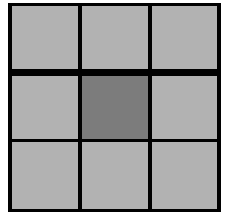


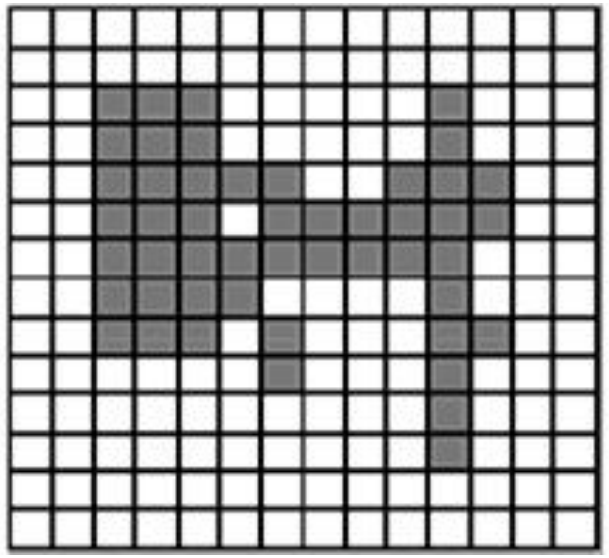
Closing Examples

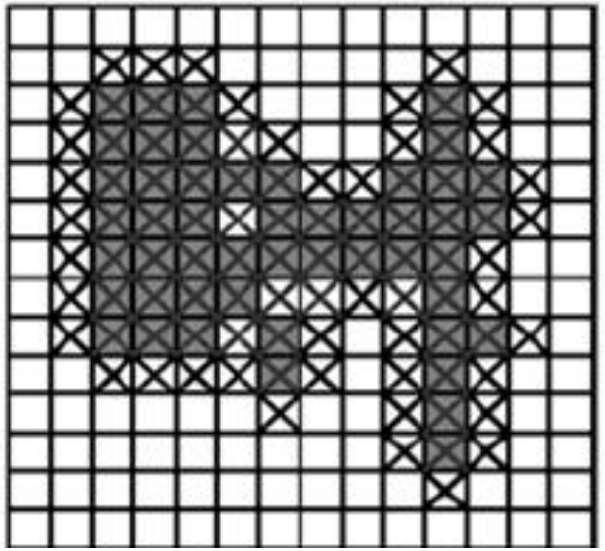




B=







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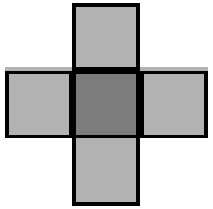
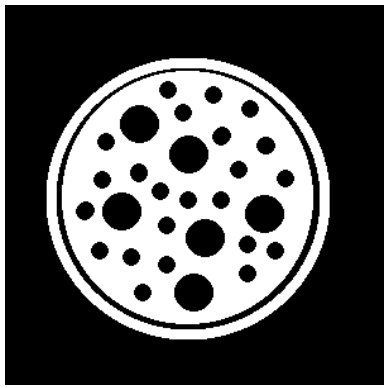
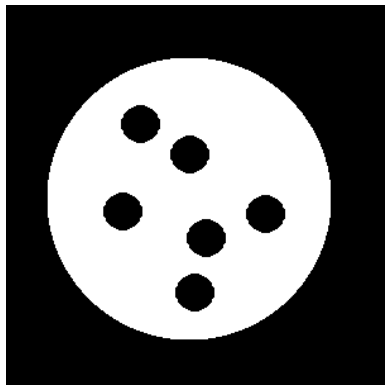


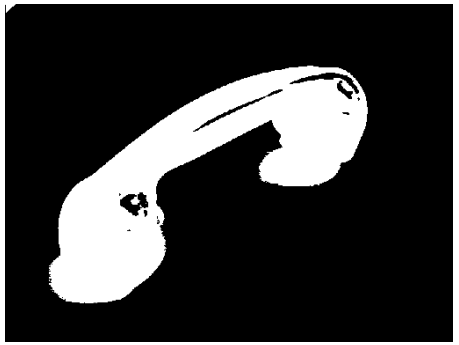
Closing Examples

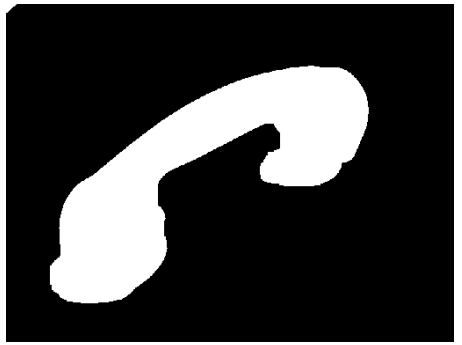
SE: disc
of size 22



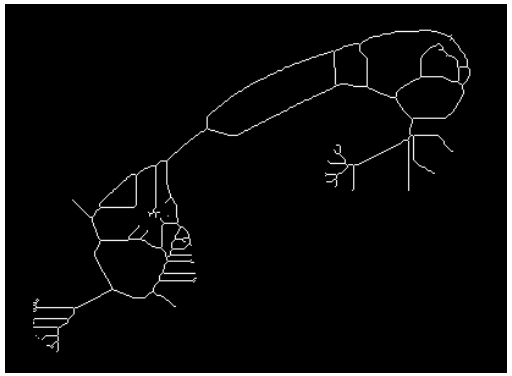


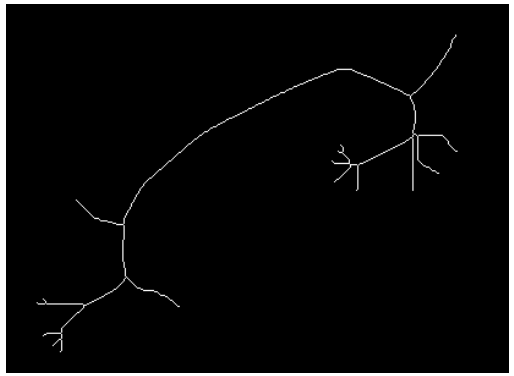
SE: disc
of size 20





Skeleton
operation looks
better for closed
image





Hit-and-miss Transform

- Hit-and-miss transform looks for particular patterns within the image.
- If the foreground and background pixels in the SE *exactly match* the foreground and background pixels in the image, then the pixel underneath the origin of the structuring element is set to the foreground color.
- Example for a Hit-and-miss Structuring Element

	1	
0	1	1
0	0	

Corner Detection

- Structuring Elements representing four corners

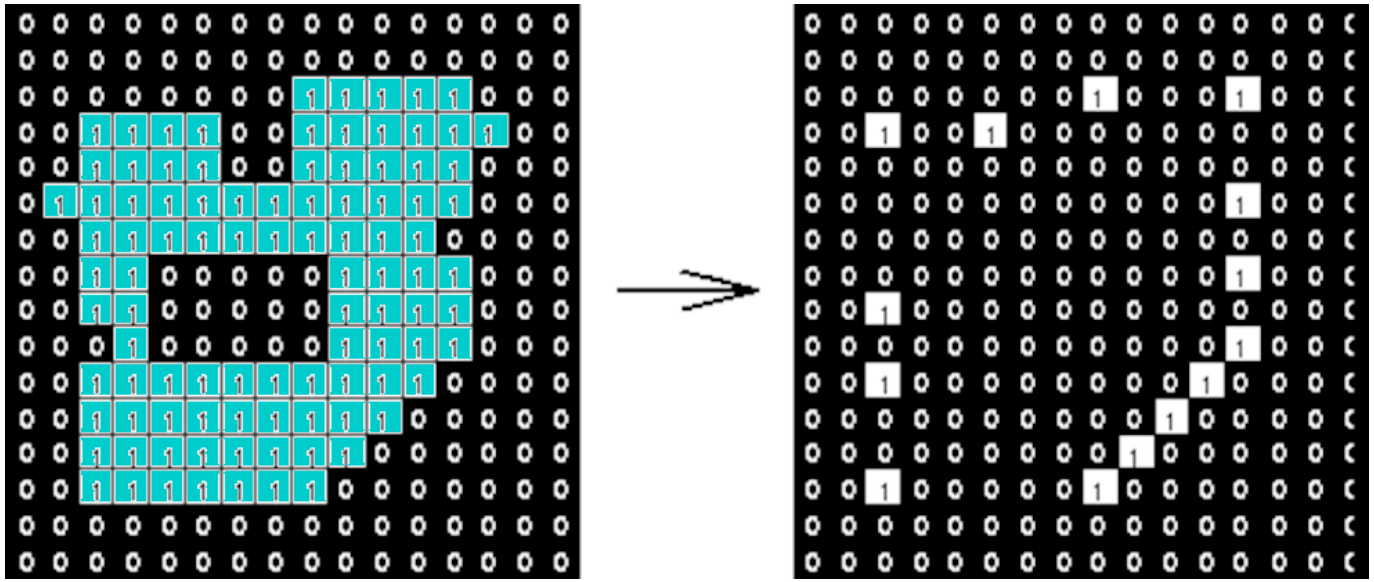
	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	

- Apply each Structuring Element
- Use OR operation to combine the four results



Thinning

- Thinning of an image A with B is defined as

$$A - HaM(A, B)$$

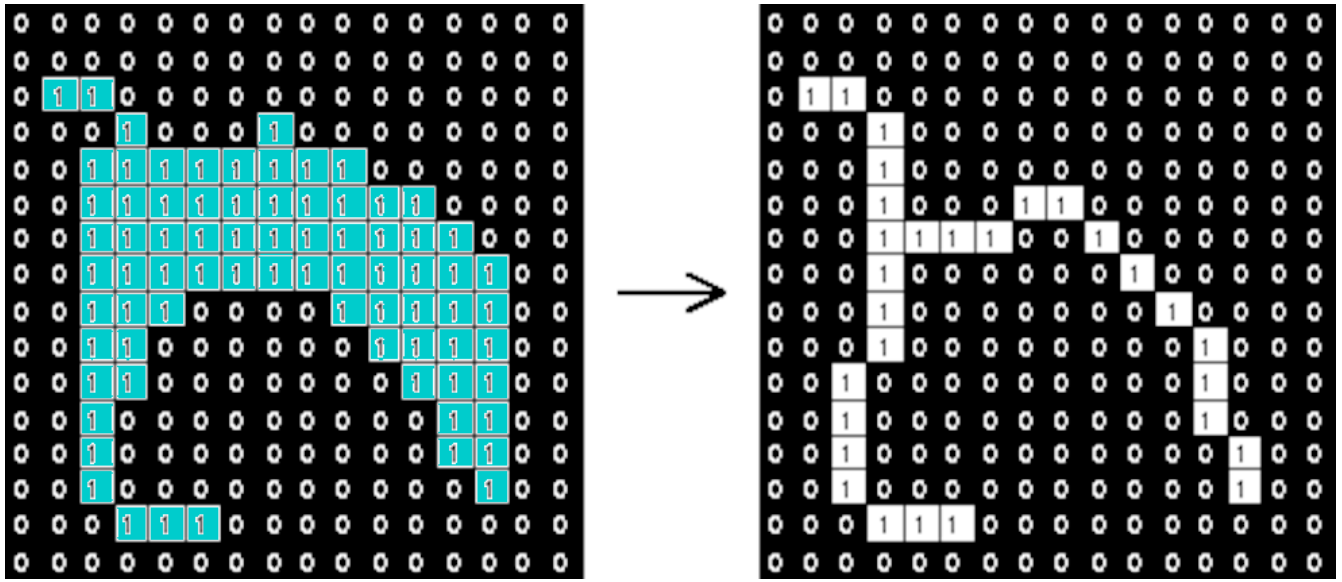
where HaM is the Hit-and-Miss operator and $0 - 1 = 0$!!

- If foreground and background fit the structuring element exactly, then the pixel at the origin of the SE is set to 0.
- Note that the value of the SE at the origin is 1 or *NaN.*

Two Hit-and-miss SE are used

0	0	0
	1	
1	1	1

	0	0
1	1	0
	1	



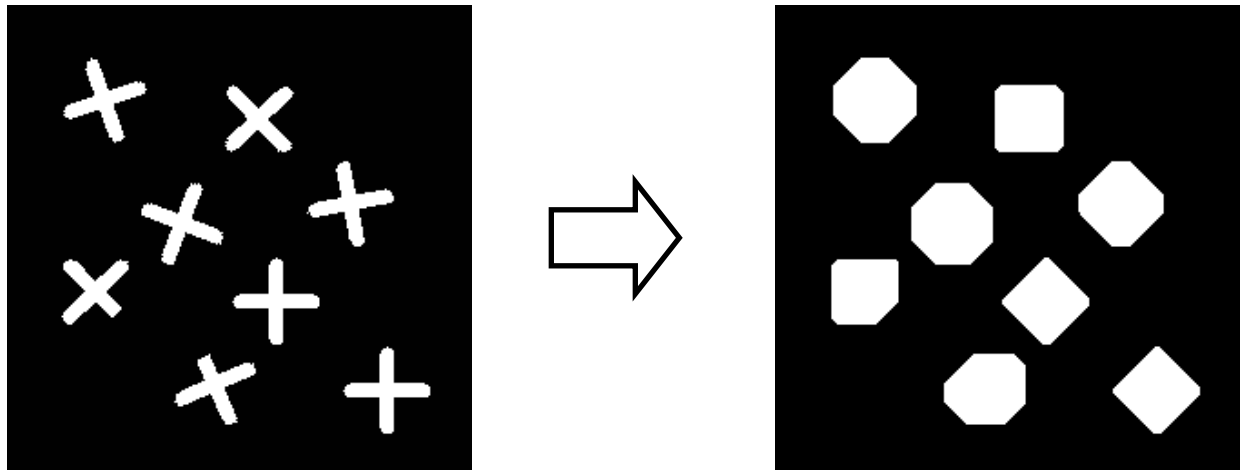
Thickening

- Thickening of an image A with B is defined as

$$A + HaM(A, B)$$

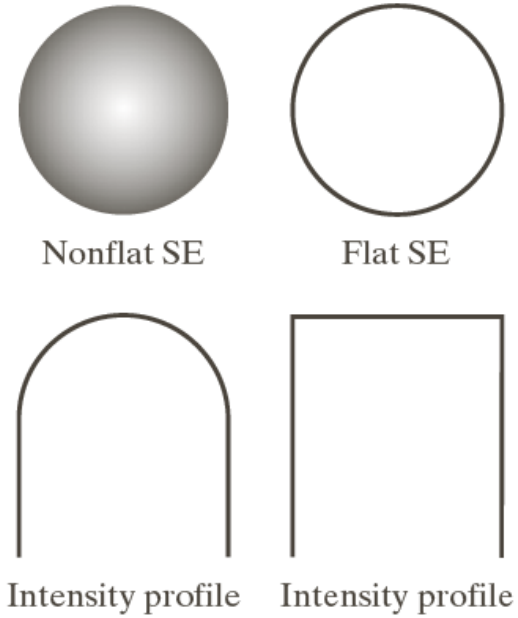
where HaM is the Hit-and-Miss operator and $1 + 1 = 1$!!

- If foreground and background match exactly the SE, then set the pixel at its origin to 1.
- Note that the value of the SE at the origin is 0 or *NaN.*

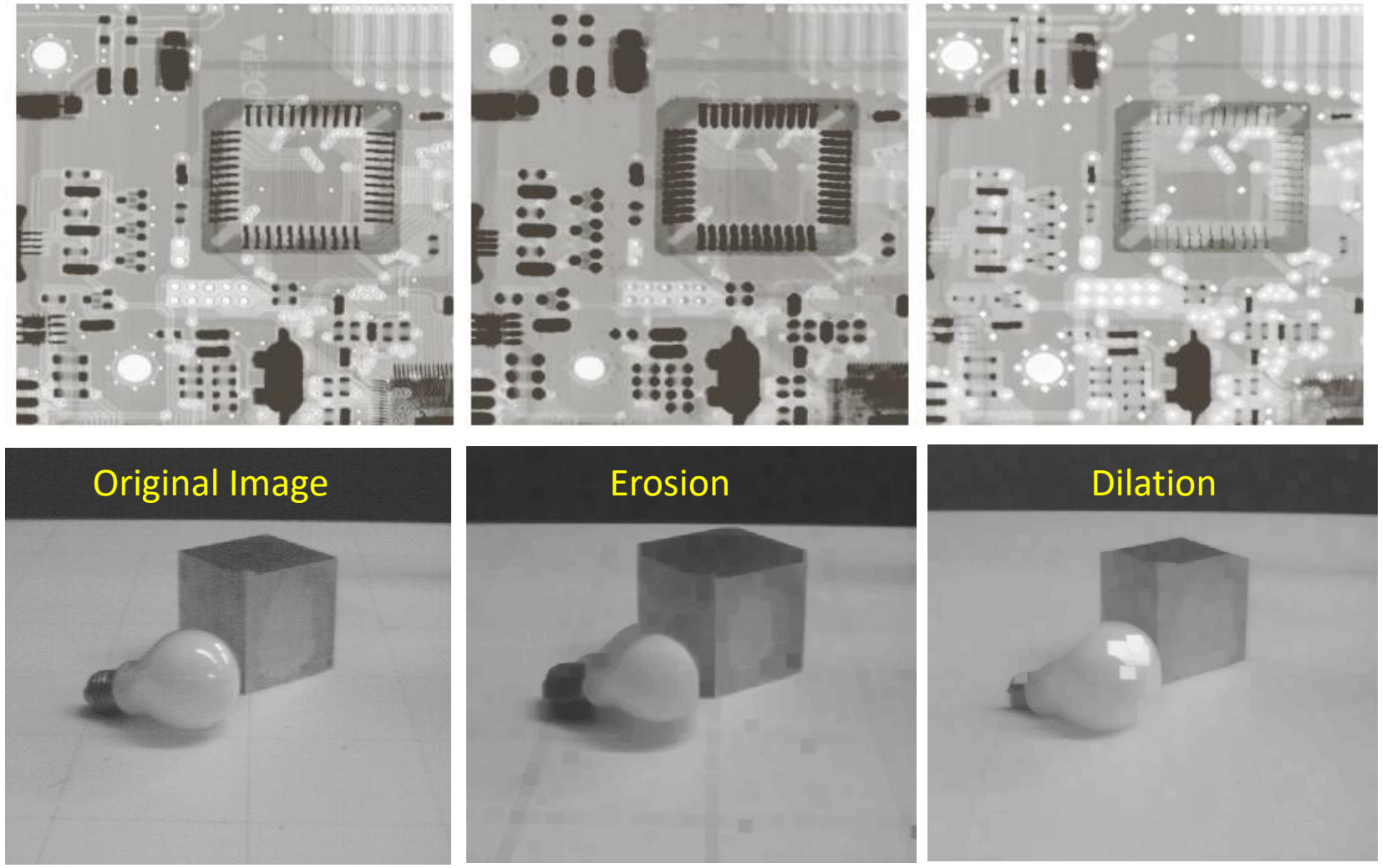


What about Grayscale Images?

- Using flat SE
  - Erosion: $[\mathrm{A} \ominus B](x,y) = \min_{(s,t)\in b}\{A(x+s, y+t)\}$
  - Dilation: $[\mathrm{A} \oplus B](x,y) = \max_{(s,t)\in b}\{A(x-s, y-t)\}$
- Using non-flat SE
  - Erosion: $[\mathrm{A} \ominus B](x,y) = \min_{(s,t)\in b}\{A(x+s, y+t) - B(s,t)\}$
  - Dilation: $[\mathrm{A} \oplus B](x,y) = \max_{(s,t)\in b}\{A(x-s, y-t) + B(s,t)\}$



Erosion and Dilation Example

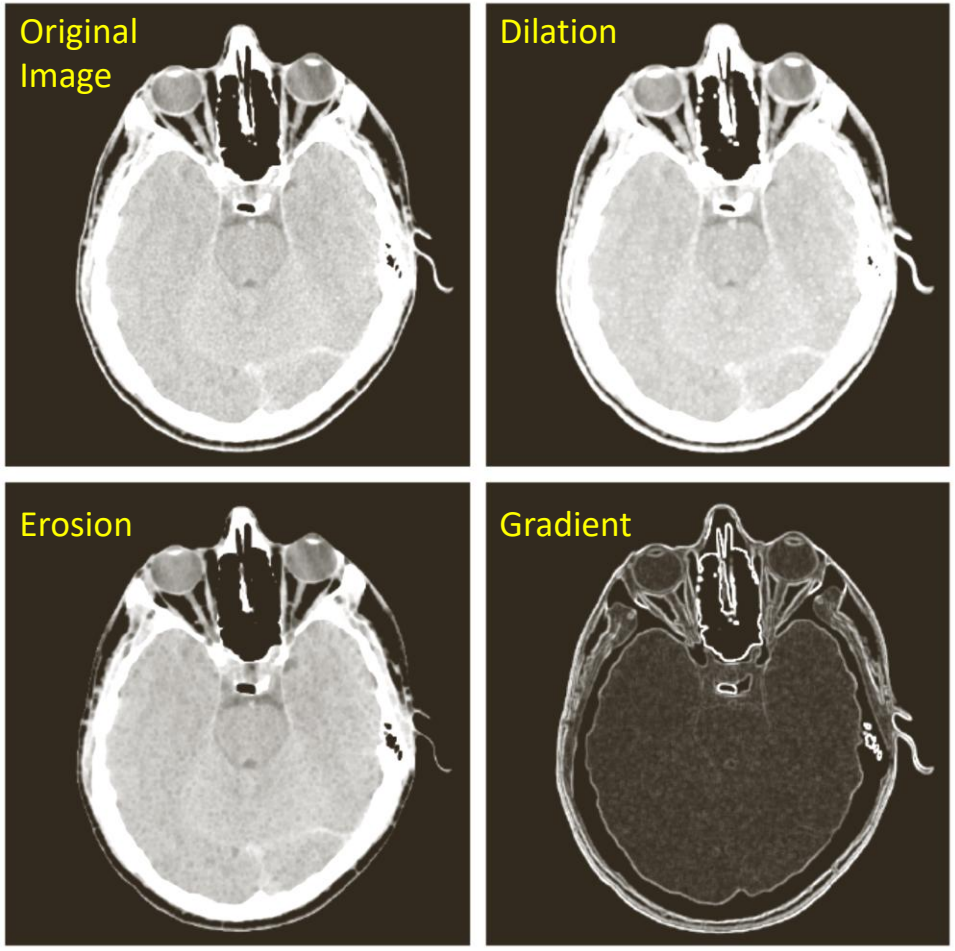


Morphological Gradient

- Dilation and erosion can be used to obtain the morphological gradient of an image

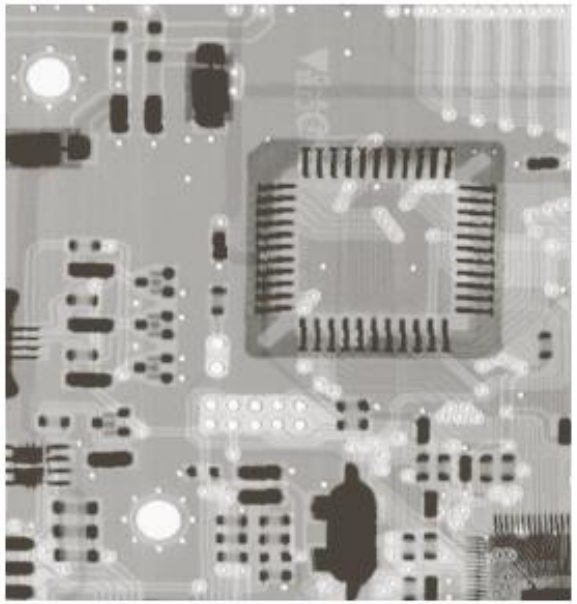
$$g = (A \oplus B) - (A \ominus B)$$

- The edges are enhanced, and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect.

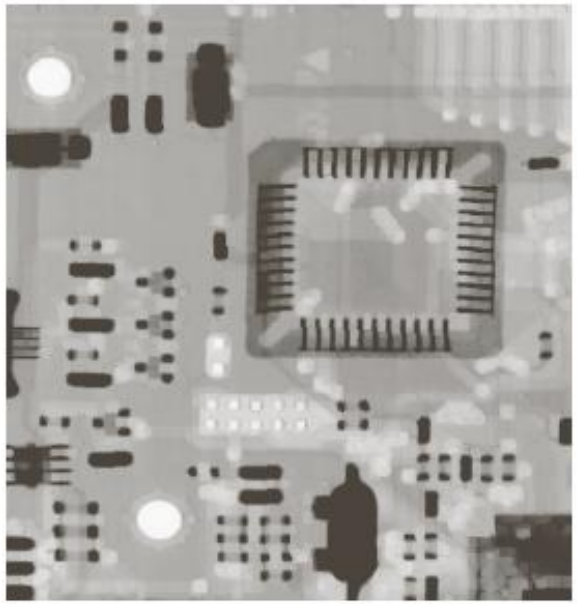


Opening and Closing

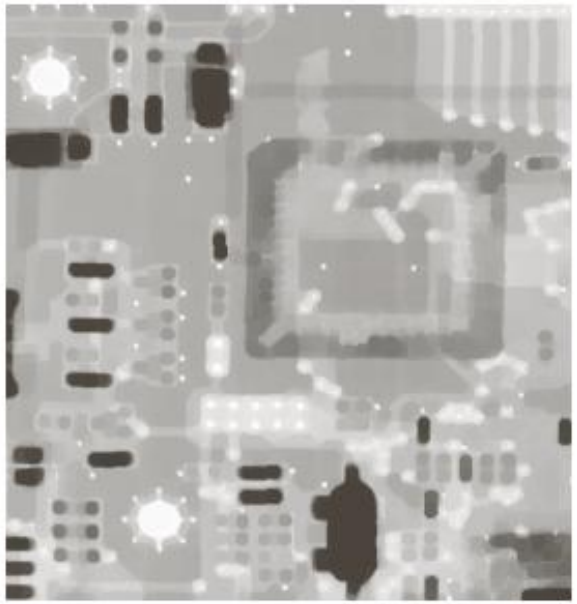
- Opening $A \circ B = (A \ominus B) \oplus B$
  - It suppresses bright details smaller than the specified SE
- Closing $A \cdot B = (A \oplus B) \ominus B$
  - It suppresses dark details
- Opening and closing are used often in combination for image smoothing and noise removal



Original image



Opening disk SE, radius=3



Closing disk SE, radius=5