Introduction to Digital Image Processing

— Wavelet Transform

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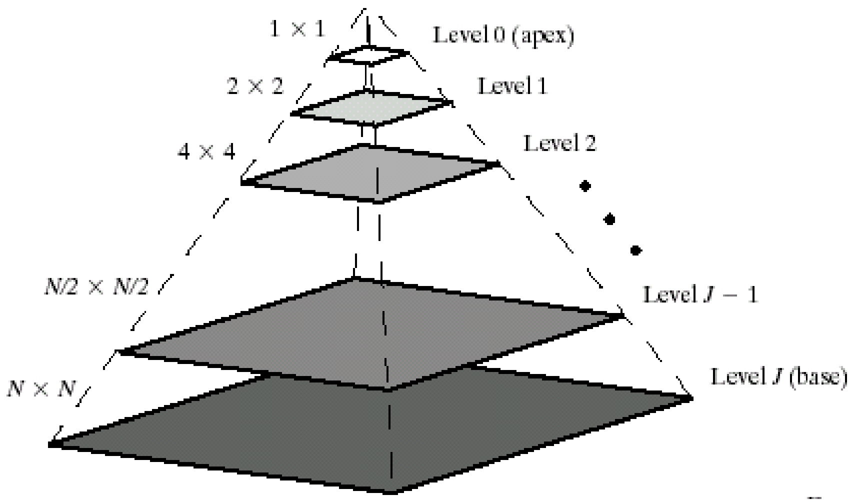
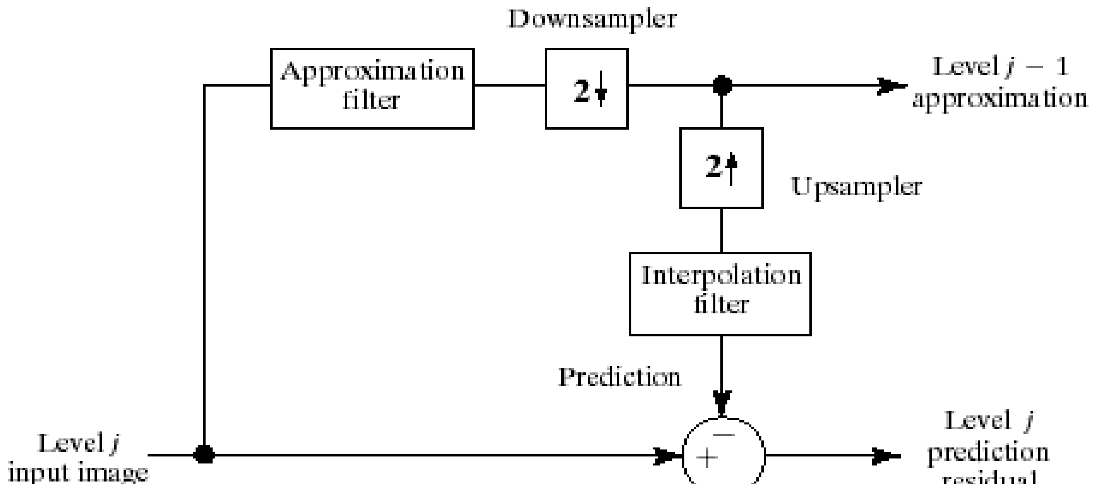
Image Pyramid Decomposition





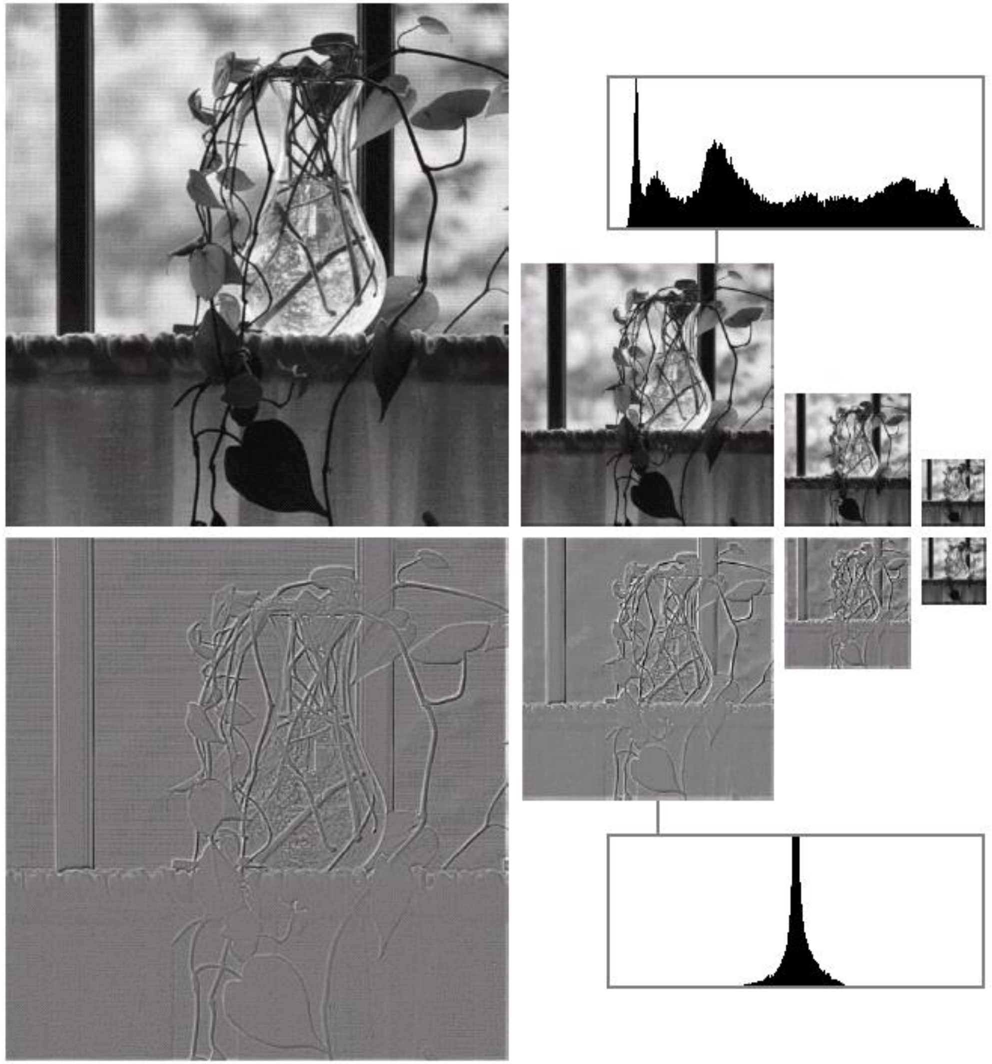


Photo by Digital Image Processing (3rd Edition), by Rafael C. Gonzalez and Richard E. Woods, Pearson, ISBN-13: 978-0131687288

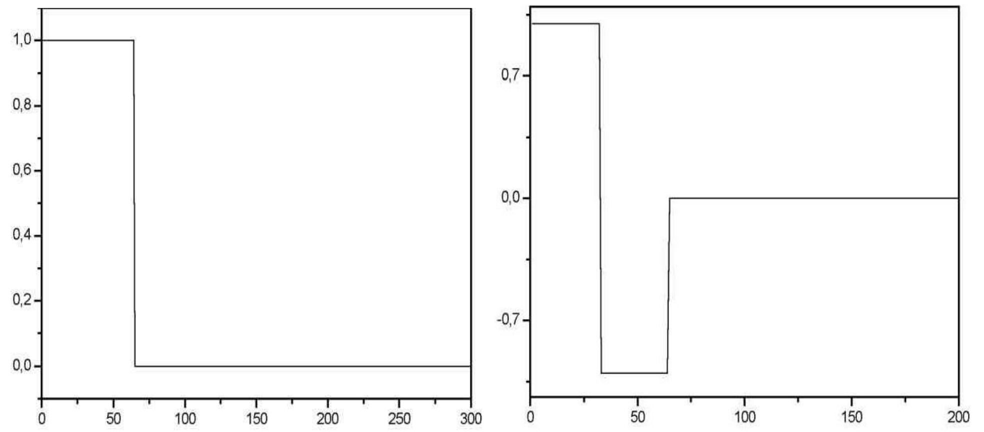
Wavelets as Signal Decomposition Basis

- The properties of a wavelet function integrates to zero

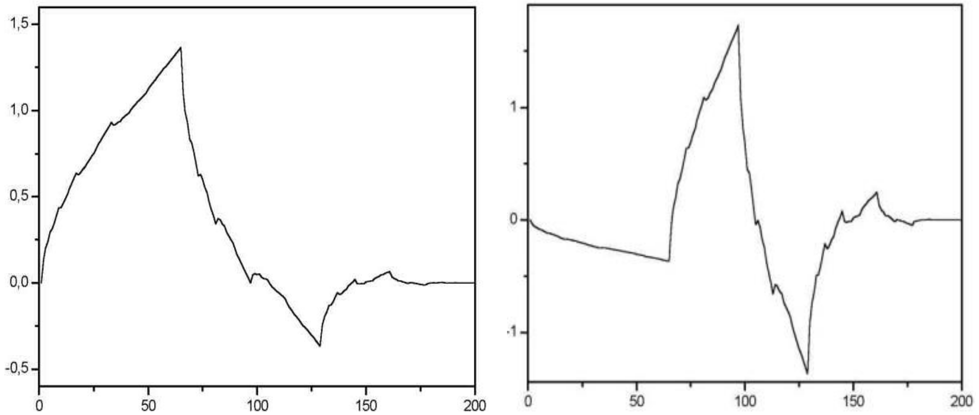
$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0$$

and has finite energy

$$\int_{-\infty}^{\infty} |\psi(t)|^2\, dt = 1$$



Haar wavelet



Daubechies wavelet

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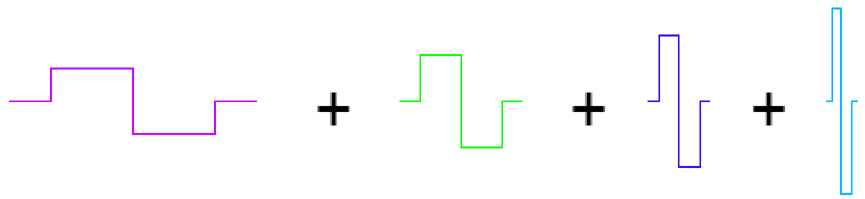
Haar Wavelets — A Numerical Example

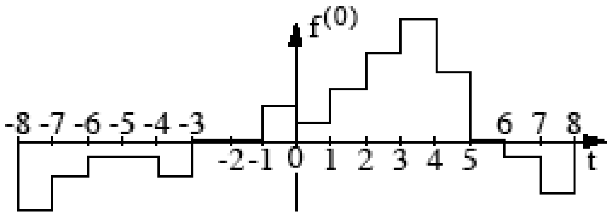
Haar wavelets



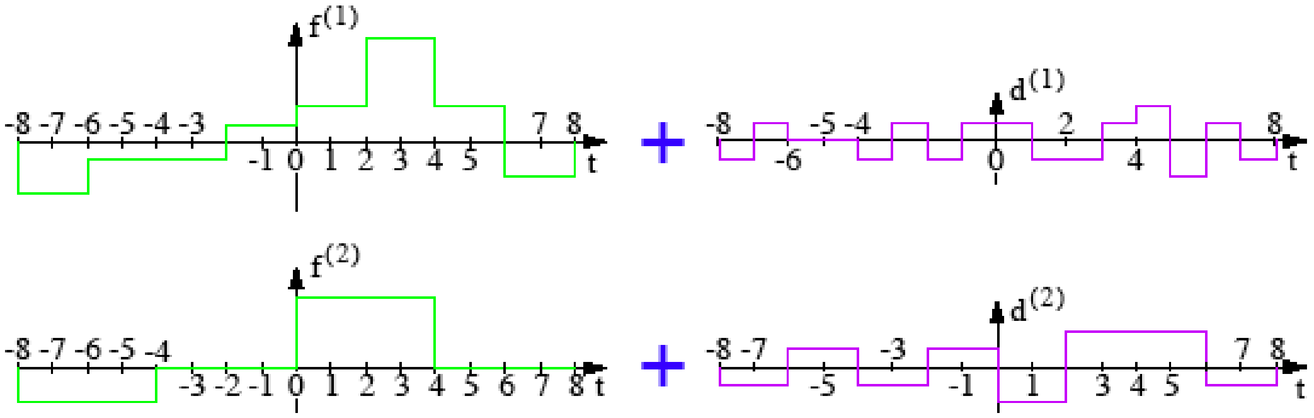
Signal (numerical)

-4 -2 -1 -1 -2 0 0 2 1 3 5 7 4 0 -1 -4

Signal profile

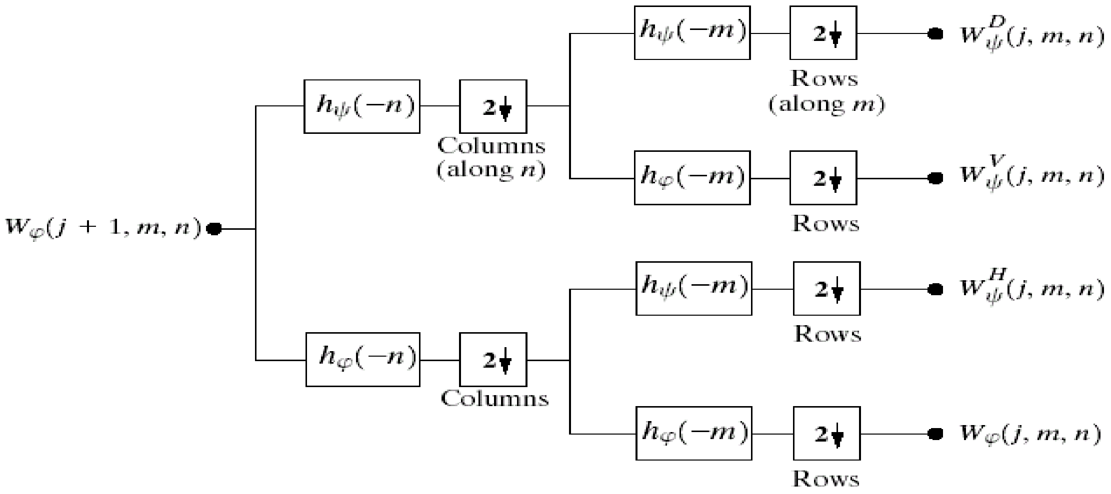


Haar wavelet decomposition

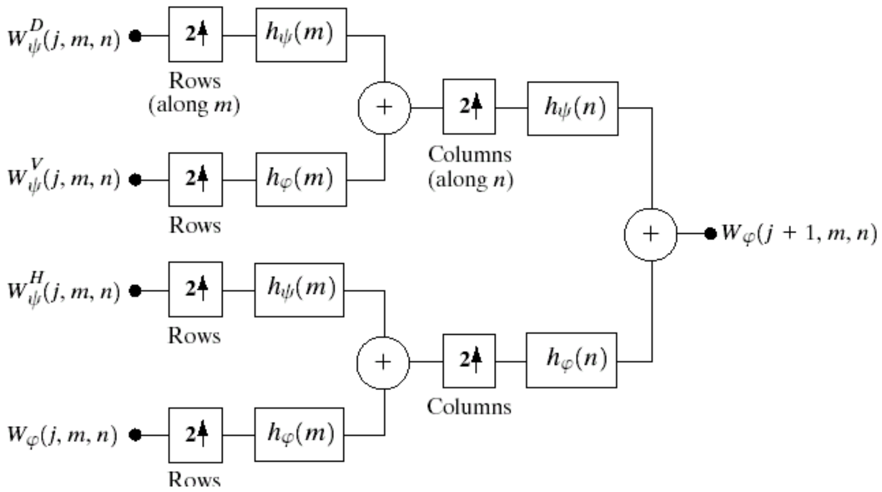


Wavelet Analysis and Synthesis

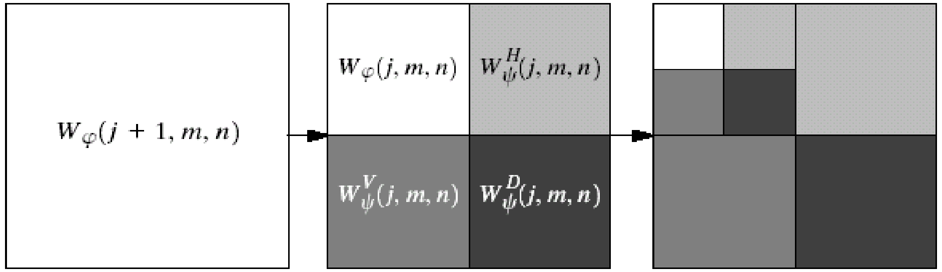
Wavelet Decomposition



Wavelet Reconstruction



Arrangement of Wavelet coefficients



What happens if the size of the image is not a power of two?

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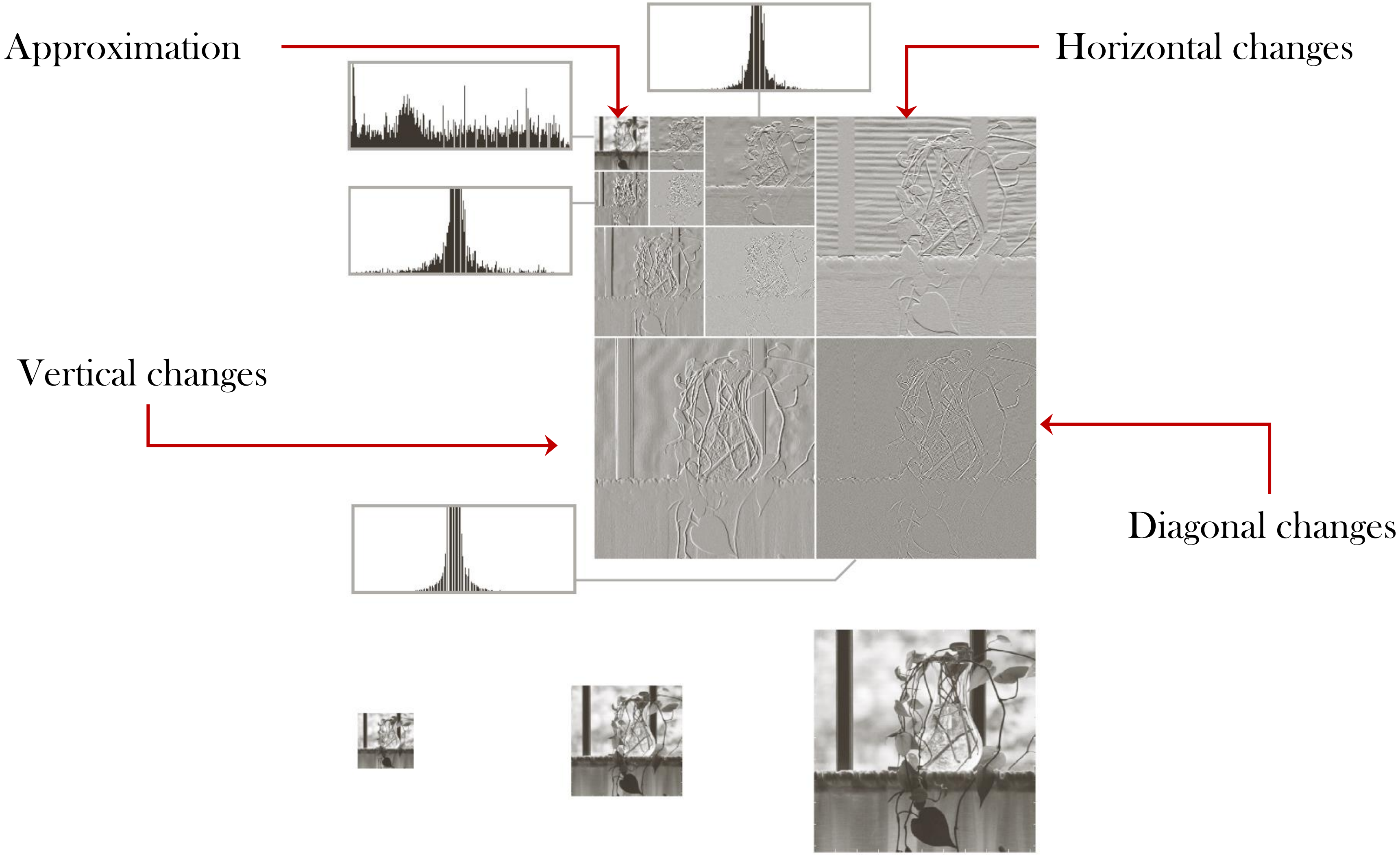
Wavelet Subband Arrangement



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FFT versus Wavelet Transforms

- Basis functions
  - FFT : sinusoid functions
  - Wavelet transforms: small waveforms, i.e., wavelets

- Coefficients
  - FFT offers frequency information
  - Wavelet offers frequency and temporal/spatial information

An Application of Wavelet Transform – Wavelet Shrinkage

- Suppose we want to recover an image I^* from a noisy observation I

$$I \; = \; I^* + n$$

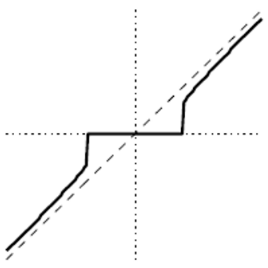
- Wavelet shrinkage removes noise by damping or thresholding in the wavelet domain, the estimate of signal is

$$w = \mathcal{W}(I) \;\; \rightarrow \;\; \hat{I} = \mathcal{W}^{-1}\,(\,T(w, \lambda)\,)$$

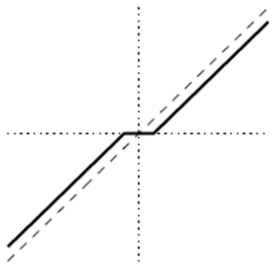
- A thresholding operation attenuates noise energy by suppressing the small coefficients while maintaining signal energy by keeping the large coefficients unchanged

Hard thresholding: $T(w;\lambda) = \begin{cases} w & if\ |w| > \lambda \\ 0 & otherwise \end{cases}$

Soft thresholding: $T(w;\lambda) = \begin{cases} w - \lambda & if\ w > \lambda \\ w + \lambda & if\ w < -\lambda \\ 0 & otherwise \end{cases}$



Hard thresholding



Soft thresholding

Estimating the Threshold for Wavelet Shrinkage

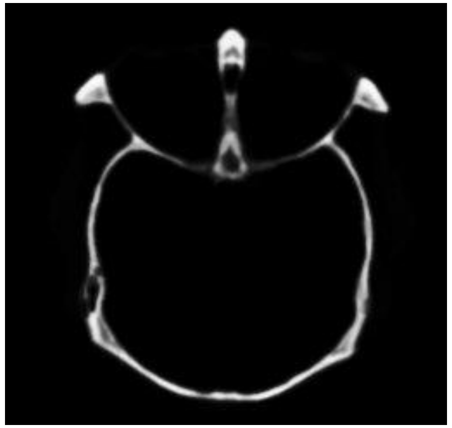
- **VisuShrink**: $\lambda = \sigma\sqrt{2\log(N)}$, where N is the number of pixels
  - where σ is estimated by $\left(\frac{median(|w_i|)}{0.6745}\right)^2$
  - The threshold λ is usually high resulting in overly smoothing
- **SUREshrink**: a different threshold is computed for each detail subband
  - The threshold minimizes the unbiased estimate of the risk and is computed as follows

$$\lambda_j(w_j) = N_j - 2 \cdot \mathbf{1}(i: |w_j| \leq t) + \sum_{j=1}^{N} min(|w_j|, \lambda)^2$$

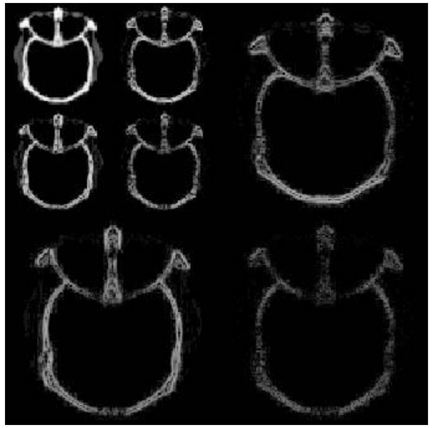
where N_j is the number of pixels, $\mathbf{1}(\cdot)$ is the indicator function that computes the count, $\lambda = \sigma\sqrt{2\log(N)}$

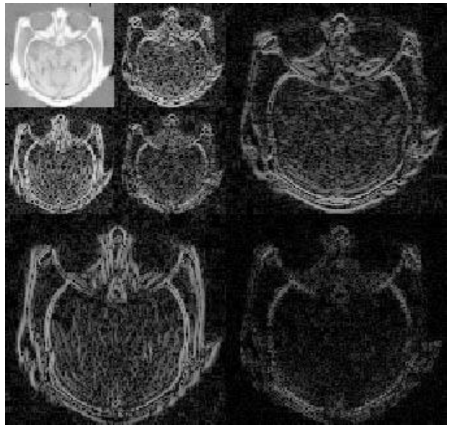
Wavelet-based Image Fusion

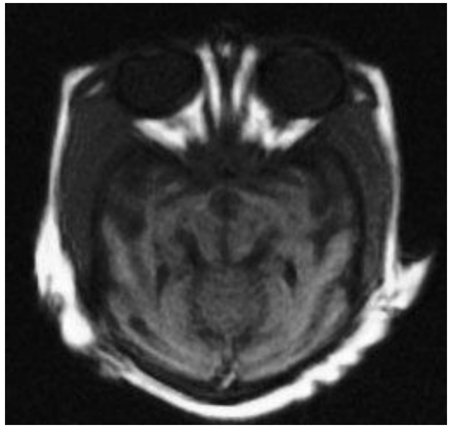
1. Decompose images using wavelet transform
2. Combine coefficients with the following rules
   a. Combine the approximation subbands using average
   b. Select the maximum coefficients among the corresponding detail subbands to put in the composite
3. Perform inverse wavelet transform on the composite wavelet coefficient matrix.



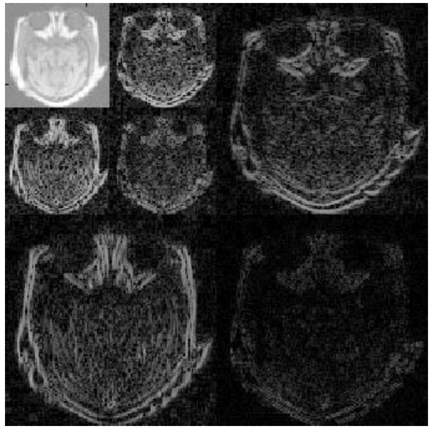
CT image

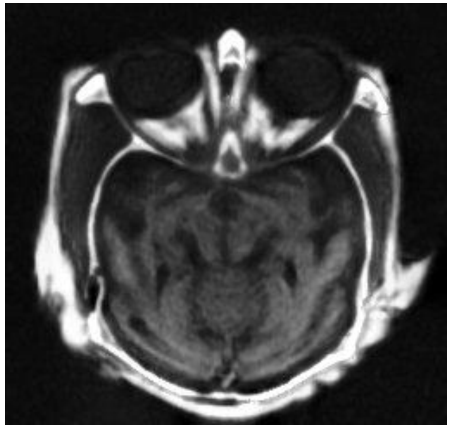






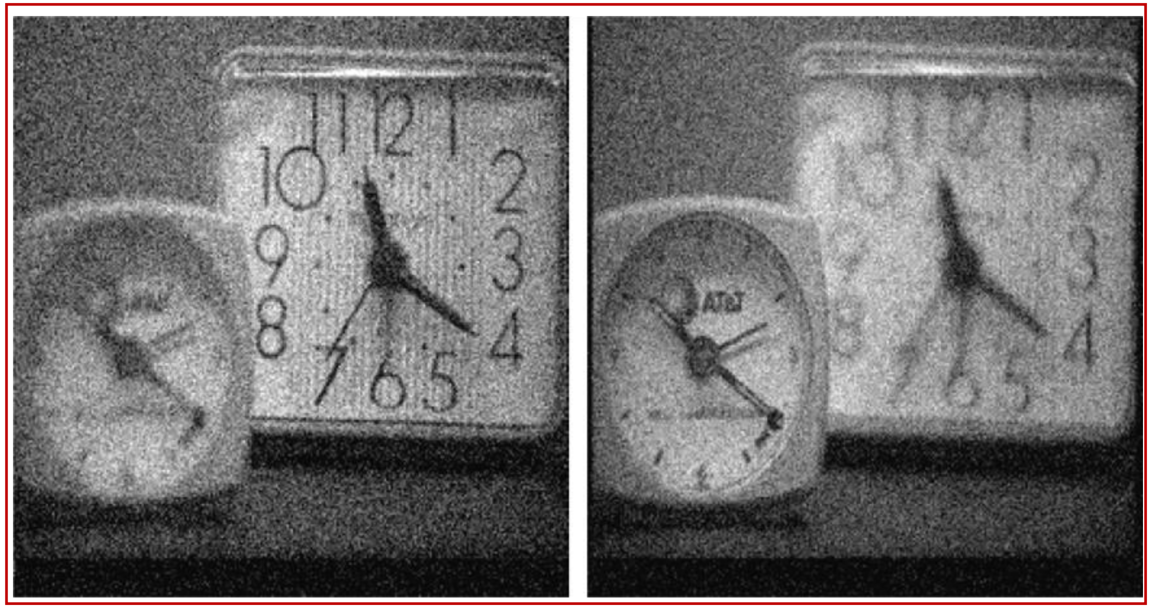
MRI image



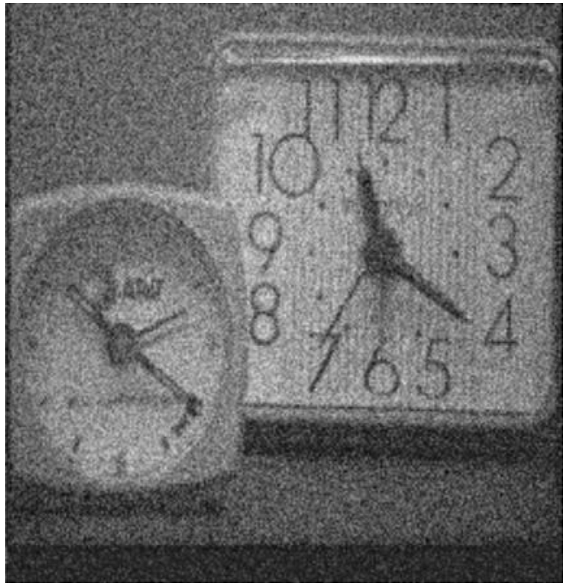


Fused image

What about Fusing Noisy Images?



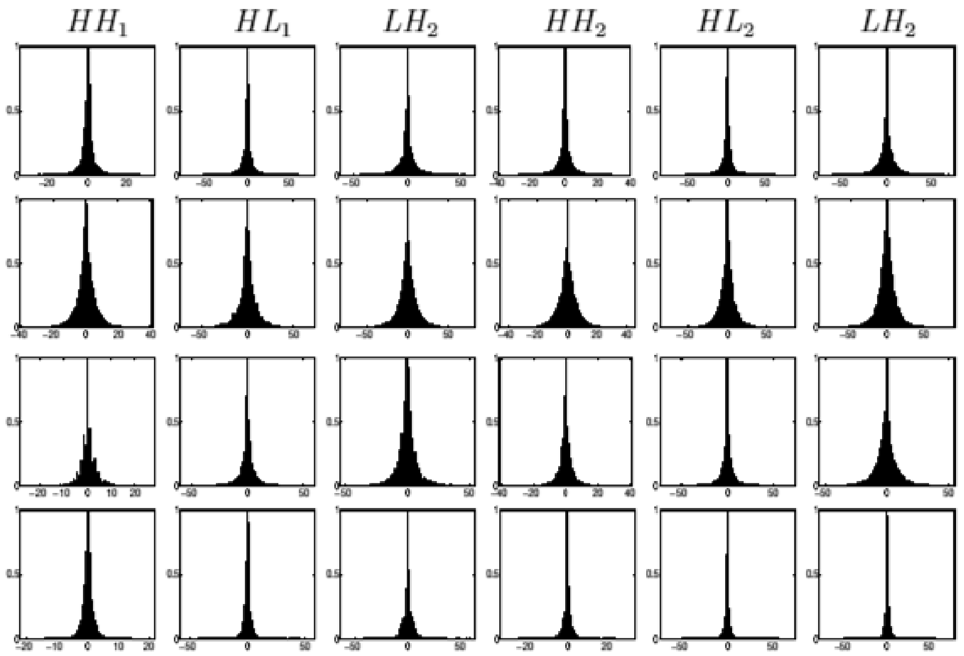
Source



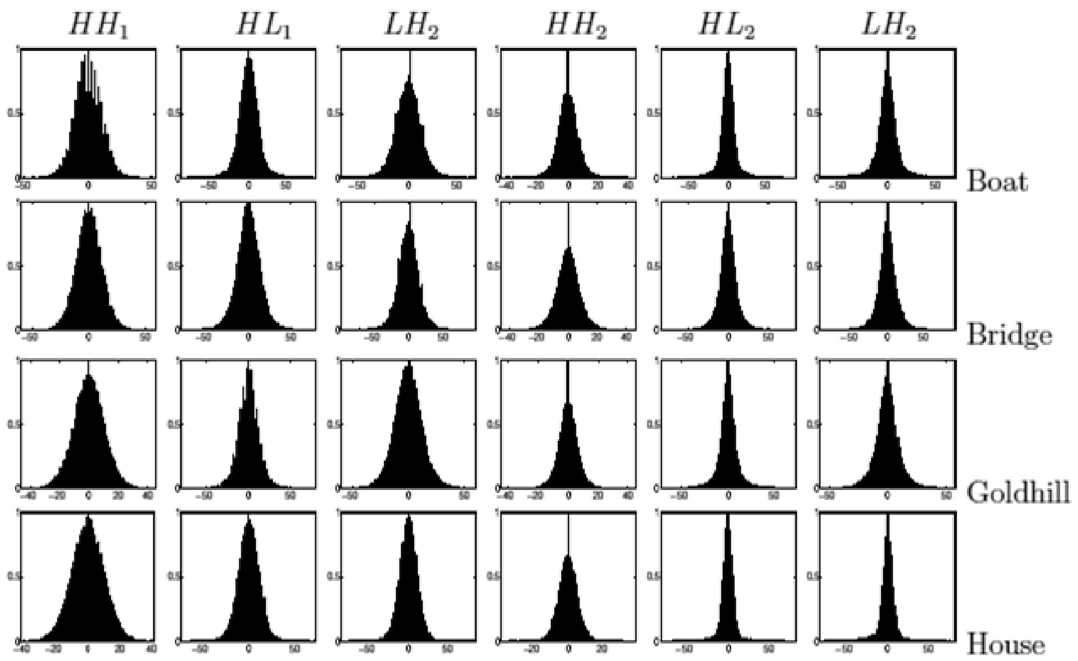
Result

- Noise components are aggregated just like image features.
- They are literally **enhanced**!

What Noise Affects Wavelet Coefficients

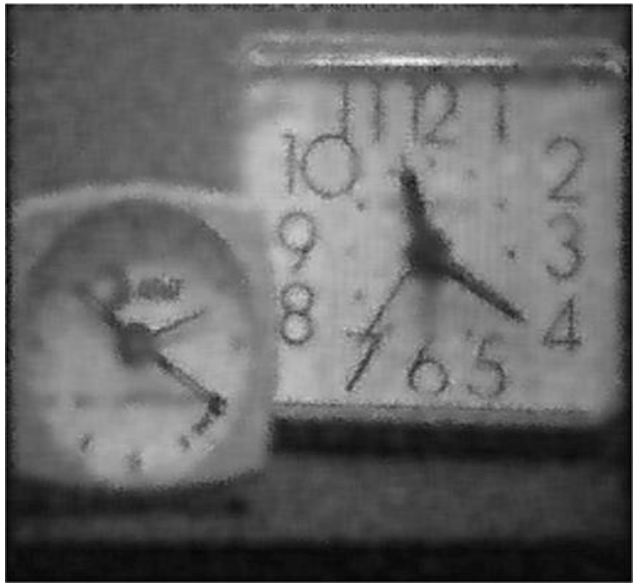


Clean Subband
Marginal Distribution



Noisy Subband
Marginal Distribution

Post Processing vs. In Situ Denoising



Fuse images then denoise

Adaptive Fusion (aware of noise)

X. Yuan, et al., “A Wavelet-based Noise-aware Method for Fusing Noisy Imagery”, IEEE International Conference on Image Processing, San Antonio, TX, September 16-19, 2007

Noise-aware Image Fusion

1. Decompose images with wavelet transformation
2. Compute the subband noise: method a. $\left(\frac{median(|w_i|)}{0.6745}\right)^2$, method b. Sample the coefficients of small magnitude and compute the variance.
3. Estimate the threshold λ that separate signal dominant coefficients and noise dominant coefficients
4. Combine SDC,
   - Compute subband variance
   - Combine SDC

$$\hat{S}^k = \frac{1}{N} \sum_{I_j^k \in SIC} \frac{\sigma_{I_j^k}^2 - \sigma_\varepsilon^2}{\sigma_{I_j^k}^2} I_j^k$$

5. Combine NDC,
   - Select the threshold ε
   - Perform consistency verification
   - Combine NDC

$$\hat{S}^k = \begin{cases} E(I_\tau) & C(I_\tau) < \varepsilon \\ 0 & Otherwise \end{cases}$$

6. Synthesize the fused image from the subband composite.

X. Yuan, et al., "A Wavelet-based Noise-aware Method for Fusing Noisy Imagery", IEEE International Conference on Image Processing, San Antonio, TX, September 16-19, 2007

JPEG2000 — Why Another Compression Standard?

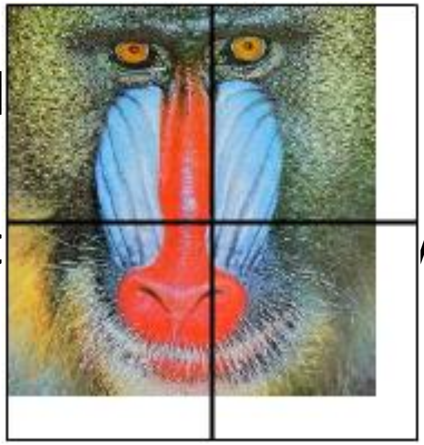
- Low bit-rate compression: At low bit-rates (e.g., below 0.25 bpp) the distortion in JPEG becomes unacceptable.
- Large images: JPEG does not compress images greater than 64K by 64K without tiling.
- Single decompression architecture: JPEG has 44 modes, many of which are application-specific and are not used by most of the JPEG decoders.
- Transmission in noisy environments: JPEG quality suffers dramatically when bit errors are encountered.
- Computer-generated imagery: JPEG is optimized for natural images and performs badly on computer-generated images
- Compound documents: JPEG fails to compress bi-level (text) imagery.

JPEG2000 Overview

1. The source image is decomposed into **color components** (RGB $\rightarrow$ YCrCb).
2. The images are (optionally) divided into rectangular **tiles**.
   - The tile component is the basic unit of the original or reconstructed image.
3. A wavelet transform is applied on each tile.
   - The tile is decomposed into different resolution levels.
4. The decomposition levels describe the frequency characteristics of local areas rather than across the entire image.
   - The sub-bands of coefficients are quantized and collected into rectangular arrays of code blocks.
5. The bit planes of the coefficients in a code block are entropy coded.
   - The encoding can be done in such a way that certain regions of interest (ROI) can be coded at a higher quality than the background.
6. Markers are added to the bit stream to allow for error resilience.
7. The code stream has a main header that describes the image and various decomposition and coding styles
   - used to locate, extract, decode, and reconstruct the image with the desired resolution, fidelity, region of interest, or other characteristics.

JPEG2000 Algorithm (steps 1, 2)

Step 1: Image tiling



- Image can be larger than the amount of memory available to the codec.
- Partition of the original image into rectangular non-overlapping blocks (til **compressed independently**
- All operations, including component mixing, wavelet transform, quantizat y coding are **performed independently on the image tiles**.
- Tiling affects the image quality
    - Smaller tiles create more tiling artifacts

Step 2: DC-level shifting

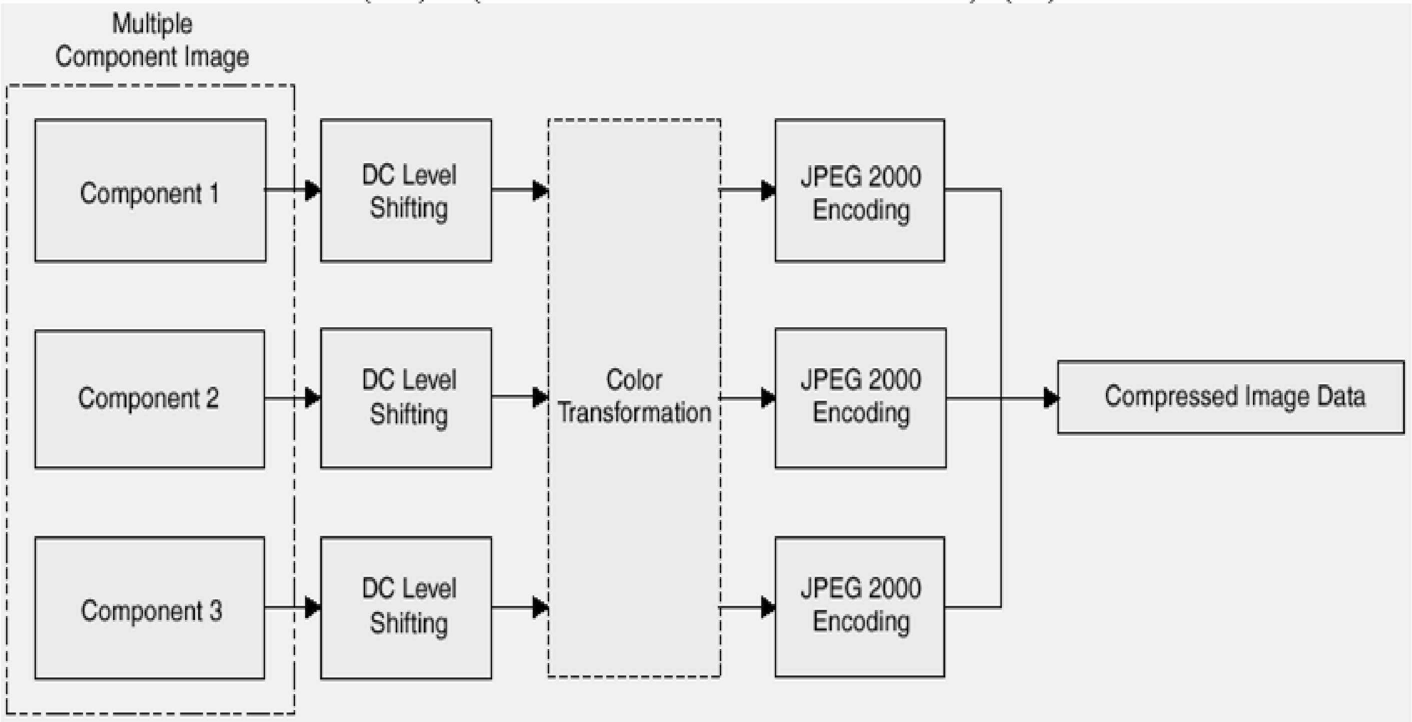
- The codec expects its input sample data to have a nominal dynamic range that is **approximately centered about zero** ($0 \sim 255 \rightarrow -128 \sim 128$)
- If the sample values are unsigned, the nominal dynamic range of the samples is adjusted by subtracting a bias from each of the sample values (2^{P-1} , P is the component's precision)

JPEG2000 Algorithm (3)

Step 3: Component transformation

- Maps data from RGB to YCrCb (Y, Cr, Cb - less statistically dependent; compress better)
- Component transformations improve compression and allow visually relevant quantization

$$\begin{pmatrix} Y \\ C_b \\ C_r \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.16875 & -0.33126 & 0.5 \\ 0.5 & -0.41869 & -0.08131 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$



JPEG2000 Algorithm (4, 5)

Step 4: Wavelet transform

- In JPEG2000, multiple levels of the DWT are performed.
- JPEG2000 supports from 0 to 32 levels.
- For natural images, a decomposition level between 4 to 8 is usually used

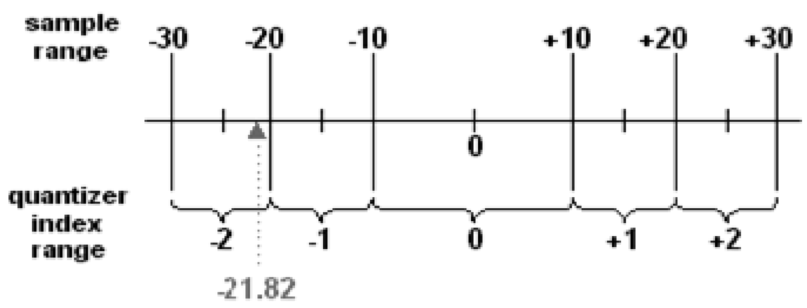
Step 5: Quantization

- The wavelet coefficients are quantized using a uniform quantizer.
- For each subband b, quantizer step size Δ_b is used to quantize the coefficients

$$q = sign(y)\left\lfloor\frac{|y|}{\Delta_b}\right\rfloor$$

- Given a quantizer step size of 10 and an input value of 21.82, the quantizer index is determined as follows:

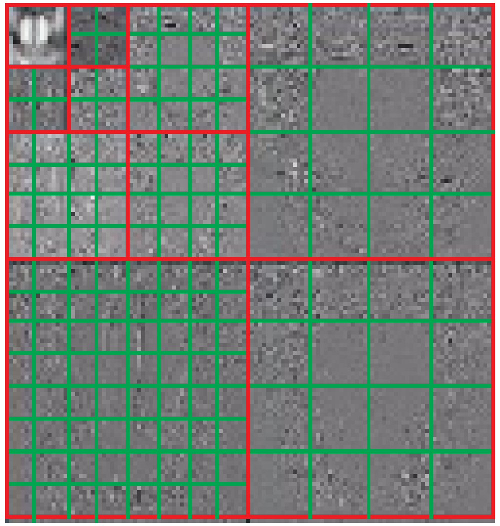
$$\text{Quantizer Index} = -\left\lfloor\frac{21.82}{10}\right\rfloor = -2$$



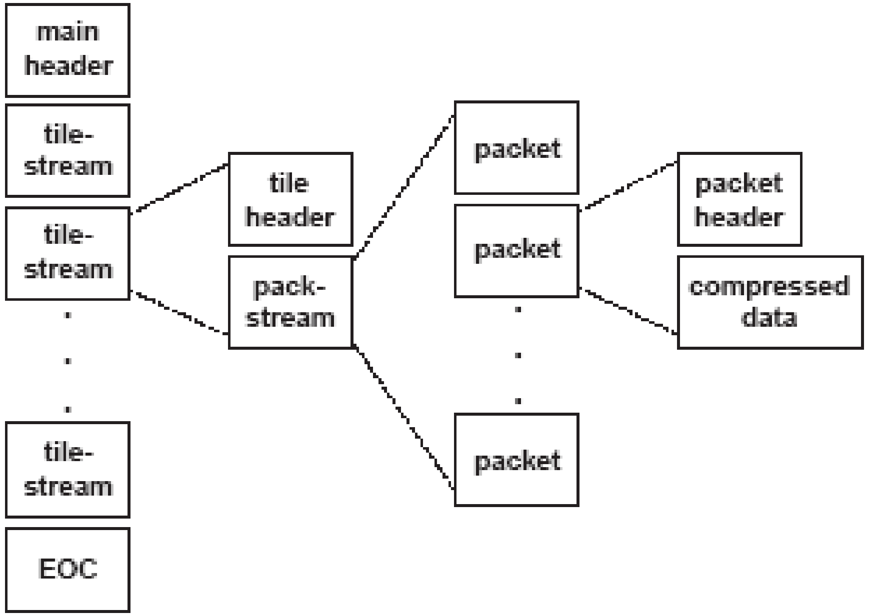
JPEG2000 Algorithm (6)

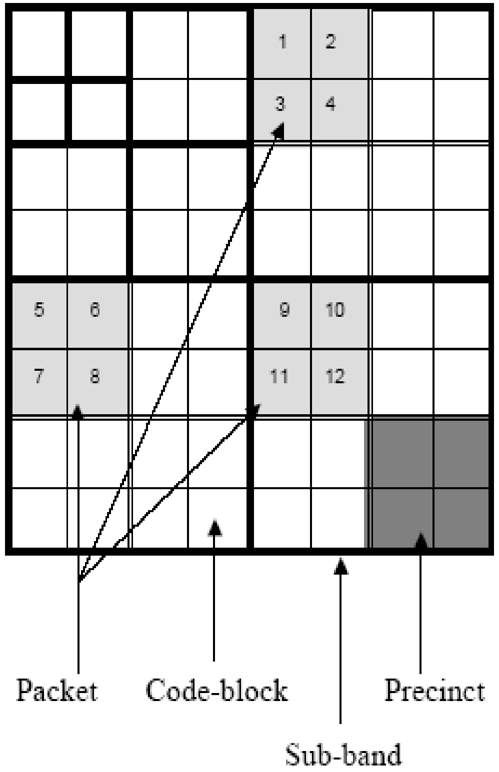
Step 6: Coefficient coding

- Wavelet coefficients are arranged into rectangular blocks within each sub-band, called code blocks.
- Code blocks are then coded (following entropy coding) independently.



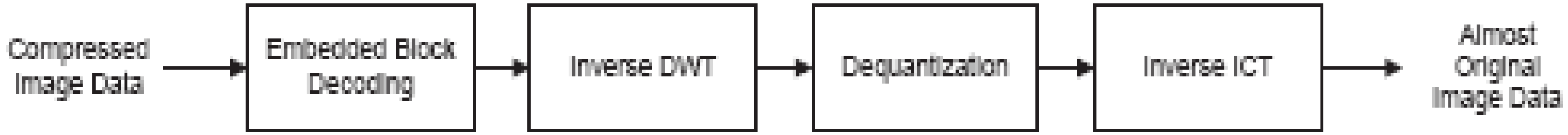
JPEG2000 File Stream





JPEG2000 Decoding

- The decoder basically performs the opposite operations of the encoder



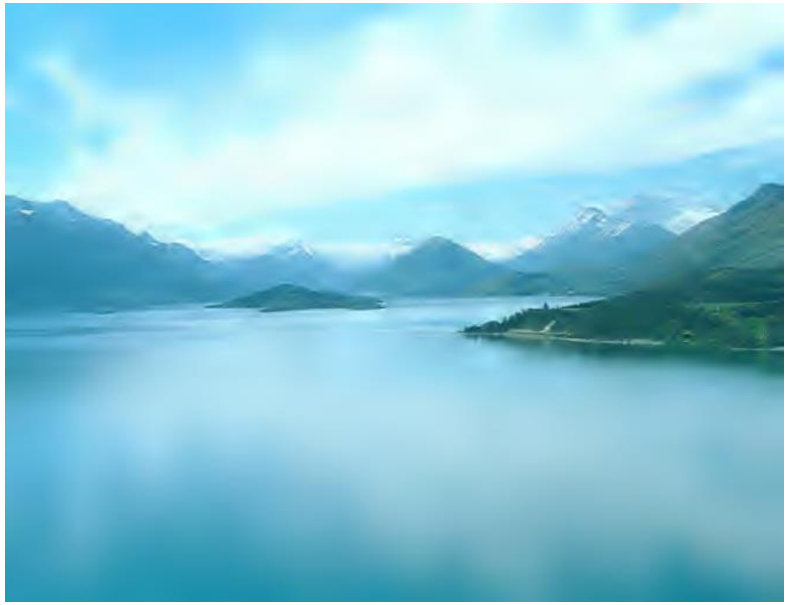
- The code stream is received by the decoder according to the progression order stated in the header.
- The coefficients in the packets are then decoded and dequantized, and the reverse ICT is performed

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 1.4021 \\ 1.0 & -0.3441 & -0.7142 \\ 1.0 & 1.7718 & 0.0 \end{bmatrix} \begin{bmatrix} Y \\ C_r \\ C_b \end{bmatrix}$$

Compressed Images using JPEG and JPEG 2000

Original (979 KB)

JPEG (6.21 KB)



JPEG 2000 (1.83 KB)