Introduction to Digital Image Processing

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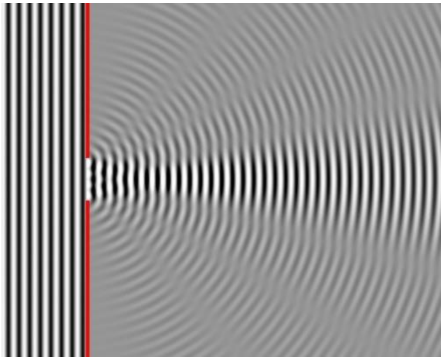
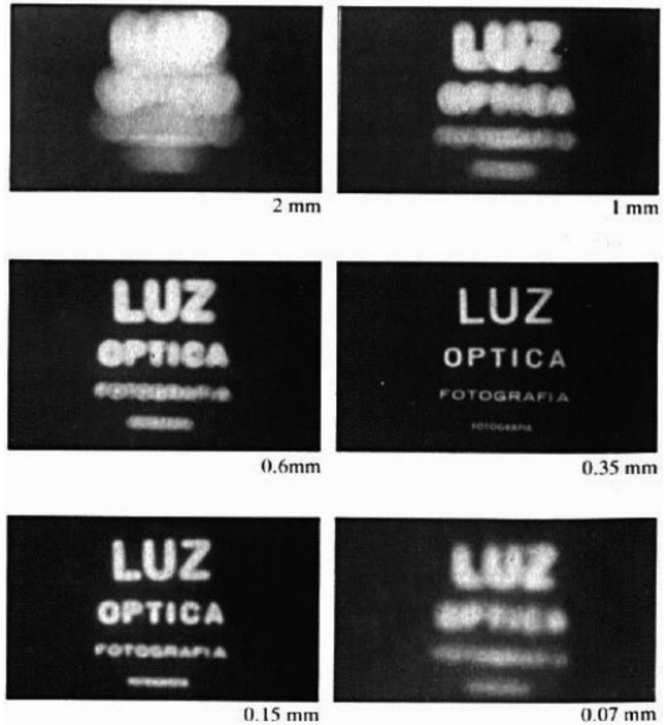
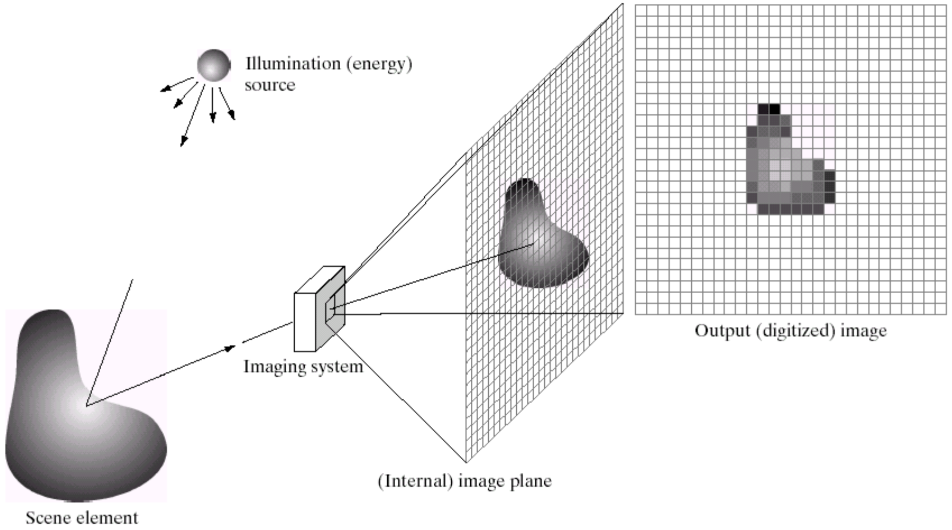
Image Acquisition

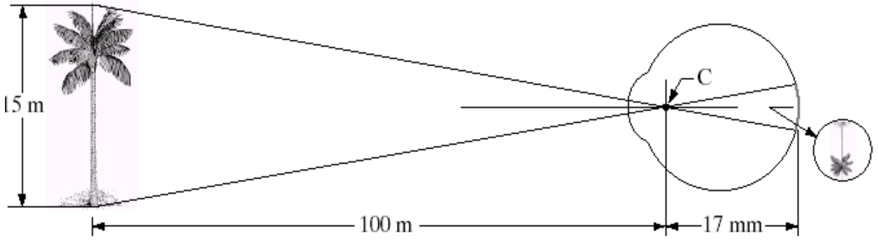
- A lens focuses light onto the film
- The thin lens model:
  - Rays passing through the center do not deviate

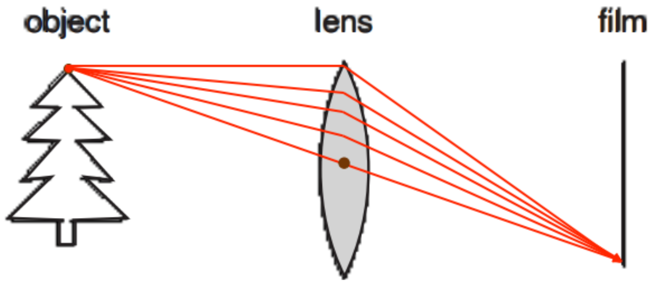


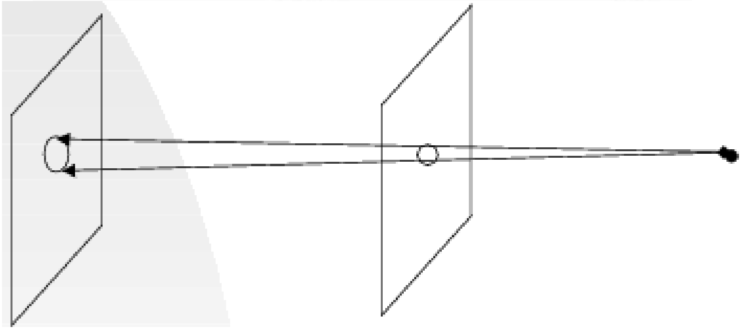
Diffraction

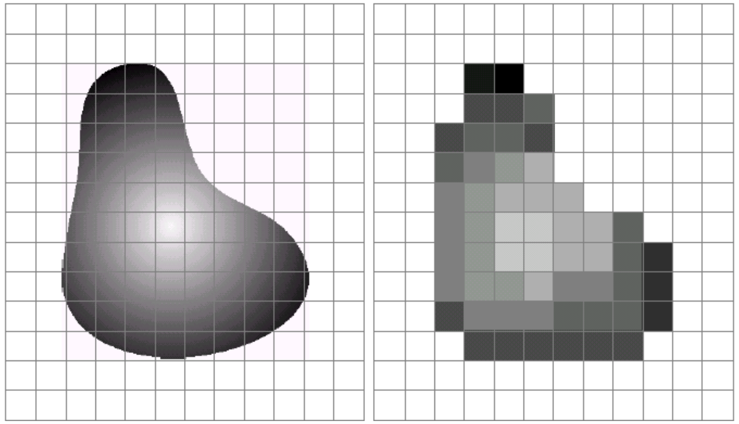








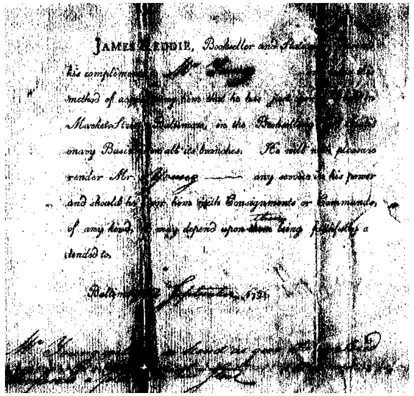




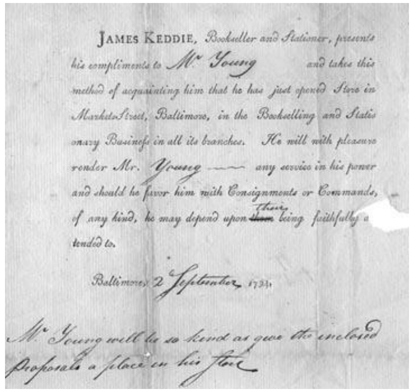
What is an Image Pixel

- Digital Images are composed of elements called **pixels**.
  - A pixel is the basic component of every digital image.
  - Each pixel consists of one (or more) numerical value that represents the color of a specific spot in the image.
  - They are organized into a grid to convey image contents.
- Each pixel is a string of binary code.
  - Each 0 or 1 is called a bit.
  - The number of bits in a pixel determines the size of the color palette, which is called color depth.
- If a pixel has more than one value, each value of all pixels forms an image plane known as a channel.
- Given an image I, a pixel is denoted as I(x, y), where x and y are the coordinates or the indices of the row and column of the pixel.

1-bit, black and white



8-bit grayscale

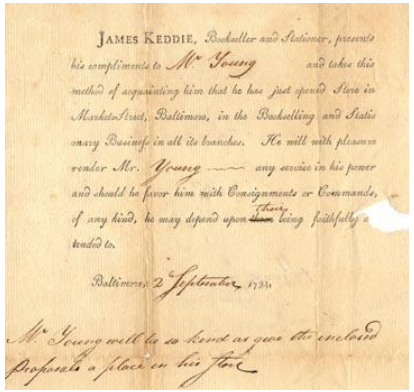


JAMES KEDDIE, Bookseller and Stationer, presents his compliments to Mr Young and takes this method of acquainting him that he has just opened Store in Market-Street, Baltimore, in the Bookselling and Stationary Business in all its branches. He will with pleasure render Mr. Young —— any service in his power and should he favor him with Consignments or Commands, of any kind, he may depend upon ~~them~~ their being faithfully attended to.

Baltimore, 2 September, 179[illegible]

Mr Young will be so kind as give the enclosed proposals a place in his Store

24-bit color



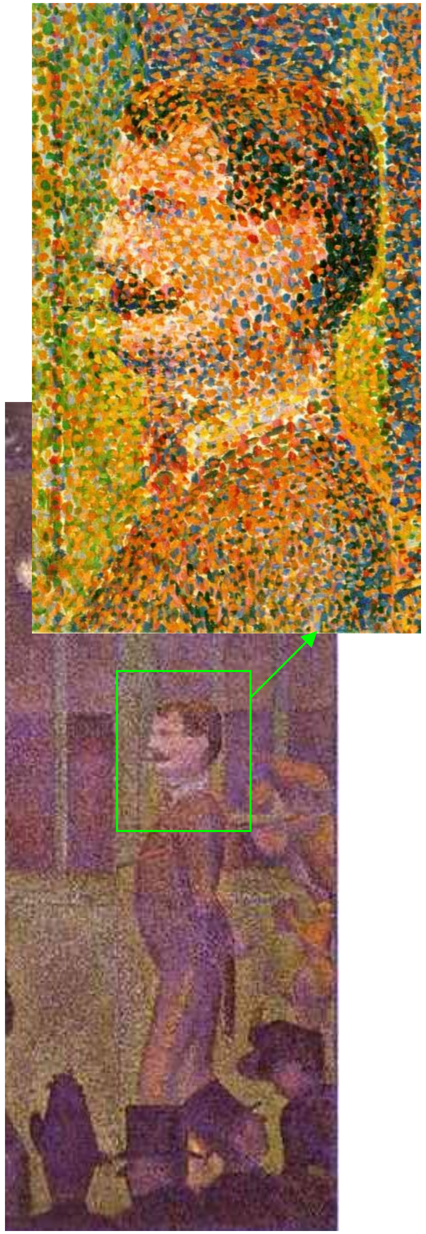
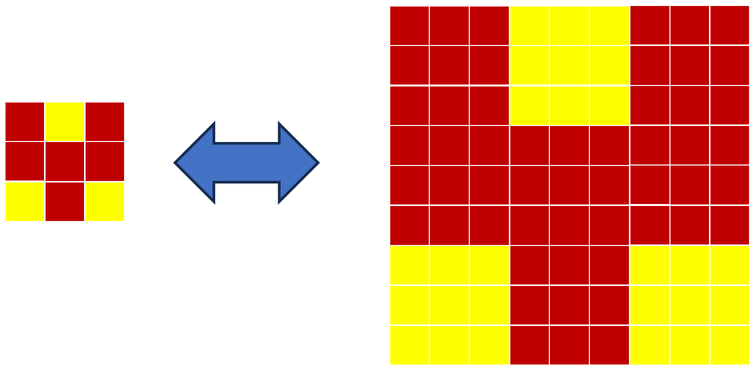


Image Resizing

- A digital image is saved as a matrix in computer memory spaces.
- Resizing an image often requires interpolation
- The interpolation process fills the voids (pixels without a value) generated from enlarging an image.
    - Zero-order interpolation (replication)
    - First-order interpolation (linear)
    - Third-order interpolation (cubic)
- Linear interpolation
    - The closer to a pixel, the higher the weight is assigned

$$I(n+a)=(1-a)\times I(n)+a\times I(n+1), \quad 0<a<1$$

- When an image is resized smaller, interpolation is also used to provide a smooth color transition between adjacent pixels.



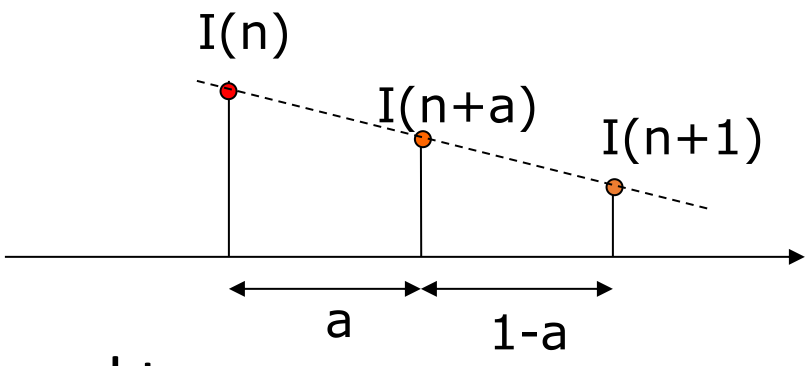


Image Histogram

- The histogram of an image with gray levels in the range [0, L-1] is a discrete function

$$h(r_k) = n_k$$

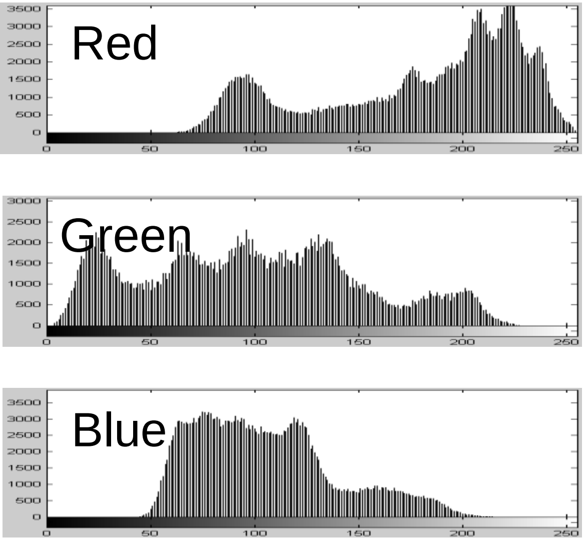
where n_k is the count of pixels that have the gray level of r_k.

- The algorithm for creating the histogram of an image uses a single loop

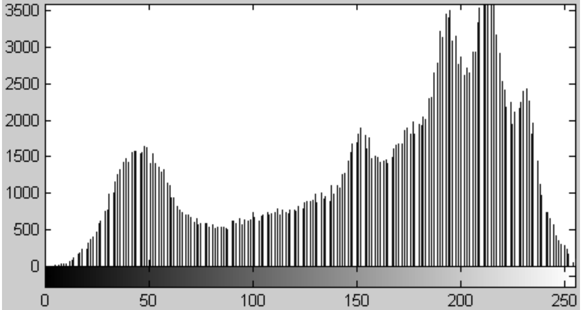
Given an image A, its value range is [0 L-1].

1. Create a one-dimensional array h of size L with an initial value of zero
2. Loop through all pixels in the image A
3. For a pixel value v, increase the value at h(v) by one
4. Continue until all pixels in the image A are visited.

Histograms of a color image



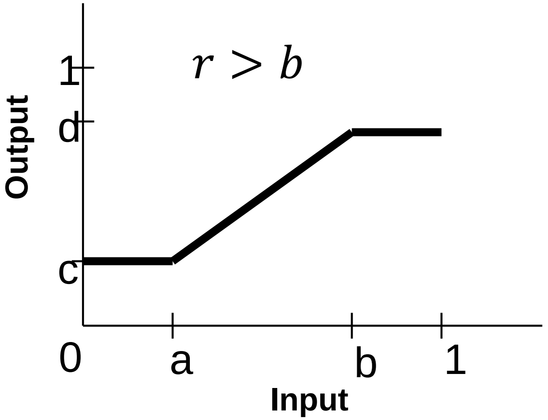
Histograms of a grayscale image

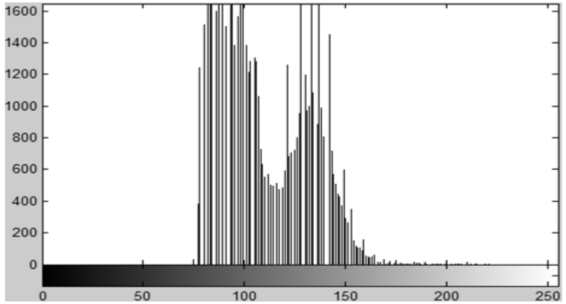


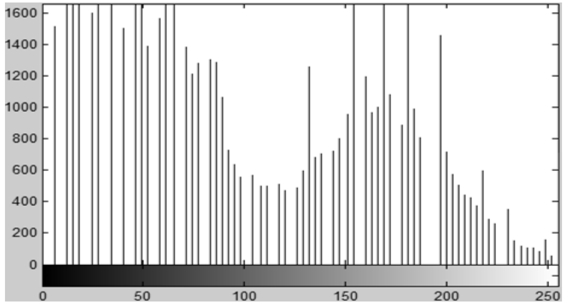
Histogram Stretching

- Image contrast is the gray level difference between neighboring pixels.
  - A larger gray level difference of an image often results in a greater contrast.
- The histogram stretching maps the value of all pixels to a new value that spans the full grayscale range.
- The mapping function can be a linear function or a nonlinear function.

$$s = \begin{cases} c & r < a \\ (r-a)\left(\dfrac{d-c}{b-a}\right)^{\gamma} + c & a \leq r \leq b \\ d & r > b \end{cases}$$







Histogram Equalization

- The goal is to maximize the usage of the full brightness range
  - This usually results in an enhanced image, with an increase in the dynamic range of pixel values.
  - The maximum contrast is achieved when the image histogram follows a uniform distribution.
- Assume continuous gray level r of an image and r is in the range of [0, 1].
- The enhanced image has a gray level s through a transformation T(·):

$$s = T(r), \qquad 0 < r < 1$$

- The transformation function satisfies the following two criteria:
  - T(·) is monotonically increasing in the range of [0, 1];
  - $0 < T(r) < 1$ and $0 < r < 1$

Histogram Equalization – Continuous Case

- Given two random variables r and s, their probability density functions are $p_r(r)$ and $p_s(s)$, respectively.
  - We want to compute the transformation function $T(r)$ that maps $p_r(r)$ to $p_s(s)$
- If $p_s(s)$ and $T(r)$ are known, $p_s(s)$ is computed as follows:

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

- Since we know $p_s(s) = 1$, we have

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = 1$$

- Multiply both sides with ds ($ds > 0$), the above equation becomes

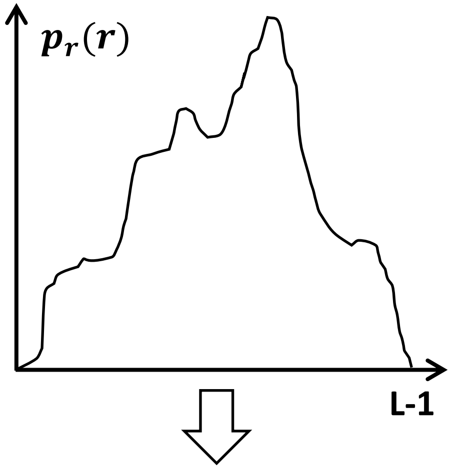
$$p_s(s)ds = p_r(r)dr = ds$$

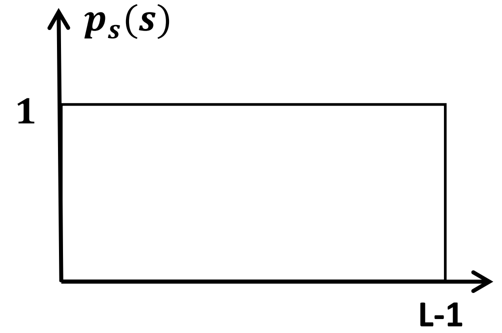
- Since $s = T(r)$, we have

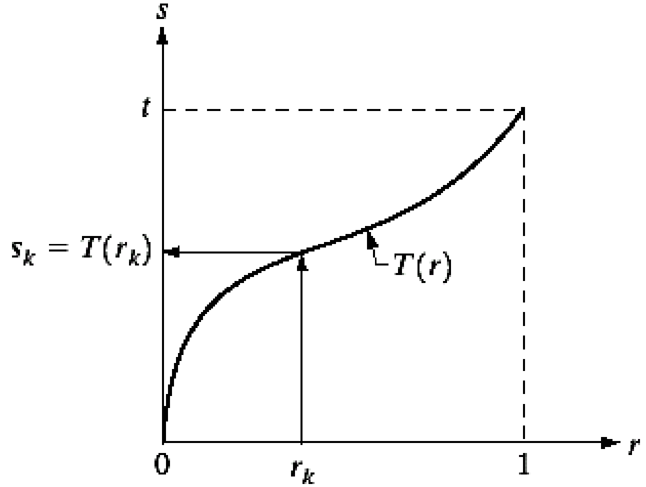
$$dT(r) = p_r(r)dr$$

- Take integration on both sides and we get

$$s = T(r) = \int_0^r p_r(w)dw$$







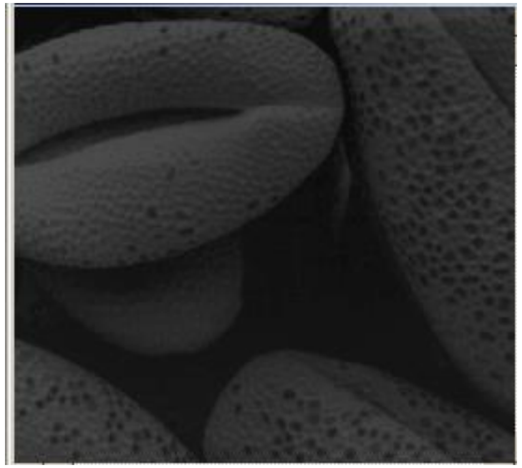
Histogram Equalization – Discrete Case

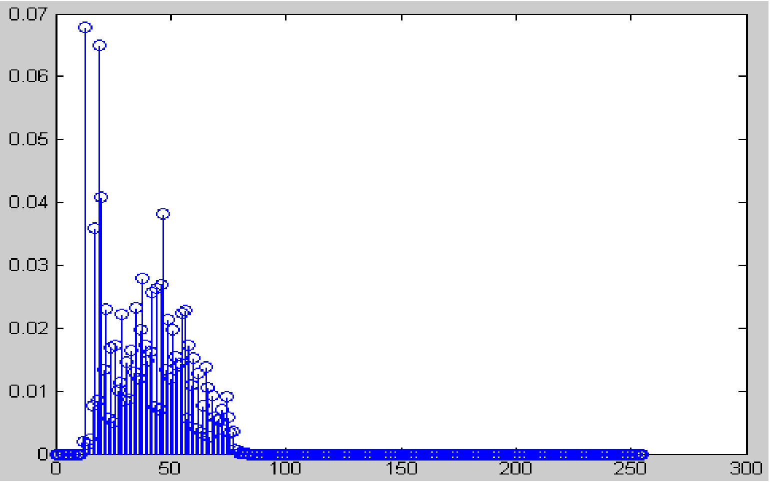
- In the discrete case, a uniform distribution in the output can hardly be achieved.
- Approximation is usually used:

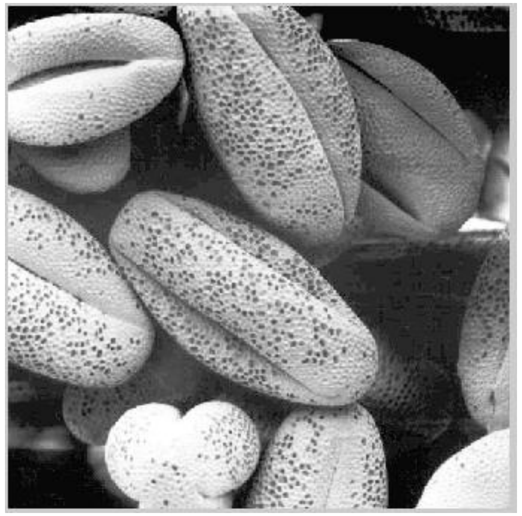
$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0,1,2,\ldots,L-1$$

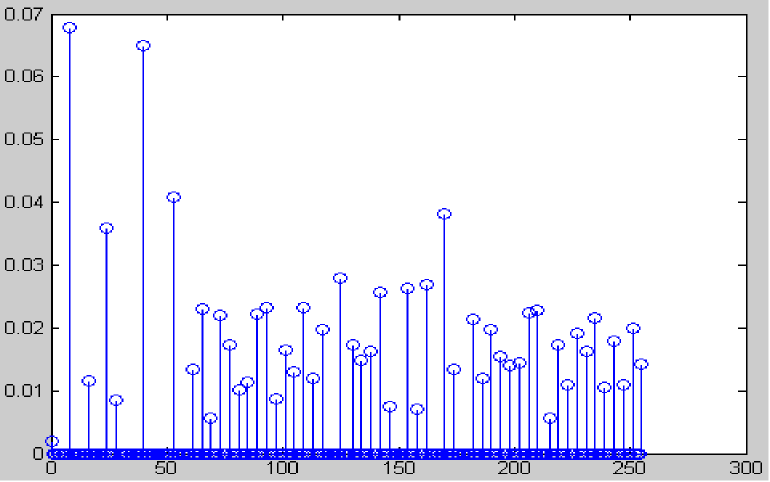
- The discrete version of the transformation function is computed as follows:

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$







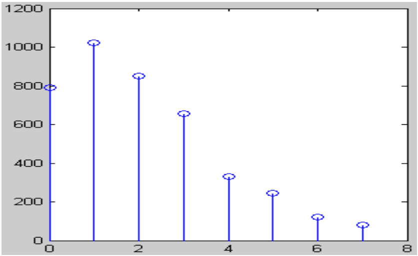


A Numerical Example

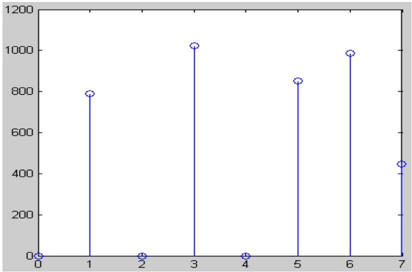
- Consider an 8-level 64 x 64 image with gray values (0, 1, …, 7). The normalized gray values are (0, 1/7, 2/7, …, 1).

Following the transformation: $s = \sum_{j=0}^{k} \frac{n_j}{n}$

k	r_k	n_k	$p(r_k) = n_k/n$
0	0	790	0.19
1	1/7	1023	0.25
2	2/7	850	0.21
3	3/7	656	0.16
4	4/7	329	0.08
5	5/7	245	0.06
6	6/7	122	0.03
7	1	81	0.02



k	s_k	n_k	$p(s_k) = n_k/n$
0	1/7	790	0.19
1	3/7	1023	0.25
2	5/7	850	0.21
3	6/7	985	0.24
4	1	448	0.11



$$s_0 = T(r_0) = \sum_{j=0} p_{in}(r_j) = p_{in}(r_0) = 0.19 \rightarrow 1/7$$

$$s_1 = T(r_1) = \sum_{j=0}^{1} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) = 0.44 \rightarrow 3/7$$

$$s_2 = T(r_2) = \sum_{j=0}^{2} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + p_{in}(r_2) = 0.65 \rightarrow 5/7$$

$$s_3 = T(r_3) = \sum_{j=0}^{3} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + \cdots + p_{in}(r_3) = 0.81 \rightarrow 6/7$$

$$s_4 = T(r_4) = \sum_{j=0}^{4} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + \cdots + p_{in}(r_4) = 0.89 \rightarrow 6/7$$

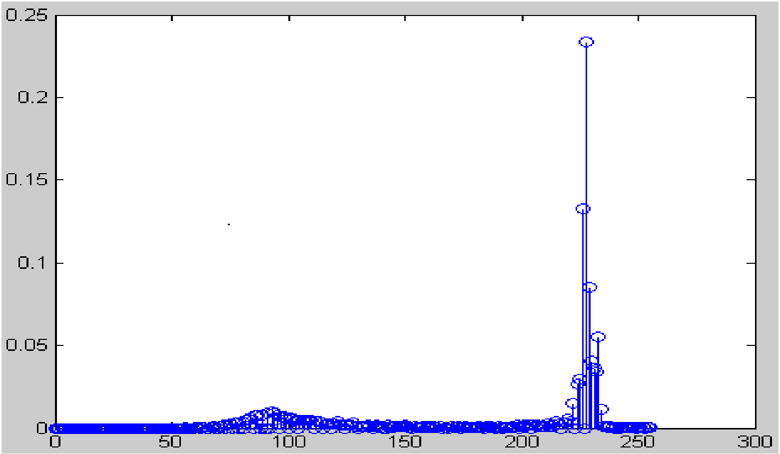
$$s_5 = T(r_5) = \sum_{j=0}^{5} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + \cdots + p_{in}(r_5) = 0.95 \rightarrow 1$$

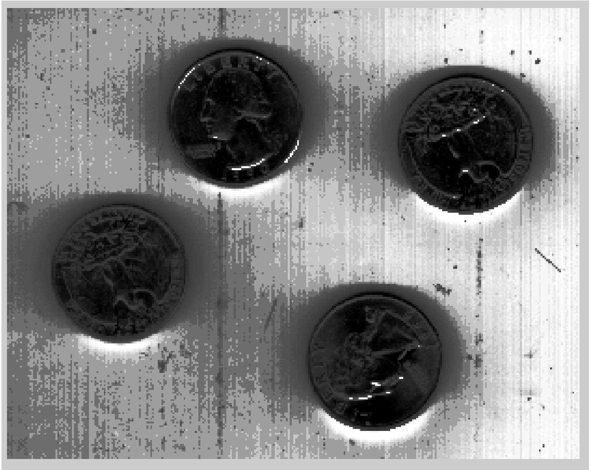
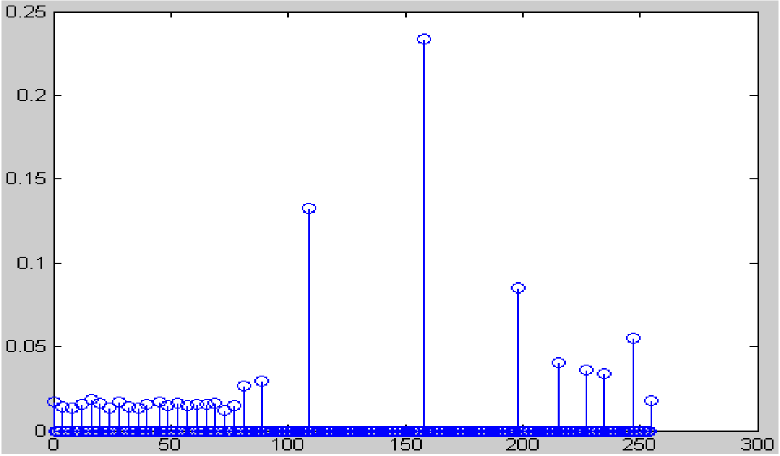
$$s_6 = T(r_6) = \sum_{j=0}^{6} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + \cdots + p_{in}(r_6) = 0.98 \rightarrow 1$$

$$s_7 = T(r_7) = \sum_{j=0}^{7} p_{in}(r_j) = p_{in}(r_0) + p_{in}(r_1) + \cdots + p_{in}(r_7) = 1.00 \rightarrow 1$$

Issues of Histogram Equalization

- Histogram equalization offers advantages of full automation and makes full range of color.
- It may not always produce desirable results
  - when image histogram has long tails.
- It can produce false edges and regions. It can also increase image “graininess.”

Histogram Matching

- Histogram matching is used to specify the shape of the histogram that we wish an image to have.
  - We can specify the target histogram by giving a sample image.
- In the continuous case, if we map the histograms of two images (input and target) into the uniform distribution following the histogram equalization, we can have two transformation functions T and G for input and target images, respectively:

$$s = T(r) = \int_0^r p_{in}(w)dw \quad \text{and} \quad v = G(z) = \int_0^z p_{tg}(w)dw$$

- We apply the inverse transformation of the target image, G^{-1}, to the input image, such that the histogram of the input image matches the histogram of the target image:

$$\mathrm{z} = H(r) = G^{-1}(T(r))$$

Histogram Matching – Discrete Case

- Obtain the histogram of the input image, $p(r_k)$.
- Map gray level r_k into s_k following the histogram equalization scheme

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$

- Obtain the transformation function G from the target image using

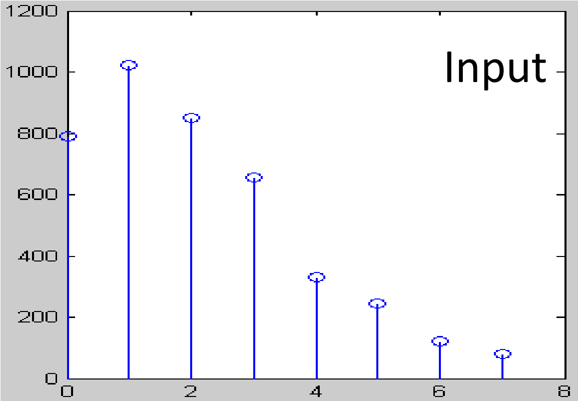
$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j) = s_k$$

- Pre-compute z_k for each value of s_k using the iterative scheme defined in connection with

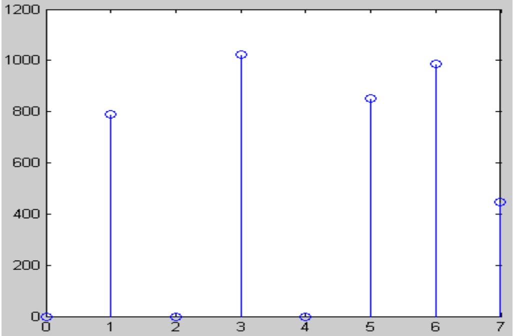
$$(G(\hat{z}) - s_k) \geq 0$$

- For each pixel in the input image, if the value of that pixel is r_k, map it to the corresponding level s_k; then map s_k into the final level z_k.

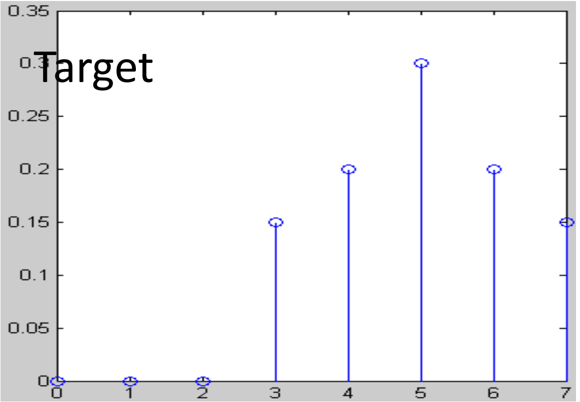
A Numerical Example



k	r_k	n_k	$p(r_k) = n_k/n$
0	0	790	0.19
1	1/7	1023	0.25
2	2/7	850	0.21
3	3/7	656	0.16
4	4/7	329	0.08
5	5/7	245	0.06
6	6/7	122	0.03
7	1	81	0.02



$r_j \rightarrow s_k$	n_k	$p(s_k)$
$r_0 \rightarrow s_0 = 1/7$	790	0.19
$r_1 \rightarrow s_1 = 3/7$	1023	0.25
$r_2 \rightarrow s_2 = 5/7$	850	0.21
$r_3, r_4 \rightarrow s_3 = 6/7$	985	0.24
$r_5, r_6, r_7 \rightarrow s_4 = 1$	448	0.11



k	z_k	$p_{\text{out}}(z_k)$
0	0	0.00
1	1/7	0.00
2	2/7	0.00
3	3/7	0.15
4	4/7	0.20
5	5/7	0.30
6	6/7	0.20
7	1	0.15

Computing Mapping Function G

$$v_0 = G(z_0) = \sum_{j=0}^{0} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) = 0.00 \rightarrow 0$$

$$v_1 = G(z_1) = \sum_{j=0}^{1} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) = 0.00 \rightarrow 0$$

$$v_2 = G(z_2) = \sum_{j=0}^{2} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + p_{\text{out}}(z_2) = 0.00 \rightarrow 0$$

$$v_3 = G(z_3) = \sum_{j=0}^{3} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_3) = 0.15 \rightarrow {}^{1}\!/_{7}$$

$$v_4 = G(z_4) = \sum_{j=0}^{4} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_4) = 0.35 \rightarrow {}^{2}\!/_{7}$$

$$v_5 = G(z_5) = \sum_{j=0}^{5} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_5) = 0.65 \rightarrow {}^{5}\!/_{7}$$

$$v_6 = G(z_6) = \sum_{j=0}^{6} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_6) = 0.85 \rightarrow {}^{6}\!/_{7}$$

$$v_7 = G(z_7) = \sum_{j=0}^{7} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_7) = 1.00 \rightarrow 1$$

$$v_0 = G(z_0) = \sum_{j=0}^{0} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) = 0.00 \rightarrow 0$$

$$v_1 = G(z_1) = \sum_{j=0}^{1} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) = 0.00 \rightarrow 0$$

$$v_2 = G(z_2) = \sum_{j=0}^{2} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + p_{\text{out}}(z_2) = 0.00 \rightarrow 0$$

$$v_3 = G(z_3) = \sum_{j=0}^{3} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_3) = 0.15 \rightarrow {}^{1}\!/_{7}$$

$$v_4 = G(z_4) = \sum_{j=0}^{4} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_4) = 0.35 \rightarrow {}^{2}\!/_{7}$$

$$v_5 = G(z_5) = \sum_{j=0}^{5} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_5) = 0.65 \rightarrow {}^{5}\!/_{7}$$

$$v_6 = G(z_6) = \sum_{j=0}^{6} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_6) = 0.85 \rightarrow {}^{6}\!/_{7}$$

$$v_7 = G(z_7) = \sum_{j=0}^{7} p_{\text{out}}(z_j) = p_{\text{out}}(z_0) + p_{\text{out}}(z_1) + \cdots + p_{\text{out}}(z_7) = 1.00 \rightarrow 1$$

Gray Level Lookup Table

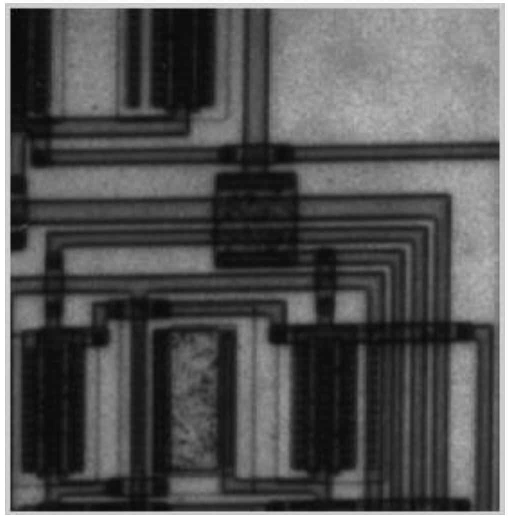
Inverse function

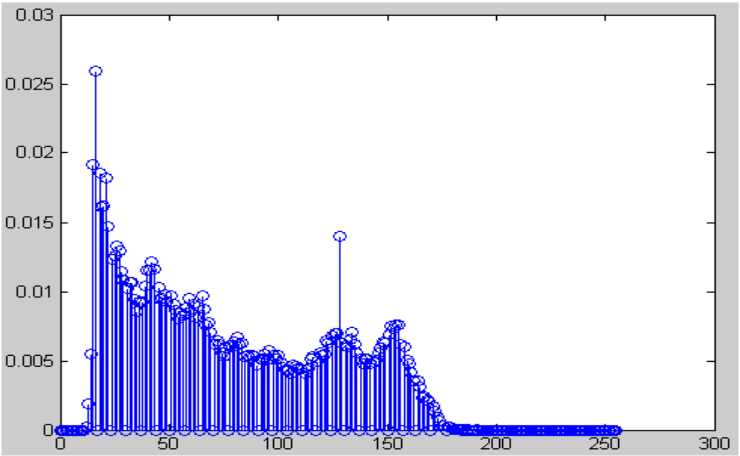
$G^{-1}(0)$	?
$G^{-1}(\frac{1}{7})$	$\frac{3}{7}$
$G^{-1}(\frac{2}{7})$	$\frac{4}{7}$
$G^{-1}(\frac{3}{7})$	?
$G^{-1}(\frac{4}{7})$	?
$G^{-1}(\frac{5}{7})$	$\frac{5}{7}$
$G^{-1}(\frac{6}{7})$	$\frac{6}{7}$
$G^{-1}(1)$	1

$r \to s$	
0	1/7
1/7	3/7
2/7	5/7
3/7	6/7
4/7	6/7
5/7	1
6/7	1
1	1

$s \to z$	
0	?
1/7	3/7
2/7	4/7
3/7	?
4/7	?
5/7	5/7
6/7	6/7
1	1

$r \to z$	
0	3/7
1/7	4/7
2/7	5/7
3/7	6/7
4/7	6/7
5/7	1
6/7	1
1	1

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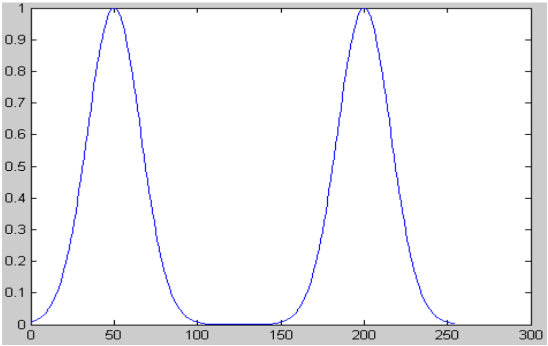
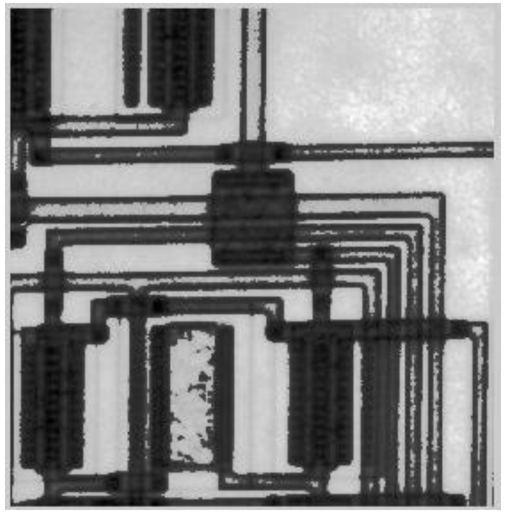
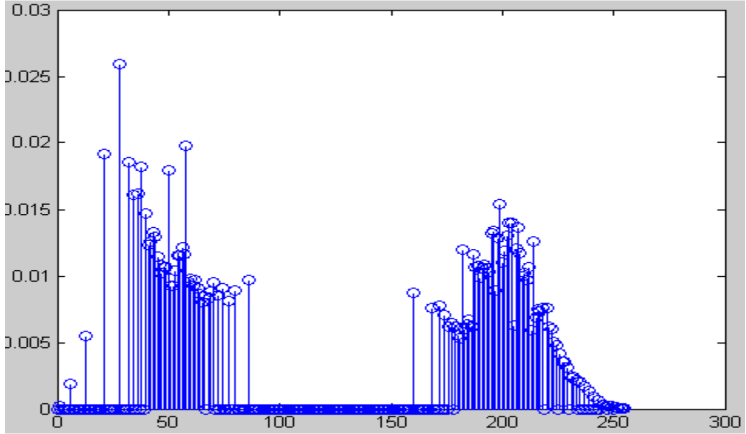
0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
200
250
300


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300




0.03
0.02
0.01
0
0
50
100
150
200
250
300