

# The dynamics of memory as a consequence of optimal adaptation to a changing body

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There are many causes for variation in the responses of the motor apparatus to neural commands. Fast-timescale disturbances occur when muscles fatigue. Slow-timescale disturbances occur when muscles are damaged or when limb dynamics change as a result of development. To maintain performance, motor commands need to adapt. Computing the best adaptation in response to any performance error results in a credit assignment problem: which timescale is responsible for this disturbance? Here we show that a Bayesian solution to this problem accounts for numerous behaviors of animals during both short- and long-term training. Our analysis focused on characteristics of the oculomotor system during learning, including the effects of time passage. However, we suggest that learning and memory in other paradigms, such as reach adaptation, adaptation of visual neurons and retrieval of declarative memories, largely follow similar rules.

Suppose that we are designing the control mechanism of an autonomous robot. We recognize that motors in various limbs will change their characteristics with use and with the passage of time. For example, with repeated use over a short period, a motor may heat up and change its response transiently until it cools. On the other hand, with repeated use over a long period, the batteries may gradually discharge and the power will only return to near its original state after a recharge. Both of these conditions will produce movement errors, requiring our controller to adapt and send updated commands to the motors to produce the desired actions. However, our controller should interpret these errors differently: errors that have a fast timescale should result in rapid adaptive changes, but should be quickly forgotten. Errors that persist for extended periods of time should result in slow adaptive changes.

The nervous system seems to face similar problems in controlling the body. Properties of our muscles change as a result of a variety of disturbances, such as fatigue<sup>1</sup>, disease, exercise and development. The states of these disturbances affect the motor gain: that is, the ratio of movement magnitude relative to the input signal. States of disturbances unfold over a wide range of timescales. Therefore, when the nervous system observes an error in performance, it faces a credit assignment problem: given that there are many possible perturbation timescales that could have caused the error, which is the most likely? We think that the solution to the problem should dictate the temporal properties of the resulting memory. That is, adaptation in response to things that are likely to be permanent should be remembered, whereas adaptation in response to things that appear transient should be forgotten.

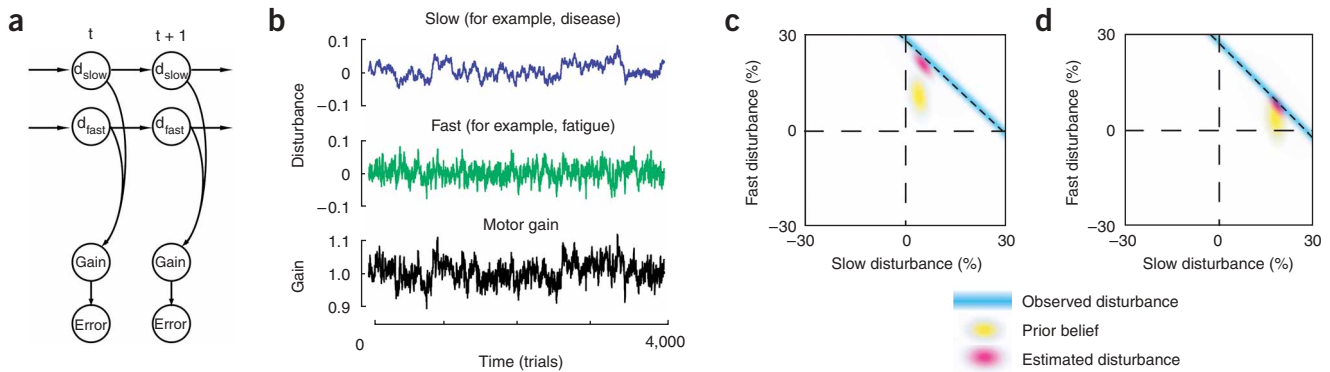
## RESULTS

Bayesian statistics allows us to formalize this problem and predict the behavior of a rational learner. Suppose that the motor plant is affected by disturbances that can come in a variety of timescales. Each disturbance will have a state, here represented as a random variable, that evolves independent of other states (**Fig. 1a**). This implies that fatigue state does not directly affect disease state. A long-timescale disturbance state, such as the general health state, will go up slowly and go down slowly (**Fig. 1b**). A short-timescale disturbance state, such as fatigue state, will change rapidly. We assume that the moment-to-moment variance of the states is higher for faster timescales than for slower timescales (although all disturbances will have the same variance over long times; see Methods for details). Finally, we assume that the various states combine linearly to affect the motor plant, resulting in perturbations to the motor gain. This motor gain defines the movement and thus the movement error. From these assumptions, the Bayesian formalism directly leads to our predictions about learning and memory.

The Bayesian learner introduced here observes the motor error (deviations from unity gain), but needs to estimate the states of the various potential disturbances. Is the error due to fatigue or something more serious? As the states of the various timescales can never be known, the learner represents its knowledge as a probability distribution. Before an observation is made, the learner has a prior belief. For example, if there are only two states, then the prior belief is characterized by a Gaussian distribution (yellow cloud in **Fig. 1c**). When the learner observes an error it has effectively measured the sum contribution of all states: this leads to a diagonal area of high likelihood (**Fig. 1c**). This measurement will be affected by noise, and so the uncertainty of the learner in its measurement is displayed as the

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**Figure 1** A generative model for changes in the motor plant and the corresponding responses of a Bayesian learner to performance errors. For illustrative purposes, here we show the results of a simulation with just two timescales. **(a)** Various disturbances ( $d$ ) evolve over time as independent random walks that linearly combine to change the motor gain. The observed error is a noisy version of the gain disturbance. **(b)** Sample disturbances and the resulting motor gain. **(c)** The Bayesian learner's belief during an experiment where a disturbance suddenly increases the gain of the motor plant. Before the learner observes the gain, it has a prior belief. The learner's belief can be represented by its current estimate of the fast and slow disturbances and its uncertainty about this estimate. This is termed a prior and is shown in yellow. In this case, the prior has a larger uncertainty along the fast state. In each trial, the learner observes the disturbance to the motor gain (in this case a 30% increase). This observation is represented by the blue line. The observation is a line and not a point because the disturbance could be due to a fast timescale with magnitude of 30%, a slow timescale with magnitude of 30% or any other point along this line. Because the learner has sensory noise, there is a probability distribution associated with its observation, and therefore the blue line is hazy. To solve the credit assignment problem, the learner integrates its observation (blue line) with the prior belief (yellow cloud) to generate a posterior estimate (red cloud). In this case, because uncertainty was greater for the faster timescales, the observation was mostly assigned to a fast timescale perturbation. **(d)** The perturbation is sustained for 30 trials. Now the learner associates the perturbation with a slow timescale.

thickness of the blue line. By combining the measurement with its prior knowledge, the learner comes up with a new estimate (the posterior, red cloud).

How did the learner solve the credit assignment problem? When the learner observes a large error on a given trial, it needs to estimate whether this is due to a fast or a slow disturbance. In this example, the first time the large disturbance happens, the system infers that the disturbance is most likely due to a fast disturbance, as the prior belief is skewed in the direction of fast timescales. That is, the yellow cloud characterizing the distribution (**Fig. 1c**) has a larger variance along the fast state. This skew arises because of the assumption that disturbances with fast timescales are affected by greater variability. Therefore, the red cloud (**Fig. 1c**) is centered on a large contribution by the fast timescale. However, if the system keeps observing large errors, this finding is best explained in terms of a slow disturbance, as a fast disturbance would be expected to quickly dissipate (**Fig. 1d**). This would explain why adaptation tends to show a rapid initial phase followed by a slower phase of performance changes.

This credit assignment also may work in more complex situations. For example, if there is a negative perturbation averaged over the last hundred trials, whereas the last three trials had positive perturbations, then the system would infer a long-timescale negative and a short-timescale positive disturbance. In this scenario, the sum of the states might be zero, indicating a motor gain of 1, but the learner knows that the various states have not returned to their baseline. This would explain why adaptation followed by a limited period of de-adaptation does not wipe out the memory.

The Bayesian learner's estimates of the contributions of each timescale, as well as the uncertainties of these estimates, are constantly changing in response to the observed outcomes of each motor command. Whenever a movement error is observed, the state estimates adapt and the uncertainty decreases. The learner thus becomes less sensitive to further errors. However, when time passes without the learner observing consequences of its actions (for example, in darkness or in sleep), the disturbances are expected to get smaller, as each

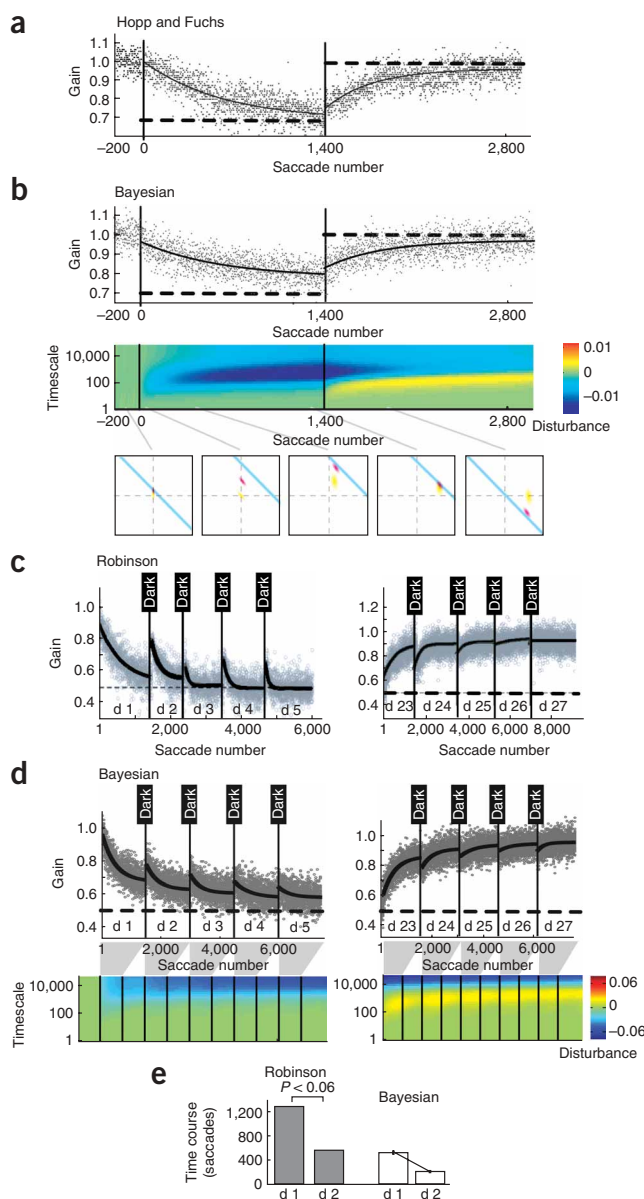
disturbance tends to vanish over its own timescale. Therefore, the learner's beliefs will change even when it cannot observe motor error. However, when the learner is prevented from observing motor errors, its uncertainty increases. This makes the interesting prediction that the learner will be more sensitive to errors that follow a period of sensory deprivation, and will therefore learn at a faster rate after a period of darkness or sleep as compared with its learning rate before that period.

This is the statistically optimal way for handling the motor errors to estimate the gain of the motor plant. Here we show that this simple computational framework is able to account for a large body of behavioral data.

### Short- and long-term effects of saccadic gain adaptation

Motor adaptation has been extensively studied in the context of saccades. Saccades are rapid eye movements that shift the direction of gaze from one target to another. The eyes move so fast that visual feedback cannot usually be used during the movement<sup>2</sup>. For that reason, any changes in the properties of the oculomotor plant that are not compensated for would lead to inaccurate saccades<sup>3</sup>. It has been observed that if saccades overshoot the target, the motor gain (that is, the ratio of eye displacement to target displacement) tends to decrease, and if they undershoot, the gain tends to increase. For example, when motor gain decreases to an amount smaller than 1, the nervous system must send a stronger command to produce a movement of the correct size. The saccadic jump model<sup>4</sup> is a way to probe such adaptation<sup>5</sup>: while the subject moves its eyes toward a target, the target is moved. For a monkey, the rate of adaptation to this disturbance is similar to the rate of adaptation in response to the weakening of eye muscles<sup>6,7</sup>, suggesting that the error is interpreted as a change in the eye plant. Using this model, it is possible to probe the mechanism that is normally used to adapt to ongoing changes of the oculomotor plant.

In a number of previous experiments<sup>4</sup>, investigators have examined how monkeys adapt their saccadic gain. The gain changes over time so that saccades progressively become more precise (**Fig. 2a**). The rate of adaptation typically starts fast and then progressively gets slower. This



**Figure 2** Short-term and long-term behavior in response to saccadic gain changes. **(a)** Short-term training. Each dot represents one saccade and the thick lines are exponential fits to the intervals [0;1,400] and [1,400;2,800]. A target was displayed starting at saccade number 0, and as soon as the saccade started, the target jumped back by 30%. The adaptation that would negate this target jump is indicated as horizontal dashed lines. This manipulation ended at saccade number 1,400, beyond which were washout trials. Reprinted from ref. 3 with permission. **(b)** The same plot is shown for the saccades during the simulated experiment for the Bayesian learner. The color plot shows the learner's estimates of the state of each disturbance (we assumed 30 different states, ranging from very short to very long). Colors indicate the estimate of the mean of each disturbance, before updating with the new feedback; negative disturbances are denoted as blue, whereas positive disturbances are denoted as red. The color plot shows the values for all saccades, including the saccades simulated during darkness. The sum of the various states is the expected gain of the motor plant with respect to unity. The subplots below this figure show the belief of the Bayesian learner during the initial stages of gain decrease and then after 30 trials, approximated by two timescales. **(c)** Long-term training. In this experiment<sup>11</sup>, the saccadic gain was reduced over many days of training. At the end of each training session the monkey was blindfolded and held in darkness for the remainder of the day. Note that the rate of relearning on day 2, following darkness, is faster than initial rate of learning. Black lines show exponential fits to the data. **(d)** The same plot for the Bayesian learner along with a color plot showing the estimate of the learner of the disturbance at each timescale. **(e)** Comparison of the saccadic gain-change time course obtained by fitting an exponential function to the set of all saccades during the day.

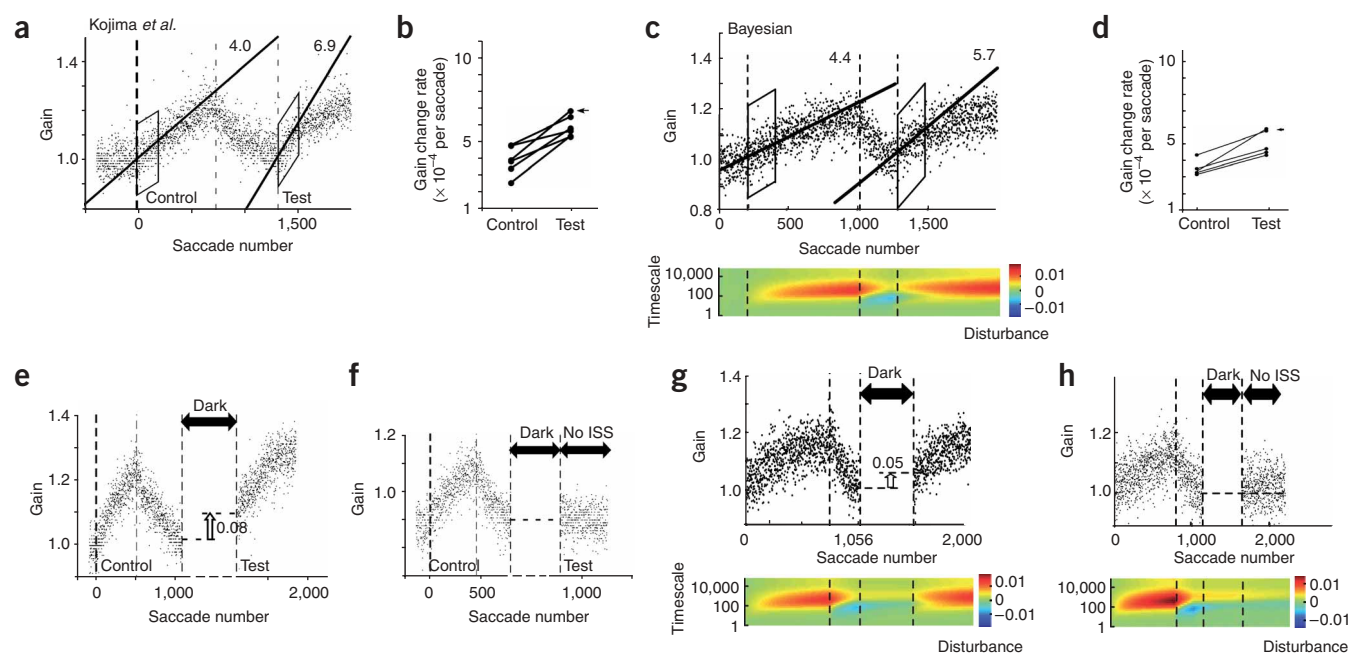
states are positive, while the slower states are negative. Adaptation followed by de-adaptation may not wash out the system.

Our model not only accounts for relatively brief periods of adaptation that are typically involved in laboratory settings, it also accounts for behavior during long-term periods of training. For example, let us consider a recent experiment<sup>11</sup> where the saccadic gain adaptation was set to  $-50\%$ . The monkey adapted for about 1,500 saccades every day for a consecutive 21-d period, and then after several days of washout trials, de-adapted back to a gain of unity. Notably, the monkey wore goggles that blocked vision after training each day. Multiple effects are visible in the data (**Fig. 2c**). First, we note that there were several timescales during adaptation: there was a fast (100 saccades) and a slow (10 d) timescale. Second, we note that the starting point of performance on each day is a bit higher than the final performance in the previous day. Third, relearning rates are affected by the periods of darkness. For example, the learning rate on the second day was much faster than that on the first day. Finally, during the gain-down adaptation (days 1–22), performance following darkness has decayed toward a gain of unity. However, during wash-out (days 23–27), the decay is toward a gain of 0.5. That is, the system appears to ‘forget’ in different directions during the two phases of learning.

Our model's behavior (**Fig. 2d**) was markedly similar given that we used the same parameters that we inferred from the single session adaptation in a previous experiment<sup>3</sup> (**Fig. 2b**). The quantification of the system's estimates of disturbances shows that the system did indeed infer longer timescales and stronger deviations over time (deeper blues at longer timescales in **Fig. 2d**, color plot). After the initial period of gain-down adaptation training, a period of darkness followed. This means that the monkey was allowed to make saccades, but was not allowed to observe the sensory consequences of its actions (effectively, the noise on the observation is set to infinity). During the darkness period, the learner became uncertain about its beliefs about the states of the motor system. Increased uncertainty means that new observations about motor gain are relatively more precise than old information, which in turn leads to faster learning when the blindfolds are removed. Consequently, although both the monkey and the model forget some of

is a classic pattern that is reflected in numerous motor adaptation paradigms, including reaching<sup>8–10</sup>. The same patterns are seen for the Bayesian learner (**Fig. 2b**). When the gain rapidly changes, credit is mostly assigned to fast states because the uncertainty is greater for them, resulting in rapid adaptation. Between trials, fast states decay rapidly, but this decay is smaller in the slower states. If the perturbation is maintained, the relative contribution of the fast states diminishes in comparison with that of the slow states (the blue bank becomes darker and shifts to longer timescales in **Fig. 2b**). This implies that as training continues the estimate of the gain change is assigned to progressively slower timescales. In practical terms, this results in the often reported observation that, in response to a constant perturbation (that is, a step change in the apparent gain), performance of the learner shows an initially rapid rate of adaptation, followed by progressively slower rates.

When the target no longer jumps (in trial 1,400, the dashed line returns to 1 in **Fig. 2a**), saccade gains return to 1. However, note that the state estimates do not return to baseline: in trial 2,900, the faster



**Figure 3** The double reversal model. (a) The gain was first adapted up until it reaches about 1.2 with a target jump of +35%. Then it was adapted down with a target jump of −35%. Once the gain reaches unity, it was again adapted up with a positive target jump. Data from ref. 13. The box indicates the trials where the line was fitted. The number on the line indicates its slope. (b) The speed of adaptation (slope of the lines in part a) was compared between the first gain-up and the second gain-up trials in different sessions of training. The monkey showed savings in that it relearned faster, despite the apparent washout. (c) The performance of the Bayesian learner is shown, as is the estimate of the learner of the disturbance at each timescale in the color plot. (d) The rate of adaptation for the Bayesian learner. (e) In this experiment, the reversal training was followed by a period of darkness and then by gain-up adaptation<sup>13</sup>. Saccade gain shows spontaneous recovery. (f) In this experiment, the period of darkness was followed by a condition in which the target did not change position during the saccade period (that is, no intrasaccadic step, ISS)<sup>13</sup>. The animal did not show spontaneous recovery. (g) The same plot for the Bayesian learner in e along with a color plot showing the estimate of the disturbance at each timescale. (h) The same plot for the Bayesian learner in f along with a color plot showing the estimate of the disturbance at each timescale.

their learning during darkness, they learn faster during the second day than during the first day (quantified in Fig. 2b,d). Similar results were found in a recent study of ocular reflexes<sup>12</sup>.

The model explains why forgetting apparently occurred in opposite directions during the first and second halves of this experiment. Passage of time produced substantial decay in the fast states. During gain-down adaptation, this results in a forgetting toward a gain of unity because the fast states return to zero. By the end of training on day 22, only the slow states were negative, whereas all the fast states were at zero. During washout, the training caused the fast states to rapidly become positive, pushing the performance away from 0.5 and toward unity. When the gain was returned to unity, it caused the fast states to become positive (yellow in Fig. 2b). Even after 5 d of reversal, the long timescales were still strongly negative (blue in Fig. 2d). As a consequence, when time passed during the darkness period of the washout days, forgetting in the fast states now made the gain estimate drop toward 0.5.

### Double reversal training of saccades

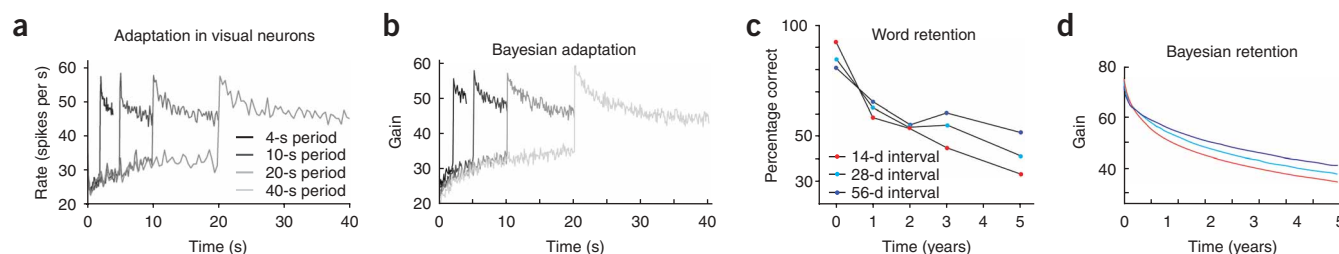
Many motor learning studies have attempted to quantify timescales of memory using an interference protocol. A common theme is a ‘double reversal’ protocol, where the direction of visual errors is changed twice (Fig. 3). For example, in a previous experiment<sup>13</sup>, the saccadic gain was initially increased, then decreased until it reached unity, and finally increased again (Fig. 3a). The animals learned faster during the second gain-up session than during the first (Fig. 3b). The reversal learning apparently reduced the estimated gain of the motor plant back to 1, yet

the monkey still had ‘saved’ some aspect of its previous gain-up training, as it showed savings.

The Bayesian model (using the same parameter values as before) explains this phenomenon (Fig. 3c). At the end of the first gain-up session, most of the gain change was associated with the slow states (they are positive, yellow in Fig. 3c, color plot). In the subsequent gain-down session, errors produced rapid changes in the fast state so that by the time the gain estimate reached unity, the fast and slow states had opposite estimates: the fast states were negative, whereas the slow states were positive. Therefore, the gain-down session did not reset the system because the latent variables stored the history of adaptation. In the subsequent gain-up session, the rate of re-adaptation was faster than that of initial adaptation (Fig. 3d) because the fast states decayed toward zero in between trials, whereas the slow states were already positive. After about 100 saccades, the speed gain from the low frequencies was over and was turned into a slowed increase as a result of the decreased error term.

In another set of experiments, the investigators<sup>13</sup> observed that after a period of darkness where the animal was not allowed to view sensory consequences of its motor commands, there was a sudden jump in performance. In these experiments, gain-up training followed gain-down training until saccade gains were restored to unity. Then the animals spent some time in the dark. Afterward, when the animal was tested in the gain-up task, saccade gain had spontaneously increased (Fig. 3e). The same effect was seen for the Bayesian learner (Fig. 3g). In the dark period, the Bayesian learner made no observations and therefore could not learn from error. However, the estimates were





**Figure 4** The Bayesian learner outside of movement settings. **(a)** The response of a neuron in the fly is shown to a visual stimulus that changed its s.d., switching between two levels (reprinted from ref. 16). **(b)** The data were modeled by a system with many timescales that drifted toward a mean of 40 spikes per s. **(c)** Declarative memory data reprinted<sup>21</sup>. The retention function is shown for word translations that had been learned with different intervals between training sessions. **(d)** The retention function of a Bayesian learner.

still affected by the passage of time: the fast states were negative and rapidly decayed toward zero, whereas the slow states were positive and only slowly decayed (Fig. 3g, color plot). The sum was a positive disturbance that after an initial transient, slowly decayed. Consequently, by the end of the dark period, the estimate had become ‘gain-up’. This effect is enhanced by fast learning following the period of darkness.

A recent model of motor adaptation<sup>14</sup> is functionally similar to the model introduced here and explains much of the data on saccadic gain adaptation (Fig. 3) using two integrators operating at different timescales. However, for the Bayesian model, not only did the passage of time during darkness produce changes in the mean of the estimates, but it also made the learner less certain of its belief. Therefore, the Bayesian model made an important prediction: extended periods of darkness should lead to faster subsequent learning. We noted this earlier in the multiday adaptation studies (Fig. 2e). However, the effect was present even in a single-day study. Here, the Bayesian model predicts that relearning will be faster when up-down adaptation is followed by a period of darkness than it will be if the darkness period is replaced with saccades in full light. Indeed, in the available data<sup>13</sup>, after darkness the gain change is much faster ( $5.8 \times 10^{-4}$  versus  $3.8 \times 10^{-4}$  for one monkey,  $9.3 \times 10^{-4}$  versus  $6.8 \times 10^{-4}$  for the other, with  $P < 0.05$  and  $P < 0.001$ , respectively). These effects of post-darkness change in rates of learning come about only if the passage of time has an influence on the uncertainty of the learner. That is, the passage of time affects the learner’s knowledge in terms of both its mean and variance, demonstrating that sensory deprivation leads to faster learning. This data is hard to explain with models that do not incorporate uncertainty<sup>14</sup>.

If the period of darkness is followed by a period without intrasaccadic target jumps (Fig. 3f), then the animal did not show spontaneous recovery. At first glance this would suggest some kind of context-dependent recall. However, the Bayesian learner showed a similar behavior (Fig. 3h) and the model explains that the effect was not due to context, but to uncertainty. At the end of the darkness period, the slow states were at a positive gain while the fast states were near zero. When darkness is followed by gain-up training, all states are more uncertain and therefore rapidly move toward a positive gain. On the other hand, when darkness is followed by unity gain training, the gain-up status of the slow states is rapidly negated by the fast states that now become negative.

### Adaptation outside the motor system

Many phenomena outside the realm of muscle properties can be expected to happen on multiple timescales. For example, the contrast of visual scenes may follow similar rules<sup>15</sup>. To adapt optimally, the nervous system might need to estimate the current level of contrast from past values. Recently, investigators measured how visual neurons

adapted to stimuli that changed on several different timescales<sup>16</sup> (Fig. 4a). It was found that adaptation timescales among the neurons were longer as the interval between switches of contrast was lengthened. The Bayesian learner showed very similar effects (Fig. 4b). Multiple timescale learning and adaptation may be optimal even for sensory phenomena.

Analogous problems of multiple-timescale inference may also be solved by the nervous system in certain cognitive tasks. For instance, in the retrieval of long-term declarative memories, numerous studies occurring over a century have explored ‘spacing effects’: a specific item will typically be remembered longer if the study trials for that item are spaced out over a long training period, rather than clustered in a short training period<sup>17–20</sup>. The spacing effect in one classic study of long-term memory for vocabulary words in a foreign language<sup>21</sup> is shown in Figure 4c. Spacing effects might seem counterintuitive if we think of forgetting as a passive decay process with a fixed time-constant, but the phenomenon should be familiar, from the often repeated (and often ignored) advice of school teachers that steady studying over a whole term leads to better retention of learning than intensive cramming right before the exam.

Our framework can explain spacing-dependent forgetting curves (Fig. 4d) as follows. Let us assume that the strength of a memory trace reflects the modulation of a ‘cognitive gain’ and that each encounter or study trial with a specific item results in a measured gain of 1 for that item. Intuitively, the model attempts to infer the changing importance of a given item, allowing that the importance of different items could rise and fall over different timescales. These timescales are reflected in the item’s past use: when experience with an item has been spaced over a long period of time, it provides evidence that the item is of long-term relevance. In contrast, an item with clustered experience or practice is more likely to be of only short-term interest.

### DISCUSSION

Traditional models of adaptation simply change motor commands to reduce prediction errors<sup>22,23</sup>. We approached the problem from a different point of view: if the CNS knows that the body is affected by perturbations that have multiple timescales, then the problem of learning in the CNS is really one of credit assignment. The rational approach would be to do three things. First, the learner should represent its knowledge of the properties of the motor system, including how disturbances of various timescales can affect it. Second, it should represent the uncertainty it has about its beliefs. Third, it should formulate the computational aim of adaptation in terms of optimally combining what it knows about the properties of the motor plant with the current observations. The experimental predictions of the presented model derive from the way knowledge about the state of the

motor plant is combined with noisy feedback into a statistically optimal estimate.

Although our work may be the first model of learning in which all three of these points are considered together, earlier studies have considered these points separately. In a previous model<sup>14</sup>, it was proposed that the brain responds to error with at least two systems: one that is highly sensitive to error but rapidly forgets, and another that has poor sensitivity to error but strong retention. That model explained savings and spontaneous recovery and demonstrated that during the period of darkness motor estimates take the form of the sum of two exponentials, one with a fast and the other with a slow time constant. Without incorporating uncertainty, it is difficult to explain the full range of phenomena described here.

The idea of multiple timescales has also been proposed in the context of connectionist learning theory<sup>24</sup> and in motor learning<sup>8,10</sup>. Connectionist models, as well as earlier motor learning models, have no systematic way of modeling uncertainty about timescales. The phenomenon of spontaneous recovery in classical conditioning<sup>25</sup> fits well into the framework presented here. In classical conditioning, it has been proposed that the nervous system should keep a measure of uncertainty about its current parameter estimates to allow for an optimal combination of new information with current knowledge<sup>26</sup>. That model included a measure of uncertainty and a mechanism mediated by neuromodulators for allowing fast changes at catastrophic moments. The multiple timescales of potential disturbances proposed here may lead to similar results, as fast timescales may take care of catastrophic fast changes. Moreover, Kalman filters have been used for systems identification in engineering to solve similar problems<sup>27</sup>. They have also been proposed to model how different joints may contribute to movements to explain generalizations from one movement to another and fast learning for high uncertainty<sup>28</sup>. Finally, even the earliest studies of oculomotor adaptation realized that the objective of adaptation is to allow precise movement with a relentlessly changing motor plant<sup>4</sup>. Our approach unifies these ideas in a coherent computational framework.

In the saccade experiments that we considered, darkness corresponded to a period of time when the animal makes eye movements, but is not allowed to observe the visual consequences of its motor commands. Because our model shows how a rational learner would update its knowledge when faced with sensory deprivation, it gives one explanation as to why there are improved rates of learning after periods of darkness. It is interesting to view sleep in a similar framework: as essentially a period in which the brain simulates movements, but is deprived of actual feedback. The post-sleep improvements in rates of learning may be partly due to an increased uncertainty regarding the states of the internal model.

There are features of adaptation that our model in its current form does not explain. For saccades, three kinds of asymmetries are observed: adaptation up is faster than adaptation down, unlearning after up adaptation is slower and spontaneous recovery is only observed in the up direction. It is an interesting question as to how these asymmetries arise. We know that our body is not symmetric with respect to strengthening and weakening of muscles. For example, we often experience errors due to rapid fatigue, but errors due to fast strengthening are really quite rare. Such an asymmetric history of perturbations can, in principle, explain both the fact that gain-down learning is slower than gain-up and the fact that spontaneous recovery is present only in the gain-up direction. The asymmetries may, however, also indicate effects stemming from suboptimal neural computation. Similarly, for the adaptation of visual neurons there exists a clear asymmetry between upward and downward adaptation.

The nervous system should also have some way of learning the importance of each possible timescale. Hierarchical Bayesian models allow a straightforward modeling of such phenomena. Moreover, the model presented here uses a simple definition of time. For example, the nervous system may model that our motor system changes less if we do not move than if we move. Such a situation may be an analog to a Kalman filter that runs fast in the presence of movement and much slower in the absence of movement. It is known that adaptation is highly context dependent<sup>29–31</sup>, and indeed we should only generalize from one situation to another situation that is similar. In this model, we cut out all properties apart from time and error magnitude to predict purely temporal adaptation phenomena.

It should be clear that we modeled the animal's learning here as if the errors were due to the behavior of the motor plant, when in fact errors were due to clever manipulations in the outside world. As long as changes in the world happen according to similar rules, or the subject does not know it is dealing with changes in the outside world, our model works well. However, it is likely that the world goes through more step-type changes than our body, in particular the world in a neuroscientist's laboratory. In that case, the nervous system has to solve an additional credit assignment problem: is the error due to a change in my body or due to a change in the world? We find it intriguing that different species may have different ways of solving this problem. Gain adaptation training in monkeys generalizes broadly to other types of saccades<sup>32</sup>, agreeing with our simple model of associating the errors to changes in the oculomotor plant. However, similar training in humans is context specific and shows more specific generalization patterns, suggesting that the credit assignment is mostly to the model of the world<sup>33</sup>. Yet humans have ample experience with changes in the world, such as the wearing of glasses, that demand specific context-dependent patterns of adaptation.

It should be emphasized that we did not model the mechanisms of any specific memory, but rather attempted to present a general model for all memories using a generalization of how the brain would learn to control the motor plant. For example, let us consider spacing-training trials. Spacing effects have been observed robustly across many timescales and stimuli<sup>34,35</sup>. Spacing effects fall out naturally and quite generally from making rational statistical inferences about the timescales over which a given piece of information's relevance is changing. This is the same kind of inference that the motor system must make about potential motor disturbances. A similar view of memory retrieval has been previously suggested<sup>36</sup>, inspired by a model for predicting library-book access, and shows how this model could predict spacing effects and other dynamical aspects of declarative memory. Our results suggest that common principles of memory and forgetting may be at work more broadly across the nervous system. Both higher-level cognitive learning and lower-level sensorimotor learning face a shared challenge of adapting their behavior to processes in the world that can unfold over different timescales.

An important question for further inquiry is how the nervous system solves problems that require multiple timescale adaptation. Our idea that the general rules for learning and memory may have arisen from time-dependent properties of the motor system gains credence from a recent observation that saccades can fatigue the eye muscles, producing a short-term adaptive response in the cerebellum<sup>37</sup>. The compensation of saccadic fatigue is determined by the adjustment of a Purkinje cell simple spike population signal. If this adaptation process is happening in the cerebellum<sup>38,39</sup>, the necessary effects could potentially be implemented directly by synapses that show long-term depression with power-law characteristics<sup>40,41</sup>. Alternatively, at least for small timescales, small groups of neurons may jointly represent the estimates

along with their uncertainties. A second question is how the nervous system infers the timescale and noise properties of the disturbances. The Bayesian learner may begin with a prior assumption about the structure of the 'generative model,' but adapts the parameters of this model as it experiences the world.

## METHODS

The Bayesian approach makes it necessary to explicitly specify all the assumptions we are making as to how the motor plant may change over time.

**Disturbances.** Our problem of learning is one of state estimation, where state refers to the state of the disturbances. Each disturbance was modeled as a random walk that was independent of all other disturbances:

$$\text{disturbance}_\tau(t+\Delta) = (1 - 1/\tau) \cdot \text{disturbance}_\tau(t) + \varepsilon_t \quad (1)$$

where  $\varepsilon_t$  was drawn from a mean zero normal distribution of width  $\sigma_\tau$ , and  $\tau$  was the timescale. The larger the value for  $\tau$ , the closer  $(1 - \tau^{-1})$  is to 1 and the longer a disturbance typically lasts. The motor gain was simply 1 plus the sum of all the disturbances:

$$\text{gain}(t) = 1 + \sum_{\tau} \text{disturbance}_\tau(t) \quad (2)$$

Equation (1) is the state update equation. The problem of state estimation is to estimate the states from measured output (see below). We do this via a Kalman filter. In our simulations each saccade is simulated as one time step of the Kalman filter. The  $\tau$  are thus defined in terms of saccades.

**Parameters.** Only those timescales will matter that are not much longer than the overall time of the experiment (because they would already have been integrated out) and that are not much shorter than the time of an individual trial (because they would average out). For that reason we chose the distribution of  $\tau$  to be 30 values exponentially scaled between 2 and  $3.3 \times 10^5$  saccades. Choosing a larger number of disturbances while correcting for the overall variance had little effect on the results. We chose 30 timescales as an approximation to a continuous distribution to allow our simulation to run quickly. Once chosen, the timescales remained fixed. The distribution of expected gains thus depended only on the distribution of  $\sigma_\tau$ , a characterization of how important disturbances were at various timescales. It seemed plausible that disturbances with a short timescale tended to be more variable than those that had a long timescale: over the timescale of about a year we can double our strength through workout; over the timescale of a week we can half our strength if we get ill; and over the timescale of a minute we can half our strength through fatigue. Each such effect seems to be similarly important, although we acknowledge that there are many more timescales. Therefore we chose  $\sigma_\tau^2 = c\tau^{-1}$ , where  $c$  is one of the two free parameters of our model. We have thus specified the prior assumption about the body that drives adaptation.

On each trial the learner made an observation about the state of the motor plant. We assumed that this observation was corrupted by noise:

$$\text{observation}(t) = \text{gain}(t) + w \quad (3)$$

where  $w$  was the observation noise with a width  $\sigma_w$ . This is the second free parameter in our model. Throughout this paper we chose  $\sigma_w = 0.05$ , which we estimated from the spread of saccade gains over typical periods of 200 saccades and  $c = (0.001)^2$ , because that yielded good fits to previously reported data<sup>3</sup>. We chose to model all data using the same set of parameters to avoid issues of over-fitting.

**Inference.** Given this explicit model, Bayesian statistics allowed for deriving an optimal adaptation strategy. MATLAB files for repeating these simulations are available online (see **Supplementary Scripts** online, which also contains parameter sensitivity analysis). We observed that the system was equivalent to the generative model of the Kalman filter<sup>42</sup> with a diagonal state transition matrix  $M = \text{diag}(1 - \tau^{-1})$ , an observation matrix  $H$  that is a vector consisting of a 1 for each of the 30 potential disturbances, and a diagonal state noise matrix of  $Q = \text{diag}(c\tau^{-1})$ . State noise was what was driving the changes of each of the disturbances. This Kalman filter represents its knowledge about disturbances by

two entities, a state vector of length 30 containing the best estimates at each timescale and a matrix  $V$  characterizing the uncertainty about that estimate. We obtained the solution that is well known from the Kalman Filter literature. We used the Kalman filter toolbox (written by K. Murphy, University of British Columbia, Vancouver, Canada) to numerically solve these equations. To model target jump experiments we simply added the displacements to the error that was being used by the Kalman learner.

Once we could estimate the gain as a function of previous observations, the learner could use the estimated gain to produce optimal movement. The corrected motor command (the used gain) is then  $m_{\text{corrected}} = m_{\text{normal}}/\text{gain}$ , where  $m_{\text{normal}}$  is the motor command that is optimal for an unperturbed motor plant. Throughout the paper, the gain is close to 1 and we thus use the simplifying approximation  $m_{\text{corrected}} = m_{\text{normal}} \times (1 - \text{gain})$ .

To model the sensory deprivation experiments (that is, darkness), we set the measurement noise for those trials to infinity. To model the experiments where the monkey spent some time in the dark (Fig. 3f), we simulated 500 saccades without any feedback. To model long-term learning in the monkey when it spent a whole night in the dark (Fig. 2d), we simulated 1,500 saccades without sensory feedback.

**Contrast adaptation of visual neurons.** The adaptation state was modeled as a muscle that had a baseline gain of 40. We chose  $c = (0.003)^2$  and  $\sigma_w = 1$  to model the data. The contrast of the input stimulus that was varied in the experiment was modeled as a motor gain change from 20 to 60. Each second was modeled as 100 time steps for the Kalman filter. Otherwise the same distribution of timescales (in terms of simulation steps) was retained and the same methods were used.

**Word learning.** To model the retention of memories, we treated each word learned as a gain perturbation of 100. We chose  $c = (0.03)^2$  and  $\sigma_w = 1$  to model the data. As words were not used in between, we assumed that apart from the learning trials the rest of the time consisted of no observations, equivalent to darkness in the saccade case. Each year was modeled by 100 time steps for the Kalman filter. The same distribution of timescales (in simulation steps) was used as in the motor case. The plotted retention function is the gain of the adapting system.

*Note: Supplementary information is available on the Nature Neuroscience website.*

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## COMPETING INTERESTS STATEMENT

The authors declare no competing financial interests.

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