CoSMo 2017
Motor Control and Decision Making Assignment

1. A utility for decisions and movements.

Eq. 1 represents the utility of an action. It is described as a function of α, the reward to be obtained, b, the scaling of effort costs with reward, m, the mass to be moved, d, the distance to be moved, T, the duration of the movement, and γ, the impulsivity of the individual.

Given the utility, J, use a pencil and paper to obtain an expression for the duration, T^*, that maximizes J.

$$J = \frac{\alpha - bmd/T}{1+\gamma T} \qquad [1]$$

2. Investigating the effects of costs and rewards on decisions and movements.

For each of the following exercises, calculate the maximum utility, J^*, the duration, T^*, at which the maximum occurs, and plot utility, J, as a function of movement duration, T. Start with these parameter values: $\alpha = 1000$, $\gamma = 1$, $b = 10$, $d = 1$, $m = 1$.

a) Reward: investigate the effect of doubling reward, α. How do J^* and T^* change with increasing reward?
b) Effort: investigate the effect of doubling the scaling on effort, b.
c) Mass: investigate the effect of doubling mass, m.
d) Impulsivity: investigate the effect of doubling impulsivity, γ.
e) your turn: try something of your own

3. Utility as a function of reach direction.
This problem has three parts. In part (a) you will calculate the effective mass of the arm as a function of reach direction. In parts (b) and (c), you will use the effective mass to calculate utility, optimal movement duration, and peak velocity as a function of reach direction.

a) Calculate effective mass of the arm as a function of movement direction.

The effective mass of the arm represents the relationship between a force vector applied to the hand and the resulting acceleration vector measured at the hand at rest. The goal in this module is to calculate and plot the effective mass of the arm in a given static configuration as a function of reach direction. The following explanation will walk you through the process. **The points at which you must explicitly do something are highlighted in bold text.**

The Arm Model

The arm model we use is a two-link planar arm with upper and lower arm segment lengths l_1 and l_2, and shoulder and elbow joint angles θ_s and θ_e. Model parameters are pulled from standard anthropometric tables:

$l_1 = 0.33$, length of upper arm, meters
$l_2 = 1.3 l_1$, length of forearm
$lc_1 = {}^{l_1}\!/\!_2$, center of mass for upper arm
$lc_2 = {}^{2l_2}\!/\!_3$, center of mass for forearm
$m_1 = 1.93$, mass of upper arm, kg
$m_2 = 1.52$, mass of forearm
$i_1 = 0.0141$, inertia of upper arm, kg.m^2
$i_2 = 0.01882$, inertia of forearm

The initial arm configuration is set to match the experimental conditions in Gordon et al. 1982:

$$\theta_s = 0.994 \quad \theta_e = 1.5724$$

Using trigonometry, we define $x = [h_x, h_y]^T$, a vector of hand position as a function of joint angles θ_s and θ_e:

$$x = \begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_s + l_2 \cos(\theta_s + \theta_e) \\ l_1 \sin\theta_s + l_2 \sin(\theta_s + \theta_e) \end{bmatrix} \qquad [2]$$

Effective Mass

To start, we determine the inertia of the arm. The inertia in this case is a 2x2 position dependent matrix:

$$\mathrm{I}(\theta) = \begin{bmatrix} \mathrm{I}_{11} & \mathrm{I}_{12} \\ \mathrm{I}_{21} & \mathrm{I}_{22} \end{bmatrix},$$

where θ is a vector that includes the instantaneous shoulder and elbow angles, $\theta = [\theta_s, \theta_e]^T$. We have

$$\begin{aligned}
\mathrm{I}_{11} &= a_3 + a_1 l_1^2 + a_4 + 2 a_2 l_1 \cos\theta_e \\
\mathrm{I}_{12} &= \mathrm{I}_{21} = a_2 + l_1 \cos\theta_e + a_4 \\
\mathrm{I}_{22} &= a_4 \\
a_1 &= m_2 \\
a_2 &= m_2 lc_2 \\
a_3 &= m_1 lc_1^2 + i_1 \\
a_4 &= m_2 lc_2^2 + i_2
\end{aligned}$$

When the arm is at rest, the inertial matrix describes the relationship between joint torques at the shoulder and elbow, and the resulting joint accelerations:

$$\tau = \mathrm{I}(\theta)\ddot{\theta} \qquad [3]$$

Our goal is to identify the relationship, $M(\theta)$, between forces at the hand as measured at rest and the resulting accelerations:

$$f = M(\theta)\ddot{x} \tag{4}$$

To calculate $M(\theta)$ we use the Jacobian matrix, $\Lambda = dx/d\theta$. To calculate the Jacobian, we return to the relationship between hand and joint coordinates (Eq. 2) and take the derivative with respect to the joint angle vector, θ:

$$\Lambda = \frac{dx}{d\theta} = \begin{bmatrix} \frac{\partial h_x}{\partial \theta_s} & \frac{\partial h_x}{\partial \theta_e} \\ \frac{\partial h_y}{\partial \theta_s} & \frac{\partial h_y}{\partial \theta_e} \end{bmatrix} \tag{5}$$

a.1) Obtain an expression for the Jacobian, Λ.

We then use the principal of virtual work and relate joint torques to forces at the hand:

$$\tau^T \Delta\theta = f^T \Delta x \tag{6}$$

Rearranging terms leads to an expression that allows us to compute the joint torques resulting from a force at the hand:

$$\begin{aligned} \tau^T &= f^T \tfrac{\Delta x}{\Delta \theta} = f^T \Lambda \\ \tau &= \Lambda^T f \end{aligned} \tag{7}$$

Combining Eq. 3 and Eq. 7, and using the following relationships:

$$\begin{aligned} \dot{x} &= \Lambda\dot{\theta} & \Rightarrow \quad & \dot{\theta} = \Lambda^{-1}\dot{x} \\ \ddot{x} &= \Lambda\ddot{\theta} + \dot{\Lambda}\dot{\theta} & \Rightarrow \quad & \ddot{\theta} = \Lambda^{-1}\left(\dot{x} - \dot{\Lambda}\dot{\theta}\right) \end{aligned} \tag{8}$$

we obtain:

$$f = \Lambda^{-1^T} \mathrm{I}(\theta) \Lambda^{-1}\left(\ddot{x} - \dot{\Lambda}\dot{\theta}\right) \tag{9}$$

The hand is at rest so the velocity term is zero, leaving us with:

$$f = \Lambda^{-1^T} \mathrm{I}(\theta) \Lambda^{-1}\ddot{x} \tag{10}$$

From Eq. 4, we see that this provides us with an expression for the mass matrix, $M(\theta)$, relating acceleration and forces measured at the hand at rest:

$$M(\theta) = \Lambda^{-1^T} \mathrm{I}(\theta) \Lambda^{-1}$$

a.2) Calculate $M(\theta)$ for the given arm configuration.

We obtain the effective mass, m, in a given reach direction by applying a unit vector acceleration in the given reach direction and calculating the length of the resultant force vector.

a.3) Calculate effective mass of the arm for reach directions $\theta_r = 0$ to 2π rad, where θ_r is measured with respect to the positive x-axis. On a polar plot, plot effective mass as a function of reach direction.

b) Using the effective mass calculated in module 3.a and the equations for utility (Eq. 1) and optimal duration in Question 1, plot optimal duration as a function of movement direction. Use these parameters in the utility equation: $\alpha = 50$, $b = 100$, $d = 0.1$, and $\gamma = 1$.

c) Plot peak movement velocity as a function of movement direction. Peak velocity can be estimated for a movement of a given duration and distance using the kinematics predicted by a minimum jerk trajectory. Use the m-file provided (minjerk.m) to calculate peak velocity for a movement of given direction and duration. Compare with results in Gordon et al. 1994. (Fig. 1A below)

d) Natalia Dounskaia and colleagues asked subjects to make 15cm out-and-back reaching movements in any direction they preferred and measured the probability with which they chose to reach in a given direction for both arms (Wang et al.2012). We have provided a .mat file (effMDounskaia.mat) with the effective mass for the left and right arm for an out-and-back movement for reaching directions thetareach=0:pi/32:2pi. Calculate the probability as a function of reach direction using Eq. 1. Use the following parameters: $\alpha = 50$, $b = 100$, $d = 0.1$, and $\gamma = 1$. Compare to Wang et al. 2012 (Fig. 1B below).

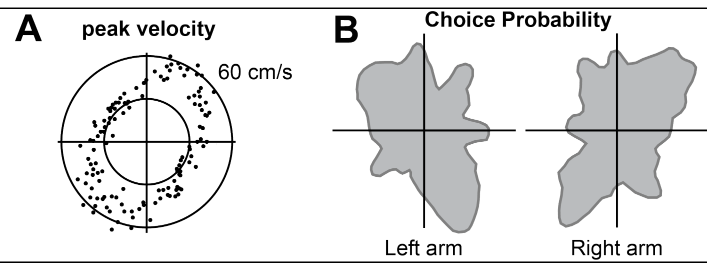

Figure 1. A: Peak velocity of a 10cm out and stop movement as a function of reach direction. Data reproduced from Gordon et al. 1994. B: For a 15cm out and back movement, probability of reaching in a given direction. Results shown for the left and right arm. Data reproduced from Wang et al. 2012.

4. Investigating the effects of costs and rewards on reaction time.

We can extend the utility framework presented in Eq. (1) to consider the effect of utility on reaction time, t, in addition to movement vigor. Reaction time can be represented as the time it takes for a variable representing evidence for a given action, to reach a threshold. If the rate at which the evidence accumulates is a normally distributed random variable, with a mean that is proportional to the utility of the action, then the probability distribution of reaction times, $p_t(t)$, can be expressed as:

$$p_t(t) = \frac{x^*}{t^2 \sigma_r \sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma_r^2} \left(\frac{x^*}{t} - \mu_r \right)^2 \right) \qquad [11]$$

The mean and standard deviation of the utility rate are μ_r and σ_r, respectively. The threshold, relative to the current state, at which movement will be initiated, is described by x^*.

Using Eq. (11), develop a model to simulate the reaction time distributions. For each of the following exercises, calculate and plot the reaction time probability distribution function, and identify the mean and the median. Check to see whether the mean is always larger than the median.

Start with these parameter values: $\alpha = 1000$, $\gamma = 1$, $b = 10$, $d = 1$, $m = 1$, $x^* = 1$, and $\sigma_r = 1$.

a) Reward: investigate the effect of doubling reward, α.
b) Effort: investigate the effect of doubling the scaling on effort, b.
c) Mass: investigate the effect of doubling mass, m.
d) Impulsivity: investigate the effect of doubling impulsivity, γ.
e) your turn: try something of your own

Compare/contrast your predictions with what you found in Question #2. Does reaction time always respond in the same manner as movement vigor?

References

Gordon, J., Ghilardi, M. F., Cooper, S. E., and Ghez, C. (1994). Accuracy of Planar Reaching Movements. II. Systematic Extent Errors Resulting From Inertial Anisotropy. Exp Brain Res 99, 112–130.

Wang, W. and Dounskaia, N. (2012). Load emphasizes muscle effort minimization during selection of arm movement direction. J Neuroeng Rehabil 9, 1–1.