

Generative Inverse Deep Reinforcement Learning for Online Recommendation

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ABSTRACT

Deep reinforcement learning enables an agent to capture users' interest through dynamic interactions with the environment. It uses a reward function to learn user's interest and to control the learning process, attracting great interest in recommendation research. However, most reward functions are manually designed; they are either too unrealistic or imprecise to reflect the variety, dimensionality, and non-linearity of the recommendation problem. This impedes the agent from learning an optimal policy in highly dynamic online recommendation scenarios. To address the above issue, we propose a generative inverse reinforcement learning approach that avoids the need of defining an elaborate reward function. In particular, we model the recommendation problem as an automatic policy learning problem. We first generate policies based on observed users' preferences and then evaluate the learned policy by a measurement based on a discriminative actor-critic network. We conduct experiments on an online platform, VirtualTB, and demonstrate the feasibility and effectiveness of our proposed approach via comparisons with several state-of-the-art methods.

CCS CONCEPTS

• **Information systems** → **Recommender systems**; • **Computing methodologies** → **Reinforcement learning**; **Neural networks**.

KEYWORDS

Deep Reinforcement Learning; Deep Learning; Recommender System

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1 INTRODUCTION

Online recommendation involves real-time interaction between user and system. It differs from traditional recommendation problems in facing a dynamic environment where user's preference may change rapidly. In recent decades, deep reinforcement learning (DRL) has become a promising solution to online recommendation, given its ability to learn optimal strategies from interactions. This is in contrast to traditional methods like multi-armed bandit, which assume users' preference to be stable and thus cannot meet the requirements for online recommendation. **DRL-based recommendation systems cover three categories: deep Q-learning based, policy gradient based, and hybrid methods.** Deep Q-learning aims to find the best step by maximizing a Q-value over all possible actions. [31] first introduced DRL into recommendation systems for news recommendation; then, [5] introduced a robust Q-learning to handle dynamic environments for online recommendation. However, **Q-learning based methods suffer the *agent stuck problem*, i.e., it requires the maximise operation over the action space, which becomes infeasible when the action space is extremely large. Policy gradient based methods can mitigate the agent stuck problem [4]. They use the average reward as a guideline, yet they may treat bad actions as good actions, making the algorithm hard to converge [20]. Hybrid methods combine the advantages of Q-learning and policy gradient. A popular hybrid method is actor-critic network [16], which adopts policy gradient on an actor network and Q-learning on a critic network to achieve nash equilibrium on both networks. Until now, actor-critic network has been widely applied to DRL-based recommendation systems [6, 19].**

Despite differences, all existing DRL-based methods rely on well-designed context-dependent reward functions, as shown in Fig 1 (a). They commonly use 'User's click or not' is the main metric to measure recommendation quality and to define the reward function. They suffer limited generalization ability because, in many cases, the reward function cannot be easily defined, due to dynamic environments and various factors that affect user's interest [26]. Besides, they generally consider both user's preference and user's actions (e.g., click-through, rating or implicit feedback) in the reward function, assuming that the reward is determined by the current chosen item and user's action is unaffected by the recommended items.

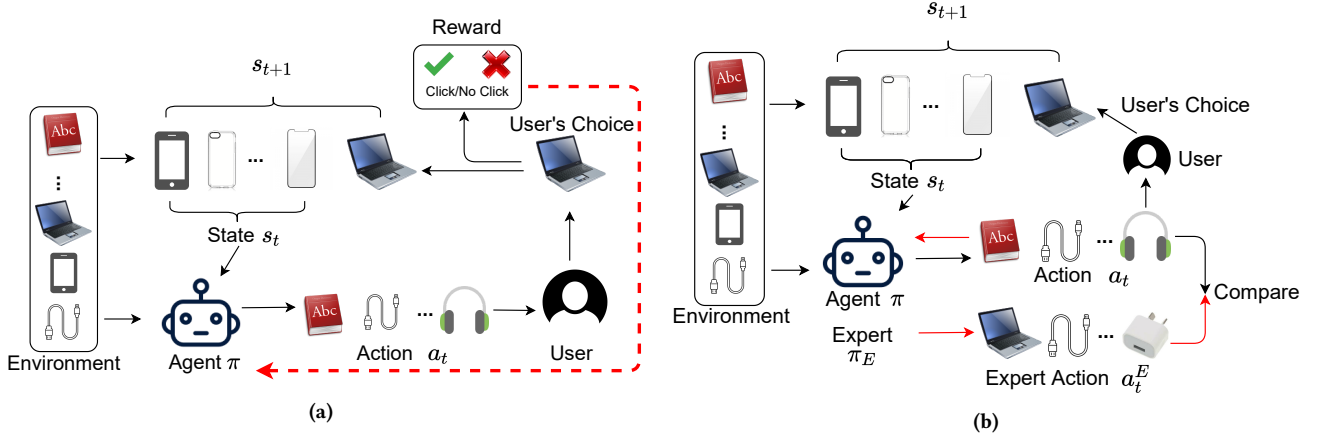


Figure 1: General workflow of current reinforcement learning based recommendation system (a) which relies on reward function to guide the agent. The inverse reinforcement learning based approach (b) which does not require the reward function guiding. We use red color to represent how those two approaches update the policy.

Those assumptions may not hold for online recommendation [26]. Another challenge comes from the time-consuming recommendation policy search using reinforcement learning [8]. The state space (i.e., all candidate items in which users might be interested) and action space (i.e., all the actions for candidate items) in a recommendation problem might be huge, and a traditional reinforcement learning based method needs to iterate all the possible combinations to figure out the best policy. Recent achievements on generative adversarial imitation learning (GAIL) [12] provide a solution to learning from expert without needing to know the exact reward function. However, GAIL is designed for traditional reinforcement learning environments that involve only static and countable states. Besides, GAIL only works well on survival or explore tasks, which have different goals from recommendation tasks.

Targeting at the above challenges, we aim to enable the agent to infer a reward function from user's behaviors via inverse reinforcement learning and learn the recommendation policy directly from user behaviors efficiently and adaptively, as shown in Fig 1 (b). We propose a generative inverse reinforcement learning approach for inferring an implicit reward function from user behaviors adaptively. The approach can measure the performance of the current recommendation policy and update the current policy with the expert policy in the discriminator, thus avoiding needing to define a reward function for online recommendation. In particular, we transform inverse deep reinforcement learning into a generator to augment a diverse set of state-action pairs. Under this generative strategy, our method can achieve better generalization ability under complex online recommendation conditions. In a nutshell, we make the following contributions:

- We propose generative inverse reinforcement learning to automatically learn reward function for online recommendation. To the best of our knowledge, this is the first work to decouple the reward function and the agent for online recommendation.

- We design a novel actor-discriminator network module, which takes a discriminator as the critic network and a novel actor-critic network as the actor network, to implement the proposed framework. The module is model-free and can be easily generalized to a variety of scenarios.
- We conduct experiments on a virtual online platform, VirtualTB, to demonstrate the feasibility and effectiveness of the proposed approach. Our proposed method achieves a higher click-through-rate than several state-of-the-art methods.

2 PROBLEM FORMULATION

Online Recommendation Online recommendation differs from offline recommendation in dealing with real-time interactions between users and the recommendation system. The system needs to analyse users' behaviors and updates the recommend policy dynamically. The objective is to find a solution that best reflects those interactions and apply it to the recommendation policy.

Reinforcement Learning-based Recommendation learns from interactions through an Markov Decision Process (MDP). Given a recommendation problem consisting of a set of users $\mathcal{U} = \{u_0, u_1, \dots, u_n\}$, a set of items $\mathcal{I} = \{i_0, i_1, \dots, i_m\}$ and user's demographic information $\mathcal{D} = \{d_0, d_1, \dots, d_n\}$, MDP can be represented as a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)$, where \mathcal{S} denotes the state space, which is the combination of the subsets of \mathcal{I} ; \mathcal{A} is the action space, which represents agent's selection during recommendation based on the state space \mathcal{S} ; \mathcal{P} is the set of transition probabilities for state transfer based on the action received; \mathcal{R} is a set of rewards received from users, which are used to evaluate the action taken by the recommendation system, with each reward being a binary value to indicate user's click; γ is a discount factor $\gamma \in [0, 1]$ for balancing the future reward and current reward.

Given a user u_0 and the state s_0 observed by the agent (or the recommendation system), which includes a subset of item set \mathcal{I} and user's demographic information d_0 , a typical recommendation iteration for user u_0 goes as follows. First, the agent makes an action a_0 based on the recommend policy π_0 under the observed

state s_0 and receives the corresponding reward r_0 . Then, the agent generates a new policy π_1 based on the received reward r_0 and determines the new state s_1 based on the probability distribution $p(s_{new}|s_0, a_0) \in \mathcal{P}$. The cumulative reward after k iterations is as follows:

$$r_c = \sum_{k=0}^{\infty} \gamma^k r_k$$

Inverse Reinforcement Learning-based Online Recommendation We propose inverse reinforcement learning without pre-defining a reward function for online recommendation. We aim to optimize the current policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$ to make recommendations that are most suitable for the user. Specifically, we model online recommendation as an MDP with a finite state set \mathcal{S} , a set of actions $\mathcal{A} = \{a_0, a_1, \dots, a_t\}$, transition probabilities $\mathcal{P}(\mathcal{S}, \mathcal{A})$, and a discount factor γ . Suppose there exist an expert policy π_E that can master any state $s \in \mathcal{S}$. The recommendation turns into the optimization problem of finding the policy π that best approximates the expert policy π_E across the cost function class \mathcal{C} , by the following objective function [1]:

$$\underset{\pi}{\text{minimize}} \max_{c \in \mathcal{C}} \mathbb{E}_{\pi} [c(s, a)] - \mathbb{E}_{\pi_E} [c(s, a)] \quad (1)$$

The cost function class \mathcal{C} is restricted to convex sets defined by the linear combination of a few basis functions $\{f_1, f_2, \dots, f_k\}$. Hence, the corresponding feature vector for the state-action pair (s, a) can be represented as $f(s, a) = [f_1(s, a), f_2(s, a), \dots, f_k(s, a)]$. The expectation $\mathbb{E}_{\pi} [c(s, a)]$ is defined as (on γ -discounted infinite horizon):

$$\mathbb{E}_{\pi} [c(s, a)] = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t) \right] \quad (2)$$

3 METHODOLOGY

The overall structure of our proposed approach is illustrated in Fig 2. It consists of two main components: recommendation policy learning, and discriminative actor-critic network for recommendation. *Recommendation policy learning* provides a theoretical analysis about how the optimal recommendation policy can be learned from the given expert policy π_E . Idealised recommendation policy π is able to recommend items to users based on their dynamic preference. *discriminative actor-critic network for recommendation* constitutes the main structure of our approach. Besides the above components, we will present the optimization method and the corresponding training algorithm, which aims to limit the update step in optimizing the loss function to ensure that a new policy achieves better performance than the old one.

3.1 Recommendation Policy Learning

Recommendation policy learning is a crucial part of the proposed method. It aims to make the learned recommendation policy π and expert policy π_E as similar as possible by training a generative adversarial network (GAN) [10].

Learn a recommendation policy π from expert policy π_E can be formulate as the problem of matching two occupancy measures [1].

The occupancy measure ρ can be defined as:

$$\rho_{\pi}(s, a) = \pi(s|a) \sum_{t=0}^{\infty} \gamma^t P(s_t = s | \pi) \quad (3)$$

Since the generator aims to generate the policy as similar to the expert policy as possible, we use the same way as described on GAIL [12] to bridge inverse reinforcement learning and GAN by making an analogy from the occupancy matching to distribution matching. To be specific, GA regularizer ψ_{GA} is introduced to regularize the loss function. we can directly measure the difference between the policy and expert policy without needing to know the reward function. However, this method may suffer the ambiguity problem that many candidate policies π_c can approximate the expert π_E . γ -discounted causal entropy $H(\pi)$ [3] is defined to resolve the ambiguity. The objective function can be expressed as:

$$\underset{\pi}{\text{minimize}} -\lambda H(\pi) + \underbrace{\psi_{GA}(\rho_{\pi} - \rho_{\pi_E})}_{D_{JS}(\rho_{\pi}, \rho_{\pi_E})} \quad (4)$$

where λ is a factor with $\lambda \geq 0$. Note that Eq.(4) has the same goal as the GAN, i.e., finding the squared metric between distributions. More specifically, we have the following equivalence for Eq.(4):

$$\underset{\pi}{\text{minimize}} -\lambda H(\pi) + \psi_{GA}(\rho_{\pi} - \rho_{\pi_E}) \equiv \min_{\pi} \max_D \mathcal{L}_{\mathcal{D}} \quad (5)$$

$$\mathcal{L}_{\mathcal{D}} = \mathbb{E}_{\pi} [\log D(s, a)] + \mathbb{E}_{\pi_E} [\log(1 - D(s, a))] - \lambda H(\pi)$$

Such a loss function would encourage agent to learn a policy π towards expert-like regions of state-action space guided by $\log D(s, a)$. However, GAIL is only suitable for survival or exploration task. That is, for survival task, the reward used on GAIL is $\log D(s, a)$. It is always negative since $D(s, a) \in [0, 1]$ which encourages agent to end current episode to stop more negative reward. For exploration task, the reward function is $-\log(1 - D(s, a))$, it is always positive and may lead to agent loop in the environment to collect more rewards. In recommendation system, agent is supposed to explore the environment and make recommendation based on observation. Hence, fully negative or positive is not suitable for recommendation. Indeed, we design a new reward function which can relief this problem and it is more suitable in recommendation task to evaluate recommendation policy:

$$R(s, a) = \log D(s, a) - \log(\max(\epsilon, 1 - D(s, a))) \quad (6)$$

Note that, in recommendation task, the agent only need to ensure the target item will be recommended; it is not essential to behave exactly same as the expert policy. Hence, we include user's feedback $r \in [0, 1]$ as part of the reward function to ensure agent will not take extra step to update itself when user click one of the recommendations.

$$R(s, a) = \log D(s, a) - \log(\max(\epsilon, 1 - D(s, a))) + r \quad (7)$$

It is worth the mention that such reward function is used to evaluate the learned policy π from the expert policy. It is distant from the reward function which used to reflect user's preference.

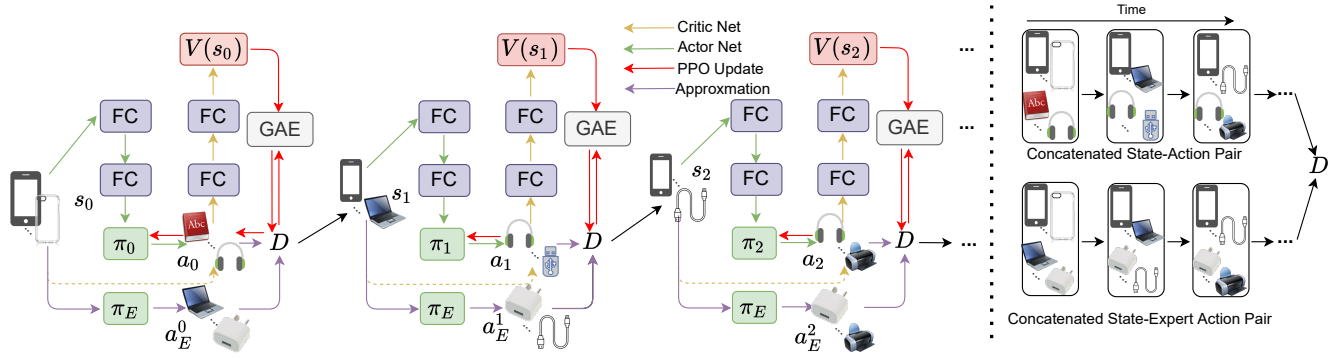


Figure 2: The proposed framework. The actor-critic network generates the policy π based on current state s_t . The discriminator takes state-action pair (s_t, a_t) and expert state-action pair (s_t, a_E^t) as input. The goal of the actor-critic network is to generate π which can let discriminator classify it as an expert policy. The discriminator aims to distinguish the policy that is generated by actor-critic network. The input of GAE is the Value from the critic network and $c(s, a)$ from discriminator (we will present its details later). We circled one update episode which happened on agent.

3.2 Discriminative Actor-Critic Network

The discriminative actor-critic network aims to make the recommendation based on the policy learned from the inverse reinforcement learning framework. Specifically, we take *advantage actor-critic network*, a variant of the actor-critic to constitute the main structure of our approach. Within this network, the actor uses the policy gradient to update the policy, and the critic uses Q-learning to evaluate the policy and provides feedback [16].

Given user's profile at timestamp t (i.e., item list $\{i_0, \dots, i_{t-2}, i_{t-1}\}$) and optional demographic information \mathcal{D} (used to generate state s_t), the environment embeds user's recent interest and features into the latent space via neural network [6, 19]. Once the actor network gets the state s_t from the environment, it feeds it to a network with two fully-connected layers with ReLU as the activation function. The final layer outputs the target policy function π parameterized by θ to be updated together with discriminator D . Then, the critic network takes the input from the actor network with current policy π_{θ_t} , which can be used for sampling to get the trajectory (s_t, a_t) . We concatenate and feed the state-action pair to two fully-connected layers with ReLU as the activation function. The critic network outputs a value $V(s_t, a_t) \in \mathbb{R}$ to be used to calculate the advantage, i.e., a value used for optimization (to be discussed later).

The discriminator D is the key component of our approach. To build an end-to-end model and better approximate the expert policy π_E , we parameterize the policy as π_{θ} and clip the output of the discriminator so that $D : \mathcal{S} \times \mathcal{A} \rightarrow (0, 1)$ with weight w . The loss function of D is \mathcal{L}_D . Besides, we use Adam [15] to optimize weight w (the optimization for θ will be introduced later). Here, the discriminator D can be treated as a local cost function to guide the policy update. Specifically, the policy will move toward expert-like regions (divided by D) in the latent space by minimizing the loss function \mathcal{L}_D , i.e., finding a point (π, D) for Eq.(5) such that the equation output is minimal.

3.3 Training and Optimization

We use the actor-critic network as a policy network to be trained jointly with the discriminator. Therefore, the actor-critic network needs to update the policy parameter θ based on the discriminator. During this process, we limit the agent's step size to ensure the new policy is better than the old one. Specifically, we use trust region policy optimization (TRPO) [23] to update the policy parameter θ and formulate the TRPO problem as follows:

$$\begin{aligned} \max_{\theta} \quad & \frac{1}{T} \sum_{t=0}^T \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} A_t \right] \\ \text{subject to } & D_{KL}^{\theta_{old}}(\pi_{\theta_{old}}, \pi_{\theta}) \leq \eta \end{aligned} \quad (8)$$

where A_t is the advantage function calculated by Generalized Advantage Estimation (GAE) [24] below:

$$A_t = \sum_{l=0}^{\infty} (\gamma \lambda_g)^l \left[-V(s_t) + \sum_{l=0}^{\infty} \gamma^l r_{t+l} \right] \quad (9)$$

where the reward r_{t+l} is the l -step's test reward at timestamp t . The reward r_{t+l} have two components which are reward returned by environment and the bonus reward calculated by Discriminator D by using $\log(D(s_t, a_t))$. Considering the massive computation load of updating the TRPO via optimizing Eq.(8), we use Proximal Policy Optimization (PPO) [25] with the objective function below, to update the policy:

$$\mathbb{E}_t \left[\min \left(\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} A_t, \text{clip} \left(\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)}, 1 - \epsilon, 1 + \epsilon \right) A_t \right) \right] \quad (10)$$

where ϵ is the clipping parameter, which represents the maximum percentage of change that can be updated at a time.

The training procedure involves two components: the discriminator and the actor-critic network. The training algorithm is illustrated in Algorithm 1. Specifically, for the discriminator, We use

Adma as the optimizer to find the gradient for Eq.(5) for weight w :

$$\mathbb{E}_{\pi} [\nabla_w \log(D_w(s, a))] + \mathbb{E}_{\pi_E} [\nabla_w \log(1 - D_w(s, a))] \quad (11)$$

4 EXPERIMENTS

Experiments are conducted on several different simulation environments includes Virtual TaoBao [27], RecoGym [22] and RecSim [13]. To best our knowledge, there are only three gym-based platforms available for online recommendation. Different from the offline experiments which the environment is built upon an existing datasets, those platforms will generate users-items interaction dynamically.

4.1 Simulation Platforms

4.1.1 VirtualTB. VirtualTB is a dynamic environment to test the feasibility of the recommendation agent. It enables a customized agent to interact with it and achieve the corresponding rewards. On VirtualTB, each customer has 11 static attributes as the demographic information and are encoded into an 88-dimensional space with binary values. The customers have multiple dynamic interests that are encoded into a 3-dimensional space and may change over the interaction process. Each item has several attributes, e.g., price and sales volume, and are encoded into a 27-dimensional space. The environment will random generate those demographic information and dynamic interests at the initialization stage, and those dynamic interests will be update episodically based on the items be recommended.

4.1.2 RecoGym. RecoGym is a simulation environment for recommender system [22]. Different from VirtualTB, RecoGym does not involve user's long-term interest. Moreover, RecoGym only contains 100 users and 10 items implies to a smaller state and action space. RecoGym is focusing on product recommendation and online advertising based on data comes from e-commerce which provides both organic and the bandit user-system interaction.

4.1.3 RecSim. RecSim is another open-sourced simulation environment for recommender system [13]. RecSim is a configurable platforms allows customized environments to fulfil unique requirements of particular aspects of user behavior and item information in sequential interactive recommendation problems. There two built-in tasks which are long-term stratification and user interest evolution. In this paper, we do no use any customized environment. those two built-in tasks are mainly used in our experiment.

4.2 Baseline methods

The proposed method is focusing on online recommendation and reinforcement learning. Hence, we select the following existing methods as baselines.

- IRecGAN [2]: An online recommendation method that employs reinforcement learning and GAN.
- PGCR [20]: A policy Gradient based method for contextual recommendation.
- GAUM [7]: A deep Q-learning based method that employs GAUM and cascade Q-learning for recommendation.
- KGRL [6]: Actor-Critic based method for interactive recommendation (i.e., a variant of online recommendation).

Note that GAUM and PGCR are not designed for online recommendation, and KGRL requires knowledge graph as side information, which is unavailable to the gym environment. Hence, we only keep the network structure and put those network into the VirtualTB platform for testing.

4.3 Evaluation Metric and Experimental Setup

The experiments are conducted in the OpenAI gym environment where the reward can be readily obtained for each episode. Since each episode may have differed numbers of steps, it leads to the difficulty in determining when users will end the session. For this reason, we choose click-through-rate to represent the performance which is defined as:

$$CTR = \frac{r_{episode}}{10 * N_{step}}$$

where 10 means that the user is interested in all the ten items that are recommended in a single page, $r_{episode}$ is the reward received per episode and N_{step} is the number of steps included in one episode.

For RecSim and RecoGym, the quality score is used to determine the performance. Quality score is pre-defined on the environment which is similar as the reward.

Algorithm 1: Training algorithm for our model

```

input: Expert Policy  $\pi_E$ , current state  $s$ 
1 Sampling expert trajectories  $(s, a_E) \sim \pi_E$ ;
2 Initialize discriminator parameter  $w_0$ ;
3 Initialize policy parameter  $\theta_0$ ;
4 Initialize clipping parameter  $\epsilon$ ;
5 for  $i = 0, 1, \dots$  do
6   Sampling trajectories  $(s, a) \sim \pi_{\theta_i}$ ;
7   Update the parameter  $w_i$  by gradient on Eq.(11);
8   for  $k = 0, 1, \dots$  do
9     Get the trajectories  $(s, a)$  on policy  $\pi_{\theta} = \pi(\theta_k)$ ;
10    Estimate advantage  $A_t$  using Eq.(9);
11    Compute the Policy Update
12     $\theta_{k+1} = \arg \max_{\theta} \text{Eq.}(10)$ 
13    Optimize these  $K$  step by Adma
14  end
15   $\theta_i \leftarrow \theta_K$ ;
16 end
```

The model is implemented by using PyTorch, and the experiments are carried out on a server which consists of two 12-core/24-thread Intel (R) Xeon (R) CPU E5-2697 v2 CPUs, 6 NVIDIA TITAN X Pascal GPUs, 2 NVIDIA TITAN RTX, with a total 768 GiB memory.

4.4 Expert Policy Acquisition

In this part, we introduce the strategy on acquiring the expert policy for VirtualTB. There is no official expert policy in VirtualTB. Obviously, it is unrealistic to manually create the expert policy from this virtual environment where the source data is not available.

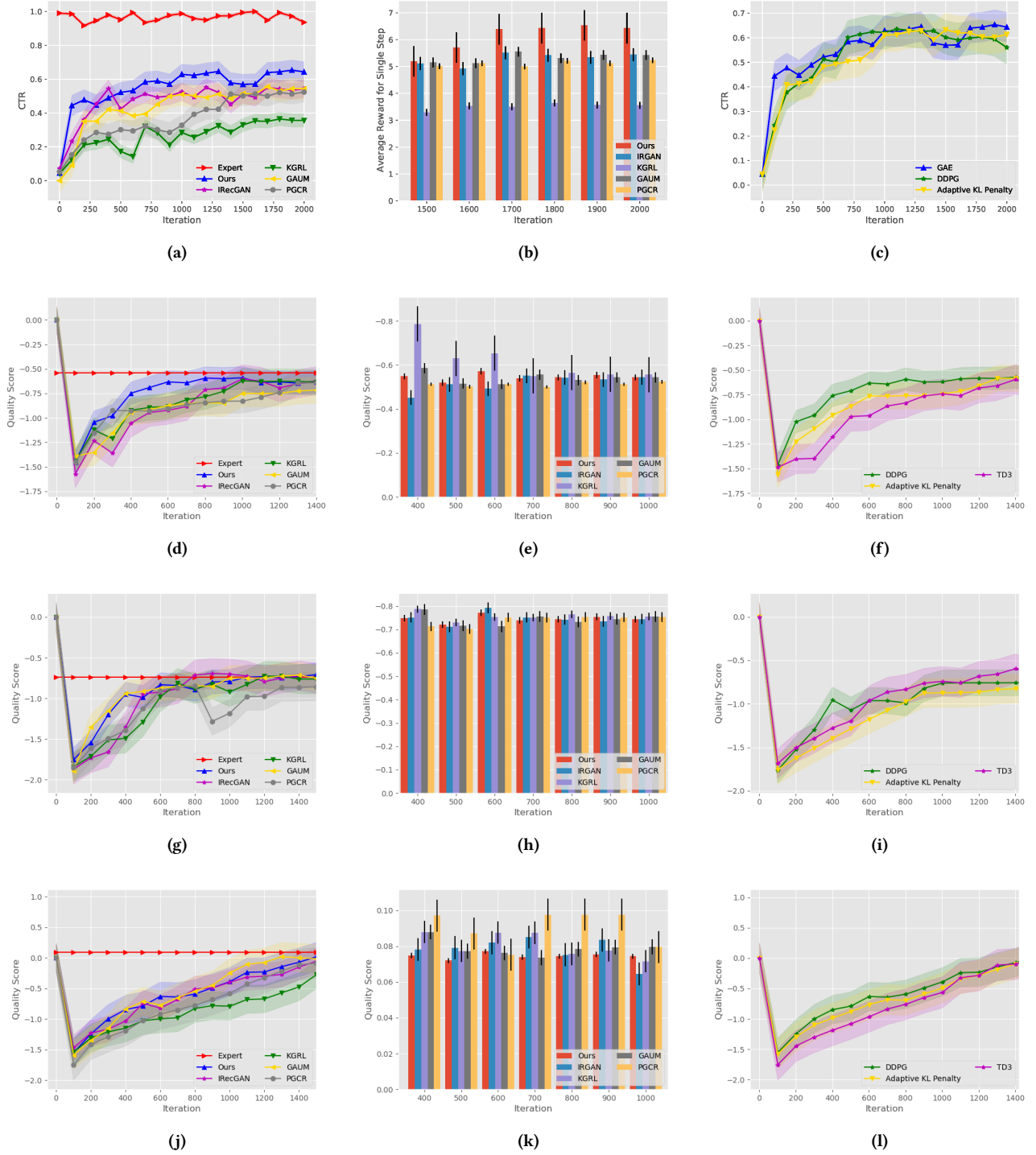


Figure 3: Experimental results where (a) is CTR with 95% confidence interval, (b) is the average reward received each step after 1500 iterations with 95% confidence interval and (c) comes from the ablation study of different algorithm in VirtualTB. For comparison, we also provide the performance from RecSim - Interest Evolution, RecSim - Long-term Engagement and RecoGym with the same order.

Table 1: CTR for Different Parameter Settings for GAE and PPO with 95% Confidence Interval in VirtualTB, RecSim - Interest Evolution, RecSim - Long-term Engagement and RecoGym.

| | | GAE: λ_g | | | | | |
|-----------------|------|--------------------------------------|--------------------|--------------------|--------------------------------------|--------------------------------------|--------------------|
| | | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
| PPO: ϵ | 0.05 | 0.630 \pm 0.063 | 0.632 \pm 0.064 | 0.633 \pm 0.062 | 0.630 \pm 0.059 | 0.626 \pm 0.060 | 0.629 \pm 0.059 |
| | 0.10 | 0.632 \pm 0.062 | 0.635 \pm 0.060 | 0.636 \pm 0.061 | 0.636 \pm 0.058 | 0.634 \pm 0.061 | 0.633 \pm 0.060 |
| | 0.15 | 0.633 \pm 0.060 | 0.635 \pm 0.061 | 0.639 \pm 0.061 | 0.640 \pm 0.057 | 0.639 \pm 0.059 | 0.638 \pm 0.061 |
| | 0.20 | 0.634 \pm 0.060 | 0.636 \pm 0.060 | 0.641 \pm 0.063 | 0.643 \pm 0.061 | 0.643 \pm 0.063 | 0.641 \pm 0.058 |
| | 0.25 | 0.631 \pm 0.061 | 0.635 \pm 0.059 | 0.636 \pm 0.060 | 0.637 \pm 0.060 | 0.636 \pm 0.061 | 0.634 \pm 0.059 |
| | 0.30 | 0.630 \pm 0.059 | 0.631 \pm 0.061 | 0.632 \pm 0.060 | 0.630 \pm 0.059 | 0.630 \pm 0.058 | 0.629 \pm 0.050 |
| | | GAE: λ_g | | | | | |
| | | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
| PPO: ϵ | 0.05 | -0.550 \pm 0.023 | -0.55 \pm 0.011 | -0.533 \pm 0.009 | -0.560 \pm 0.021 | -0.529 \pm 0.032 | -0.531 \pm 0.034 |
| | 0.10 | -0.563 \pm 0.031 | -0.564 \pm 0.020 | -0.536 \pm 0.022 | -0.564 \pm 0.034 | -0.534 \pm 0.036 | -0.533 \pm 0.031 |
| | 0.15 | -0.533 \pm 0.030 | -0.535 \pm 0.021 | -0.532 \pm 0.021 | -0.540 \pm 0.027 | -0.539 \pm 0.029 | -0.532 \pm 0.031 |
| | 0.20 | -0.530 \pm 0.019 | -0.532 \pm 0.030 | -0.531 \pm 0.019 | -0.529 \pm 0.021 | -0.530 \pm 0.033 | -0.540 \pm 0.018 |
| | 0.25 | -0.530 \pm 0.020 | -0.532 \pm 0.029 | -0.532 \pm 0.020 | -0.537 \pm 0.018 | -0.536 \pm 0.011 | -0.541 \pm 0.029 |
| | 0.30 | -0.532 \pm 0.019 | -0.532 \pm 0.021 | -0.532 \pm 0.019 | -0.530 \pm 0.019 | -0.529 \pm 0.018 | -0.526 \pm 0.019 |
| | | GAE: λ_g | | | | | |
| | | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
| PPO: ϵ | 0.05 | -0.730 \pm 0.023 | -0.742 \pm 0.024 | -0.733 \pm 0.020 | -0.750 \pm 0.019 | -0.728 \pm 0.019 | -0.729 \pm 0.019 |
| | 0.10 | -0.732 \pm 0.012 | -0.738 \pm 0.020 | -0.735 \pm 0.021 | -0.746 \pm 0.018 | -0.733 \pm 0.021 | -0.734 \pm 0.020 |
| | 0.15 | -0.733 \pm 0.020 | -0.737 \pm 0.019 | -0.734 \pm 0.021 | -0.740 \pm 0.017 | -0.735 \pm 0.019 | -0.738 \pm 0.021 |
| | 0.20 | -0.734 \pm 0.020 | -0.736 \pm 0.020 | -0.741 \pm 0.023 | -0.741 \pm 0.021 | -0.743 \pm 0.023 | -0.741 \pm 0.018 |
| | 0.25 | -0.731 \pm 0.021 | -0.736 \pm 0.019 | -0.736 \pm 0.020 | -0.737 \pm 0.020 | -0.736 \pm 0.021 | -0.734 \pm 0.019 |
| | 0.30 | -0.730 \pm 0.019 | -0.734 \pm 0.021 | -0.732 \pm 0.018 | -0.734 \pm 0.019 | -0.731 \pm 0.018 | -0.729 \pm 0.020 |
| | | GAE: λ_g | | | | | |
| | | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
| PPO: ϵ | 0.05 | 0.0532 \pm 0.008 | 0.0531 \pm 0.007 | 0.0533 \pm 0.008 | 0.0531 \pm 0.009 | 0.0533 \pm 0.008 | 0.0529 \pm 0.009 |
| | 0.10 | 0.0532 \pm 0.006 | 0.0535 \pm 0.008 | 0.0536 \pm 0.006 | 0.0536 \pm 0.007 | 0.0534 \pm 0.006 | 0.0533 \pm 0.006 |
| | 0.15 | 0.0533 \pm 0.009 | 0.0535 \pm 0.011 | 0.0539 \pm 0.061 | 0.0540 \pm 0.007 | 0.0539 \pm 0.009 | 0.0538 \pm 0.008 |
| | 0.20 | 0.0534 \pm 0.006 | 0.0536 \pm 0.012 | 0.0541 \pm 0.005 | 0.0543 \pm 0.004 | 0.0543 \pm 0.007 | 0.0541 \pm 0.008 |
| | 0.25 | 0.0531 \pm 0.007 | 0.0535 \pm 0.009 | 0.0536 \pm 0.005 | 0.0537 \pm 0.006 | 0.0536 \pm 0.005 | 0.0534 \pm 0.009 |
| | 0.30 | 0.0530 \pm 0.009 | 0.0531 \pm 0.005 | 0.0532 \pm 0.006 | 0.0530 \pm 0.005 | 0.0530 \pm 0.008 | 0.0529 \pm 0.050 |

Hence, we follow a similar strategy as in [9] to generate a set of expert policy from a pre-trained expert policy network. We design an actor-critic network \mathcal{M} with the same actor and critic network structure as our model, but without advantage. The critic network from \mathcal{M} is used to calculate the Q -value by adopting deep Q -learning. We adopts the Deep Deterministic Policy Gradients (DDPG) [18] to train \mathcal{M} . The detailed training procedure can be found on Algorithm 2.

For RecSim and RecoGym, the expert policy are acquired via a similar way. Different from VirtualTB, RecSim and RecoGym are relatively small which do not requires extra step to process. DDPG is directly applied to learn the policy and expert policies are collected when it converged.

4.5 Hyperparameters Setting

For the policy network \mathcal{M} , we set the DDPG parameters as: $\gamma = 0.95$, $\tau = 0.001$, size of hidden layer is 128, the size of reply buffer is 1,000 and the number of episode is set to 200,000. For Ornstein-Uhlenbeck Noise, scale is 0.1, $\mu = 0$, $\theta = 0.15$, $\sigma = 0.2$. For our

approach, number of episode is set to 100,000, hidden size of the advantage actor-critic network is 256, hidden size for discriminator is 128, learning rate is 0.003, factor λ is 10^{-3} , mini batch size is 5 and the epoch of PPO is 4. For the generalized advantage estimation, we set the discount factor γ to 0.995, $\lambda_g = 0.97$ and $\epsilon = 0.2$ for VirtualTB, $\lambda_g = 0.98$ and $\epsilon = 0.3$ for RecSim - Interest Evolution, $\lambda_g = 0.94$ and $\epsilon = 0.3$ for RecSim - Long-term Engagement, $\lambda_g = 0.97$ and $\epsilon = 0.2$ for RecoGym. For fair comparison, all those baseline methods are training under the same condition. For easy recognition, we set one iteration as 100 episodes in VirtualTB and one iteration as 1 episode in RecSim and RecoGym.

4.6 Results

Fully results are reported in Fig 3. Our approach generally outperforms all four existing methods. In RecSim and RecoGym, our method achieves the similar results as the baselines as those two environment are quite small. learning from expert can not significantly improve the performance. Specifically, our method gets the best result among all the baseline methods after 5500 iterations in

Algorithm 2: DDPG algorithm for expertise

```

1 Initialize actor network  $\mu$  with parameter  $\theta_\mu$  and critic
  network  $Q$  with parameter  $\theta_Q$  randomly;
2 Initialize target network  $\mu'$  and  $Q'$  with weight  $\theta_{\mu'} \leftarrow \theta_\mu$ ,
   $\theta_{Q'} \leftarrow \theta_Q$ ;
3 Initialize Replay Buffer  $\mathcal{B}$ ;
4 for  $i = 0$  to  $n$  do
5   Receive the initial state  $S_i$ ;
6   for  $t = 1$  to  $T$  do
7     Infer a action  $a_t$  according to the  $\mu(\cdot)$ ;
8     Execute the action  $a_t$  to receive a reward  $r_t$  and
       observe a new state  $S_{t+1}$ ;
9      $\mathcal{B}.\text{store}(S_t, a_t, r_t, S_{t+1})$ ;
10    Sample a random mini-batch of  $N$  transitions
       $(S_k, a_k, r_k, S_{k+1})$  from  $\mathcal{B}$ ;
11    Set  $y_i = r_t + \gamma Q'(S_{t+1}, \mu'(S_{t+1}|Q_{\mu'}))|_{\theta_{Q'}}$ ;
12    Update Critic by minimise the loss:
       $\frac{1}{N} \sum_{j=1}^N (y_i - Q(S_t, a_t|Q_\theta))^2$ ;
13    Update the Actor policy by policy gradient:
       $\frac{1}{N} \sum_{j=1}^N \nabla_a Q(S_t, a)|_{a=\mu(S_t)} \nabla_{\theta_\mu} \mu(S_t|\theta_\mu)$ ;
14    Update target network:
       $\theta_{\mu'} \leftarrow \tau \theta_\mu + (1 - \tau) \theta_{\mu'}$ ;
15     $\theta_{Q'} \leftarrow \tau \theta_Q + (1 - \tau) \theta_{Q'}$ ;
16  end
17 end

```

VirtualTB. It demonstrates the feasibility of the proposed approach. A possible reason for the poor performance of KGRL is that KGRL maintains a local knowledge graph inside the model and actively interacts with the environment. KGRL performs poorer than other baseline methods as the experiments are conducted on an online platform, which does not provide the side information for KGRL to generate its knowledge graph.

4.6.1 Impact of Key Parameters. We are interested in how the control parameter λ on GAE and the clipping parameter ϵ on PPO affect the performance. These two key parameters significantly affect the generalized advantage estimation and proximal policy optimization process. λ is used to make a compromise between bias and variance and is normally selected from $[0, 95, 1)$ with a step size of 0.01. The clipping parameter ϵ is used to determine the number of percentage that needs to be clipped, normally smaller than 0.3 to control the optimization step size. For fairness, we report CTR at iteration 2000. Observe that when $\lambda = 0.97$ and $\epsilon = 0.2$, the model achieves the best result which is 0.643 ± 0.061 . These two values are also used as our default setting. The results can be found on Table 1.

4.6.2 Ablation Study. In this part, we investigate the effect of the GAE. We use two different optimization strategies to optimize the proposed model which are DDPG and Adaptive KL Penalty Coefficient. The Adaptive KL Penalty Coefficient is the simplified

version of the PPO which can be defined as:

$$L(\theta) = \mathbb{E}_t \left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} A_t - \beta \text{KL}[\pi_{\theta_{old}}(\cdot|s_t), \pi_\theta(\cdot|s_t)] \right]$$

The update rule for β is:

$$\begin{cases} \beta \leftarrow \beta/b & d < d_{target} * a \\ \beta \leftarrow \beta * b & d \geq d_{target} * a \end{cases}$$

$$\text{where } d = \mathbb{E}_t [\text{KL}[\pi_{\theta_{old}}(\cdot|s_t), \pi_\theta(\cdot|s_t)]]$$

The parameter $a = 1.5$, $b = 2$ and $d_{target} = 0.01$ are determined empirically. The result of the ablation study can be found on Fig 3(c),(f),(i) and (l) for those four different tasks in three environments. Moreover, we also interest in the parameter impact of the a , b and d_{target} . The results in VirtualTB can be found on Table 2.

Table 2: Result when a, b, d_{target} vary in VirtualTB

| | | a | | |
|-------|---|-----|-------------------|-------------------------------------|
| | | 1 | 2 | 3 |
| 0.001 | b | 0.5 | 0.600 \pm 0.041 | 0.600 \pm 0.040 |
| | | 1.0 | 0.599 \pm 0.041 | 0.601 \pm 0.041 |
| | | 1.5 | 0.599 \pm 0.040 | 0.601 \pm 0.041 |
| | | a | | |
| | | 1 | 2 | 3 |
| 0.003 | b | 0.5 | 0.610 \pm 0.044 | 0.611 \pm 0.044 |
| | | 1.0 | 0.611 \pm 0.042 | 0.612 \pm 0.050 |
| | | 1.5 | 0.611 \pm 0.040 | 0.612 \pm 0.045 |
| | | a | | |
| | | 1 | 2 | 3 |
| 0.01 | b | 0.5 | 0.619 \pm 0.041 | 0.618 \pm 0.044 |
| | | 1.0 | 0.619 \pm 0.042 | 0.620 \pm 0.040 |
| | | 1.5 | 0.620 \pm 0.043 | 0.621 \pm 0.042 |
| | | a | | |
| | | 1 | 2 | 3 |
| 0.03 | b | 0.5 | 0.631 \pm 0.044 | 0.629 \pm 0.044 |
| | | 1.0 | 0.630 \pm 0.042 | 0.631 \pm 0.040 |
| | | 1.5 | 0.631 \pm 0.041 | 0.632 \pm 0.041 |

Based on the conclusion from [25], the value of a, b would not significantly affect the performance of the PPO process. Hence, we final decide to use $a = 2, b = 1.5, d_{target} = 0.03$ as the final parameter. Moreover, based on that, we only report the result on VirtualTB as it does not significantly affect the performane.

4.7 Discussion

This study provides a new approach for reinforcement learning based online recommendation, without needing to define the reward function. In this way, our work is feasible to be applied to various real-world recommendation scenarios where the reward function is highly domain-dependent or hard to be manually defined. The proposed method offers a fundamental support for inverse reinforcement learning based recommendation system. The proposed method can extract an adaptive reward function and automatically find the optimal strategies for generating recommendations that best fit users' interest based on a few user behaviors. Our empirical evaluation testifies its competitive performance against

several state-of-the-art reinforcement learned based methods. Our model can potentially accelerate the application of reinforcement learning in complex environments.

5 RELATED WORK

In this section, we will briefly review the related works about the deep reinforcement learning in recommender systems which includes partial observable MDP-based methods and MDP-based methods.

Firstly, partial observable MDP-based (POMDP) relevant methods will be discussed. POMDP based methods can be further divided into three categories: value function estimation, policy optimization, and stochastic sampling. Hauskrecht [11] introduce the value function estimation to estimate the upper bound of the value by using an incremental value iteration method. Poupart and Boutilier [21] further improve the upper bound by using bounded policy iteration and value-directed compression to increase the scalability of the policy optimization. Kearns et al. [14] also deal with the scalability of the policy optimization in large-scaled POMDP. In a large-scaled POMDP, the reward is sparse and the traditional sampling strategy can not achieve a good performance. Due to the high computational and representational complexity of the POMDP based methods, MDP-based methods are relatively more popular in academia. However, there still exist a work [26] uses POMDP to construct the recommender system. The main idea is that, in recommender system task, there existing lots of co-founder factors will affect user's decision. Those co-founders are not visible which implies to POMDP would be a best choice.

MDP-based DRL methods cover three approaches to recommendation: deep Q-learning based, policy gradient based, and Actor-Critic based methods. [31] adopts deep Q-learning in news recommendation and use users' historical records as states. [32] improves the structure of Deep Q-learning to achieve robust results. [20] applies policy gradient to learn optimal recommendation strategies. [28] introduces the knowledge graph into the policy gradient for sequential recommendation. However, Q-learning may get stuck because of the max operation, and policy gradient requires large-scale data to boost convergence. It only updates once per episode. Hence, Actor-Critic uses Q-value to conduct the policy gradient per step instead of per episode. [30] adopts actor-critic methods in list-wise recommendation in a simulated environment. [6, 29] utilize the knowledge graph as the side information embedded into the state-action space to improve the model's capability on an actor-critic network. Besides, [19] produces recommendations via learning a state-action embedding within the DRL framework.

Furthermore, [7] integrates the generative adversarial network with reinforcement learning structure to generate user's attribute so that more side information would be available to boost the performance of reinforcement learning based recommendation. [26] proposes a multi-agent based DRL method for environment reconstruction by taking environmental co-founders into account.

6 CONCLUSION AND FUTURE WORK

We propose a new approach InvRec for online recommendation. Our proposed approach is designed to overcome the challenges posed by an inaccurate reward function. Our proposed model is built on

advantage actor-critic network with generate adversarial imitation learning. We evaluate our method on the online platform VirtualTB in comparison with three categories of state-of-the-art methods: Deep Q-Learning based, policy gradient based, and actor-critic network based methods. The results demonstrate the feasibility and superior performance of our approach.

This study is an preliminary attempt on applying deep inverse reinforcement learning to online recommendation, and a few challenges remain unaddressed, e.g., the sample inefficiency problem for the imitation learning [17], i.e., low sample inefficiency will lead to longer training time and require a larger dataset; random sampling also would affect performance. The possible solutions include using off-policy methods instead of on-policy methods and finding an optimal sampling strategy to enable the agent to obtain the same expert trajectories to handle the states that used to occur previously.

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