COMP 4030/6030 - Spring 2017
Exam 3

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Note: Do not write on the backs; only front pages are digitized and graded.

**1.** (*20 points*) Use the Master's theorem to find the complexity of the following functions (assuming $T(1) = 1$):

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1. $T(n) = 4n + 4 \cdot T(\frac{n}{4})$ $\in \Theta(n \log n)$ $\log_4 4 = 1 = 1$
2. $T(n) = 10n^2 + 7 \cdot T(\frac{n}{3})$ $\in \Theta(n^2)$ $\log_3 7 < 2$

**2.** (*15 points*) Determine (write it out clearly) the running time equation in terms of $n$, which is the number of items in the input list $L$. Based on that equation, find the running time complexity of the function **foo**. Hint: Pay attention to the running time of Python slicing.

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```
def foo(L):
  if len(L) <= 1:
    return 1
  A = L[0: len(L)//2]
  return len(L)//2 + foo(A)
```

$T(n) = \frac{n}{2} + T(\frac{n}{2}) \in \Theta(n)$ $\log_2 1 = 0 < 1$

**3.** (*15 points*) Determine (write it out clearly) the running time equation in terms of $n$, which is the number of items between two indices *left* and *right* of the input list $L$. Based on that equation, find the running time complexity of the function **bar**. Hint: Pay attention to the running time of Python slicing.

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```
def bar(L, left, right):
  if left >= right:
    return 1
  sum = 0
  for i in range(left, right+1):
    sum = sum + i*i
  A = L[0: len(L)//2]
  return len(L)//2 + bar(A) * bar(A)
```

$T(n) = \frac{3n}{2} + 2T(\frac{n}{2})$ $\log_2 2 = 1 = 1$

$T(n) \in \Theta(n \log n)$

A = L[left: (left+right)//2]

return len(L)//2 + bar(A,0,len(A)-1) + bar(A,0,len(A)+1)

**4.** (*10 points*) Rewrite the function **bar** in the previous problem so that your revised version is faster. (Your revised function must, of course, do the same thing as **bar** does.) Use the Master's theorem to explain why your revised version is faster.

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```
def bar(L, left, right):
  if left >= right:
    return 1
  sum = 0
  A = L[left: (left+right)//2]
  return len(L)//2 + (bar(A,0,len(A)-1)*2)
```

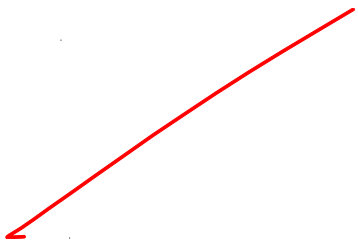
**7.** *(20 points)* The scenario for this problem is as follows. There are $n$ people and $n$ jobs. Each person has a ranking for the $n$ jobs (smaller rank is better). The goal is to assign each person to each job so that (i) every person has a job and (ii) every job is assigned to exactly one person. The input of this problem is a list of preference list. P[i] is the list of preference of person i. To be consistent the ranking and numbering start from 0 and ends at n-1. Example:

1. P[0] = [1,0,2]. This means person 0 likes job 1 the most, followed by job 0, followed by job 2.
2. P[1] = [2,1,0]. This means person 1 likes job 2 the most, followed by job 1, followed by job 0.
3. P[2] = [0,2,1]. This means person 2 likes job 0 the most, followed by job 2, followed by job 1.

In this example, the best assignment is [1,2,0], which means person 0 is assigned to job 1, person 1 is assigned to job 2, and person 2 is assigned to job 0. The total cost for this assignment is P[0][1]+P[1][2]+P[2][0] = 0, which is the best possible (a smaller rank is better implies a smaller cost is better).
Your task is to write a backtracking algorithm to find the best assignment (one with minimal total rank sum). Do that by completing the partially written code below.

```
P = get_preference()  # Assume this function is already defined.
N = len(P)
Solution = [-1]*N     # Hint: this is a permutation problem.
optimal = N*N         # N*N is worse than the worst solution.  So it's a safe initial value.

def FindJob(i):
    global optimal
    # Your code goes here
```



```
FindJob(-1)     # This will invoke the backtracking algorithm
print("Optimal solution is", optimal)
```

**5.** (*10 points*) Consider these three different algorithms:

1. Algorithm A takes as input a list $L$ of $n$ items. It makes two recursive calls on two sublists of $L$; each sublist has $\frac{n}{2}$ items. Then, Algorithm A combines the result returned by the two recursive calls in $n$ steps.
2. Algorithm B takes as input a list $L$ of $n$ items. It makes four recursive calls on four sublists of $L$; each sublist has $\frac{n}{2}$ items. Then, Algorithm B combines the result returned by the four recursive calls in only 1 constant step.
3. Algorithm C takes as input a list $L$ of $n$ items. It makes only one recursive call on a sublist of $L$ with $\frac{n}{2}$ items. Then, Algorithm C does additional processing in $n^2$ steps.

Explain which algorithm gives the fastest running time.

A $T(n) = n + 2T(\frac{n}{2}) \in \theta(n \log n)$ $\log_2 2 = 1 = 1$

B $T(n) = 1 + 4T(\frac{n}{4}) \in \theta(n)$ $\log_4 4 = 1 = 1$ 7

C $T(n) = n^2 + T(\frac{n}{2}) \in \theta(n^2)$ $\log_2 1 = 0 < 2$

Algorithm B has the fastest running time

**6.** (*20 points*) This problem is known as a *bin packing* problem. The input is a list of weights $w_1, \cdots, w_n$ and a size $C$. A valid packing is a set of weights that total up to exactly $C$. For example, given weights 3, 5, 10, 2 and $C = 7$, the output is True (because 5+2=7). If $C = 9$, the answer is False (note: no repetition). Your task is to write a backtracking algorithm to print all valid packings. Do that by completing the partially written code below.

```
Weight = get_weights()    # Assume get_weights is already defined. It returns a list of weights.
C = get_capacity()        # Assume get_capacity is already defined.
N = len(Weight)
Solution = [-1]*N

def k_pack(i):
    # Your codes go here
```

```
k_pack(-1)   # this will print out all valid packings
```