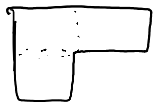
Decomposition

Thursday, February 23, 2017 9:46 AM

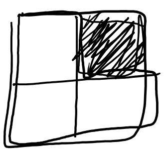
Trominoes tiling

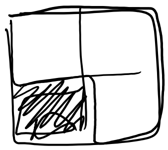


Input: $2^k \times 2^k$ grid / square with 1 empty cell. k

Output: a tiling of the square using trominoes.

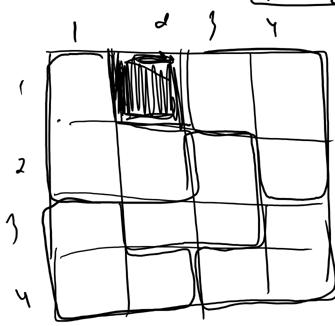
$2^1 \times 2^1$

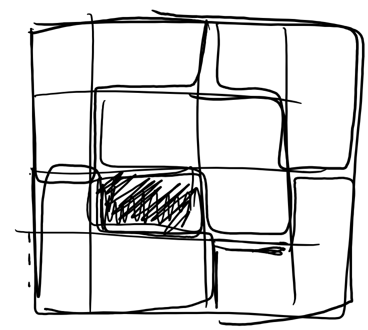




$k=1$

$K=2$ $2^2 \times 2^2$

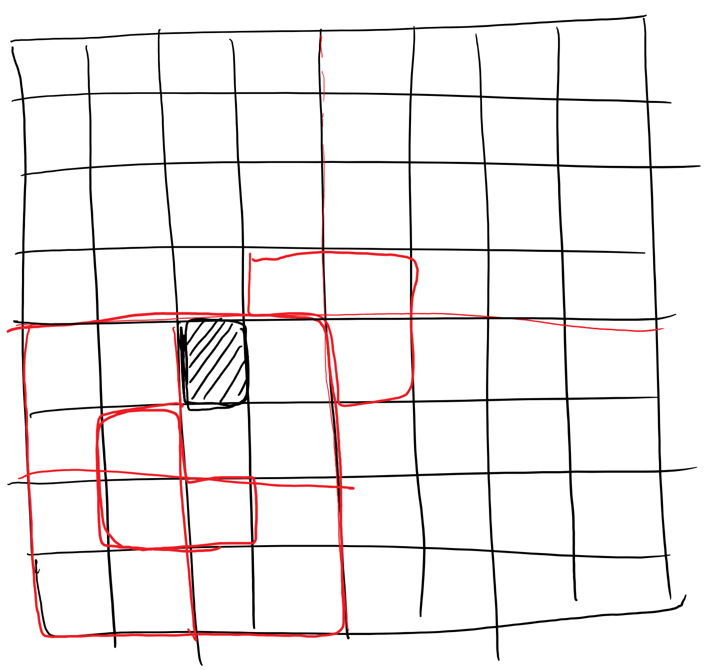




— around the empty cell

— corners

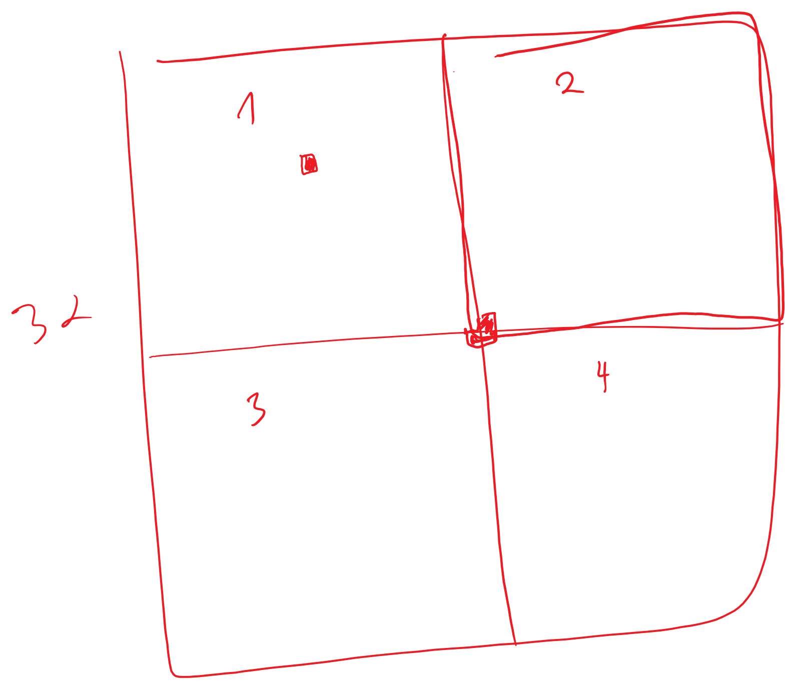
8×8



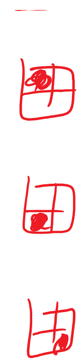
1. decompose $2^{k-1} \times 2^{k-1}$ square

2.

32



1
2
3
4
32



substitution

Tuesday, February 28, 2017 10:49 AM

$$T(n) = c \cdot n + 2T(n/2)$$
$$T(1) = c$$

$$T(n) = c \cdot n + 2\underline{T(n/2)}$$
$$T(n) = c \cdot n + 2 \cdot \left[c \frac{n}{2} + 2 \cdot T\left(\frac{n}{2^2}\right)\right]$$
$$T(n) = cn + c \cdot n + 2^2 T\left(\frac{n}{2^2}\right) = 2cn + 2^2 \boxed{T\left(\frac{n}{2^2}\right)}$$
$$T(n) = 2cn + 2^2 \cdot \left[c \frac{n}{2^2} + 2T\left(\frac{n}{2^3}\right)\right]$$
$$T(n) = 3cn + 2^3 \cdot T\left(\frac{n}{2^3}\right)$$
$$T(n) = 3cn + 2^3 \cdot \left[c \frac{n}{2^3} + 2T\left(\frac{n}{2^4}\right)\right]$$
$$T(n) = 4cn + 2^4 \cdot T\left(\frac{n}{2^4}\right)$$

$\vdots$

$$T\left(\frac{n}{2}\right) = c \cdot \frac{n}{2} + 2 \cdot T\left(\frac{n}{2^2}\right)$$
$$T\left(\frac{n}{2^2}\right) = c \cdot \frac{n}{2^2} + 2T\left(\frac{n}{2}\right)$$
$$T\left(\frac{n}{2^2}\right) = c \frac{n}{2^2} + 2T\left(\frac{n/2^2}{2}\right)$$

$$T(x) = c \cdot x + 2T\left(\frac{x}{2}\right)$$
$$T\left(\frac{n}{2^2}\right) = c \cdot \frac{n}{2^2} + 2T\left(\frac{n/2^2}{2}\right)$$

$$T\left(\frac{n}{2^3}\right) = c \cdot \frac{n}{2^3} + 2T\left(\frac{n}{2^4}\right)$$

after k steps $\qquad$ $\log_2 n$ steps

$$T(n) = kcn + 2^k T\left(\frac{n}{2^k}\right)$$

we get to 1 (smallest case) when $\frac{n}{2^k} = 1$

or $n = 2^k$ or $\boxed{\log_2 n = k}$

$$T(n) = \log_2 n \cdot c \cdot n + 2^{\log_2 n} \cdot T(1)$$
$$T(n) = \log_2 n \cdot c \cdot n + n \cdot c \in \Theta(n \log n)$$
$$= c \cdot n \log n + c \cdot n$$

partition

Thursday, March 2, 2017 10:27 AM

my strategy

First = L[0]

L

left = 0
right = len(L) - 1

Goal

$\boxed{10}$

L[left]

[10 7, 20, 3, 0, 9, ~~10~~, 7, 18, 50]

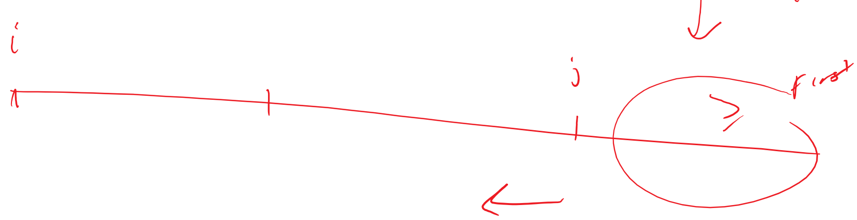
i i i i i i i j j j

9 i 10 j j

50

16,

for x in range(len(A), -1, -1)



40

i

j

substitution

Tuesday, March 14, 2017 9:49 AM

$T(1) = 1$

$T(n) = n + 2T(n/2)$ — original equation

$T(n) = n + 2\left[\frac{n}{2} + 2T(n/2^2)\right]$

$T(n) = n + n + 2^2\, \boxed{T(n/2^2)}$

$T(n) = n + n + 2^2\left[\frac{n}{2^2} + 2T(n/2^3)\right]$

$T(n) = n + n + n + 2^3\, T(n/2^3)$

$T(n) = n + n + n + 2^3\left[\frac{n}{2^3} + 2T(n/2^4)\right]$

$T(n) = n + n + n + n + 2^4\, T(n/2^4)$

$T(n/2) = \frac{n}{2} + 2T(n/2^2)$

$T(n/2^2) = \frac{n}{2^2} + 2T(n/2^3)$

$T(n/2^3) = \frac{n}{2^3} + 2T(n/2^4)$

If we this k times; when w stop

when $T\left(\frac{n}{2^k}\right) = T(1)$

$n = 2^k$

$\boxed{\log_2 n = k}$

$T(n) = \underbrace{n + n + n + n + \cdots + n}_{k \text{ times}} + 2^k\, T\left(\frac{n}{2^k}\right)$

$T(n) = n \cdot k + 2^k \cdot T(1)$

$T(n) = n \cdot \log_2 n + n \cdot T(1) \in \Theta(n \log n)$

sum

Tuesday, March 14, 2017 10:01 AM

arithematic sum

geometric sum

$$\boxed{1+2+3+\dots+m = \frac{m(m+1)}{2}}$$

$\boxed{\text{Arithmetic sum}}$

$$1+2+3+4+5 = \frac{5\cdot 6}{2} = 15$$

$\boxed{\text{Geometric sum}}$

$$1+a+a^2+\dots+a^k = \frac{a^{k+1}-1}{a-1} \in \Theta(a^k)$$

$$(a-1)[LHS] \stackrel{?}{=} (a-1)\cdot[RHS]$$

$$\left.\begin{array}{l} a+a^2+a^3+\dots+a^k+a^{k+1} \\ -1-a-a^2-a^3 \quad -a^k \end{array}\right\} LHS$$

$$\boxed{a^{k+1}-1} \quad = (a-1)\cdot RHS$$

$$1+2+2^2+\dots+2^{10} = \frac{2^{11}-1}{2-1} \in \Theta(2^{10})$$

$a>1$

$\Theta(2^{11})$ - $\cancel{\Theta}$

$2^{11} = c\cdot 2^{10}$

$$1+\frac{3}{4}+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+\dots+\left(\frac{3}{4}\right)^{10} = \frac{\left(\frac{3}{4}\right)^{11}-1}{\frac{3}{4}-1}$$

$$= -4\cdot\left[\left(\frac{3}{4}\right)^{11}-1\right]$$

$$= 4\cdot\left(1-\left(\frac{3}{4}\right)^{11}\right)$$

$$< 4$$

When $a<1$

$$\boxed{1+a+a^2+\dots+a^k \in \Theta(1)}$$

$a=1$

$1 + a + a^2 + a^3 + \ldots a^k = 1 + 1 + 1 \ldots + 1$

$= k+1 \in \theta(k)$

$a=1 \quad \sum_{i=0}^{k} a^i \in \theta(k)$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} < 2$

$a<1 \quad \sum_{i=0}^{k} a^i \in \theta(1)$

$a>1 \quad \sum_{i=0}^{k} a^i \in \theta(a^k)$

substitution

Tuesday, March 14, 2017 10:16 AM

$$\boxed{T(n) = n + T\left(\frac{n}{2}\right)} \in \Theta(n)$$

$T\left(\frac{n}{2}\right)$

$$T\left(\frac{n}{2}\right) = \boxed{\frac{n}{2} + T\left(\frac{n}{2^2}\right)}$$

$$T\left(\frac{n}{2}\right) = \frac{n}{2} + T\left(\frac{n}{2} \cdot \frac{1}{2}\right)$$

$$T(n) = n + \left[\frac{n}{2} + T\left(\frac{n}{2^2}\right)\right]$$

$$T(n) = n + \frac{n}{2} + T\left(\frac{n}{2^2}\right)$$

$$T\left(\frac{n}{2^2}\right) = \frac{n}{2^2} + T\left(\frac{n}{2^3}\right)$$

$$T(5) = 5 + T\left(\frac{5}{2}\right)$$

$$f(x) = x^{10} - \frac{2x}{5}$$

$$T(n) = n + \frac{n}{2} + \frac{n}{2^2} + T\left(\frac{n}{2^3}\right)$$

$$T(n) = n + \frac{n}{2} + \frac{n}{2^2} + \frac{n}{2^3} + T\left(\frac{n}{2^4}\right)$$

$$\boxed{T\left(\frac{n}{2^3}\right) = \frac{n}{2^3} + T\left(\frac{n}{2^4}\right)}$$

$$T\left(\frac{n}{2^4}\right) = \frac{n}{2^4} + T\left(\frac{n}{2^5}\right)$$

$$T(n) = n + \frac{n}{2} + \frac{n}{2^2} + \frac{n}{2^3} + \frac{n}{2^4} + T\left(\frac{n}{2^5}\right)$$

$$\frac{n}{2^k} = 1$$

$$k = \log_2 n$$

When we stop after k-1 steps,

$$T(n) = n + \frac{n}{2} + \frac{n}{2^2} + \cdots + \frac{n}{2^{k-1}} + T\left(\frac{n}{2^k}\right)$$

$$T(n) = n\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{k-1}}\right) + T(1)$$

$k = \lg_2 n$

Geometric sum $a = 1/2$

$$T(n) = c \cdot n + c_1 \in \Theta(n)$$

substitution

Tuesday, March 14, 2017 10:36 AM

$$T(n) = n + 4T(n/2) \quad \in \Theta(n^2)$$

$$T\left(\frac{n}{2}\right) = \frac{n}{2} + 4T\left(\frac{n}{2^2}\right)$$

$$T\left(\frac{n}{2^2}\right) = \frac{n}{2^2} + 4T\left(\frac{n}{2^3}\right)$$

$$T(n) = n + 4\left[\frac{n}{2} + 4T\left(\frac{n}{2^2}\right)\right]$$

$$T(n) = n + 2n + 4^2 T\left(\frac{n}{2^2}\right)$$

$$T(n) = n + 2n + 4^2\left[\frac{n}{2^2} + 4T\left(\frac{n}{2^3}\right)\right]$$

$$T(n) = n + 2n + \frac{4^2}{2^2}n + 4^3 T\left(\frac{n}{2^3}\right)$$

$$T(n) = n + 2n + 2^2 n + 4^3 T\left(\frac{n}{2^3}\right)$$

$$T(n) = n + 2n + 2^2 n + 2^3 n + 4^4 T\left(\frac{n}{2^4}\right)$$

k steps

$$T(n) = n\left[\underbrace{1 + 2 + 2^2 + 2^3 + \dots + 2^{k-1}}\right] + 4^k T(1)$$

$$\frac{n}{2^k} = 1$$

$$k = \log_2 n$$

$$T(n) = n \cdot \left(\frac{2^k - 1}{2 - 1}\right) + 4^k$$

$$T(n) = n \cdot \left(\frac{2^{\log_2 n} - 1}{}\right) + 4^{\log_2 n}$$

1

$$T(n) = n(n-1) + n^2 = 2n^2 - n \in \Theta(n^2)$$

$$4^{\log_2 n} = (2^2)^{\log_2 n}$$

$$= (2^{\log_2 n})^2 =$$

$$T(n) = n + 3T\left(\frac{n}{2}\right)$$

$$T(n) = n^2 + 2T\left(\frac{n}{2}\right)$$

Substitution

Tuesday, March 14, 2017 10:48 AM

$T(1) = 1$

$T(n) = n + T(n-1)$

$T(n) = n + (n-1) + T(n-2)$

$T(n) = n + (n-1) + (n-2) + T(n-3)$

$T(n) = n + (n-1) + (n-2) + \ldots + 2 + 1 + T(0)$

$T(n) = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \in \theta(n^2)$

$T(n-1) = n - 1 + T(n-2)$

$T(n) \in \theta(\ ?\ )$

$T(0) = 1$ $\theta(1)$

$T(1) = 1$

$T(n) = 2 \cdot T(n-1)$

$T(n-1) = 2T(n-2)$

$T(n) = 2 \cdot 2 \cdot T(n-2) = 2^2 \cdot T(n-2)$

$T(n) = 2^2 \cdot 2 \cdot T(n-3)$

$T(n) = 2^3 \cdot T(n-3)$

$\vdots$

$T(n) = 2^k \cdot T(n-k)$

$\in \theta(2^n)$

$2^n \cdot T(0)$

$k = n$

$T(n) = 2^n$