## Statistical Learning I

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Assignment 1: Chapter 2 (2,7,8), Chapter 3 (3,4,5,8,9,10,13)

## Chapter 2:

- 2)
- a) regression, inference: quantitative output of CEO salary based on CEO firm's features
- n: the top 500 firms in the US
- p: profit, number of employees, industry
- b) classification, prediction: predicting a new product whether is a success or failure
- n: 20 similar products previously launched
- p: price charge, marketing budget, competition price, 10 other variables
- c) regression, prediction: predicting % change in US dollars
- n: all weekly data of 2012 (52 weeks)
- p: % change in US market, % change in German market, % change in British market

7)

_a)					
Obs	$X_1$	$X_2$	$X_3$	Y	Distance to (0,0,0)
1	0	3	0	Red	3
2	2	0	0	Red	2
3	0	1	3	Red	~3.2
4	0	1	2	Green	~2.2
5	-1	0	1	Green	~1.4
6	1	1	1	Red	~1.7

- b) Prediction: Green, the obs #5 is the closet neighbor for K=1
- c) Prediction: Red, the obs #2, #5, #6 are the closet neighbor for K=3 (Red, Green, Red)
- d) The best value for K would be small, if the Bayes decision boundary for this problem highly non-linear. Since a small K would be flexible for non-linear boundary, whereas a large K would try to fit a more linear boundary it takes more points into consideration.
- 8) a) >college = read.csv(file="College.csv")
  - b)> rownames(college) = college[,1]
  - > fix(college)
  - > college = college[,-1]
  - > fix(college)
  - c)
  - i)> summary(college)

Private Apps Accept Enroll Top10perc
No:212 Min.: 81 Min.: 72 Min.: 35 Min.: 1.00
Yes:565 1st Qu.: 776 1st Qu.: 604 1st Qu.: 242 1st Qu.:15.00

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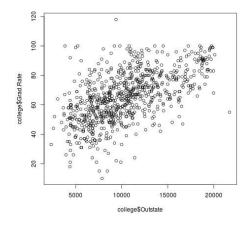
- ii)> jpeg("pairs.jpg")
- > pairs(college[,1:10])
- > dev.off()

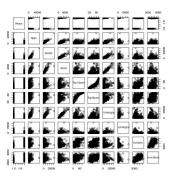
# iii)> jpeg("boxplots.jpg")

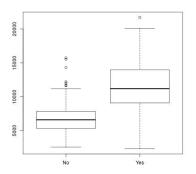
- > plot(college\$Private, college\$Outstate)
- > dev.off()
- iv)> Elite = rep("No", nrow(college))
- > Elite[college\$Top10perc>50] = "Yes"
- > Elite = as.factor(Elite)
- > college = data.frame(college, Elite)
- > summary(college\$Elite)

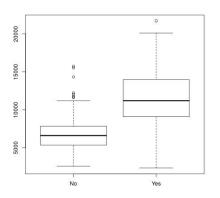
No Yes 699 78

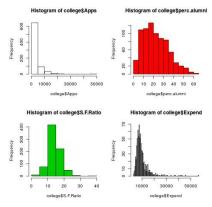
- > jpeg("new-boxplots.jpg")
- > plot(college\$Private, college\$Outstate)
- > dev.off()
- v)> jpeg("hist.jpg")
- > par(mfrow=c(2,2))
- > hist(college\$Apps)
- > hist(college\$perc.alumni, col=2)
- > hist(college\$S.F.Ratio, col=3, breaks=10)
- > hist(college\$Expend, breaks=100)
- > dev.off()
- vi)> plot(college\$Top10perc, college\$Grad.Rate)
- > jpeg("discovery.jpg")
- > plot(college\$Outstate, college\$Grad.Rate)
- > dev.off()
- > # High tuition correlates to high graduation rate.











## Chapter 3:

- 3) Y = 50 + 20(gpa) + 0.07(iq) + 35(gender) + 0.01(gpa \* iq) 10 (gpa \* gender)
  - a) male (gender = 0):  $50 + 20 X_1 + 0.07 X_2 + 0.01(X_1 * X_2)$  female (gender = 1):  $50 + 20 X_1 + 0.07 X_2 + 35 + 0.01(X_1 * X_2) 10 (X_1)$  Once the GPA is high enough, males earn more on average. => iii is correct.

```
b) Y(Gender = 1, IQ = 110, GPA = 4.0)
= 50 + 20 * 4 + 0.07 * 110 + 35 + 0.01 (4 * 110) - 10 * 4
= 137.1
```

- c) False. We must examine the p-value of the regression coefficient to determine if the interaction term is statistically significant or not.
- a) I would expect the polynomial regression to have a lower training RSS than the linear regression because it could make a tighter fit against data that matched with a wider irreducible error (Var(epsilon)).
  - b) Converse to (a), I would expect the polynomial regression to have a higher test RSS as the overfit from training would have more error than the linear regression.
  - c) Polynomial regression has lower train RSS than the linear fit because of higher flexibility: no matter what the underlying true relationshop is the more flexible model will closer follow points and reduce train RSS. An example is shown on Figure 2.9 from Chapter 2.
  - d) There is not enough information to tell which test RSS would be lower for either regression given the problem statement is defined as not knowing "how far it is from linear". If it is closer to linear than cubic, the linear regression test RSS could be lower than the cubic regression test RSS. Or, if it is closer to cubic than linear, the cubic regression test RSS could be lower than the linear regression test RSS. It is dues to bias-variance tradeoff: it is not clear what level of flexibility will fit data better.

```
5) y_i \land = x_i * B \land

B \land = sigma(x_i * y_i) / sigma(x_i^2)

y_i \land = x_i * sigma(x_i * y_i) / sigma(x_i^2) = y_i * sigma(x_i * x_i / x_i^2)

a_{i'} = (x_{i'} * x_i) / sigma(x_{i'}^2)
```

8) a) > Auto = read.csv("Auto.csv", header=T, na.strings="?") > Auto = na.omit(Auto)

> summary(Auto)

mpg cylinders displacement horsepower weight
Min.: 9.00 Min.: 3.000 Min.: 68.0 Min.: 46.0 Min.: 1613
1st Qu.:17.00 1st Qu.:4.000 1st Qu.:105.0 1st Qu.: 75.0 1st Qu.:2225
Median: 22.75 Median: 4.000 Median: 151.0 Median: 93.5 Median: 2804
Mean: 23.45 Mean: 5.472 Mean: 194.4 Mean: 104.5 Mean: 2978
3rd Qu.:29.00 3rd Qu.:8.000 3rd Qu.:275.8 3rd Qu.:126.0 3rd Qu.:3615
Max.: 46.60 Max.: 8.000 Max.: 455.0 Max.: 230.0 Max.: 5140

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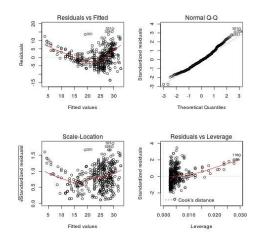
Residual standard error: 4.906 on 390 degrees of freedom Multiple R-squared: 0.6059, Adjusted R-squared: 0.6049 F-statistic: 599.7 on 1 and 390 DF, p-value: < 2.2e-16

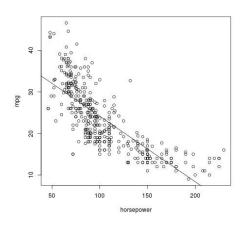
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' '1

- i) Yes, there is a relationship between horsepower and mpg as deterined by testing the null hypothesis of all regression coefficients equal to zero. Since the F-statistic is far larger than 1 and the p-value of the F-statistic is close to zero we can reject the null hypothesis and state there is a statistically significant relationship between horsepower and mpg.
- ii) To calculate the residual error relative to the response we use the mean of the response and the RSE. The mean of mpg is 23.4459. The RSE of the lm.fit was 4.906 which indicates a percentage error of 20.9248%. The R² of the lm.fit was about 0.6059, meaning 60.5948% of the variance in mpg is explained by horsepower.
- iii) The relationship between mpg and horsepower is negative. The more horsepower an automobile has the linear regression indicates the less mpg fuel efficiency the automobile will have.

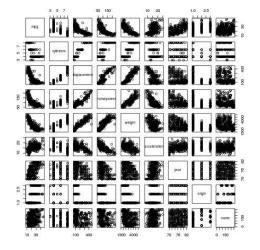
```
iv) > predict(lm.fit, data.frame(horsepower=c(98)), interval="confidence")
    fit
                lwr
                               upr
1 24.46708
                23.97308
                               24.96108
> predict(lm.fit, data.frame(horsepower=c(98)), interval="prediction")
   fit
               lwr
                                upr
1 24.46708
                14.8094
                               34.12476
c) > jpeg("lm-hp-mpg.jpg")
> plot(horsepower, mpg)
> abline(lm.fit)
> dev.off()
d) > jpeg(" diagnostic-plots.jpg")
> par(mfrow=c(2,2))
> plot(lm.fit)
> dev.off()
```

Based on the residuals plots, there is some evidence of non-linearity.





9) a) > pairs(Auto) > jpeg(" pairs-auto.jpg") > pairs(Auto) > dev.off()



### b) > cor(subset(Auto, select=-name))

mpg cylinders displacement horsepower weight
mpg 1.0000000 -0.7776175 -0.8051269 -0.7784268 -0.8322442
cylinders -0.7776175 1.0000000 0.9508233 0.8429834 0.8975273
displacement -0.8051269 0.9508233 1.0000000 0.8972570 0.9329944
horsepower -0.7784268 0.8429834 0.8972570 1.0000000 0.8645377
weight -0.8322442 0.8975273 0.9329944 0.8645377 1.0000000
acceleration 0.4233285 -0.5046834 -0.5438005 -0.6891955 -0.4168392
year 0.5805410 -0.3456474 -0.3698552 -0.4163615 -0.3091199
origin 0.5652088 -0.5689316 -0.6145351 -0.4551715 -0.5850054
acceleration year origin

year origin 0.4233285 0.5805410 0.5652088 mpg cylinders -0.5046834 -0.3456474 -0.5689316 displacement -0.5438005 -0.3698552 -0.6145351 horsepower -0.6891955 -0.4163615 -0.4551715 weight -0.4168392 -0.3091199 -0.5850054 acceleration 1.0000000 0.2903161 0.2127458 year 0.2903161 1.0000000 0.1815277 origin 0.2127458 0.1815277 1.0000000

```
c) > lm.fit1 = lm(mpg~.-name, data=Auto) > summary(lm.fit1)
```

#### Call:

 $lm(formula = mpg \sim . - name, data = Auto)$ 

### Residuals:

Min 1Q Median 3Q Max -9.5903 -2.1565 -0.1169 1.8690 13.0604

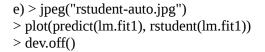
#### Coefficients:

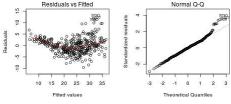
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ''1

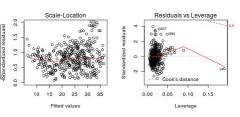
Residual standard error: 3.328 on 384 degrees of freedom Multiple R-squared: 0.8215, Adjusted R-squared: 0.8182 F-statistic: 252.4 on 7 and 384 DF, p-value: < 2.2e-16

- i) Yes, there is a relatioship between the predictors and the response by testing the null hypothesis of whether all the regression coefficients are zero. The F -statistic is far from 1 (with a small p-value), indicating evidence against the null hypothesis.
- ii) Looking at the p-values associated with each predictor's t-statistic, we see that displacement, weight, year, and origin have a statistically significant relationship, while cylinders, horsepower, and acceleration do not.
- iii) The regression coefficient for year, 0.7508, suggests that for every one year, mpg increases by the coefficient. In other words, cars become more fuel efficient every year by almost 1 mpg / year.
- d) > jpeg("diagnostic-plots-auto.jpg")
  > par(mfrow=c(2,2))
- > plot(lm.fit1)
- > dev.off()

The fit does not appear to be accurate because there is a discernible curve pattern to the residuals plots. From the leverage plot, point 14 appears to have high leverage, although not a high magnitude residual.







There are possible outliers as seen in the plot of studentized residuals because there are data with a value greater than 3.

```
e) > lm.fit2 = lm(mpg~cylinders*displacement+displacement*weight) > summary(lm.fit2)
```

#### Call:

lm(formula = mpg ~ cylinders \* displacement + displacement \* weight)

### Residuals:

Min 1Q Median 3Q Max -13.2934 -2.5184 -0.3476 1.8399 17.7723

### Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 5.262e+01 2.237e+00 23.519 < 2e-16 \*\*\*

cylinders 7.606e-01 7.669e-01 0.992 0.322

displacement -7.351e-02 1.669e-02 -4.403 1.38e-05 \*\*\*
weight -9.888e-03 1.329e-03 -7.438 6.69e-13 \*\*\*
cylinders:displacement -2.986e-03 3.426e-03 -0.872 0.384
displacement:weight 2.128e-05 5.002e-06 4.254 2.64e-05 \*\*\*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' '1

Residual standard error: 4.103 on 386 degrees of freedom Multiple R-squared: 0.7272, Adjusted R-squared: 0.7237

F-statistic: 205.8 on 5 and 386 DF, p-value: < 2.2e-16

From the correlation matrix, I obtained the two highest correlated pairs and used them in picking my interaction effects. From the p-values, we can see that the interaction between displacement and weight is statistically significant, while the interaction between cylinders and displacement is not.

```
f) > lm.fit3 = lm(mpg~log(weight)+sqrt(horsepower)+acceleration+I(acceleration^2)) > summary(lm.fit3)
```

### Call:

 $lm(formula = mpg \sim log(weight) + sqrt(horsepower) + acceleration + I(acceleration^2))$ 

## Residuals:

Min 1Q Median 3Q Max -11.2932 -2.5082 -0.2237 2.0237 15.7650

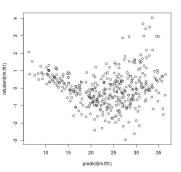
#### Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
```

(Intercept) 178.30303 10.80451 16.503 < 2e-16 \*\*\* log(weight) -14.74259 1.73994 -8.473 5.06e-16 \*\*\* sqrt(horsepower) -1.85192 0.36005 -5.144 4.29e-07 \*\*\* acceleration -2.19890 0.63903 -3.441 0.000643 \*\*\* I(acceleration^2) 0.06139 0.01857 3.305 0.001037 \*\*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ''1

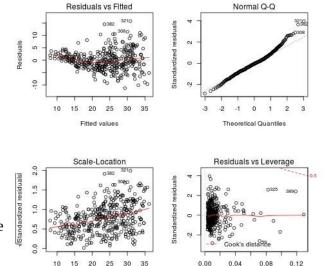


Residual standard error: 3.99 on 387 degrees of freedom Multiple R-squared: 0.7414, Adjusted R-squared: 0.7387 F-statistic: 277.3 on 4 and 387 DF, p-value: < 2.2e-16

f) > jpeg("c3-9f.jpg") > par(mfrow=c(2,2)) > plot(lm.fit3) > dev.off()

- > jpeg("c3-9f1.jpg")
- > plot(predict(lm.fit3), rstudent(lm.fit3))
- > dev.off()

Apparently, from the p-values, the log(weight), sqrt(horsepower), and acceleration<sup>2</sup> all have statistical significance of some sort. The residuals plot has less of a discernible pattern than the plot of all linear regression terms. The studentized residuals displays potential outliers (>3). The leverage plot indicates more than three points with high leverage.



predict(Im.fit3)

- 1) the residuals vs fitted plot indicates heteroskedasticity (unconstant variance over mean) in the model.
- 2) The Q-Q plot indicates somewhat unnormality of the residuals.

So, a better transformation need to be applied to our model. From the correlation matrix in 9a., displacement, horsepower and weight show a similar nonlinear pattern against our response mpg. This nonlinear pattern is very close to a log form. So in the next attempt, we use log(mpg) as our response variable. The outputs show that log transform of mpg yield better model fitting (better  $R^2$ , normality of residuals).

Income

## 10) > library(ISLR)

Sales

> summary(Carseats)

CompPrice

Min.: 0.000 Min.: 77 Min.: 21.00 Min.: 0.000 1st Qu.: 5.390 1st Qu.:115 1st Qu.: 42.75 1st Qu.: 0.000 Median: 7.490 Median: 125 Median: 69.00 Median: 5.000 Mean: 7.496 Mean: 125 Mean: 68.66 Mean: 6.635 3rd Qu.: 9.320 3rd Qu.:135 3rd Qu.: 91.00 3rd Qu.:12.000 Max. :16.270 Max. :175 Max. :120.00 Max. :29.000 Population Price ShelveLoc Age Education Min.: 10.0 Min.: 24.0 Bad: 96 Min.: 25.00 Min.: 10.0 1st Qu.:139.0 1st Qu.:100.0 Good : 85 1st Qu.:39.75 1st Qu.:12.0 Median: 272.0 Median: 117.0 Medium: 219 Median: 54.50 Median: 14.0 Mean :264.8 Mean :115.8 Mean :53.32 Mean :13.9

Advertising

 3rd Qu.:398.5
 3rd Qu.:131.0
 3rd Qu.:66.00
 3rd Qu.:16.0

 Max. :509.0
 Max. :191.0
 Max. :80.00
 Max. :18.0

. . . .

```
a) > attach(Carseats)
> lm.fit = lm(Sales~Price+Urban+US)
> summary(lm.fit)
```

#### Call:

lm(formula = Sales ~ Price + Urban + US)

#### Residuals:

Min 1Q Median 3Q Max -6.9206 -1.6220 -0.0564 1.5786 7.0581

### Coefficients:

Estimate Std. Error t value Pr(>|t|)
(Intercept) 13.043469 0.651012 20.036 < 2e-16 \*\*\*
Price -0.054459 0.005242 -10.389 < 2e-16 \*\*\*
UrbanYes -0.021916 0.271650 -0.081 0.936
USYes 1.200573 0.259042 4.635 4.86e-06 \*\*\*
---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' '1

Residual standard error: 2.472 on 396 degrees of freedom Multiple R-squared: 0.2393, Adjusted R-squared: 0.2335 F-statistic: 41.52 on 3 and 396 DF, p-value: < 2.2e-16

b) Price: The linear regression suggests a relationship between price and sales given the low p-value of the t-statistic. The coefficient states a negative relationship between Price and Sales: as Price increases, Sales decreases.

UrbanYes: The linear regression suggests that there isn't a relationship between the location of the store and the number of sales based on the high p-value of the t-statistic.

USYes: The linear regression suggests there is a relationship between whether the store is in the US or not and the amount of sales. The coefficient states a positive relationship between USYes and Sales: if the store is in the US, the sales will increase by approximately 1201 units.

```
c) Sales = 13.04 + -0.05 Price + -0.02 UrbanYes + 1.20 USYes
```

d) Predictors reject null hypothesis: Price and USYes, based on the p-values, F-statistic, and p-value of the F-statistic.

```
e) > lm.fit2 = lm(Sales ~ Price + US) > summary(lm.fit2)
```

#### Call:

lm(formula = Sales ~ Price + US)

### Residuals:

Min 1Q Median 3Q Max -6.9269 -1.6286 -0.0574 1.5766 7.0515

### Coefficients:

Estimate Std. Error t value Pr(>|t|) (Intercept) 13.03079 0.63098 20.652 < 2e-16 \*\*\*

```
Price -0.05448 0.00523 -10.416 < 2e-16 ***
USYes 1.19964 0.25846 4.641 4.71e-06 ***
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' '1

Residual standard error: 2.469 on 397 degrees of freedom Multiple R-squared: 0.2393, Adjusted R-squared: 0.2354 F-statistic: 62.43 on 2 and 397 DF, p-value: < 2.2e-16

f) Based on the RSE and  $R^2$  of the linear regressions, they both fit the data similarly, with linear regression from (e) fitting the data slightly better.

### g) > confint(lm.fit2) 2.5 %

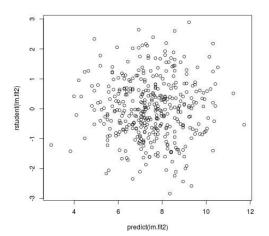
2.5 % 97.5 % (Intercept) 11.79032020 14.27126531 Price -0.06475984 -0.04419543 USYes 0.69151957 1.70776632

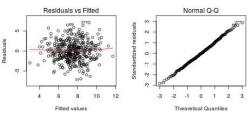
h) > jpeg("c310h.jpg") > plot(predict(lm.fit2), rstudent(lm.fit2)) > dev.off()

All studentized residuals appear to be bounded by -3 to 3, so not potential outliers are suggested from the linear regression.

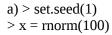
> jpeg("c310h1.jpg")
> par(mfrow=c(2,2))
> plot(lm.fit2)
> dev.off()

There are a few observations that greatly exceed (p+1)/n(p+1)/n (0.0076) on the leverage-statistic plot that suggest that the corresponding points have high leverage.





13)

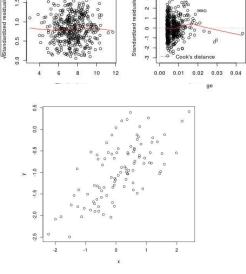


b) > eps = rnorm(100, 0, sqrt(0.25))

c) > y = -1 + 0.5\*x + eps  
y has length 100, 
$$b_0$$
 = -1,  $b_1$  = 0.5

d) > jpeg("13d")
> plot(x, y)
> dev.off()

A linear relationship between x and y with a positive slope, with a variance as is to be expected.



```
e) > lm.fit = lm(y\sim x)
> summary(lm.fit)
Call:
lm(formula = y \sim x)
Residuals:
  Min
          1Q Median
                        3Q
                              Max
-0.93842 -0.30688 -0.06975 0.26970 1.17309
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ''1
Residual standard error: 0.4814 on 98 degrees of freedom
Multiple R-squared: 0.4674, Adjusted R-squared: 0.4619
F-statistic: 85.99 on 1 and 98 DF, p-value: 4.583e-15
The linear regression fits a model close to the true value of the coefficients as was constructed. The
model has a large F-statistic with a near-zero p-value so the null hypothesis can be rejected.
f) > jpeg("13f.jpg")
> plot(x, y)
> abline(lm.fit, lwd=3, col=2)
> abline(-1, 0.5, lwd=3, col=3)
> legend(-1, legend = c("least square", "population regression"), col=2:3, lwd=3)
> dev.off()
g) > lm.fit_sq = lm(y\sim x+I(x^2))
                                                   0.5
> summary(lm.fit sq)
                                                   0.0
Call:
lm(formula = y \sim x + I(x^2))
Residuals:
          1Q Median
                        3Q
                              Max
-0.98252 -0.31270 -0.06441 0.29014 1.13500
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.97164  0.05883 -16.517 < 2e-16 ***
       I(x^2)
        Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ''1
```

Residual standard error: 0.479 on 97 degrees of freedom Multiple R-squared: 0.4779, Adjusted R-squared: 0.4672

F-statistic: 44.4 on 2 and 97 DF, p-value: 2.038e-14

```
h) > set.seed(1)
> eps1 = rnorm(100, 0, 0.125)
> x1 = rnorm(100)
> y1 = -1 + 0.5*x1 + eps1
> lm.fit1 = lm(y1\sim x1)
> summary(lm.fit1)
Call:
lm(formula = y1 \sim x1)
Residuals:
  Min
         1Q Median
                         3Q
                               Max
-0.29052 -0.07545 0.00067 0.07288 0.28664
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
x1
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 0.1128 on 98 degrees of freedom
Multiple R-squared: 0.9479, Adjusted R-squared: 0.9474
F-statistic: 1782 on 1 and 98 DF, p-value: < 2.2e-16
> jpeg("13h.jpg")
> plot(x1, y1)
> abline(lm.fit1, lwd=3, col=2)
> abline(-1, 0.5, lwd=3, col=3)
> legend(-1, legend = c("least square", "population regression"), col=2:3, lwd=3)
> dev.off()
As expected, the error observed in R<sup>2</sup> and RSE decreases considerably.
i) > set.seed(1)
                                                       0.0
> eps2 = rnorm(100, 0, 0.5)
> x2 = rnorm(100)
y^2 = -1 + 0.5 \times x^2 + eps^2
> lm.fit2 = lm(v2\sim x2)
> summary(lm.fit2)
Call:
lm(formula = y2 \sim x2)
Residuals:
  Min
         1Q Median
                         3Q
                               Max
-1.16208 -0.30181 0.00268 0.29152 1.14658
Coefficients:
```

x2

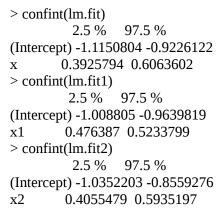
```
---
```

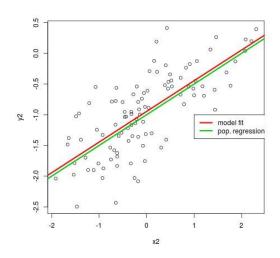
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ''1

Residual standard error: 0.4514 on 98 degrees of freedom Multiple R-squared: 0.5317, Adjusted R-squared: 0.5269

F-statistic: 111.2 on 1 and 98 DF, p-value: < 2.2e-16

```
> jpeg("13i.jpg")
> plot(x2, y2)
> abline(lm.fit2, lwd=3, col=2)
> abline(-1, 0.5, lwd=3, col=3)
> legend(-1, legend = c("model fit", "pop. regression"), col=2:3, lwd=3)
> dev.off()
```





All intervals seem to be centered on

approximately 0.5, with the second fit's interval being narrower than the first fit's interval and the last fit's interval being wider than the first fit's interval.