

Aufgabe 1.

- a) The runtime of the Algorithm 1 for the shortest path problem equals $\mathcal{O}(n \cdot m)$, where n equals the number of vertices and m equals the number of edges.

Proof. The initialization of the lengths runs in $\mathcal{O}(n)$. For the worst case scenario, for every length update a new shorter path will be revealed, until every vertex is finally evaluated ($\mathcal{O}(n)$). To find the newly revealed shortest path, it takes $\mathcal{O}(m)$ comparisons. Hence, the algorithm will run in $\mathcal{O}(n \cdot m)$ in the worst case and in $\mathcal{O}(\max\{n, m\})$ in best case. \square

- b) $A <_{\text{aymp}} B <_{\text{aymp}} E <_{\text{aymp}} C <_{\text{aymp}} D$

Proof. • $A <_{\text{aymp}} B$

$$2^{\sqrt{\log n}} = 2^{\log n^{\frac{1}{2}}} = 2^{\frac{1}{2} \log n} = \sqrt{2}^{\log n}$$

Also, the following holds. Let f be a monotone increasing function and $c > 1$. Then:

$$\lim_{\infty} \frac{c^f}{f} > 1$$

- $B <_{\text{aymp}} E$

$$2^{\sqrt{\log n}} < 2^{\sqrt{\log n \cdot \log n}} = n < 28n$$

- $E <_{\text{aymp}} C$

$$\log n^{\log n} = (2^{\log \log n})^{\log n} = n^{\log \log n}$$

Therefore, every linear function out of $\mathcal{O}(n)$ will be outgrown by C .

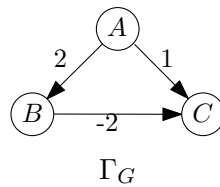
- $C <_{\text{aymp}} D$

$$\mathcal{O}(2^{\log \log n}) \subset \mathcal{O}(2^n)$$

\square

Aufgabe 2.

- 1.
2. Consider the drawing of the following graph:



Here, the PQ will look as follows:

$$PQ = [(A, 0), (C, \infty), (B, \infty)]$$

`Extract_Min` returns and deletes $(A, 0)$. Its neighbours will get updated to PQ' :

$$PQ' = [(C, 1), (B, 2)]$$

Next, C will be extracted since it is the minimum. But no edge from C is left out of PQ' . The result will be:

$$d[A] = 0$$

$$d[B] = 2$$

$$d[C] = 1 \neq 0$$

Therefore not the correct answer.

Aufgabe 3.

Aufgabe 4.

- a) The height of a Fibonacci heap is in $\mathcal{O}(n)$ in the worst case.

Proof. Consider a chain of increasing elements L . Then, The first element can be interpreted as a root list with one element which has one child. This child has only one child and so on. Every element points to itself in a linked list. Therefore, L can be interpreted as a valid Fibonacci Heap where the height is in $\mathcal{O}(|L|)$. \square