Smoothening of Kandinsky Drawings

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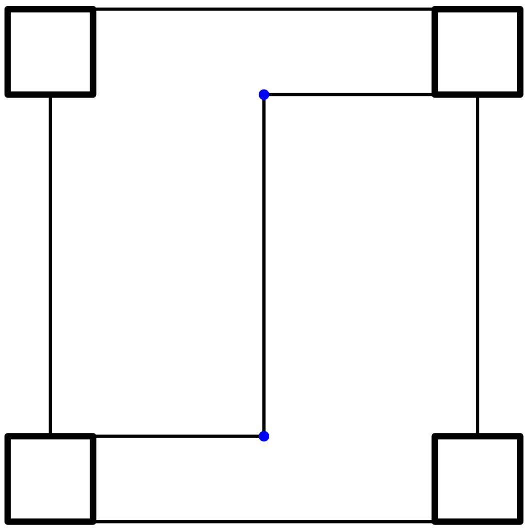
Kandinsky drawings

- Well-established orthogonal graph drawing model



Kandinsky
Complexity 3

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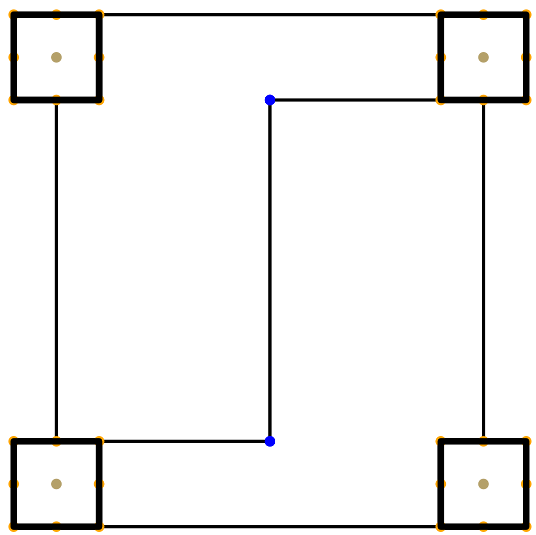

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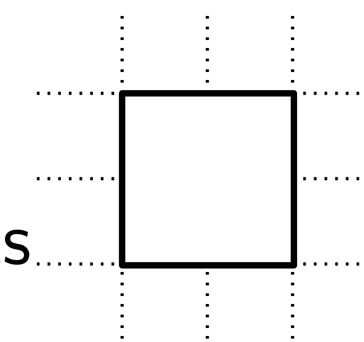
- underlying grid embedding
- vertices as boxes of uniform size, inherit ports



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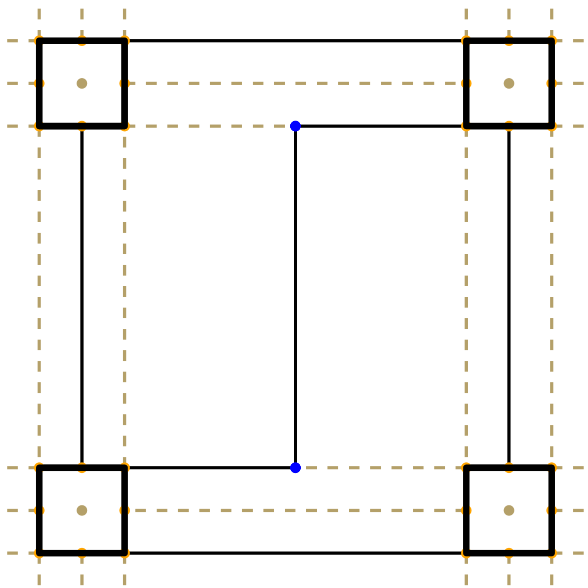

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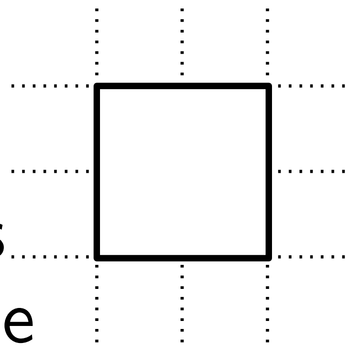
- underlying grid embedding
- vertices as boxes of uniform size, inherit ports
- polyedges as non-empty line segment sequence



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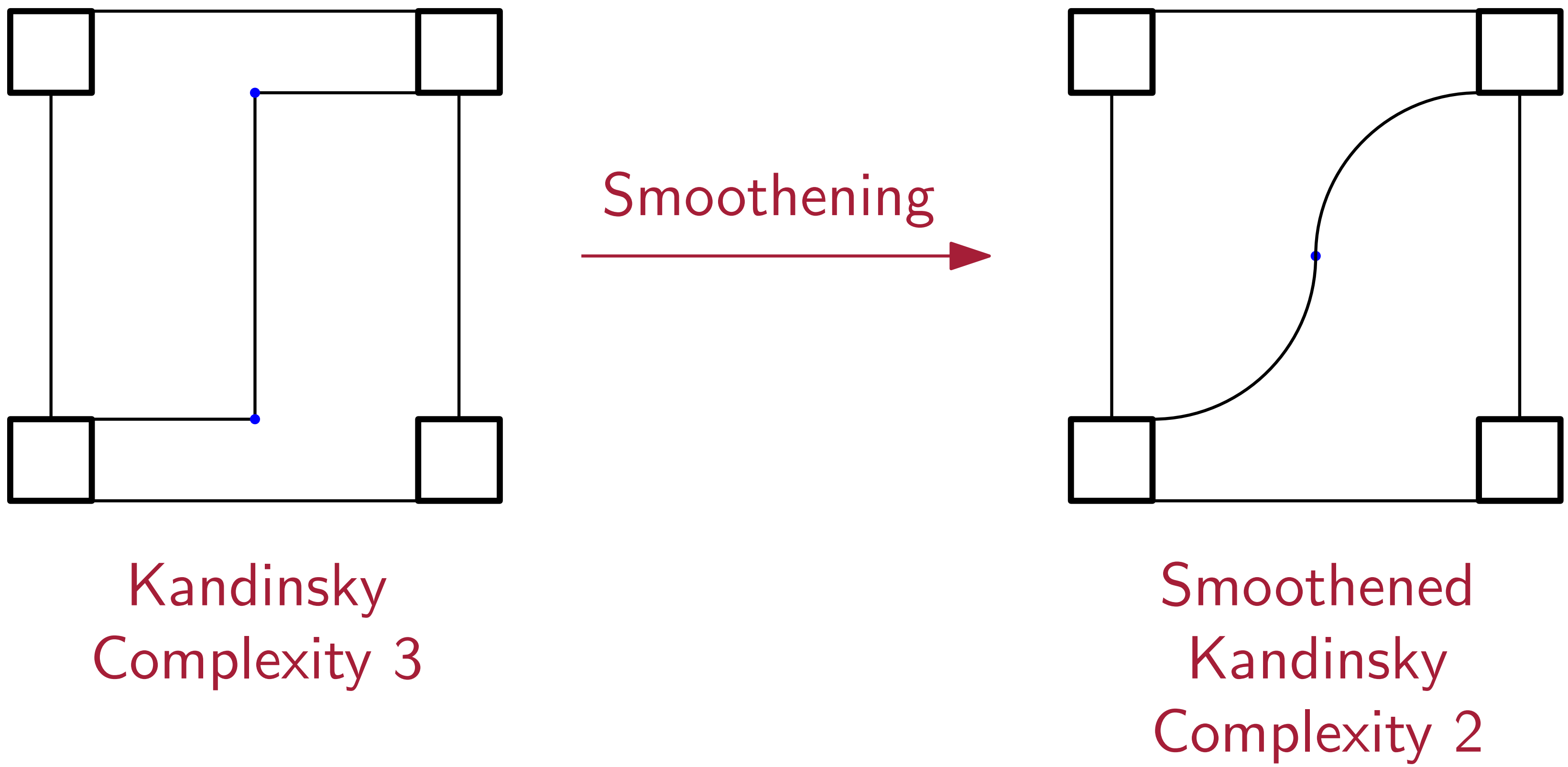
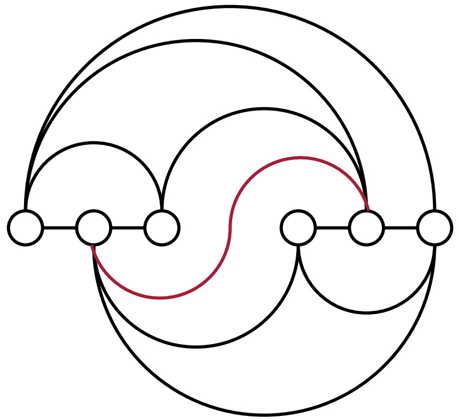


- new segments - quarter / semi circular arcs
- Clarity + Aesthetics combined

Previous results

- **Kandinsky drawings**
  - admit a complexity-2 smoothened drawing in $\mathcal{O}(n^2)$ area inspired by book embeddings [Bekos et al. 2013]



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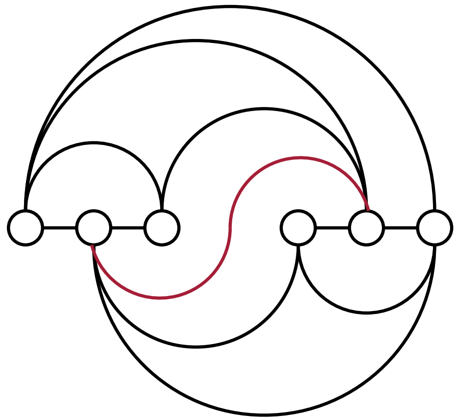

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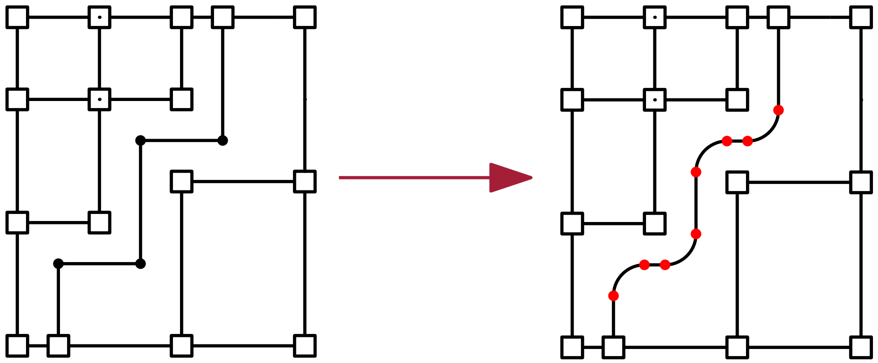

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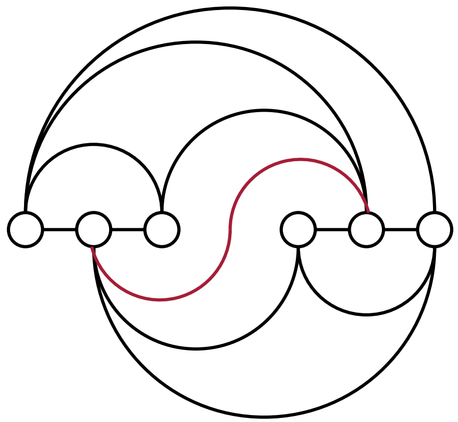

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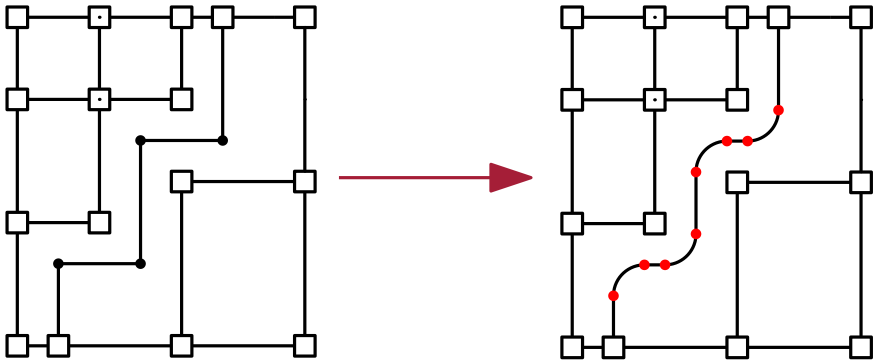

- **Disadvantages**
  - Either shape alternation or high complexity increase

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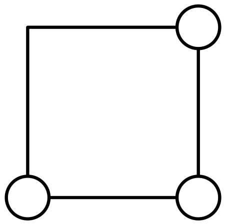

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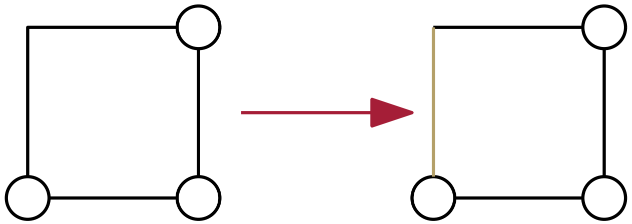

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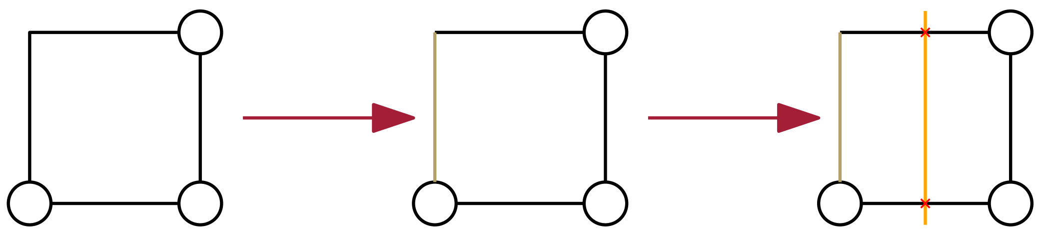

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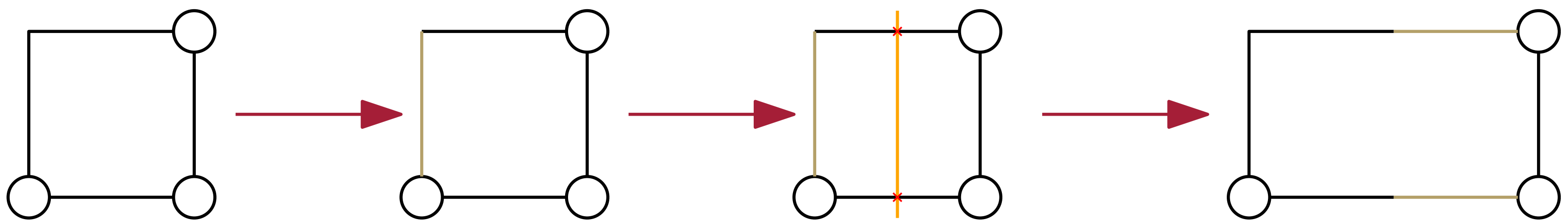

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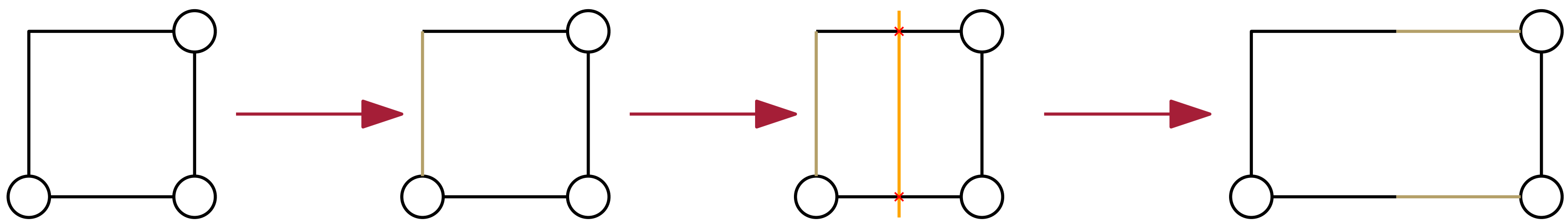
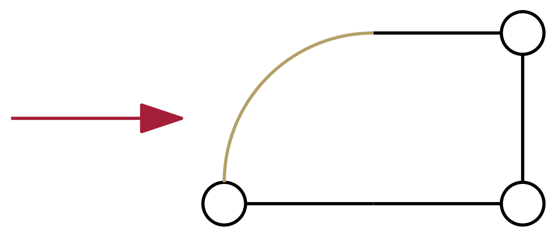


- quarter circular arc substitution at vertical segments



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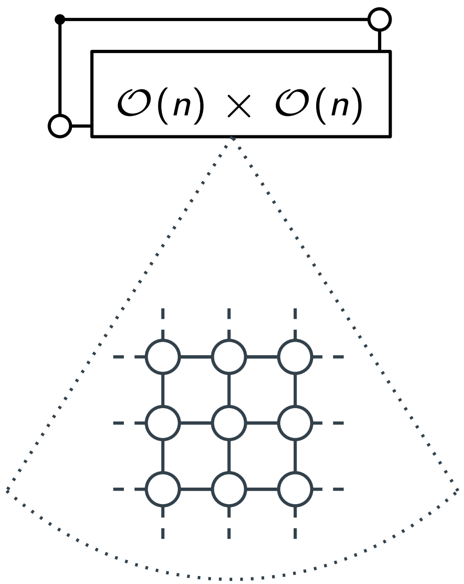

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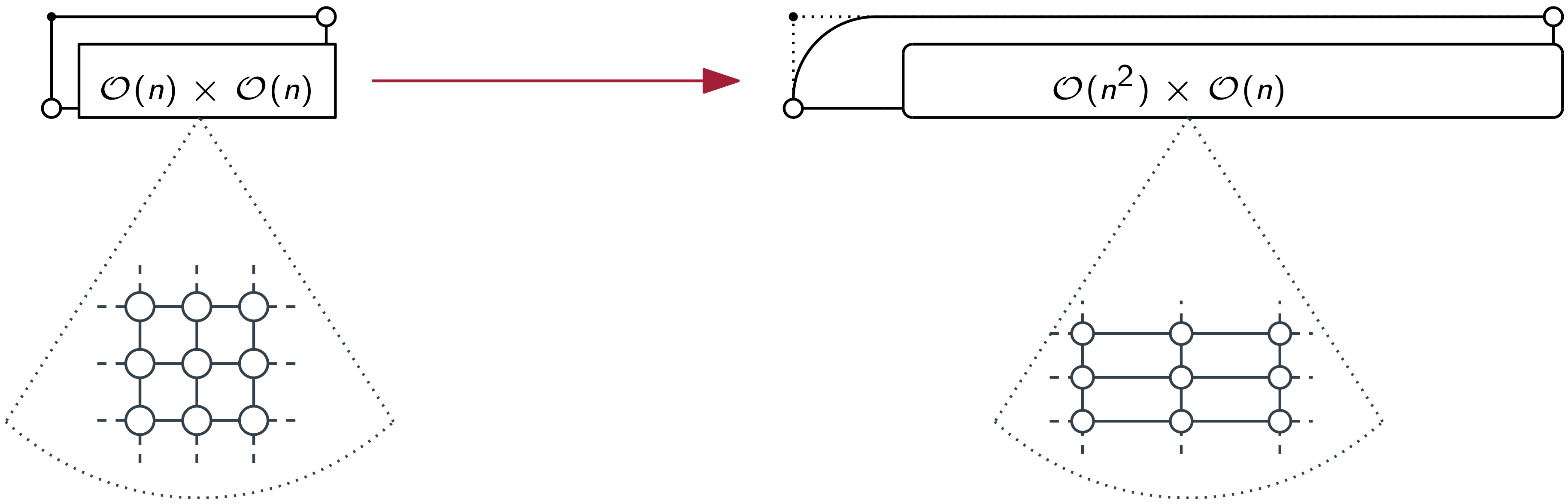

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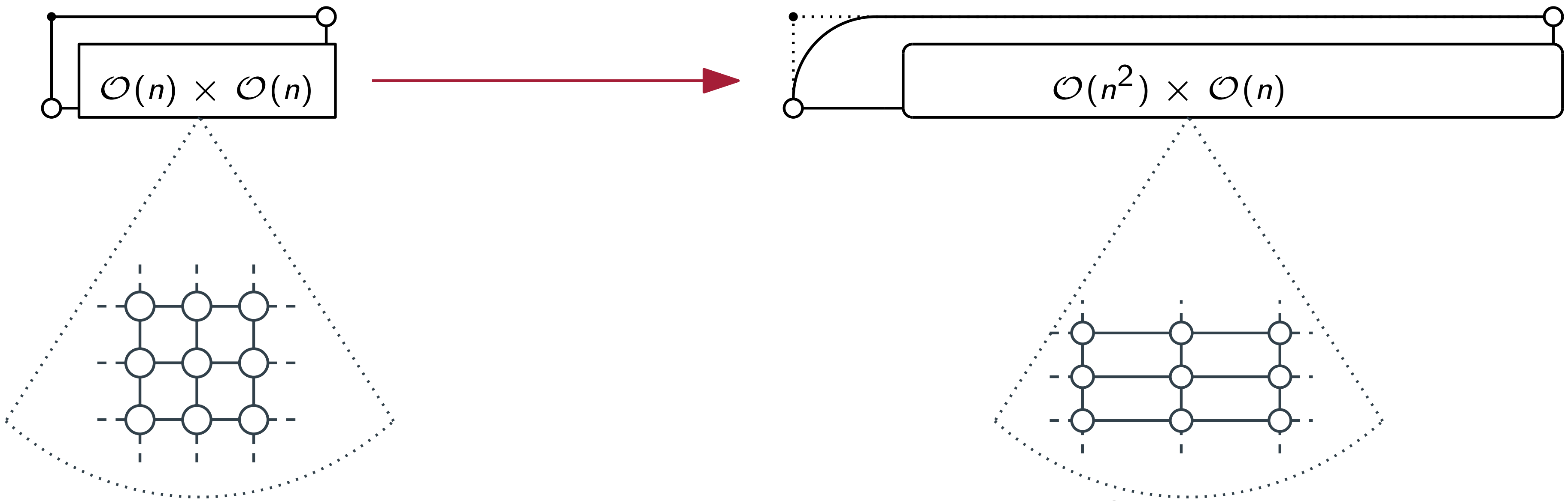

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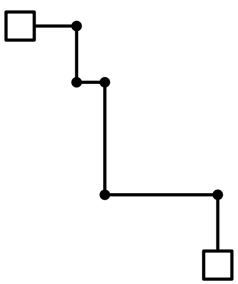

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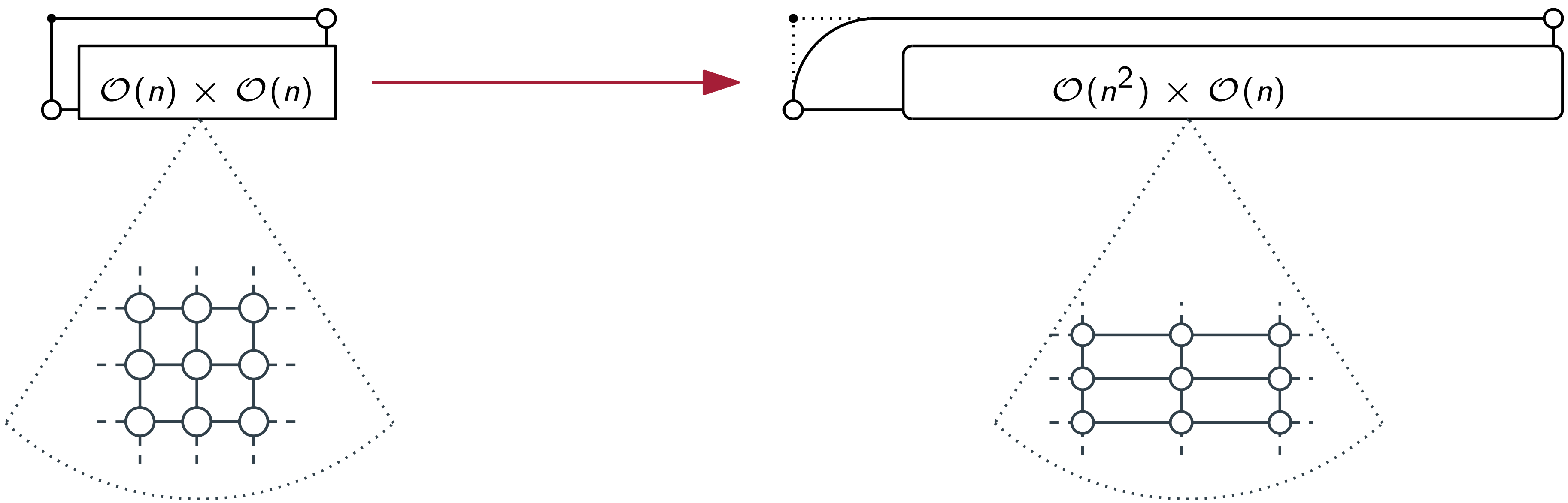

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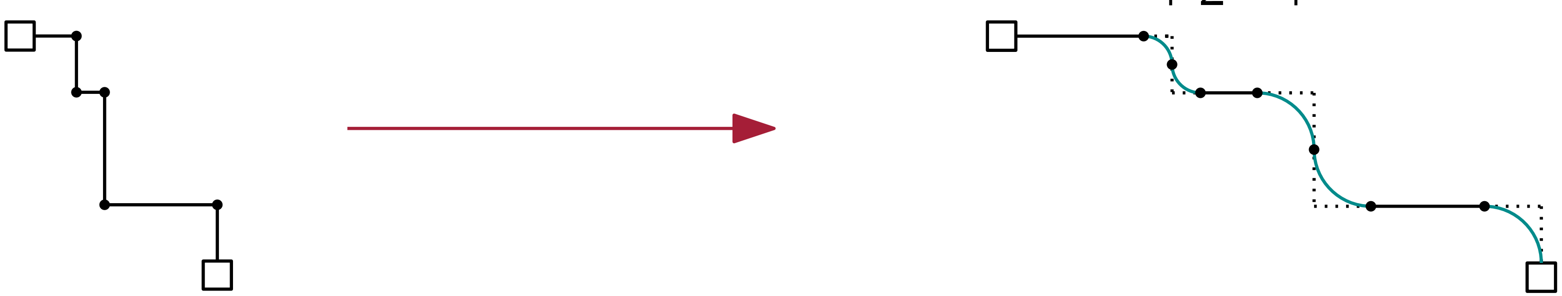

Smoothening of Kandinsky drawings

- Outline
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    - Does the stretching technique smoothening work for Kandinsky drawings of arbitrary degree?
    - Area bounds?
    - Complexity bounds?

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  - **Saving measures**
    - Area saving measures
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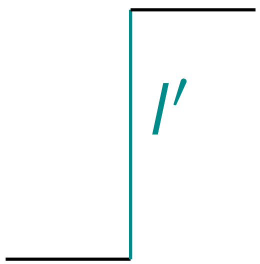

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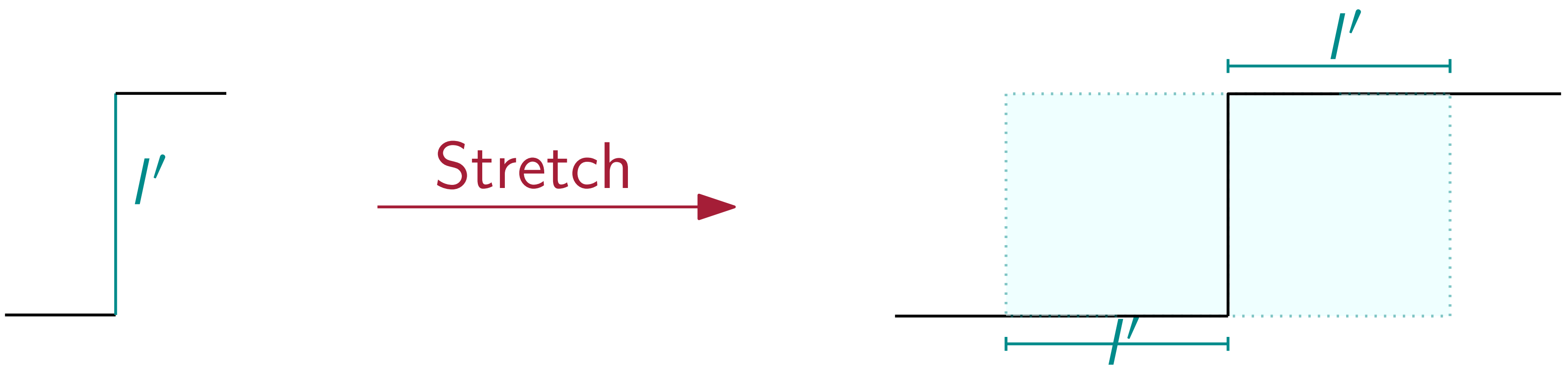

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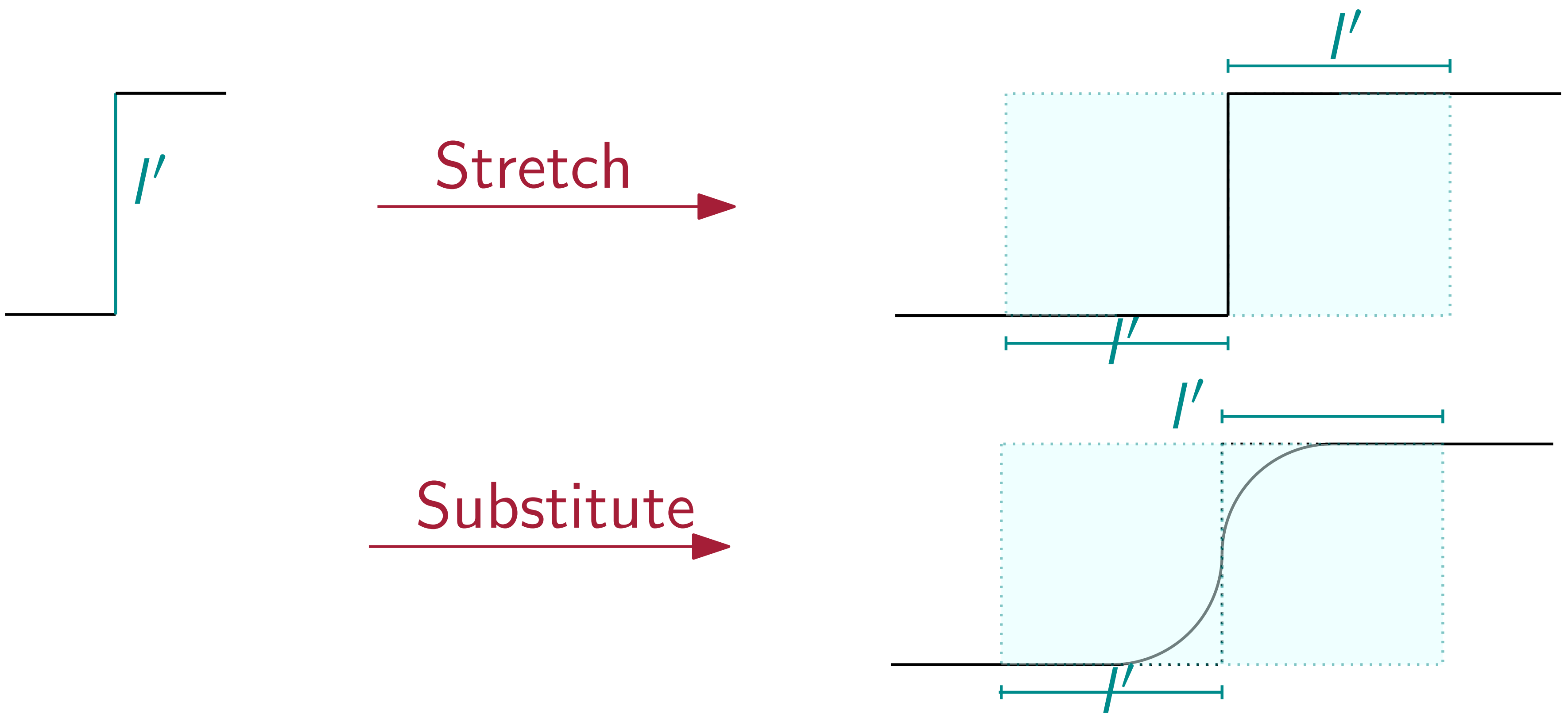

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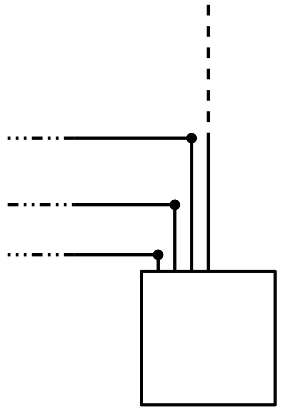

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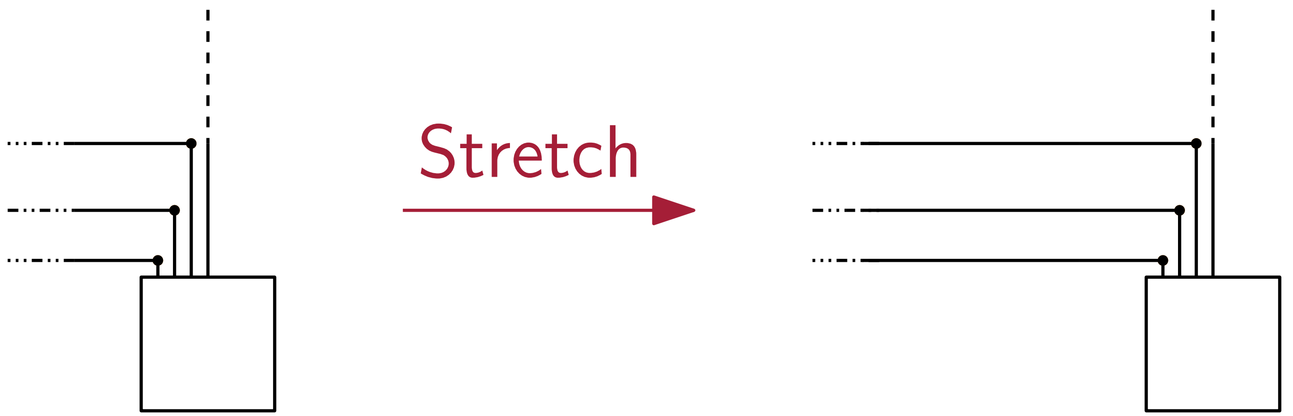

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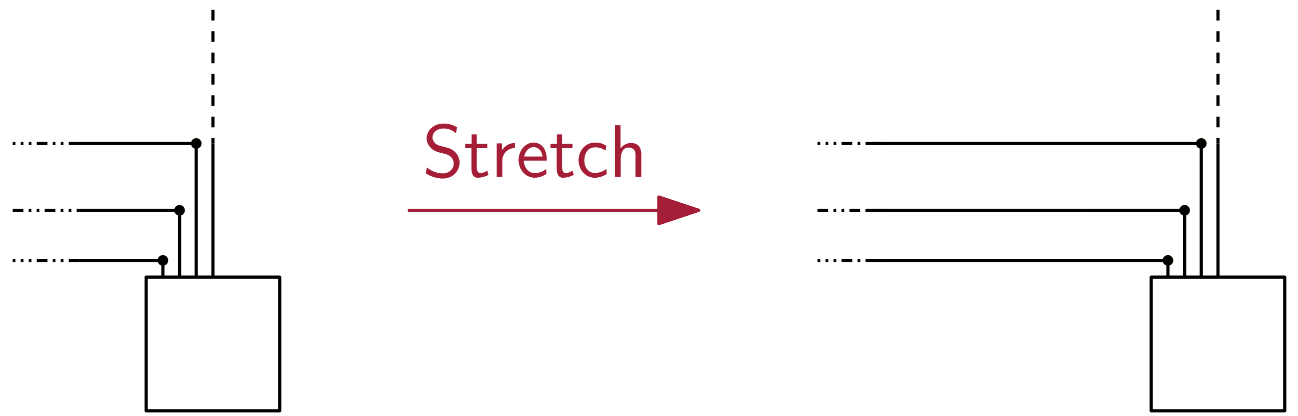

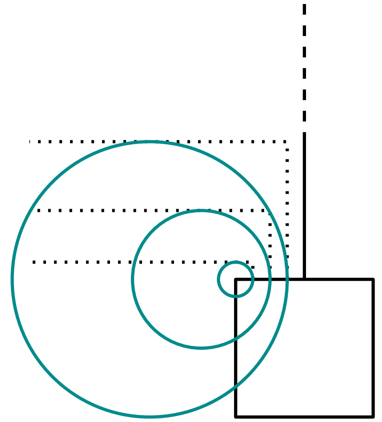

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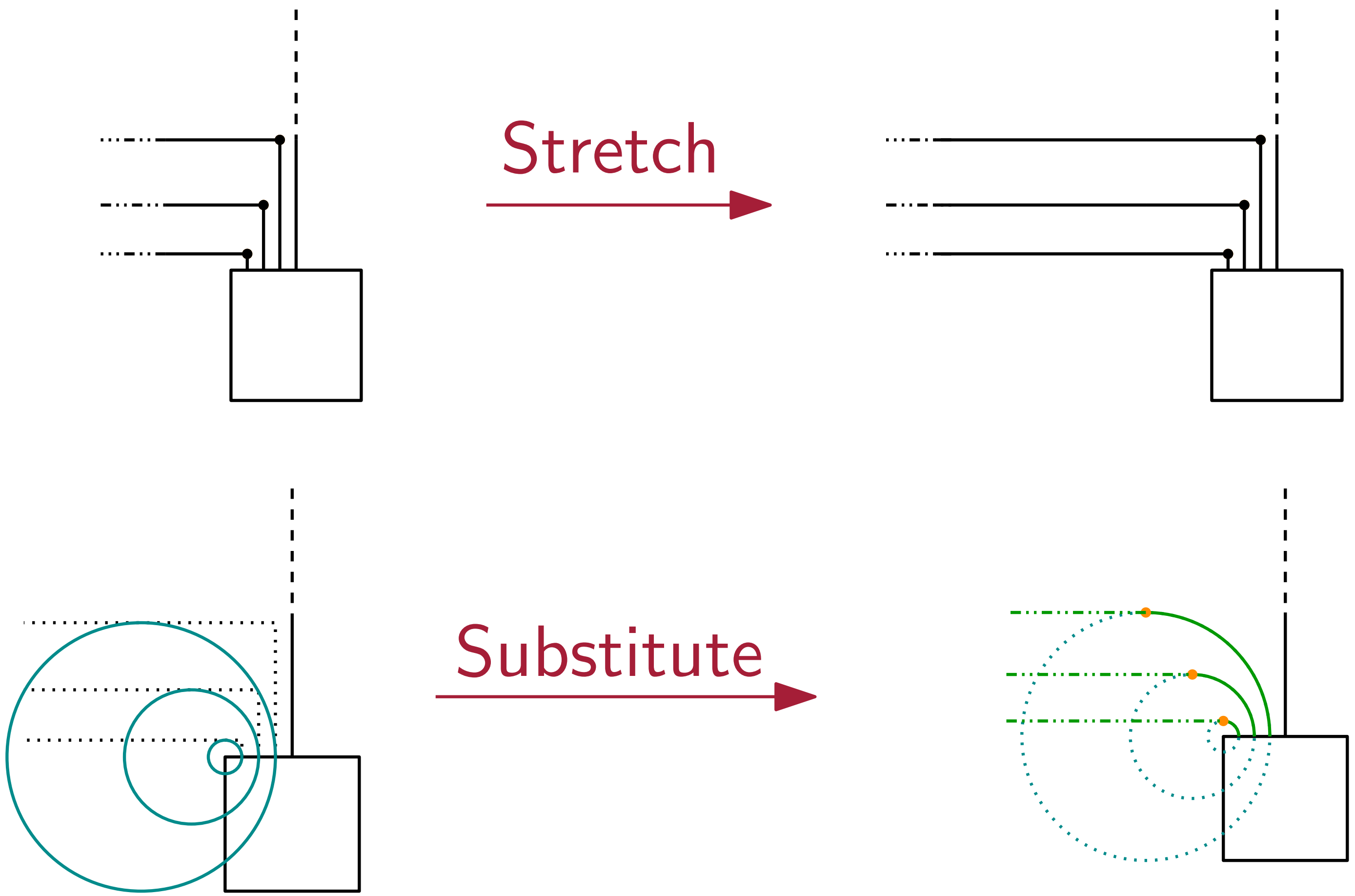

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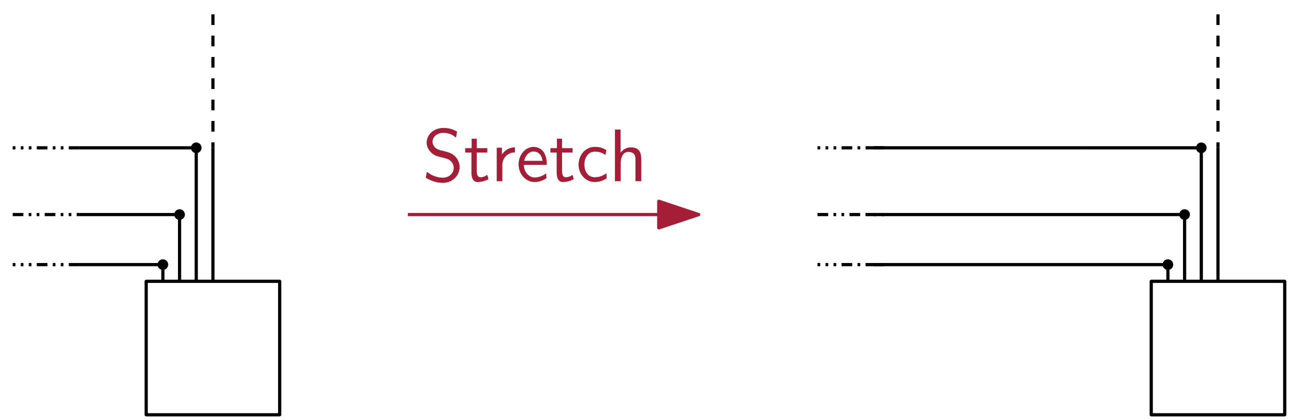

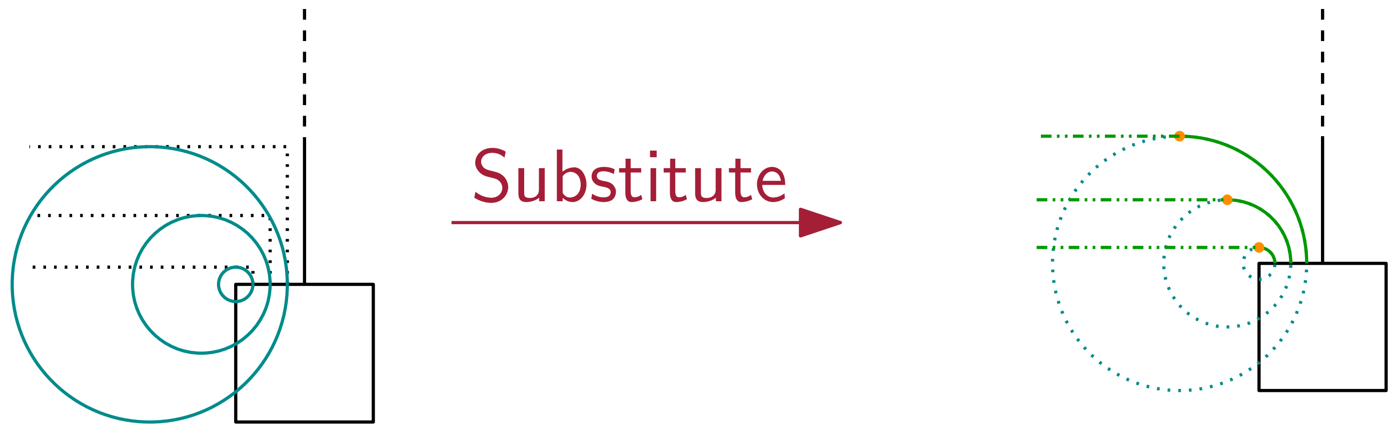

- Single vertical segment between two vertices - do nothing

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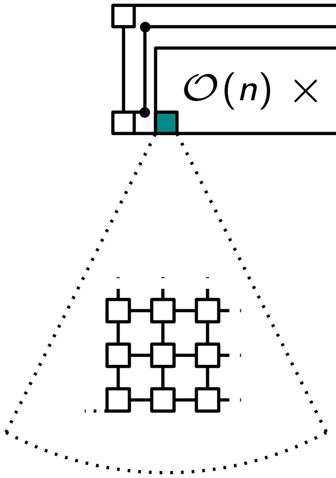

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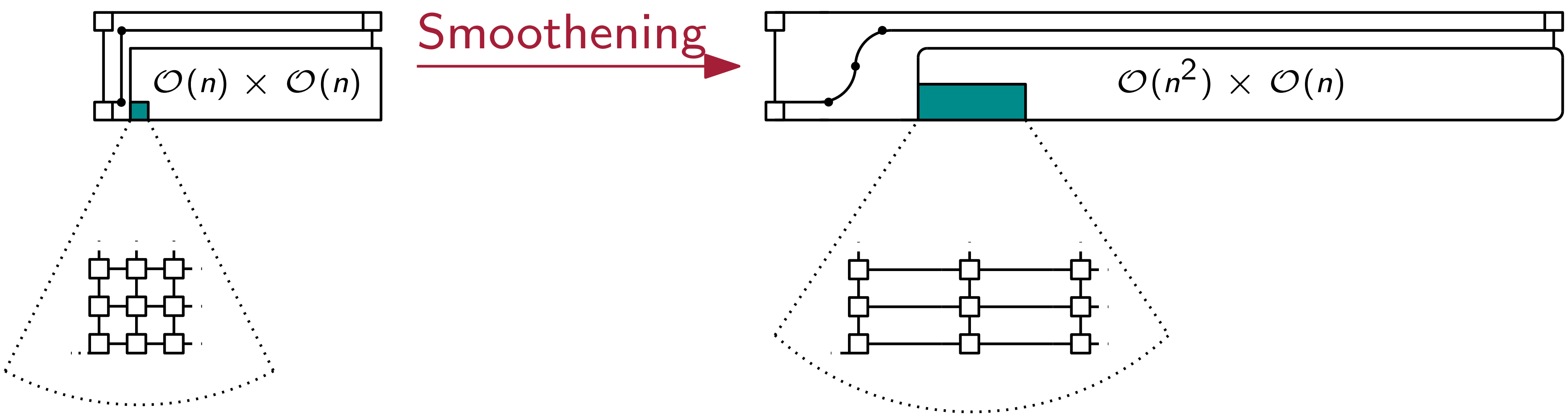

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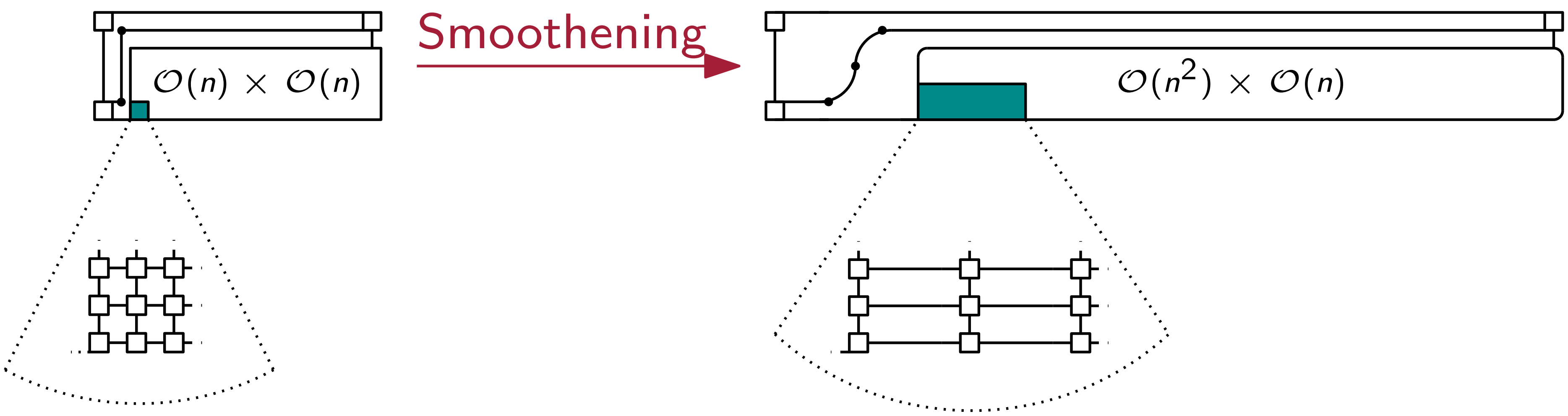

- Consider the complexity increase from three to four

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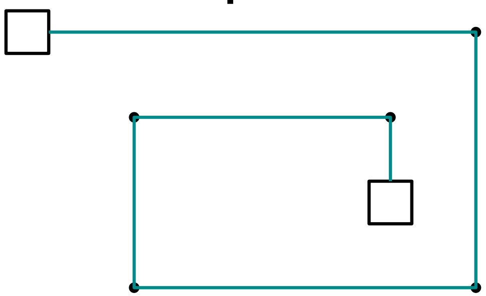

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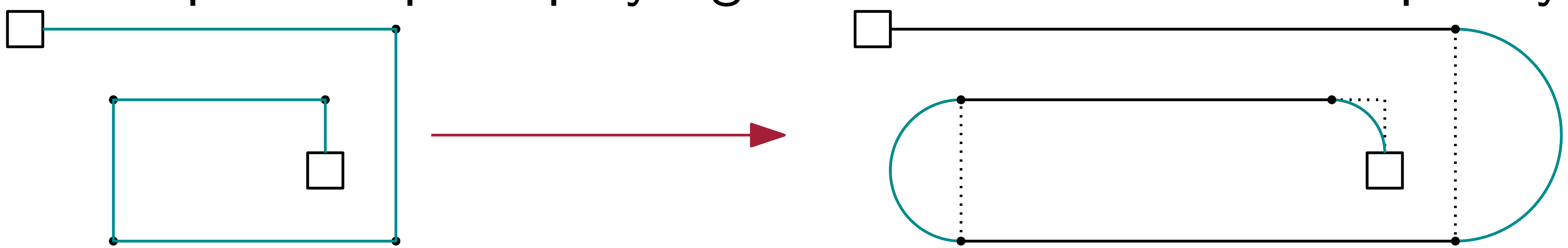

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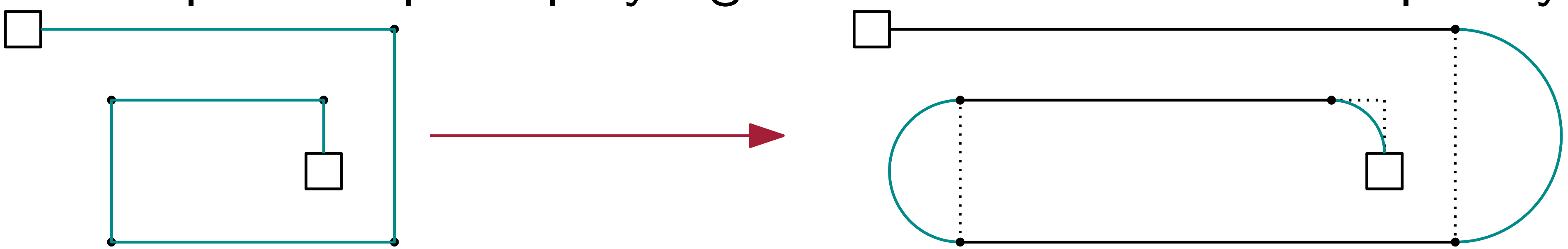
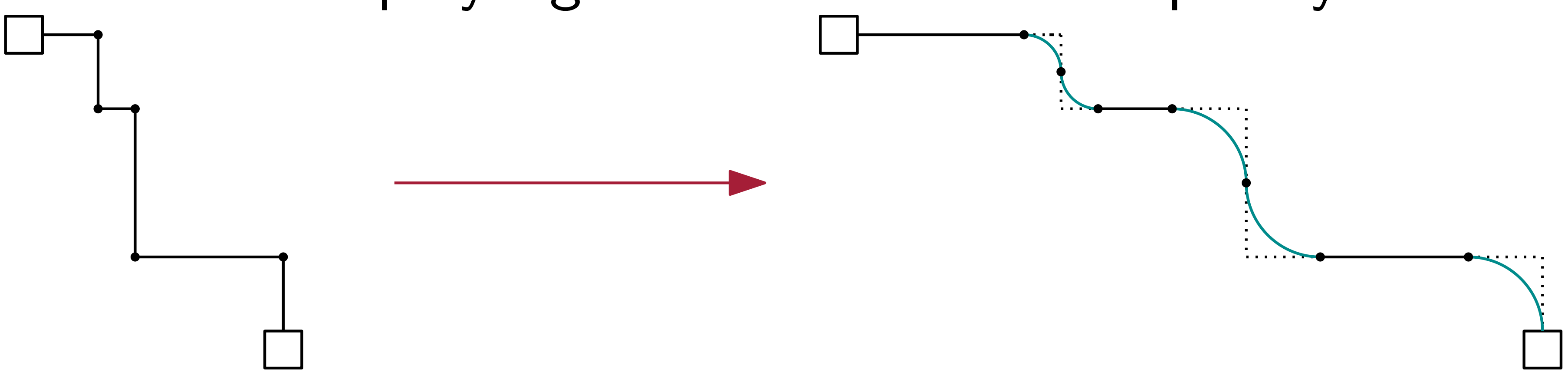


  - “staircase“ polyedges **do** increase in complexity



Defining “good“ and “bad“ parts

- Orthogonal polyedge e given
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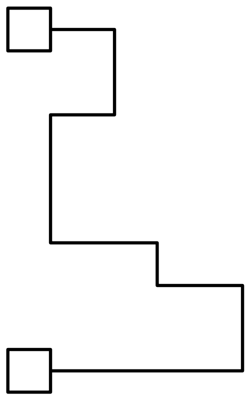

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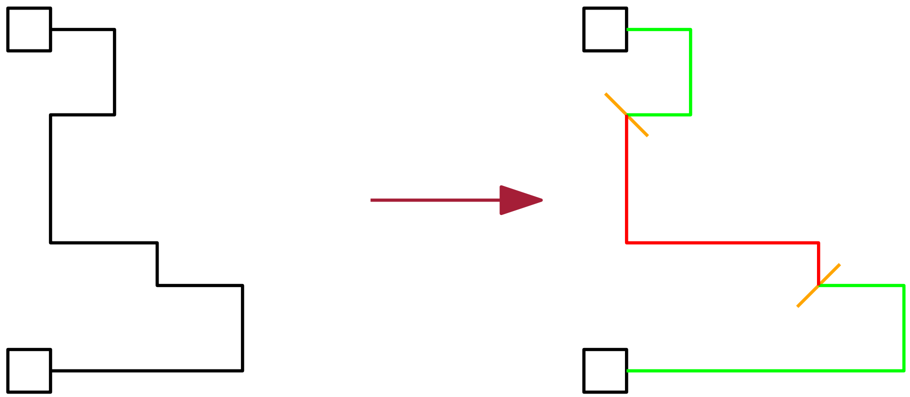

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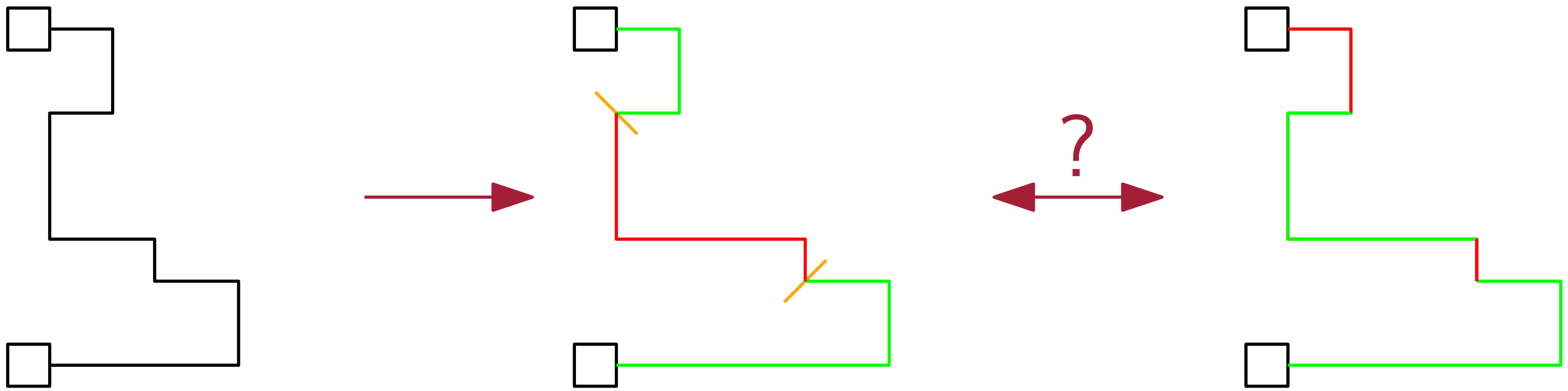

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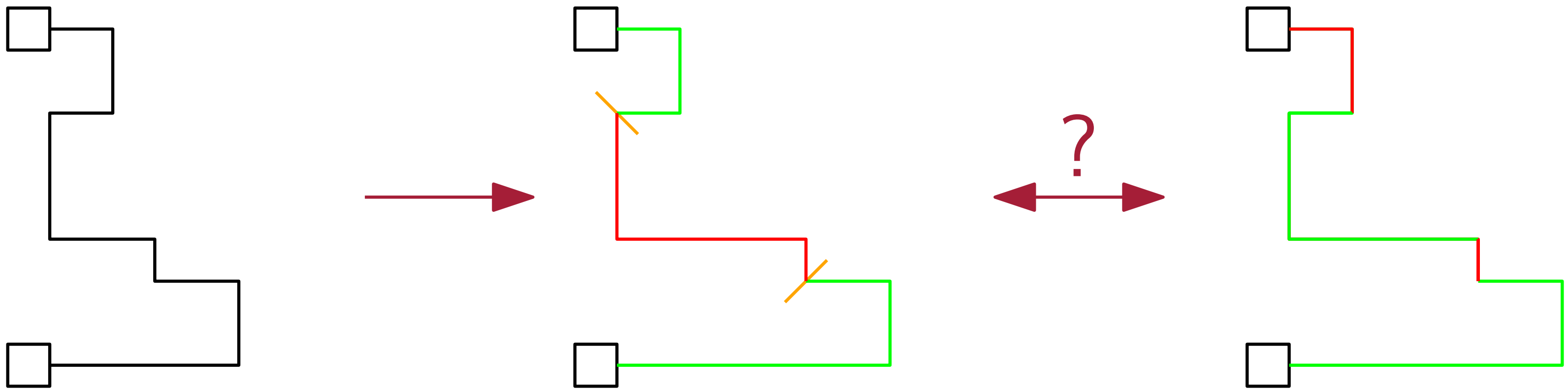
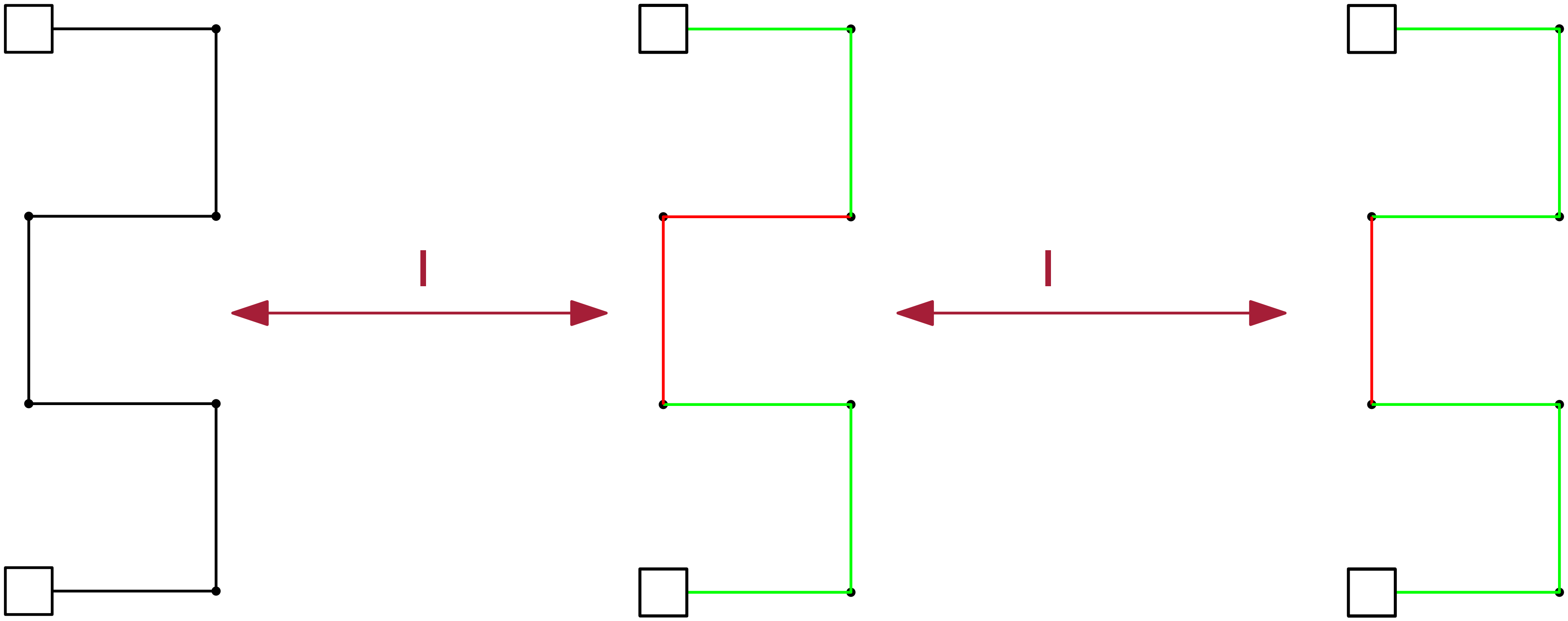


    - Good part $\triangleq$ spiral shapes $\triangleq$ uniform fragment
    - Bad part $\triangleq$ zig-zag shapes $\triangleq$ alternating fragment
  - What fragmentation is the “most accurate“?
  - How can we utilize the partitioning for complexity investigaion?

Fragmentation properties

- Multiple valid fragmentations can describe one polyedge

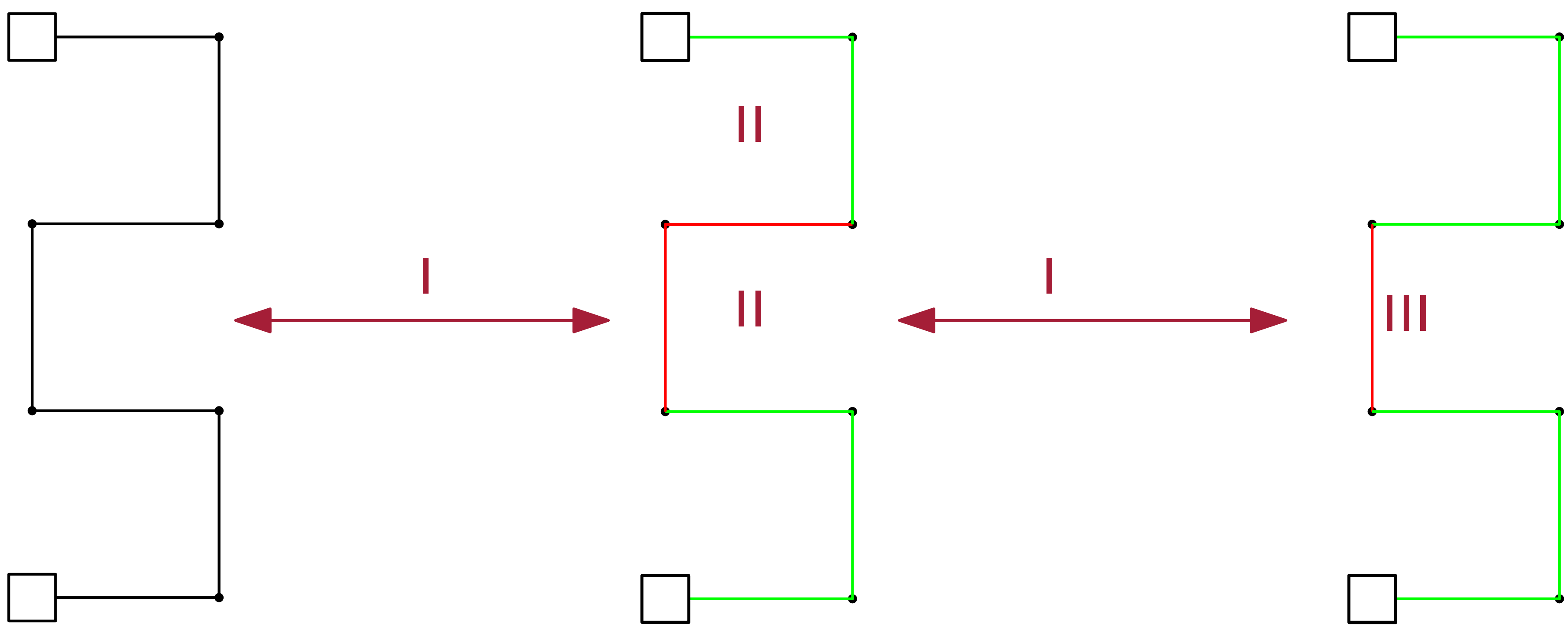


Fragmentation properties

I Multiple valid fragmentations can describe one polyedge

II Fragments of length up to 2 are both uniform *and* alternating

III Two fragments are *incompatible*
$\Leftrightarrow$ they are not further mergable



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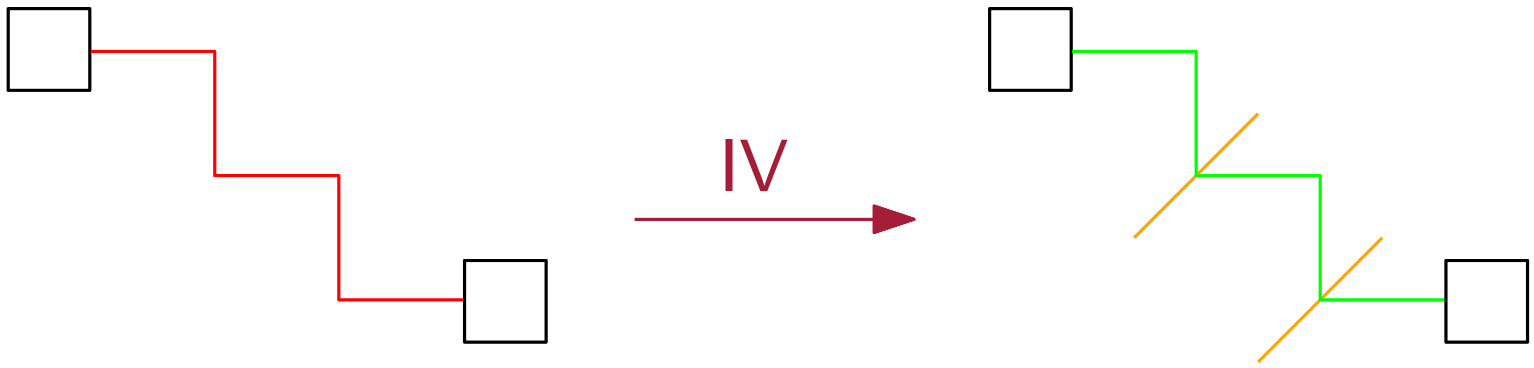

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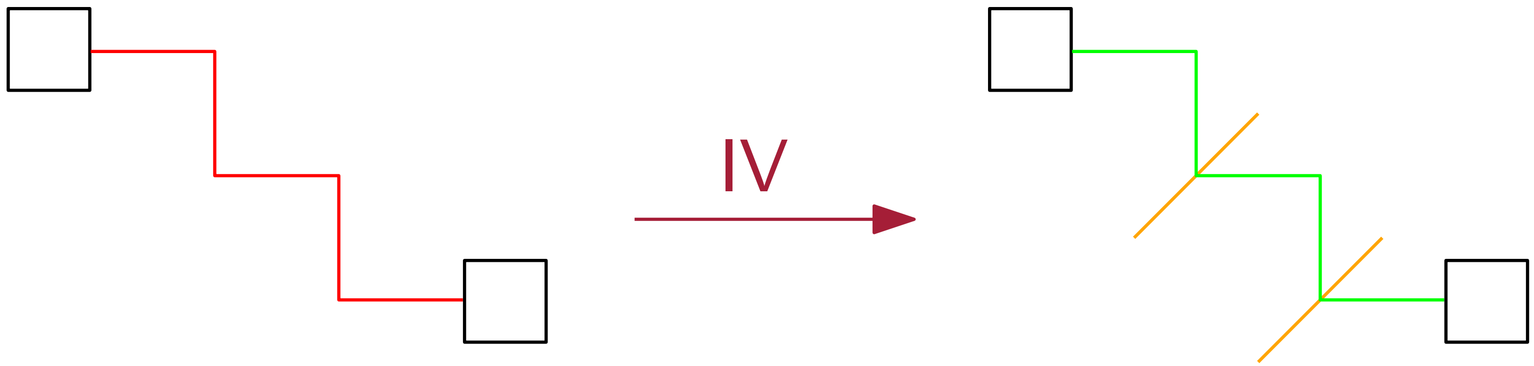
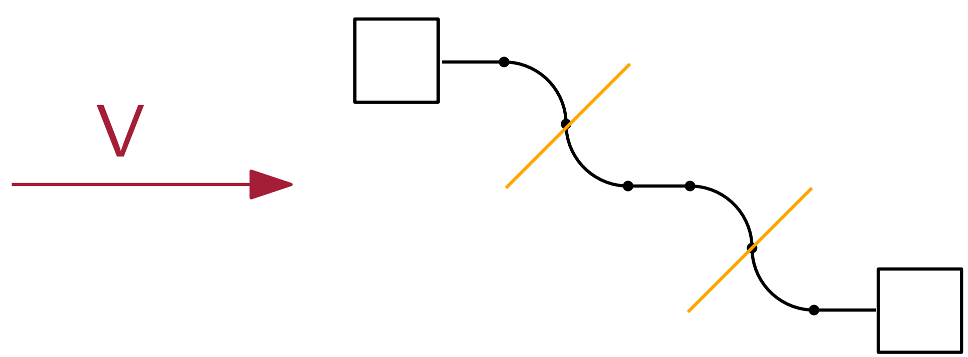


V Incompatible fragments increase the complexity by 1 in the smoothened drawing



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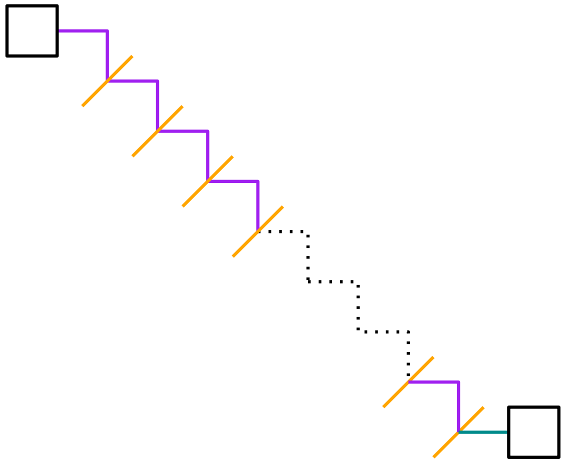
- Uniform-only fragmentations describe “smoothening behaviour“ accurately, linear runtime algorithm
- $k \triangleq$ complexity of input orthogonal polyedge
  - $1 \leq$ fragmentation length $\leq \lceil \frac{k}{2} \rceil$
  - worst case: staircase polyedge
    - “chopped“into fragments of length at most 2
    - k odd:
      - fragmentation length: $\lceil \frac{k}{2} \rceil$



Complexity Investigation

- Uniform-only fragmentations describe “smoothening behaviour“ accurately, linear runtime algorithm
- $k \triangleq$ complexity of input orthogonal polyedge
  - $1 \leq$ fragmentation length $\leq \left\lceil \frac{k}{2} \right\rceil$
  - worst case: staircase polyedge
    - “chopped“into fragments of length at most 2
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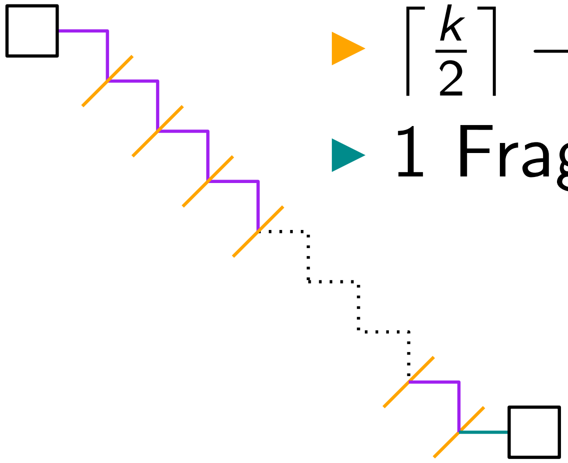

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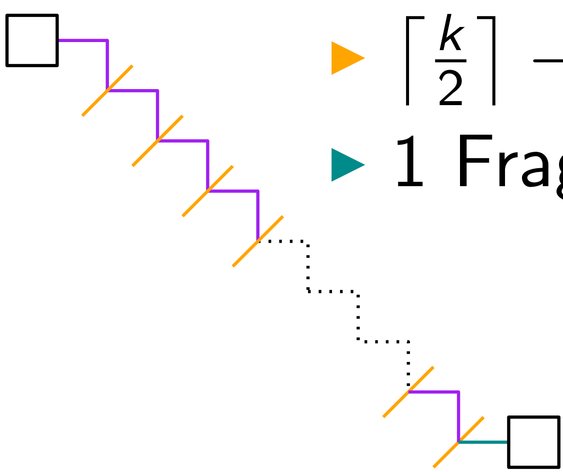

- $\left(\sum_{i=1}^{\left\lceil \frac{k}{2} \right\rceil - 1} 2 \right) + 1 + \left\lceil \frac{k}{2} \right\rceil - 1 = \left\lceil \frac{3k}{2} \right\rceil - 1$

(complexity upper bound)

What about Kandinsky drawings?

- Upper bound holds for low complexity polyedges
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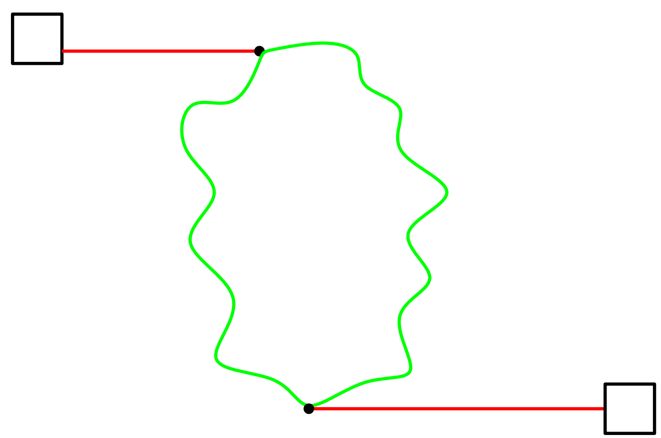
- Large polyedges in Kandinsky may inherit two bends which are not deducable
- Polyedge spiral-shaped in-between those bends



- In this case: complexity increase from k to $k+2$

Overall results

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  - worst case area consumption: quadratic in width size
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  - Edge complexity increase:

Original Kandinsky k	Smoothened Kandinsky k'
≤ 5	$\lceil \frac{3k}{2} \rceil - 1$
≥ 5	$k + 2$

Saving measures - Complexity

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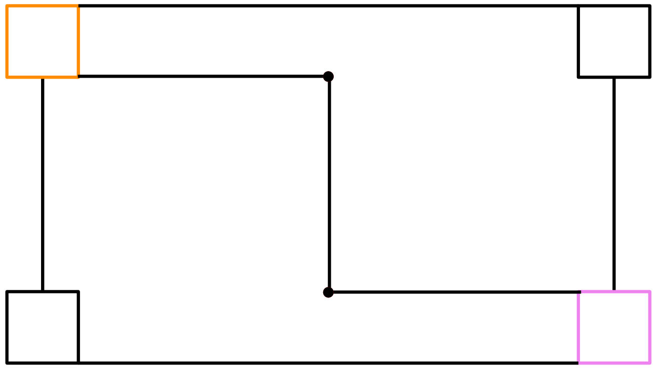
  - May reduce the edge complexity by one



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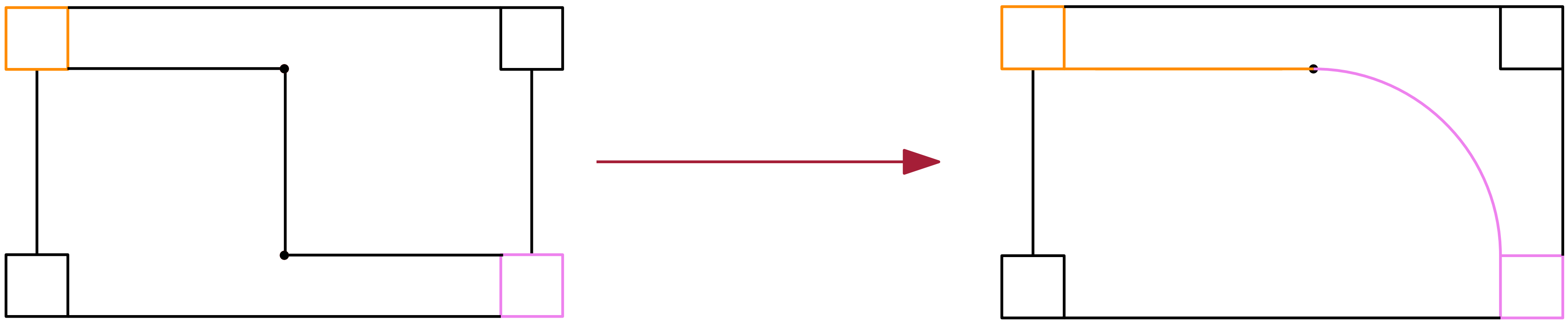

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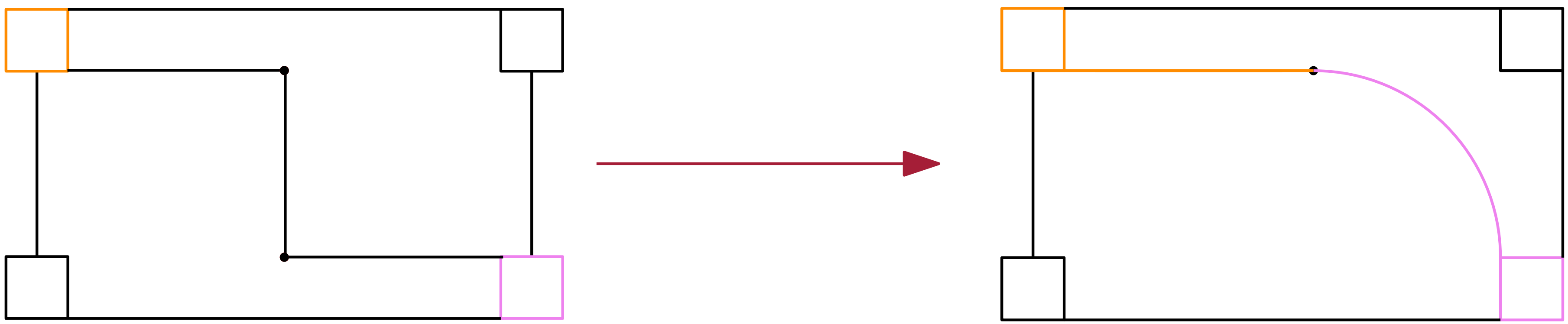

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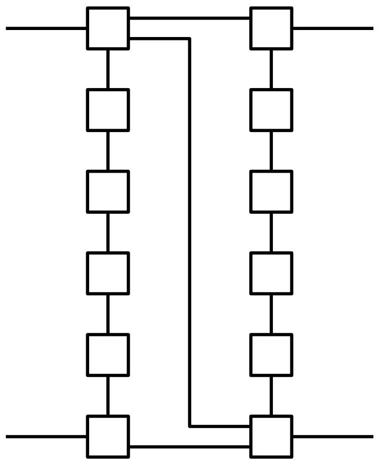

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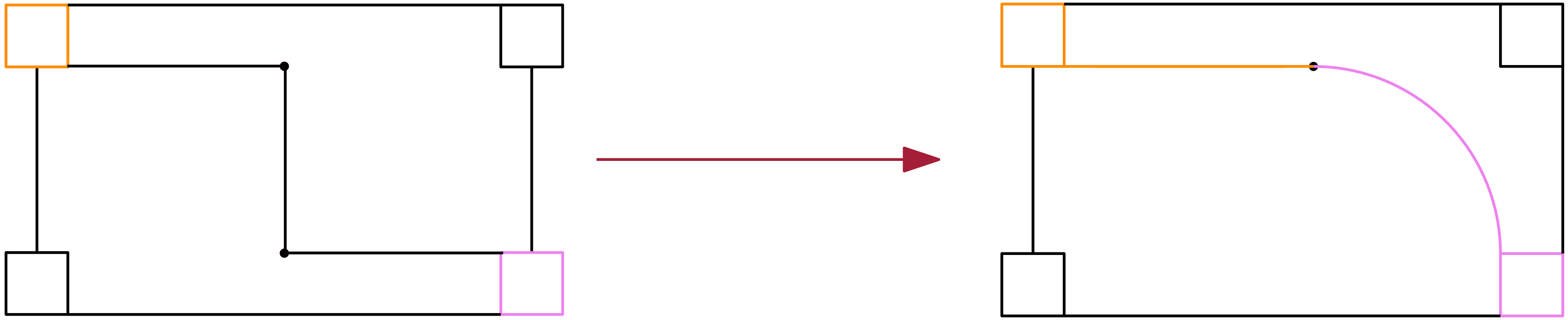

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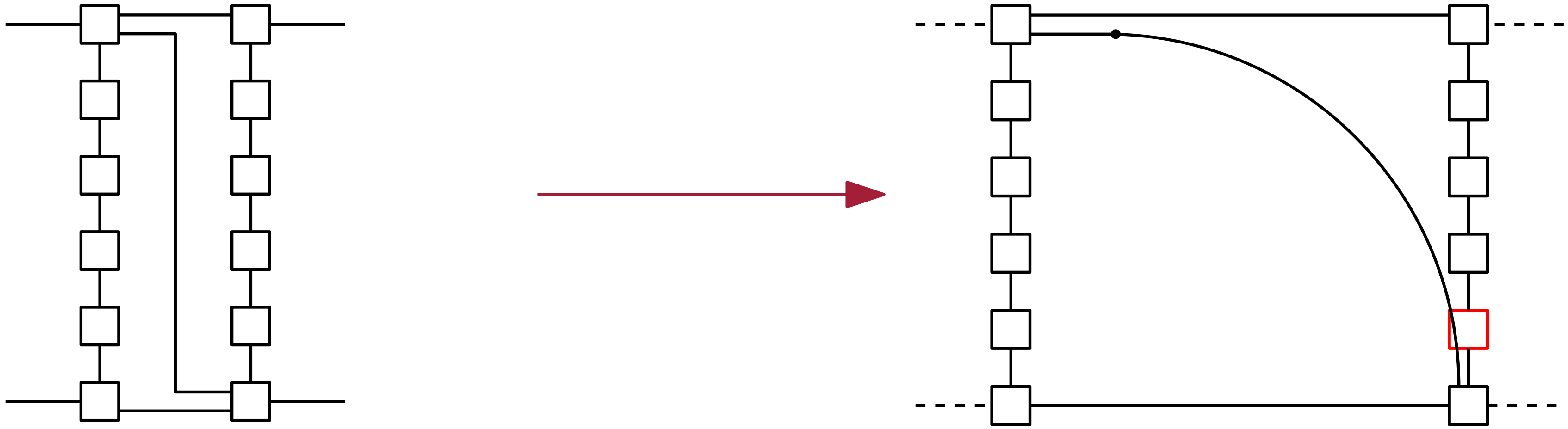

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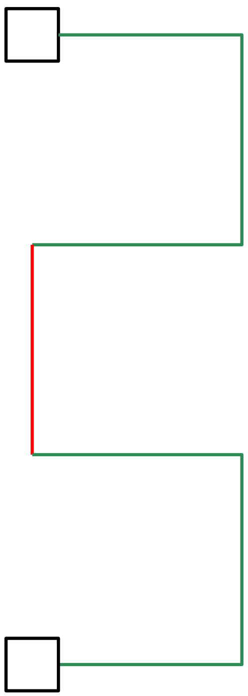

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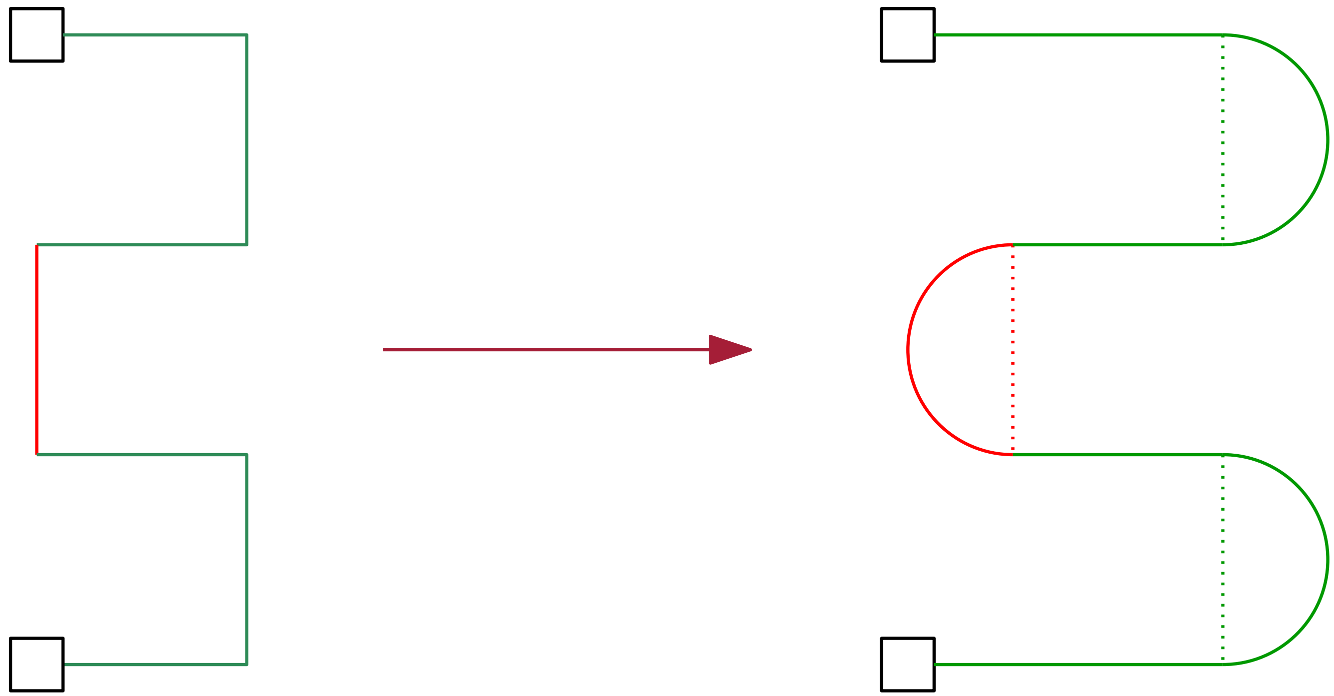

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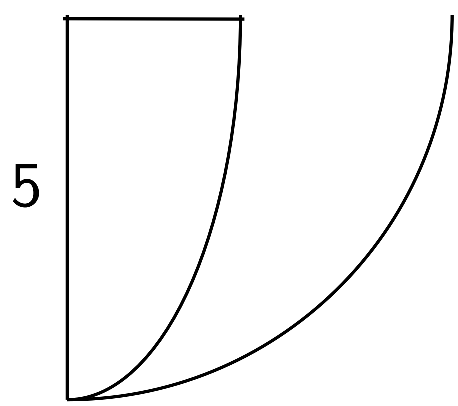

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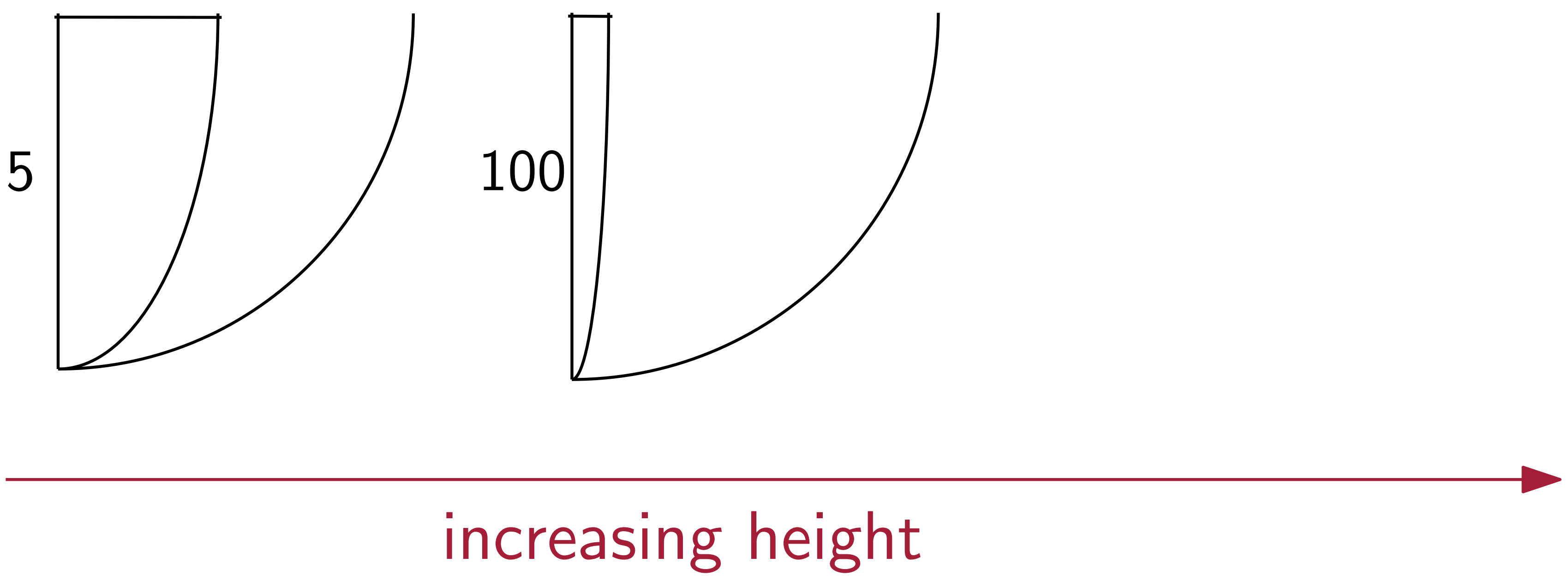

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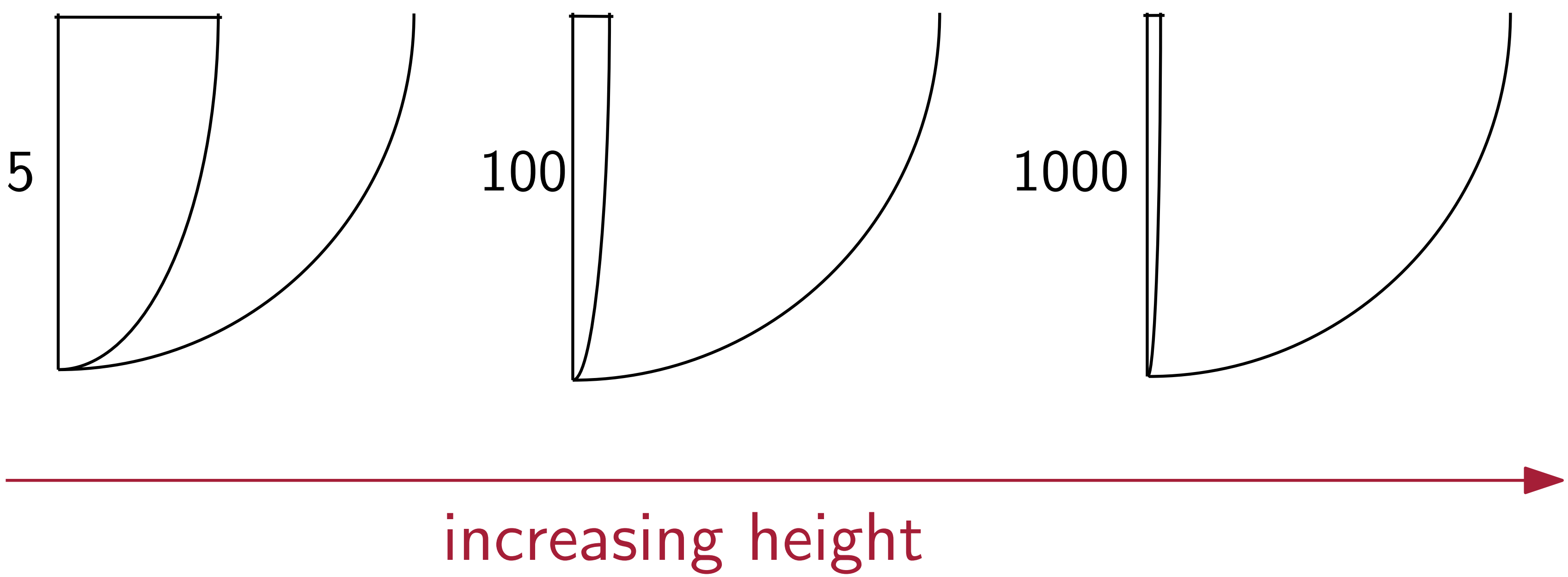

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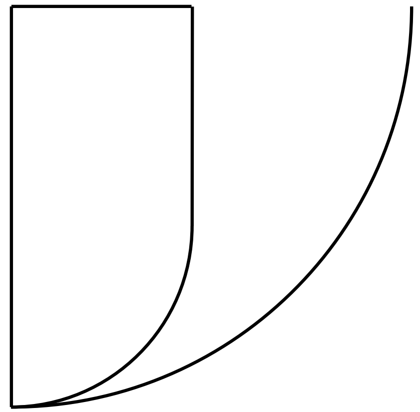

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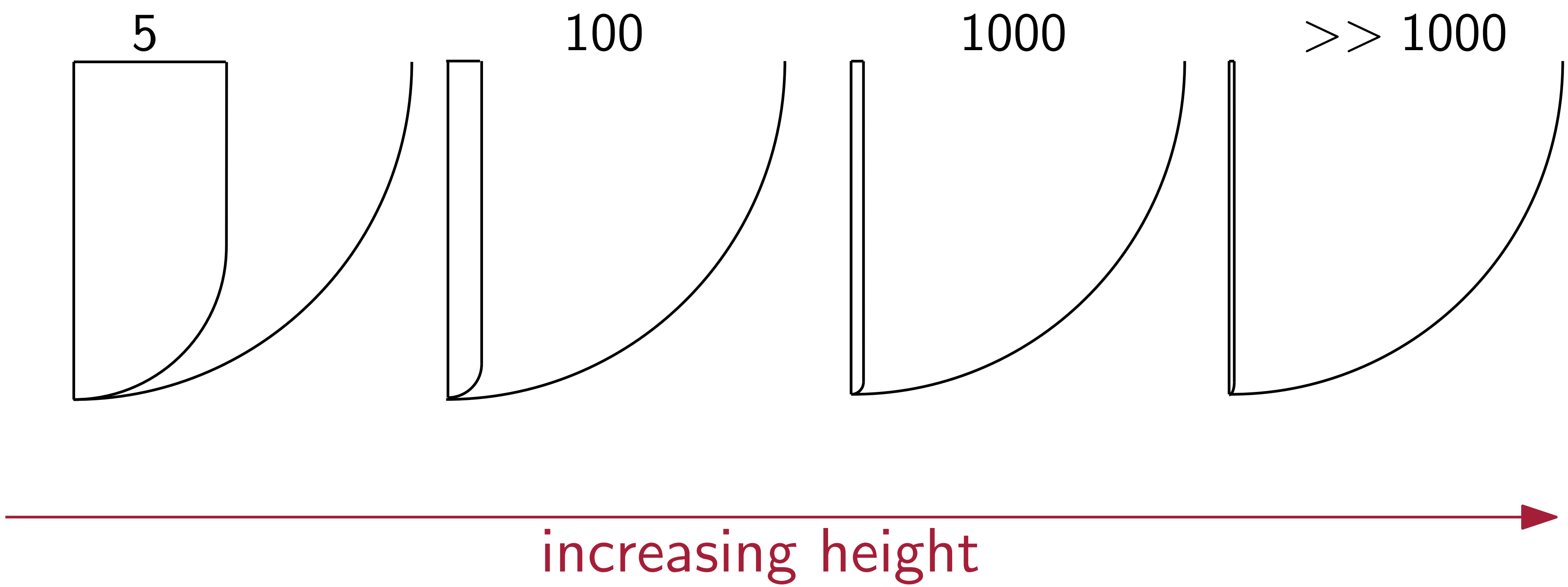

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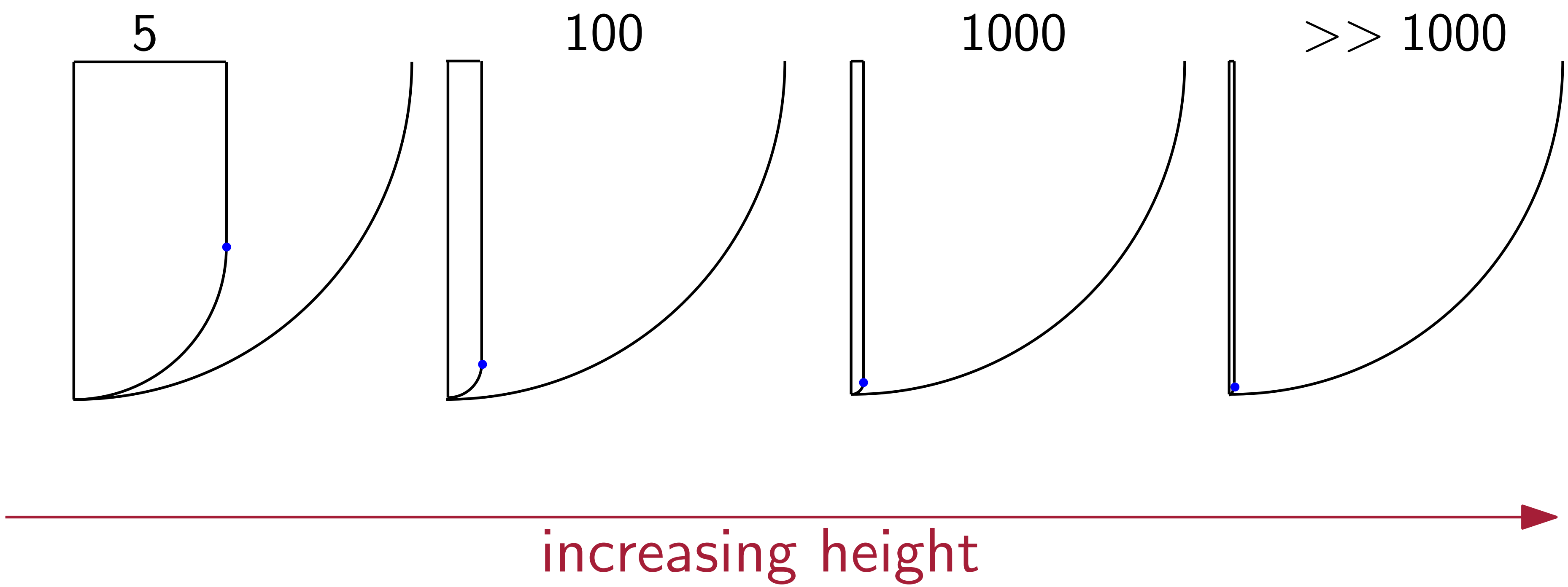
      - Higher readability, but high complexity increase



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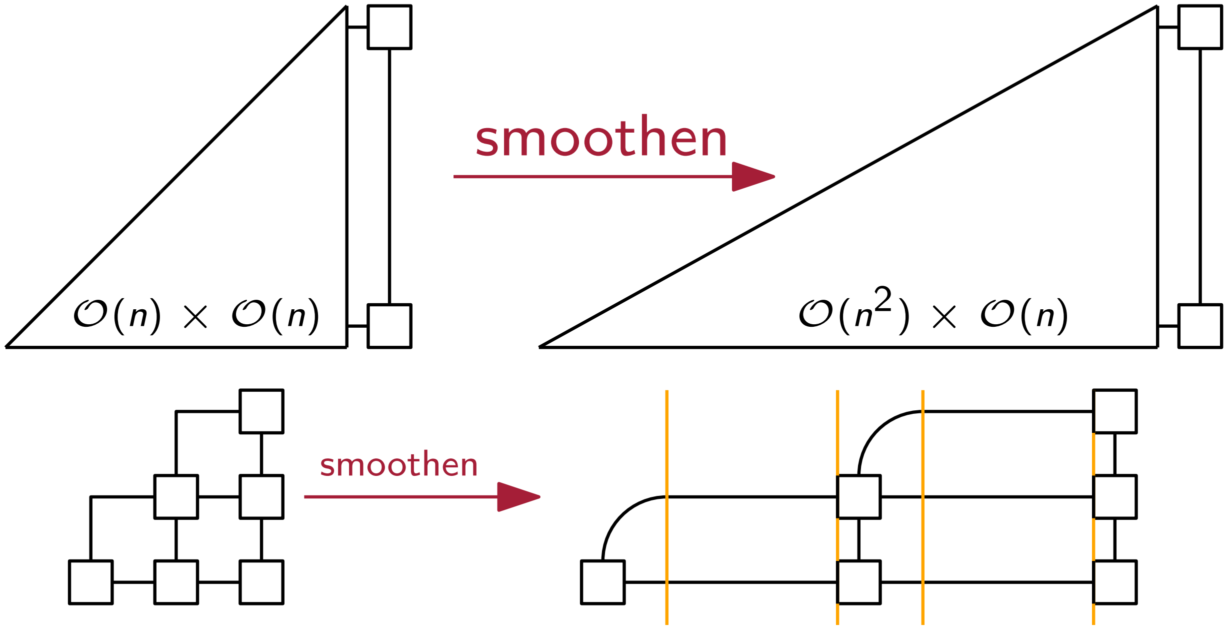

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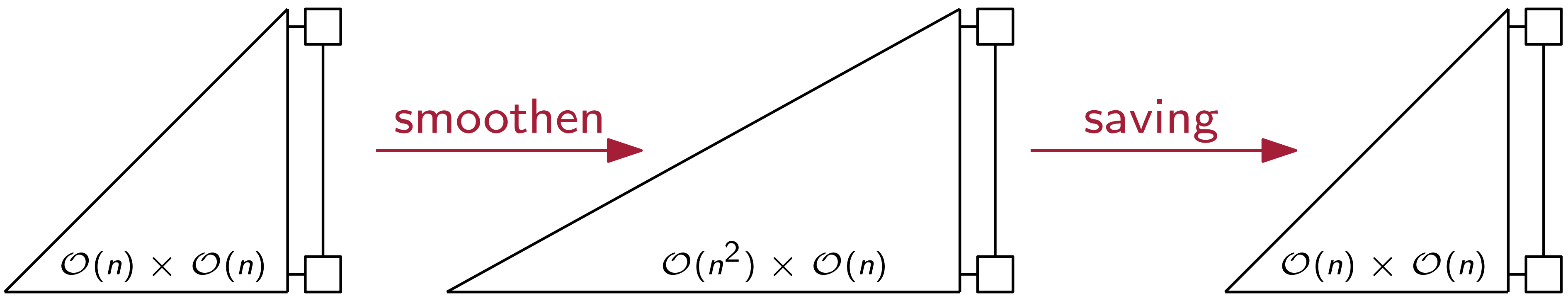

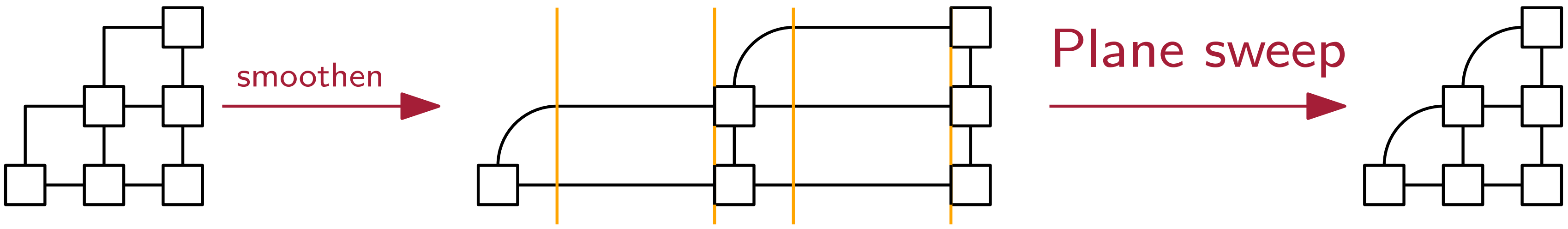

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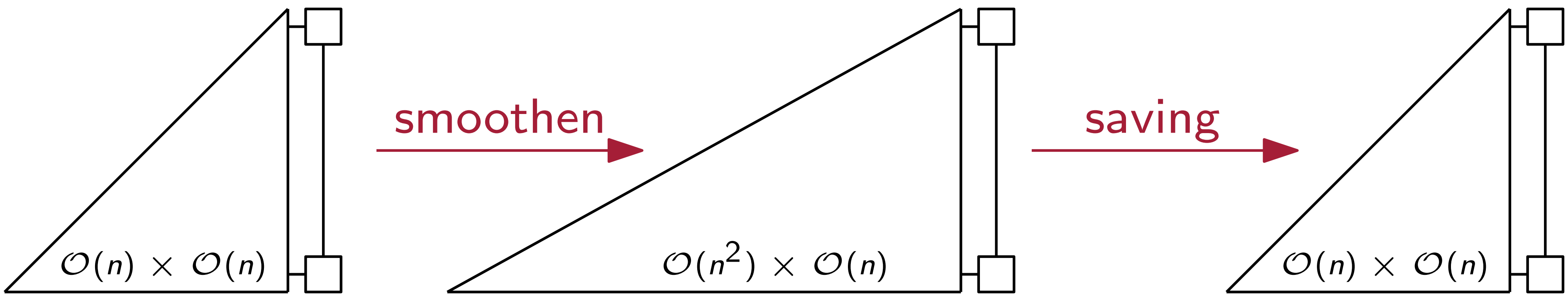
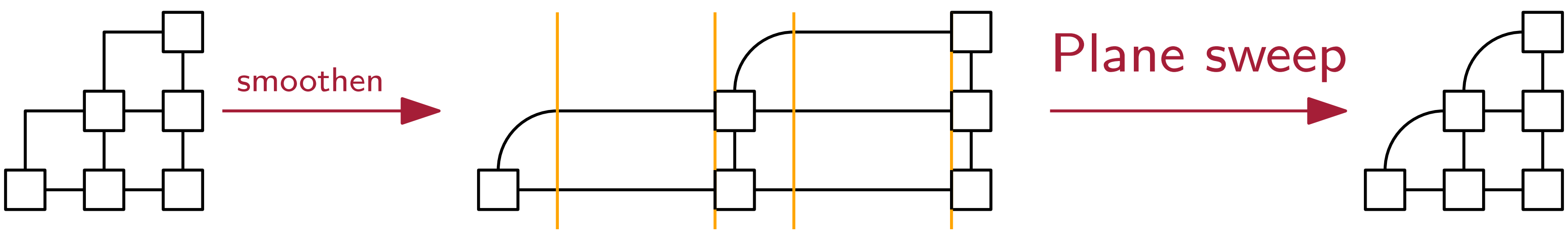
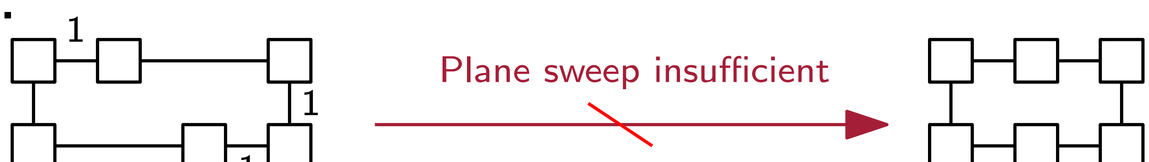




- But...



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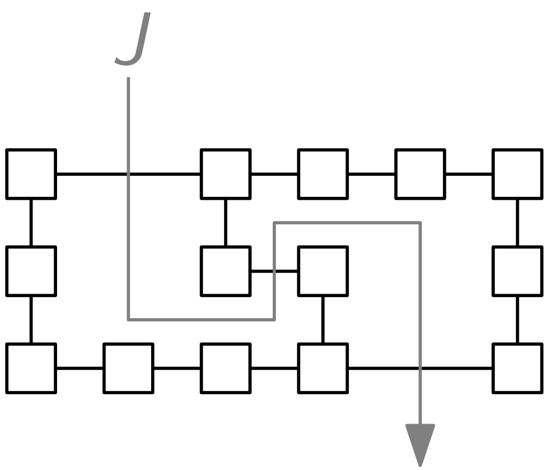

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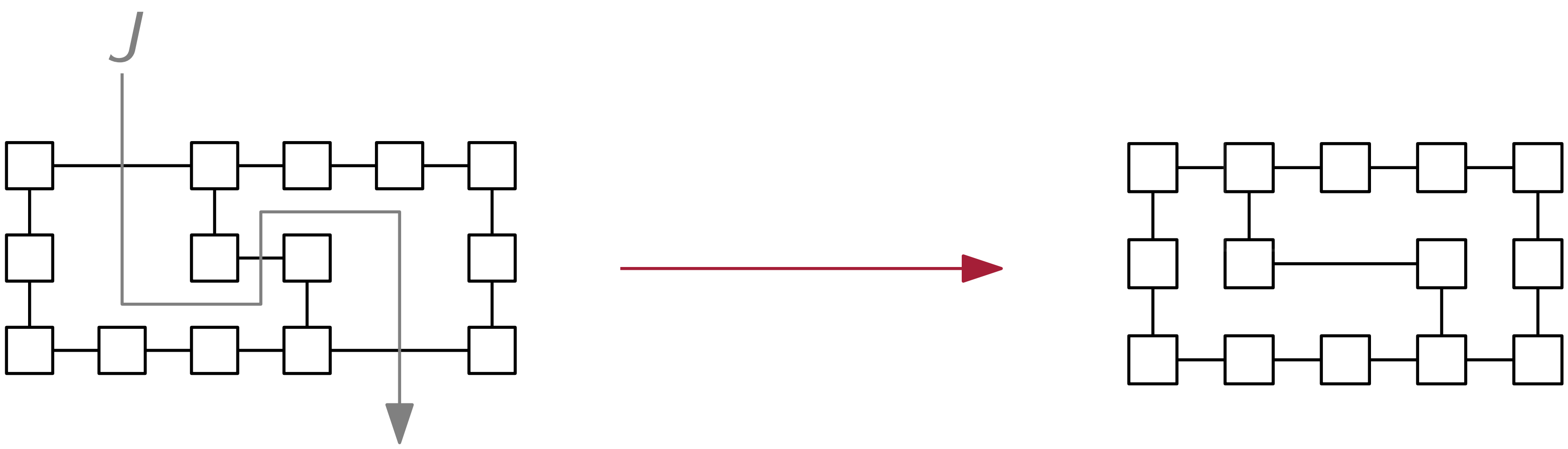

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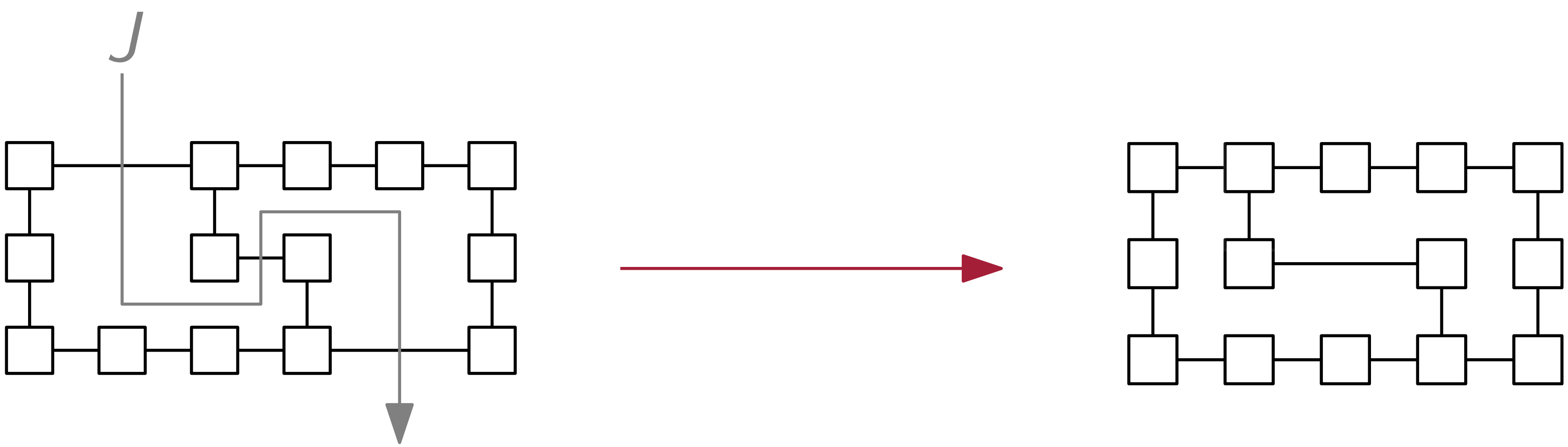

- Can save area either horizontally or vertically
- Can be modified for Smoothened Kandinsky drawings

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4M Algorithm - Moving modification

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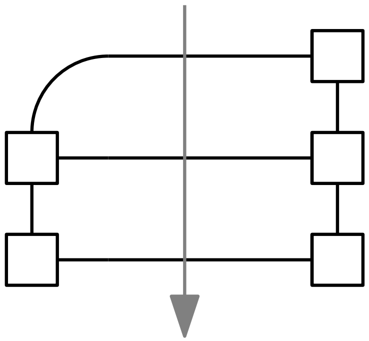

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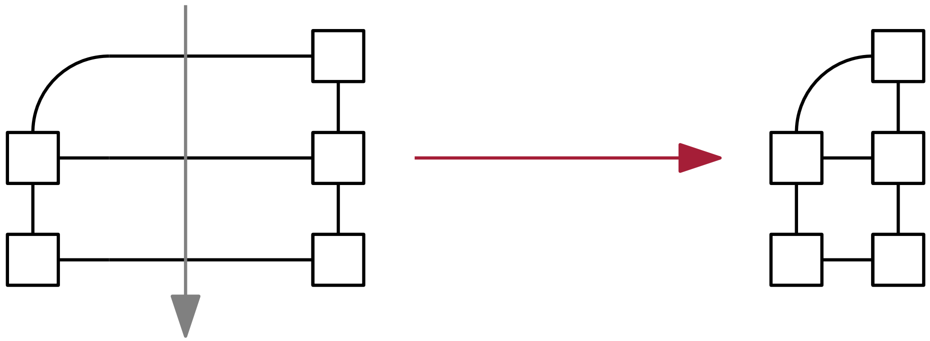

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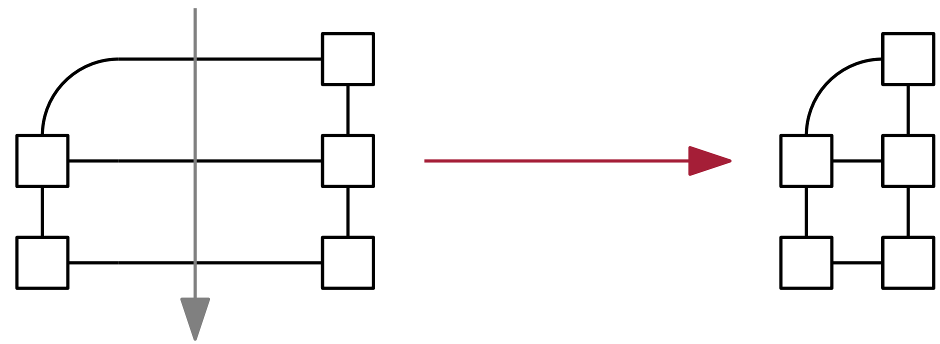

  - Else cross circular arc
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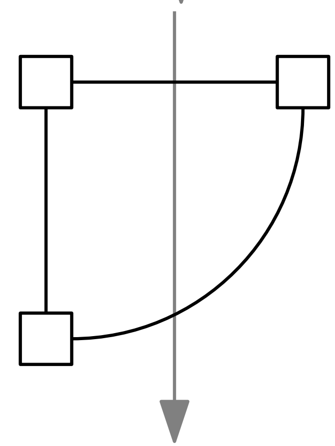

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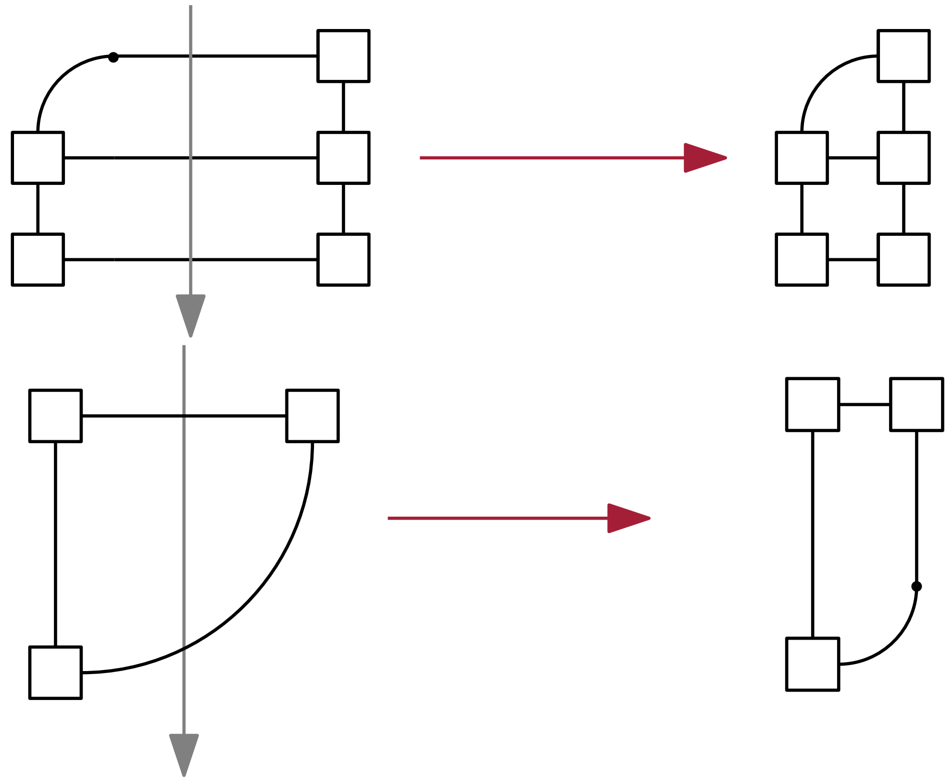
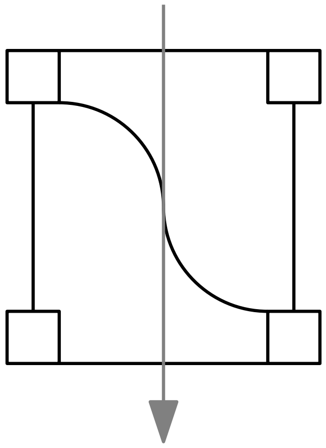


  - Quarter circular arcs substituted
    - downwards: smaller circular arc, line segment
    - upwards: same-sized circular arc, line segment

4M - Example

- Horizontal saving...



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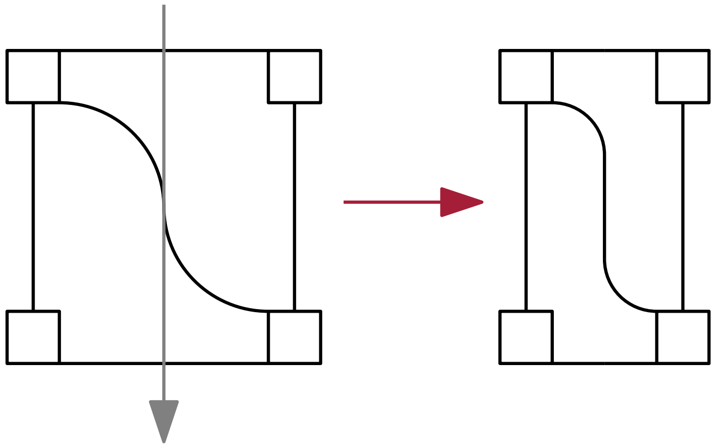

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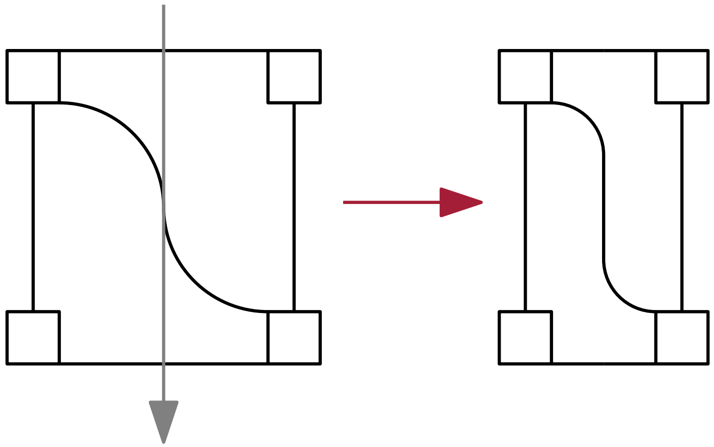
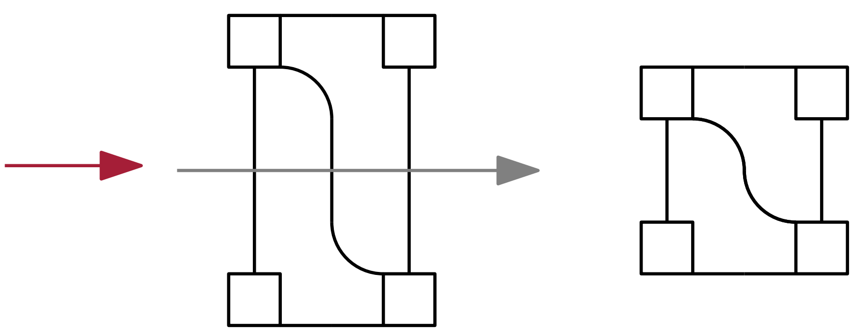


- Followed by vertical saving



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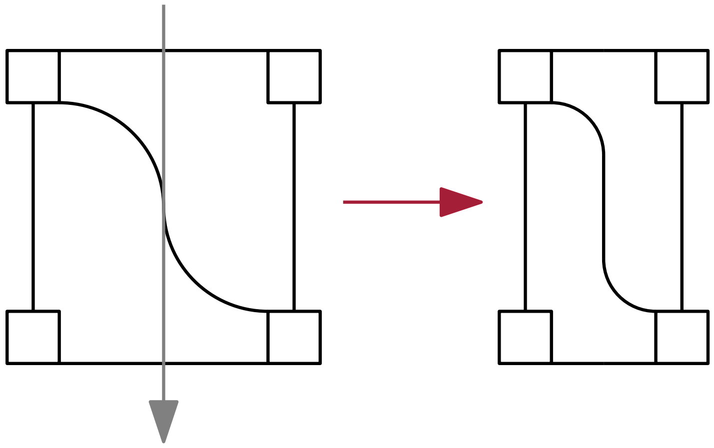

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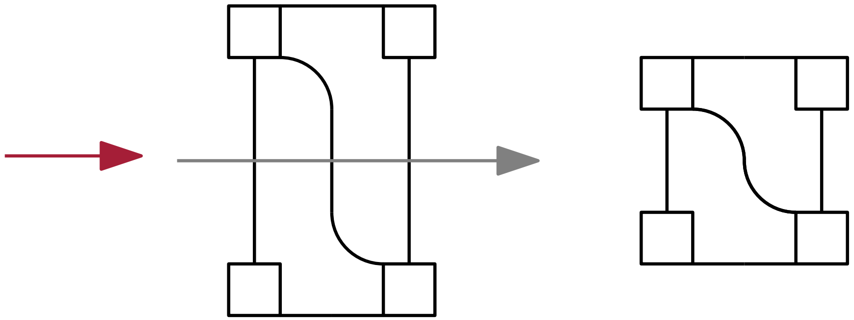
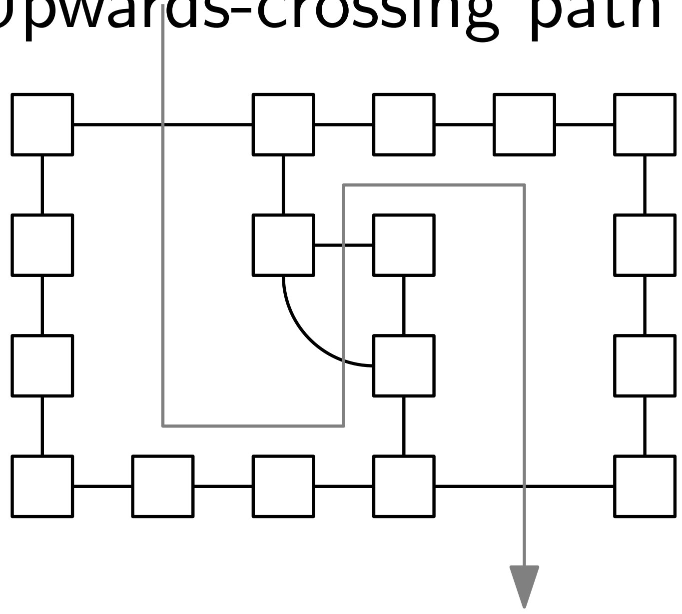


- Upwards-crossing path



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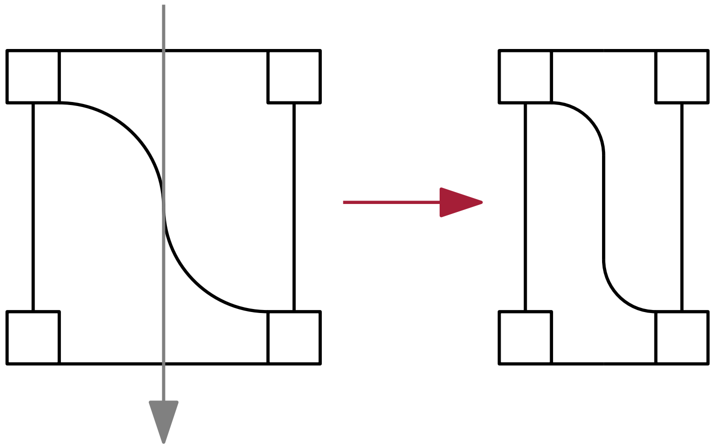

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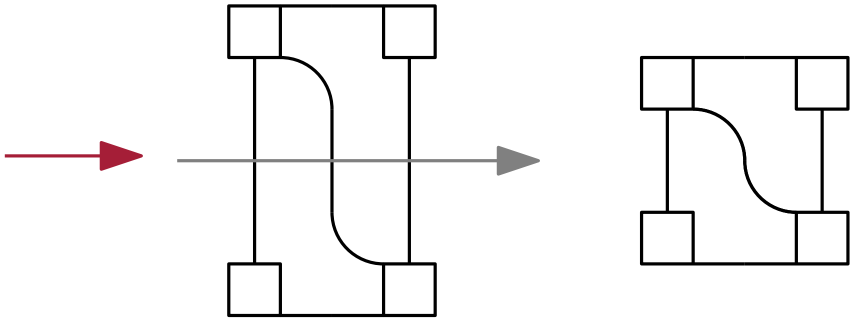

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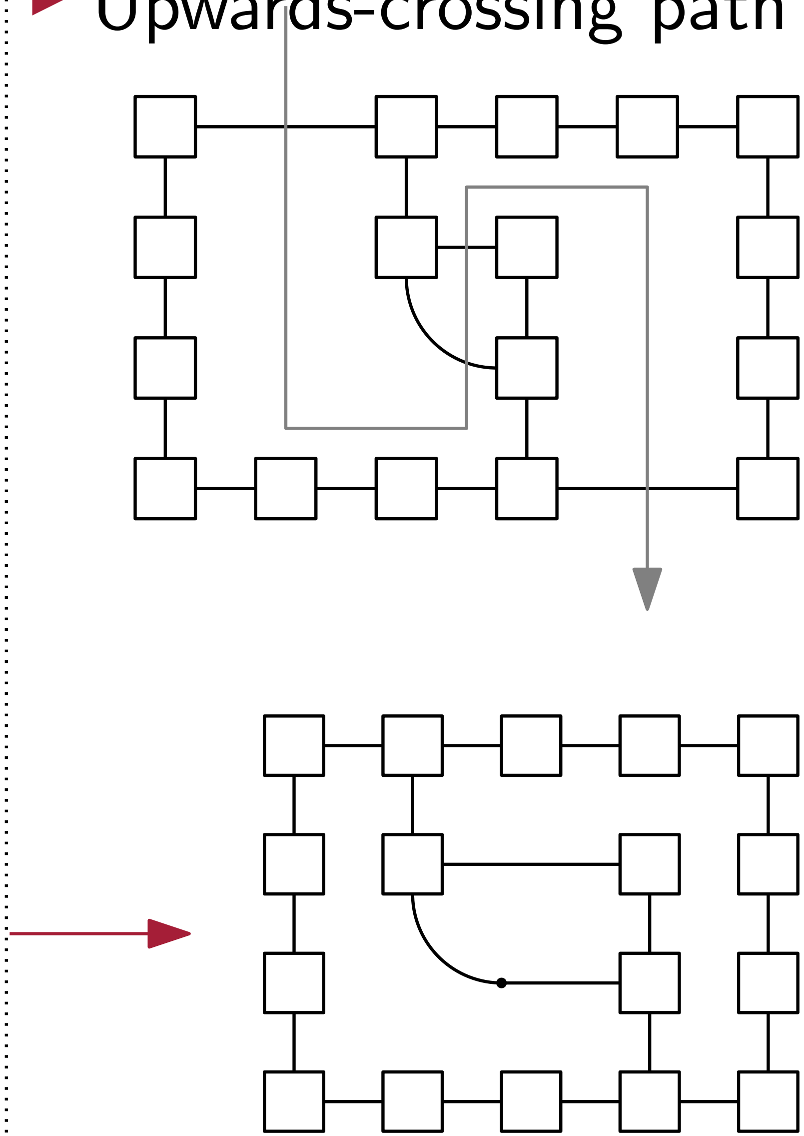

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- Port reassignment may decrease complexity
  - hard to determine, whether possible or not

Open Problems

- **Implementation**
  - Useful for heuristics
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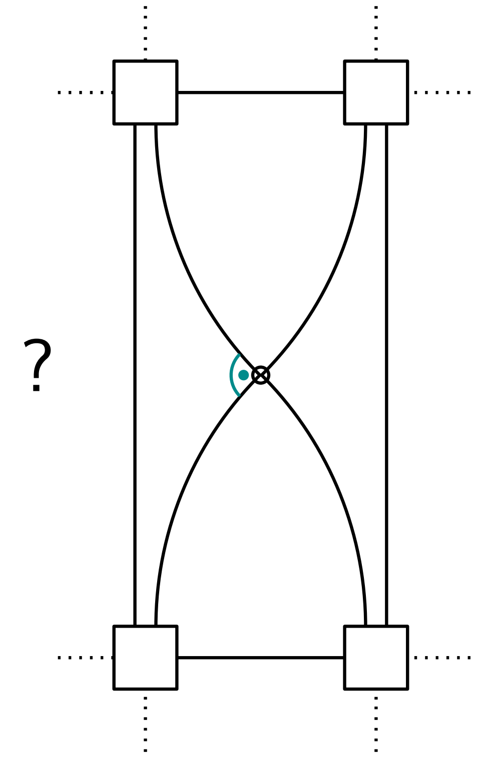

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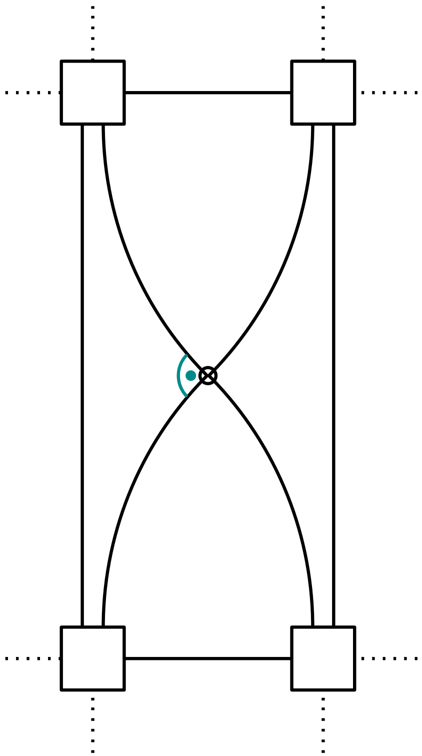

- **Further saving approaches**

Questions?

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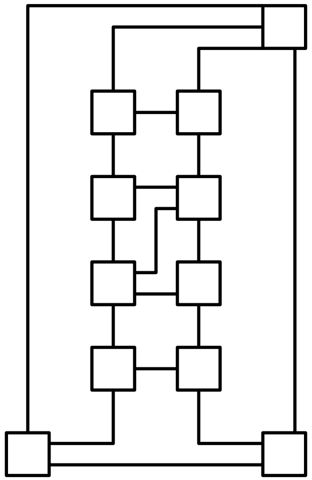
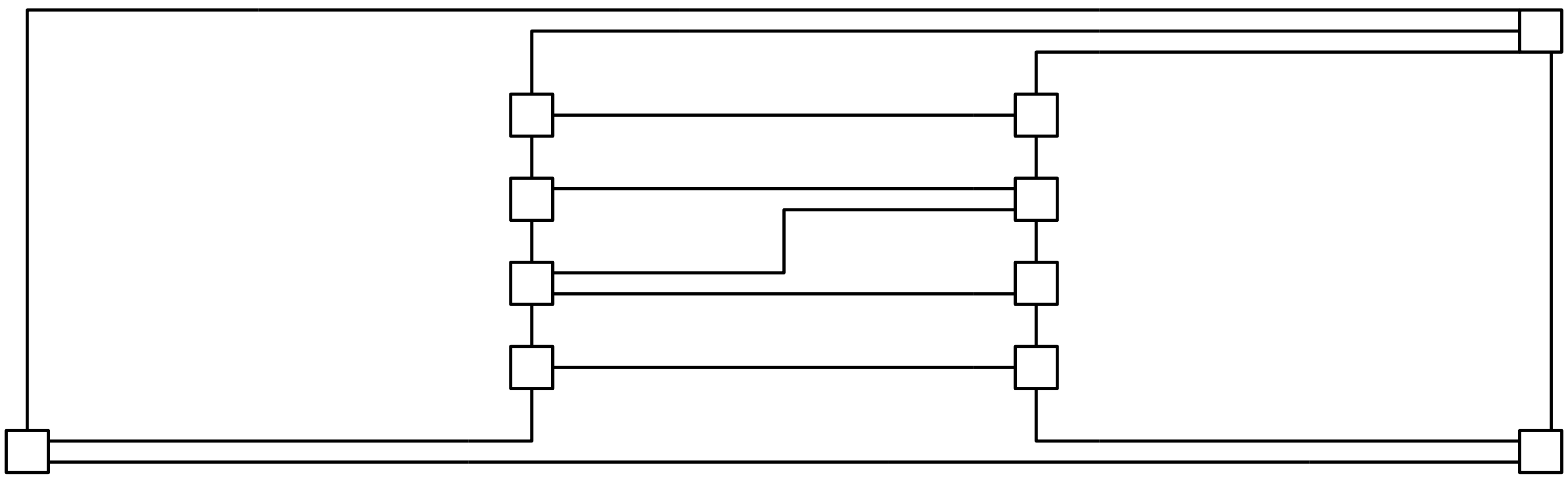
Appendix - Let's see, how it should look

- Example
  - Input drawing



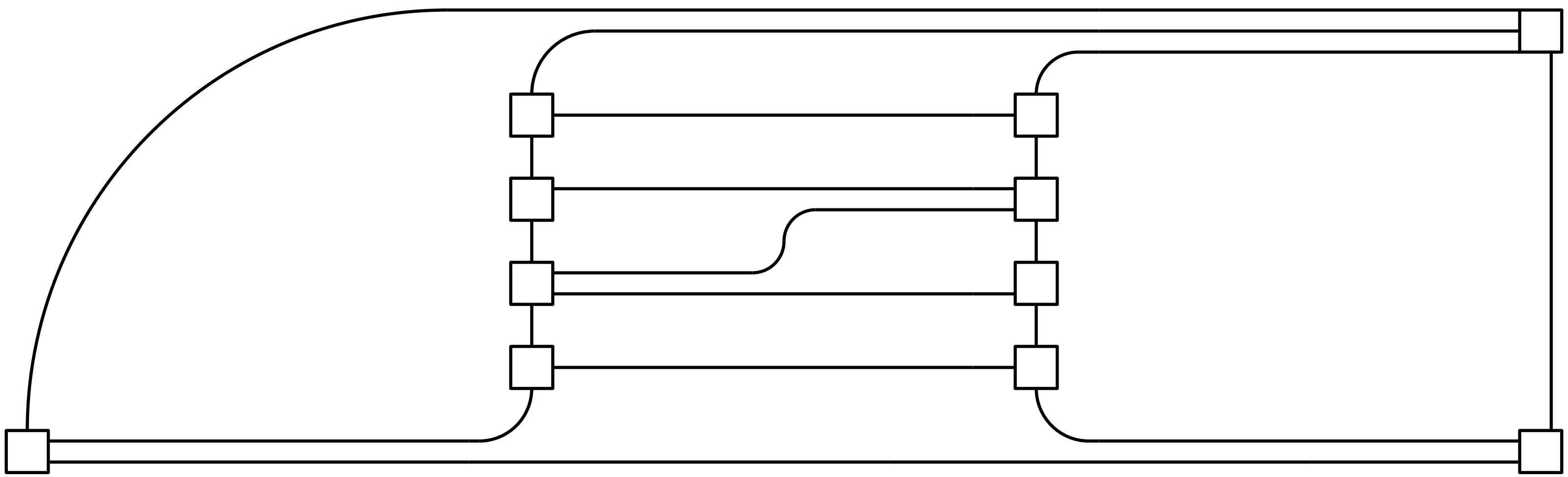
Appendix

- Example
  - Stretched by the longest vertical segment



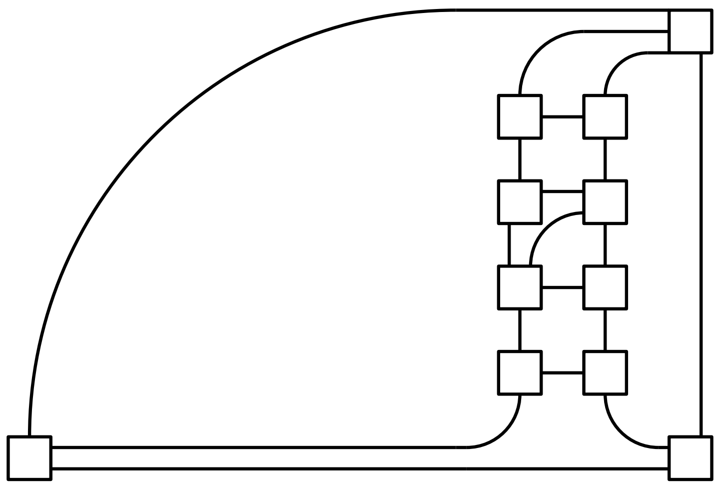
Appendix

- Example
  - Substituted



Appendix

- Example
  - Optimized



Appendix

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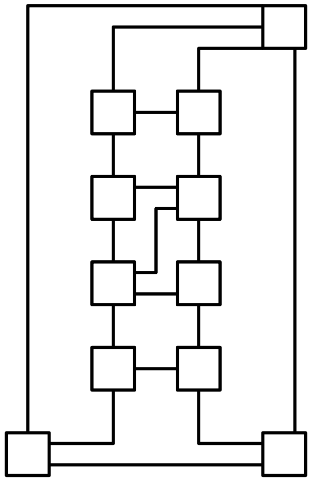
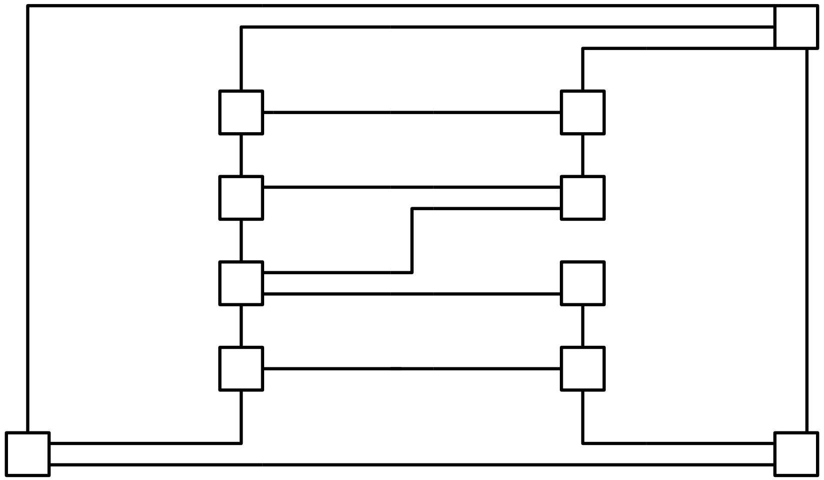


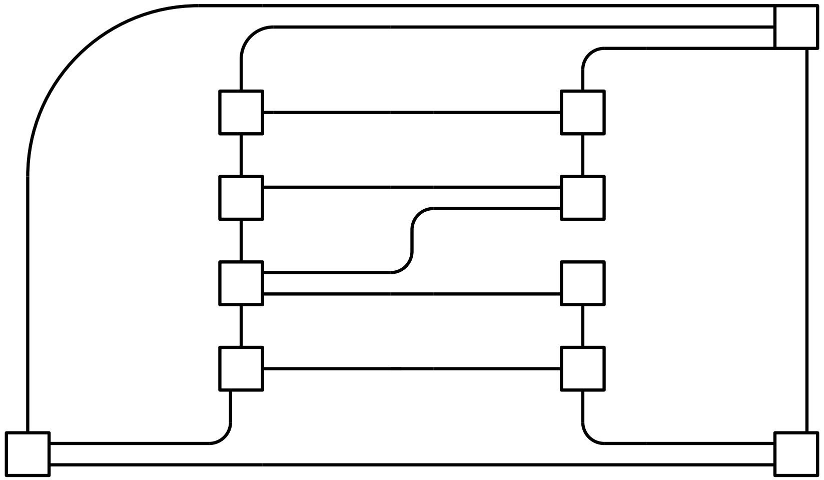
Appendix

- Example
  - Stretched by the sqrt of longest vertical segment



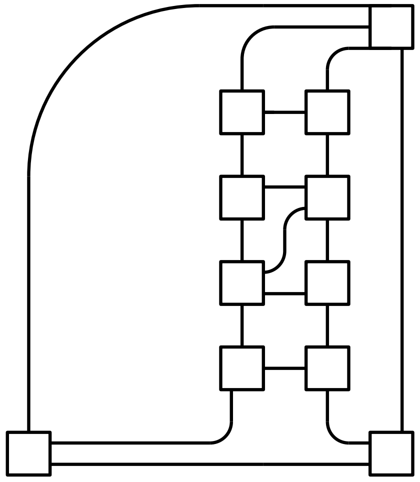
Appendix

- Example
  - Substitution of circular arcs with vertical segments



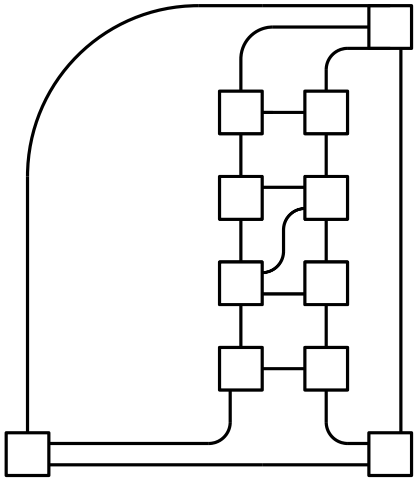
Appendix

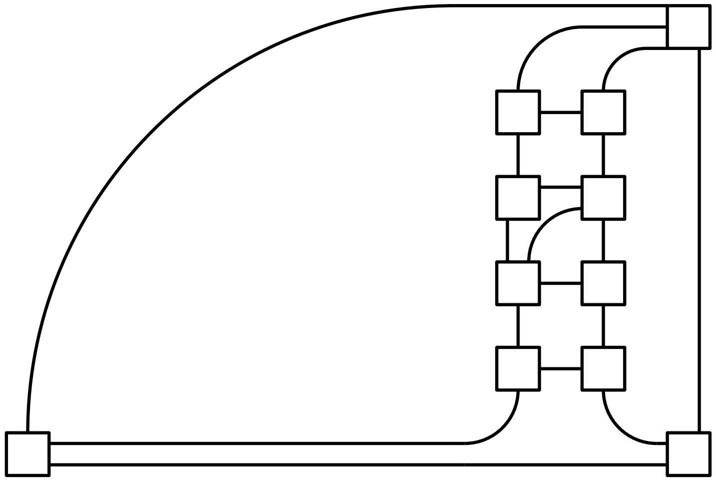
- Example
  - Optimization



Appendix

- Example
  - Comparism





Thank you!

:)