The goal is to have our usual parabola function which takes tightness, phaseShift, and verticalShift but also have another function which can generate a wave by providing a period and peak height.

$$y = a(x-h)^2 + k$$

$$a\ is\ known\ as\ 'tightness'$$

$$h\ is\ known\ as\ 'phaseShift'$$

$$k\ is\ known\ as\ 'verticalShift'$$

Given period p and peak q, we want to calculate a, h, k with assumptions that we are passing through origin and are peaking at $\frac{p}{2}$. k is the vertical shift, which is also the peak height for parabolas. We know k is q. h is the phase shift, but if we know the distance between the intercepts (p), and that one of the intercepts is at $x = 0$, then the phase shift must be half of the period, $\frac{p}{2}$. We know h. We only need to find a

$$Convert\ to\ standard\ form$$

$$y = a(x^2 + h^2 - 2hx) + k$$

$$y = ax^2 - 2ahx + k + ah^2$$

$$This\ is\ in\ the\ form$$

$$y = ax^2 + bx + c$$

$$\therefore$$

$$a = a$$

$$b = -2ah$$

$$c = k + ah^2$$

First assumption: We pass through the origin, meaning one of the roots of this quadratic equation is $x = 0$. We can solve for quadratic roots using the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$We\ are\ stating\ that\ x = 0$$

$$0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$0 = -b \pm \sqrt{b^2 - 4ac}$$

$$b = \pm\sqrt{b^2 - 4ac}$$

$$Square\ both\ sides. It\ obscures\ the \pm but\ that\ is\ ok\ as\ we\ only\ want\ one\ solution$$

$$b^2 = b^2 - 4ac$$

$$Substitute\ our\ values\ for\ a, b, c$$

$$4a^2h^2 = 4a^2h^2 - 4a(k + ah^2)$$

$$ah^2 = ah^2 - k - ah^2$$

$$ah^2 = -k$$

$$a = -\frac{k}{h^2}$$

$$a = -\frac{k}{\frac{p^2}{2^2}}$$

$$a = -\frac{4k}{p^2}$$

$$\therefore$$

$$Given\ period\ p, peak\ q$$

$$a = -\frac{4q}{p^2}, \qquad h = \frac{p}{2}, \qquad k = q$$