subfiles

Iowa State University

COM S 311L
Homework 1

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1 Problem 1

a

$$f(n) = 2n + 300$$
$$g(n) = 100n + 2$$

Using rules of limits of equations:

$$\begin{aligned}\lim_{n\to\infty}\frac{f(n)}{g(n)} &\equiv \lim_{n\to\infty}\frac{2n+300}{100n+2}\\ &\equiv \lim_{n\to\infty}\frac{2n}{100n}\\ &= \frac{2}{100} = \frac{1}{50}\end{aligned}$$

Since $\lim_{n\to\infty}\frac{f(n)}{g(n)} = C$ where $C \in \mathbb{R}$, $f(n) \in \Theta(g(n))$

b

$$f(n) = n^{\frac{3}{4}}$$
$$g(n) = n^{\frac{2}{3}}$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{n^{\frac{3}{4}}}{n^{\frac{2}{3}}} \\
&= \lim_{n\to\infty} n^{\frac{3}{4}-\frac{2}{3}} \\
&= \lim_{n\to\infty} n^{\frac{1}{12}} = 0
\end{aligned}
$$

Since $\lim_{n\to\infty}\frac{f(n)}{g(n)} = 0,\ f(n) \in O(g(n))$.

c

$$
\begin{aligned}
f(n) &= 10n^2 + (log_2(n))^3 \\
g(n) &= n + log_2(n)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{10n^2 + (log_2(n))^3}{n + log_2(n)} \\
&= \lim_{n\to\infty}\frac{10n^2}{n} \\
&= \lim_{n\to\infty} 10n = \infty
\end{aligned}
$$

Since $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \infty,\ f(n) \in \Omega(g(n))$

d

$$
\begin{aligned}
f(n) &= 37n\log(10^6 n) \\
g(n) &= nlog(n)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \frac{37n\log(10^6 n)}{n\log(n)} \\
&= \lim_{n\to\infty}\frac{37(\log(10^6)+\log(n))}{\log(n)} \\
&= \lim_{n\to\infty}\frac{37*\log(10^6)}{\log(n)} + 1 \\
&= 1
\end{aligned}
$$

Since $\because \exists C \in \mathbb{R}^+ S.T. \lim_{n\to\infty}\frac{f(n)}{g(n)} = C,\ f(n) \in \Theta(g(n))$

e

$$
\begin{aligned}
f(n) &= n^2\log n \\
g(n) &= n^{2.0001}(log(n)^2
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{n^2\log n}{n^{2.0001}(\log(n))^2} \\
&= \lim_{n\to\infty}(\frac{n^2}{n^{2.0001}}\frac{\log(n)}{\log(n)*\log(n)}) \\
&= \lim_{n\to\infty}\frac{1}{n^{0.0001}log(n)} \\
&= 0 \\
&\implies f(n)\in O(g(n))
\end{aligned}
$$

f

$$
\begin{aligned}
f(n) &= (\log(n))^2 \\
g(n) &= \frac{n}{\log(n)}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}(\frac{\log(n)*\log(n)}{\frac{n}{\log(n)}}) \\
&= \lim_{n\to\infty}(\frac{(\log(n))^3}{n}) \\
&= \lim_{n\to\infty}\frac{1}{n} \\
&= 0 \\
&\implies f(n)\in O(g(n))
\end{aligned}
$$

g

$$
\begin{aligned}
f(n) &= \frac{n}{\log(n)} \\
g(n) &= (\log n)^{\log(n)}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \frac{\frac{n}{\log(n)}}{(\log(n))^{\log(n)}} \\
&= \lim_{n\to\infty}\frac{n(\log(n))^{-1}}{(\log(n)^{\log(n)}} \\
&= \lim_{n\to\infty}(n(\log(n))^{\log(n)-1}) \\
&= \lim_{n\to\infty}(n(\log(n))^{\log(n)-\log_2(2)} \\
&= \lim_{n\to\infty} n(\log(n))^{\log(n/2)} \\
&= \infty \\
&\implies f(n)\in\Omega(g(n)) \;\;\&\;\; f(n)\notin(O(g(n))
\end{aligned}
$$

h

$$
\begin{aligned}
f(n) &= (\log(n))^3 \\
g(n) &= \sqrt[3]{n}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{(\log(n))^3}{\sqrt[3]{n}} \\
&= \lim_{n\to\infty}\frac{3(\log(n))^2\frac{1}{n\ln 2}}{\frac{1}{3}n^{-\frac{2}{3}}} \text{ (By L'Hopital's Law)} \\
&= \lim_{n\to\infty}\frac{3n^{\frac{2}{3}}*3*(\log(n))^2}{n\ln(2)} \\
&= \lim_{n\to\infty}\frac{9(\sqrt[3]{n}\log(n))^2}{n\ln(2)} \\
&= \lim_{n\to\infty}\frac{9*2(\sqrt[3]{n}log(n))(\sqrt[3]{n}\frac{1}{n\ln(2)}+\frac{1}{3}log(n)n^{-\frac{2}{3}})}{\ln(2)} \text{ (By L'Hopital's Law)} \\
&= 18\lim_{n\to\infty}\frac{\frac{n^{\frac{2}{3}}log(n)}{n\ln 2}+n^{-\frac{1}{3}}(\log(n))^2}{\ln(2)} \\
&= 0 \\
&\implies f(n)\in O(g(n))
\end{aligned}
$$

i

$$
\begin{aligned}
f(n) &= 3^{\log_2(n)} \\
g(n) &= n^{\log_2(3)}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{3^{\log_2(n)}}{n^{\log_2(3)}}\\
&= \frac{3^{\frac{\log_3(n)}{\log_3(2)}}}{n^{\log_2(3)}}\\
&= \frac{3^{\log_3(n)(\frac{1}{\log_3(2)})}}{n^{\log_2(3)}}\\
&= \lim_{n\to\infty}\frac{n^{\log_3(2)}}{n^{\log_2(3)}}\\
&= \lim_{n\to\infty} n^{\log_3(2)-\log_2(3)}\\
&\because \log_3(2) < 1 \ \ \& \ \ \log_2(3) > 1 \log_3(2) - \log_2(3) < 0\\
&\therefore \exists C \in \mathbb{R}^+ : n^{\log_3(2)-\log_2(3)} = n^{-C}\\
&\ \& \ \lim_{n\to\infty} n^{-C} = 0\\
&\implies f(n) \in O(g(n)) \because \lim_{n\to\infty}\frac{f(n)}{g(n)} = 0
\end{aligned}
$$

j

$$
f(n) = 3^n
$$
$$
g(n) = n^4 2^n
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}\frac{3^n}{n^4 2^n}\\
&\lim_{n\to\infty}\frac{1.5^n}{n^4}\\
&= \infty\\
&\implies f(n) \in \Omega(g(n))
\end{aligned}
$$

k

$$
f(n) = n!
$$
$$
g(n) = 2^n
$$

$$
\begin{aligned}
\lim_{n\to\infty}\frac{f(n)}{g(n)} &= \lim_{n\to\infty}(\frac{n!}{2^n}\\
&= \infty\\
\implies f(n) \in \Omega(g(n))
\end{aligned}
$$

l

$$
\begin{aligned}
f(n) &\triangleq 2^{(\log(n))^2} \\
g(n) &\triangleq (\log(n))^{(\log(n))}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty} \frac{f(n)}{g(n)} &= \lim_{n\to\infty} \frac{2^{(\log(n))^2}}{(log(n))^{\log(n)}} \\
&= \lim_{n\to\infty} \frac{2^{(\log(n)\log(n)}}{(\log(n))^{\log(n)}} \\
&= \lim_{n\to\infty} \frac{(2^{\log(n)})^{(\log(n)}}{(\log(n))^{(\log(n)}} \\
&\because \forall n > 2 \log(n) > 1, 2^{\log(n)} > \log(n) \text{ for large n} \\
&\implies \lim_{n\to\infty} 2^{\log(n)\log(n)} > \lim_{n\to\infty} (\log(n))^{(\log(n)} \\
&\because \lim_{n\to\infty} \log(n)^{\log(n)} = \infty, \lim_{n\to\infty} \frac{(2^{\log(n)})^{\log(n)})}{(\log(n))^{\log(n)}} = \infty \\
&\implies f(n) \in \Omega(g(n))
\end{aligned}
$$

2 Problem 2

Solve the following problems using formal definitions of O, Ω, and Θ

a

Show $\sum_{i=1}^{n} i^2 \in O(n^3)$