Project 1

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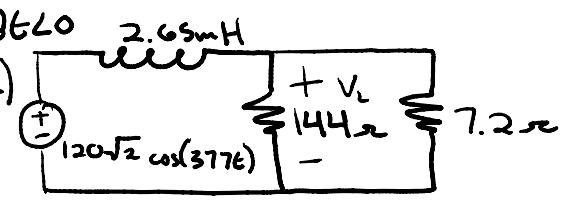
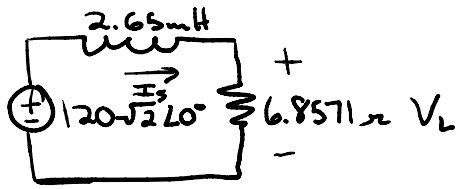
1 @ $t<0$

a)

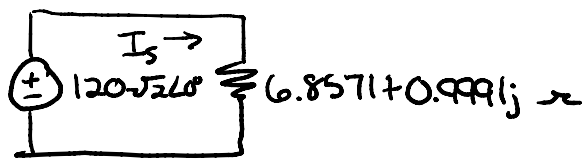




$Z_L = (2.65\times10^{-3})(377)j$

$Z_L = 0.9991j\ \Omega$

$Z_{eq} = 6.8571 + 0.9991j$



$$I_s = \frac{V_s}{Z_{eq}} = \frac{120\sqrt{2}\angle 0}{6.8571+0.9991j} = 24.4902\angle -8.2894\ A$$

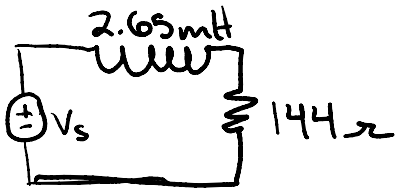
$$\boxed{I_s = 24.4902\angle -8.2894\ A}$$

$I_s(t) = 24.4902\cos(377t - 8.2894°)\ A$

$I_s(0) = 24.4902\cos(0 - 8.2894)$

$\boxed{I_s(0) = 24.2343A}$

b)



$V_s = V_L + V_R$

$V_s = LI' + RI$

$120\sqrt{2}\angle 0° = 2.65\times10^{-3}\frac{di}{dt} + 144i$

$$\left(\frac{120\sqrt{2}\angle 0° - 144i}{2.65\times10^{-3}}\right)dt = i(t+\Delta t)$$

Discretized equation

c) $V_L = (I_s)(R_1)$

$$I_s = \frac{120\sqrt{2}\angle 0°}{Z_{eq}}$$

$I = 120\sqrt{2}\angle 0° = 1.7845\angle -0.3075°$

```
clear;
clc;

R = 144;
L = 2.65e-3;
tau = L/R;
delta_t = tau/1000;
w = 377;

t_max = (2*pi)/w;
t = [0:delta_t:5*t_max];

v_s = 120*sqrt(2)*cos(w*t);
i_s = [24.23431];

for n=1:1:length(t)-1

  i_s(n+1) = (delta_t/L)*(v_s(n)-i_s(n)*R) + i_s(n);

end

v_L = i_s*R;

tiledlayout(2,1);

nexttile;
plot(t,v_L);
title('Voltage over R1');
xlabel('Time (s)');
ylabel('Voltage (v)');
grid;

nexttile;
plot(t,i_s);
title('Source Current');
xlabel('Time (s)');
ylabel('Current (A)');
grid;

figure(2);
plot(t,v_s);
title('Source Voltage');
xlabel('Time (s)');
ylabel('Voltage (v)');
grid;
```

Leq

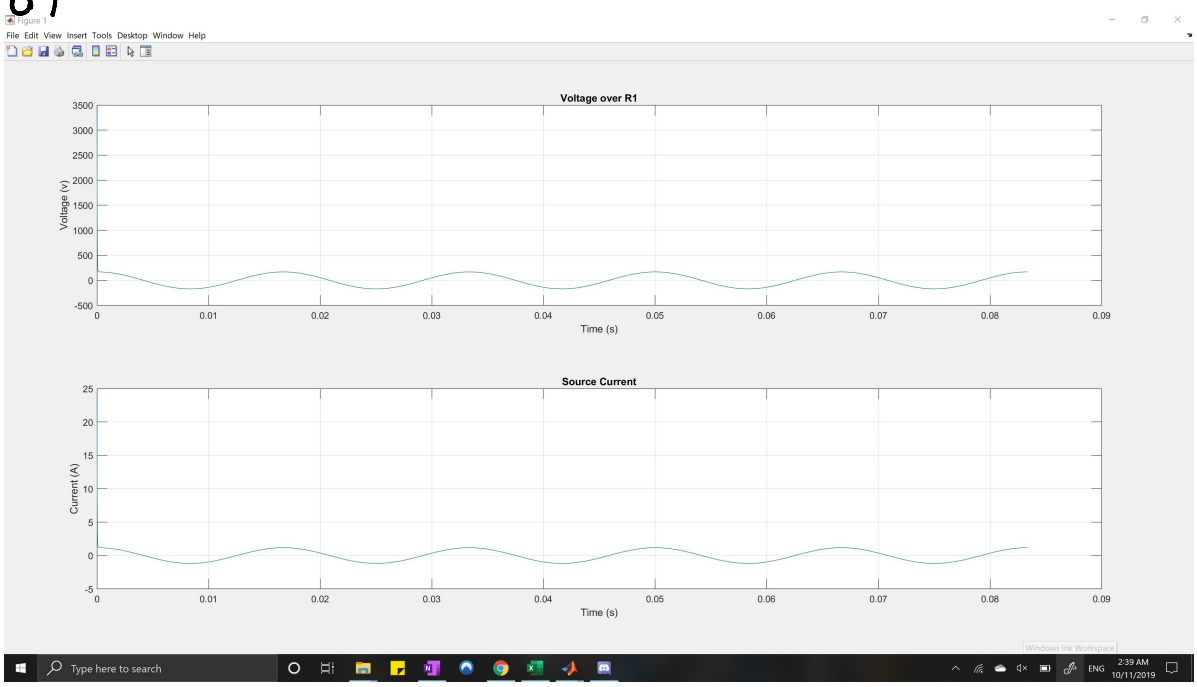
Discretized equation

$$I_s = \frac{120\sqrt{2}\angle 0^\circ}{144 + .9991j} = 1.1785\angle -0.3975^\circ$$

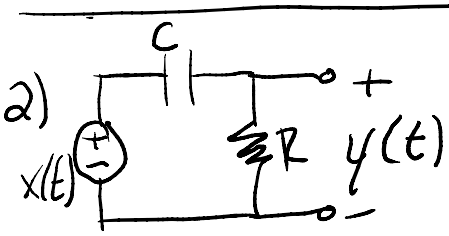
$$V_L = (1.1785\angle -0.3975^\circ)(144)$$
$$V_L = 169.7015\angle -0.3975^\circ$$
$$\boxed{V_L(t) = 169.7015\cos(377t - 0.3975)\ V}$$

The time step was chosen because it was the best to show the entire equation

d)



2)



a&b)

$$y(t) = I \cdot R$$

$$I = C\frac{d(x(t) - y(t))}{dt}$$

$$y(t) = RC\left(\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right)$$

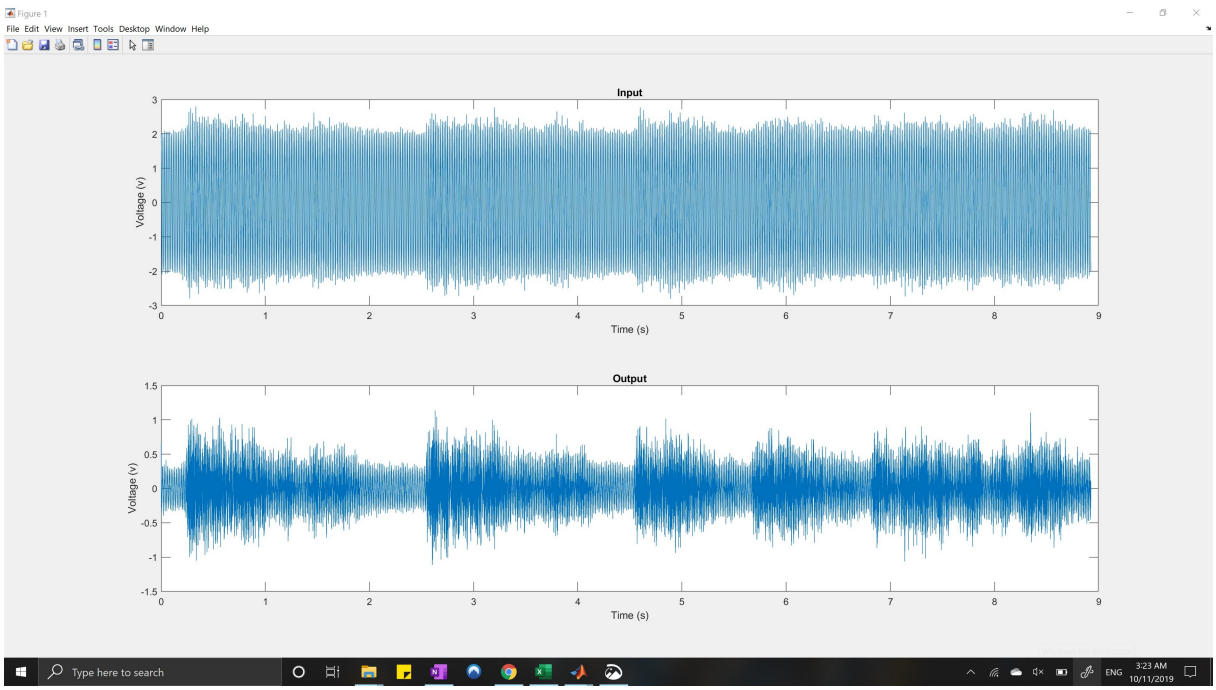
$$y(t) = RC\left(\frac{x(t+\Delta t) - x(t)}{\Delta t} - \frac{y(t+\Delta t) - y(t)}{\Delta t}\right)$$

$$y(t) = RC\left(\frac{x(t+\Delta t)-x(t)}{\Delta t} - \frac{y(t+\Delta t)-y(t)}{\Delta t}\right)$$

$$-\frac{y(t)}{RC} + \frac{x(t+\Delta t)-x(t)}{\Delta t} = \frac{y(t+\Delta t)-y(t)}{\Delta t}$$

$$\boxed{-\frac{y(t)}{RC}\Delta t + x(t+\Delta t) - x(t) + y(t) = y(t+\Delta t)}$$

c & d)




```
clear;
clc;

[y,Fs]=audioread('problem2.wav');
x=y*3;

R = 1;
C = 318.3e-6;
dt = 1/Fs;
t=dt:dt:length(x)*dt;
t=transpose(t);

for n=1:1:length(x)-1

    y(n+1)=y(n)-x(n)+x(n+1)-(dt/(R*C))*y(n);

end

v_max = max(y);

subplot(2,1,1);
plot(t,x);
title('Input');
ylabel('Voltage (v)');
xlabel('Time (s)');

subplot(2,1,2);
plot(t,y);
title('Output');
ylabel('Voltage (v)');
```

```
subplot(2,1,2);
plot(t,y);
title('Output');
ylabel('Voltage (v)');
xlabel('Time (s)');

sound(y,Fs);
```

3) a) $y_c(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$

$s^2 + 2s + 4 = 0$

$s = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2} = \frac{-2 \pm \sqrt{-12}}{2}$

$s = \frac{-2 \pm j\sqrt{12}}{-2}$ $\boxed{\text{Underdamped}}$

$s_1 = -1 + 1.7321j$
$s_2 = -1 - 1.7321j$

$y_c(t) = C_1 e^{(-1+1.7321j)t} + C_2 e^{(-1-1.7321j)t}$
$= C_1 e^{-t} e^{1.7321jt} + C_2 e^{-t} e^{-1.7321jt}$

$\boxed{y_c(t) = e^{-t}(C_1 \cos(1.7321t) + C_2 \sin(1.7321t))}$

b) $\frac{dx_1}{dt} = x_2$

$x_1(t+\Delta t) - x_1(t) = x_2 \Delta t$

$x_1(t+\Delta t) = x_2(\Delta t) + x_1(t) \rightarrow 1$

$\frac{d^2 y_1}{dt^2} + 2\frac{dy_1}{dt} + 4y_1 = 0$

$\frac{dy_2}{dt} + 2y_2 + 4y_1 = 0$

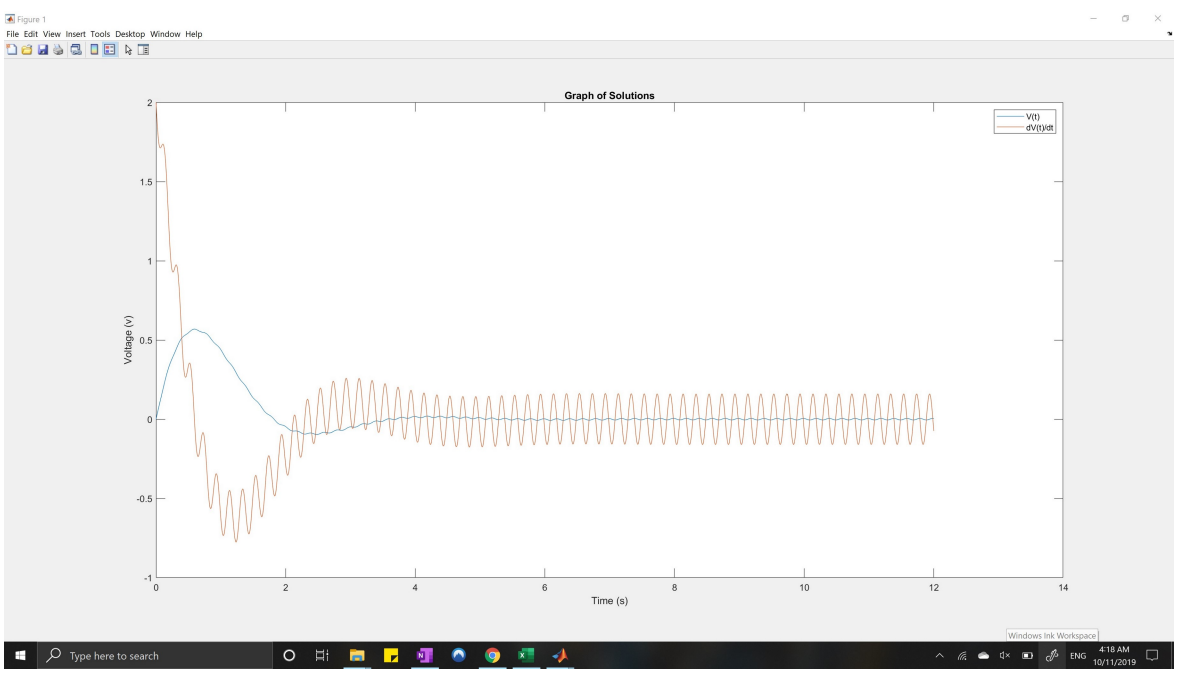
$\frac{y_2(t+\Delta t) - y_2(t)}{\Delta t} = -2y_2(t) - 4y_1(t)$

$y_2(t+\Delta t) = \Delta t(-2y_2(t) - 4y_1(t)) + y_2(t) \rightarrow 2$

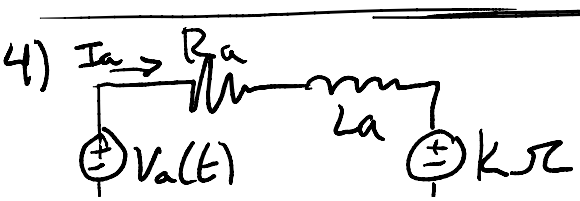
c)

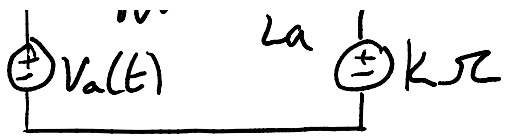
```
clear;
clc;

x1 = [0];
x2 = [2];

T = (2*pi)/10;
dt = T/1000;
stop = round((4*3)/dt);
t = [0:1:stop]*dt;

v_in = 5*cos(10*pi*t + 30*(180/pi));

for n = 1:1:stop

   x1(n+1) = x2(n)*dt + x1(n);
   x2(n+1) = dt*(v_in(n) - 2*x2(n) - 4*x1(n)) + x2(n);

end

plot(t, x1);
hold on
plot(t,x2);

title('Graph of Solutions');
xlabel('Time (s)');
ylabel('Voltage (v)');
hold off

legend ('V(t)', 'dV(t)/dt');
```

e)



4)





a) KVL: $-V_a(t) + I_a(R_a) + L\frac{dI_a}{dt} + K\omega = 0$

$$K\omega = V_a(t) - I_aR_a - L\frac{dI_a}{dt}$$

$$\omega = \frac{V_a(t) - I_aR_a - L\frac{dI_a}{dt}}{K}$$

$$\frac{d\omega}{dt} = \frac{dV_a(t)}{dt} - \frac{dI_a}{dt}R_a - L\frac{d^2I_a}{dt^2}$$

$$J\left(\frac{dV_a(t)}{dt\,K} - \frac{dI_a}{dt}\frac{R_a}{K} - \frac{d^2I_a}{dt^2}\frac{L}{K}\right) = KI_a - B\omega$$

$$J\cdot V_a'(t) + BV_a(t) = JL\frac{d^2I_a}{dt^2} + (JR+BL)\frac{dI_a}{dt} + K^2I_a + BI_aR_a$$

b)

$$\boxed{\begin{aligned} X_2(t+\Delta t) &= \Delta t\left(\frac{V_{in}(t)}{2\times10^{-6}} - 338X_2(t) - 421X_1(t)\right) + X_2(t) \\ X_1(t+\Delta t) &= X_2(t)\Delta t + X_1(t) \end{aligned}}$$

Mechanical Equation:

$$\frac{d}{dt}\omega(t)\cdot J = Ki_a - B\omega$$

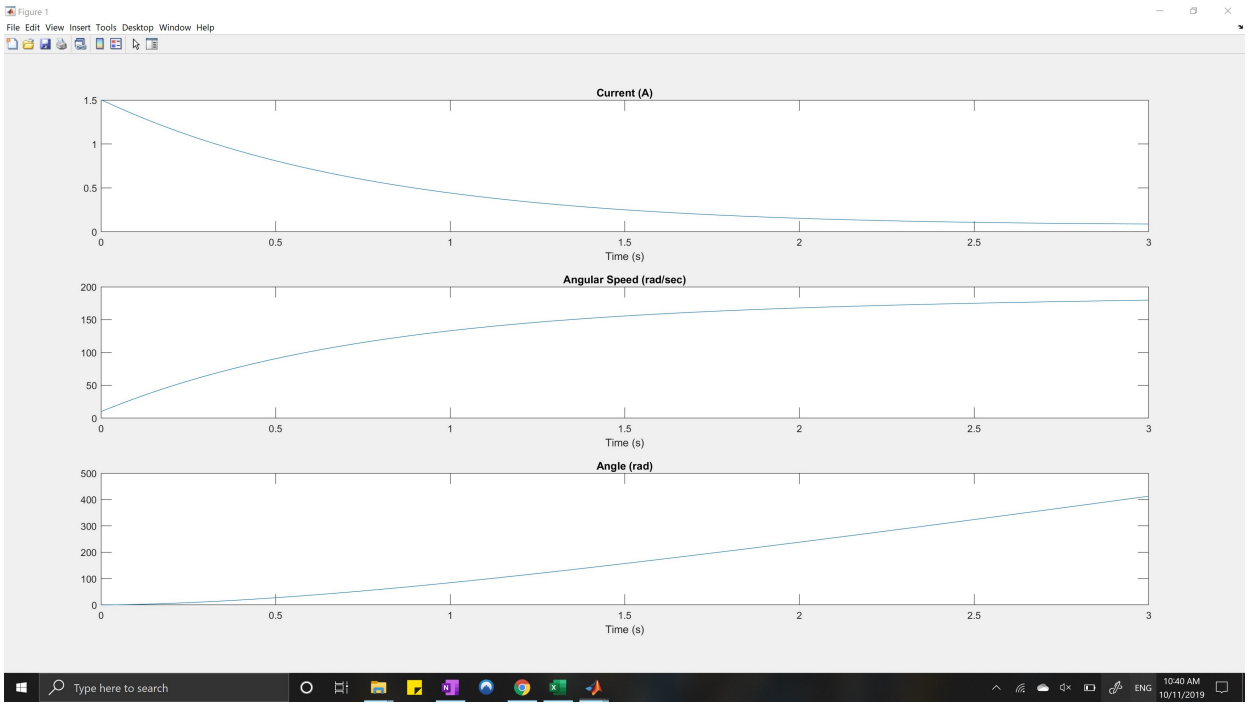
$$\boxed{\omega(t+\Delta t) = \left(\frac{Ki_a - B\omega(t)}{J}\right)\Delta t + \omega(t)}$$

Position controller:

d) $\frac{d\theta}{dt} = \omega$

$$\boxed{\theta(t+\Delta t) = \Delta t\,\omega(t) + \theta(t)}$$

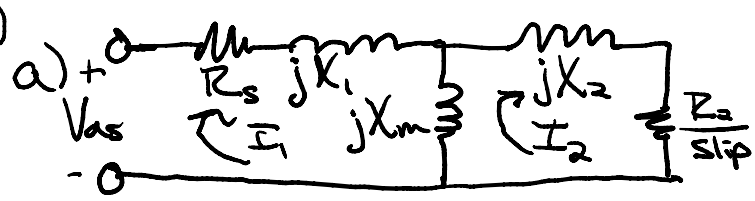
```
clear;
clc;

K = 0.029;
J = 2e-4;
beta = 0.5e-5;
L = 0.01;
R = 3.38;

x1 = 1.5;
x2 = 0;

dt = .2/1000;
stop = round(3/dt);
t = [0:1:stop]*dt;

v_in = (0.002*t + 0.00005)/0.0002;

for a = 1:1:stop

   x1(a+1) = dt*x2(a) + x1(a);
   x2(a+1) = dt*(v_in(a) - 338*x2(a) - 421*x1(a)) + x2(a);

end

omega = [10];

for b = 1:1:stop

   omega(b+1) = dt*((K*x1(b)) - (beta*omega(b)))/J + omega(b);

end

theta = [0];

for c = 1:1:stop

   theta(c+1) = dt*omega(c) + theta(c);

end

tiledlayout(3,1)

nexttile;
plot(t,x1);
title('Current (A)');
xlabel('Time (s)');

nexttile;
plot(t,omega);
title('Angular Speed (rad/sec)');
xlabel('Time (s)');

nexttile;
plot(t,theta);
title('Angle (rad)');
xlabel('Time (s)');
```



f) Lower $K_p\theta$ adjustments means we are closer to an overdamped system. Logically this makes sense due to a smaller factor causing a variable to come out smaller.

5)

a)



Mesh: $I_1 R_s + I_1 jX_1 + jX_m(I_1 - I_2) = V_{as}$

$I_1(R_s + jX_1 + jX_m) + I_2(-jX_m) = V_{as} \rightarrow (1)$

$jX_m(I_2 - I_1) + jX_2 I_2 + \frac{R_2}{slip} I_2 = 0$

$I_1(-jX_m) + I_2(jX_m + jX_2 + \frac{R_2}{slip}) = 0 \rightarrow (2)$

$$\boxed{\begin{bmatrix} (R_s + jX_1 + jX_m) & (-jX_m) & V_{as} \\ (-jX_m) & (\frac{R_2}{slip}) + jX_m + jX_2 & 0 \end{bmatrix}}$$

b) $J\frac{d\,r(t)}{dt} = T_m - B\,r(t)^2$

$$r(t+\Delta t) - r(t) = \frac{\Delta t\left(T_m - \beta r(t)^2\right)}{J}$$

$$\boxed{r(t+\Delta t) = \frac{\Delta t\left(T_m - \beta r(t)^2\right)}{J} + r(t)}$$

c & d)

```
clear;
clc;

Rs = 0.262;
X1 = 1.206;
Xm = 54.02;
X2 = 1.206;
R2 = 0.187;
w = 2*pi*60;

J = 63.87;
beta = 0.2565;

dt = 0.001;
tmax = 10;
t = [0:dt:tmax];

Vas = 2300;
omega = [0];
omega_max = 2*pi*30;

for n = 1:1:length(t)-1

   slip = (omega_max - omega(n))/omega_max;

   A = [(-j*Xm),((R2/slip)+j*X2+j*Xm),0;(Rs+j*X1+j*Xm),(-j*Xm),Vas];
   C = rref(A);

   i_1(n+1) = abs(C(1,3));
   i_2(n+1) = abs(C(2,3));

   Tm = (3*((i_2(n))^2)*(R2/slip))/(omega_max);
   omega(n+1) = (dt*(Tm - (beta*(omega(n)^2))))/J + omega(n);

end

figure;
tiledlayout(2,1);

nexttile;
plot(t,omega);
title('Motor Speed ');
xlabel('Time (s)');

nexttile;
plot(t,i_1);
title('Current I_1');
xlabel('Time (s)');
```

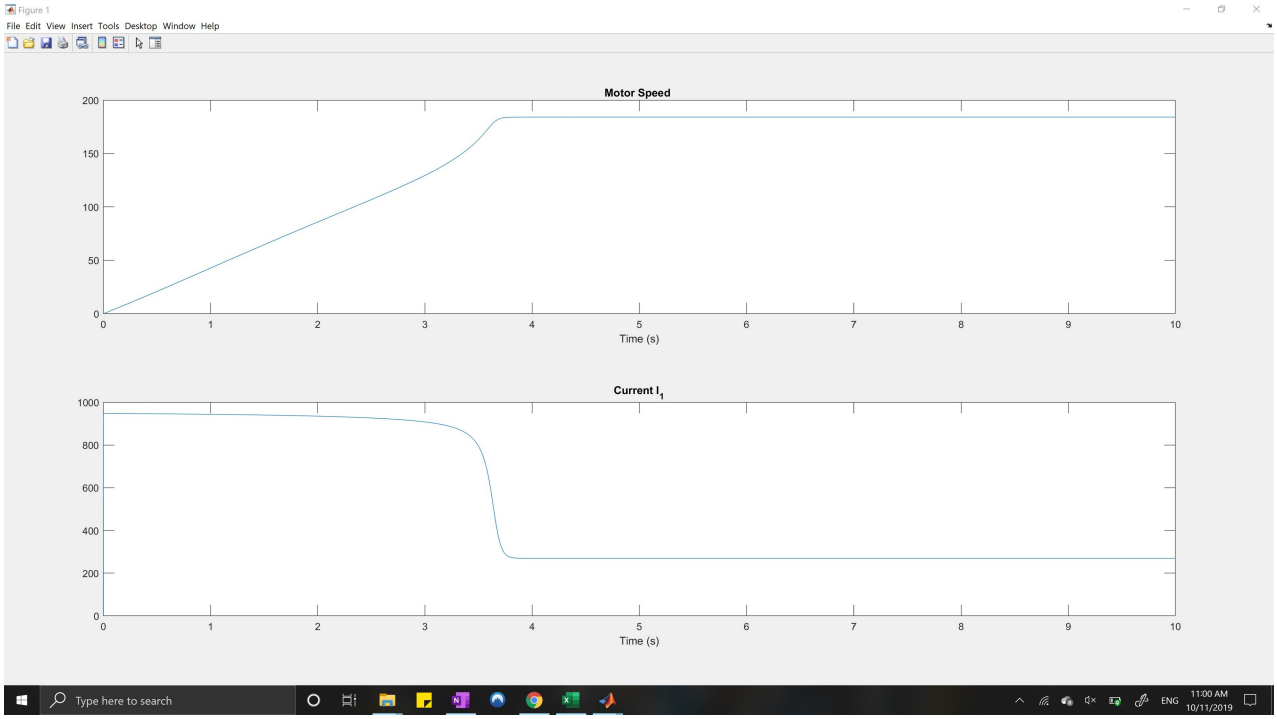


Figure 1
File Edit View Insert Tools Desktop Window Help
Motor Speed
200
150
100
50
0
0 1 2 3 4 5 6 7 8 9 10
Time (s)
Current I1
1000
800
600
400
200
0
0 1 2 3 4 5 6 7 8 9 10
Time (s)
Type here to search
ENG
11:00 AM
10/11/2019