Test 11C
<u>Part I</u>

1. **d** We are looking for a difference in the proportion cars choosing each of the three lanes, so the alternative hypothesis is that the population distribution of the categorical variable is not equal for at least one of the three values (the alternative hypothesis includes the possibility that the two proportions are equal to each other, but different from the third).

2. **c** $\chi^2 = \sum_{i=1}^{3}\left(\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}\right)$

3. **d** Choice a is about tests of means, choices b and c make reference to observed counts, which are not involved in conditions required for chi-square procedures.

4. **c** For a chi-square test of homogeneity, the null hypothesis is that the distribution of a categorical variable is the same for two or more different populations. There are three random samples in this setting from three distinct populations.

5. **d** Degrees of freedom for any chi-square test involving a 2-way table is $(number\ of\ rows - 1)(number\ of\ columns - 1)$.

6. **e** Expected counts in a two-way table $= \frac{row\ total \times column\ total}{grand\ total}$.

7. **a** Since the component for midnight/within specifications is a small fraction of the total chi-square statistic (0.090), the observed and expected counts must be close to equal.

8. **e** Each component of a chi-square statistic is $\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$ for the given cell.

9. **b** The 2 x 3 table produces a chi-square statistic with 2 degrees of freedom, thus the *P*-value is much less that 0.05, providing strong evidence against the null hypothesis that accident type and age are independent.

10. **a** A chi-square test on a 2 x 2 table is equivalent to a two-sided two-proportion z-test, and the χ^2 distribution with df = 1 is the square of the standard normal distribution. Thus if $z = 3.07, \chi^2 = (3.07)^2 = 9.42$, and the resulting *P*-value is the same.

Part II

11. State: We are testing the hypothesis H_o: The proportion of allergy sufferers born during each season of the year matches the proportion of the general population born during each season, against H_a: The proportion of allergy sufferers born during each season of the year does not match the proportion of the general population born during each season. We will use a significance level of $\alpha = 0.05$. Plan: The procedure is a chi-square goodness-of-fit test. Conditions: *Random*: the data come from a simple random sample of 500 people who are allergic to dust mites. *10%*: There are surely at least 10 x 500 people who suffer from dust mite allergies. *Large counts*: Expected counts are: Winter: 150; Spring: 110; Summer: 120; Fall: 120—all are at least 5.

Do: $\chi^2 = \frac{(117-150)^2}{150} + \frac{(105-110)^2}{110} + \frac{(145-120)^2}{120} + \frac{(133-120)^2}{120} = 14.10$, $df = 3$; P-value $= 0.0028$.

Conclude: Since the P-value is much smaller than $\alpha = 0.05$, we can reject H_0. There is good evidence that the proportion of allergy sufferers born during each season of the year does not match the proportion of the general population born during each season.

12. (a) Since age group is the explanatory variable, we calculate conditional distributions for each age group. See table below. Age does seem to be associated with differences in which service people would give up. Younger people are more likely to give up Cable TV, older people are more likely to give up their cell phone. People in the 25- 50 age group are more likely that the other two groups to give up home internet. (b) State: We are testing the hypothesis H_o: There is no association between age group and which service people would give up, against H_a: There is an association between age group and which service people would give up. We will use a significance level of $\alpha = 0.05$. Plan: Since the problem refers to a single sample, age group and "which service" should be considered two variables measured in a single population, which makes this a chi-square test for independence/association. Conditions: *Random*: the data come from a SRS of people who own all three devices. *Large counts*: All expected counts are at least 5 (see expected counts table below). *10%*: The population is clearly at least 10 x 120 = 1200. Do: Using a calculator, $\chi^2 = 21.50$; $df = 4$; $P-\text{value} \approx 0.00025$. (From Table C, P-value is below 0.0005). Conclude: Since the P-value is much smaller than $\alpha = 0.05$, we can reject H_0. There is convincing evidence of an association between age group and which service people would give up for three months.

Conditional distribution for part (a)

		Cable TV	Home internet	Cell phone
Age Group	Under 25	0.75	0.16	0.09
	25 – 50	0.51	0.33	0.15
	Over 50	0.24	0.28	0.48

Expected counts for part (b)

		Cable TV	Home internet	Cell phone
Age Group	Under 25	16.3	8.3	7.4
	25 – 50	19.9	10.1	9.0
	Over 50	14.8	7.5	6.7

Test 11D
<u>Part I</u>

1. **d** Since the data consists of two variables measured on individuals from a single population, this is a chi-square test for independence. Thus the null hypothesis is that there is no association between the variables; that the variables are independent.

2. **b** I is false, since the components for "College degree or more" are the lowest. II is true, since the "High School or less" cell have high components. III is false because the expected count for this cell is $\frac{250 \cdot 281}{856} = 82.1$, making the observed count of 67 lower, not higher.

3. **a** The appropriate χ^2 distribution has $(3-1)\cdot(2-1) = 2$ degrees of freedom. In Table C, the test statistic's value of 10.09 lies between the $p = 0.01$ column's value of 9.21 and the $p = 0.005$ column's value of 10.60. [Alternatively, a calculator produces an upper tail value of 0.00644.]

4. **e** Since the chi-square statistics is always positive, it must be true that $P(\chi^2 \geq 0) = 1$. All the other statements can be confirmed as false by examining graphs of chi-square distributions in the text.

5. **b** If sales of all colors are the same, we expect $0.25 \cdot 76 = 19$ cases of each color to be sold.

6. **e** A χ^2 test for a 2 x 2 table is equivalent to a test of $H_0: p_1 - p_2 = 0$ against the two-sided alternative.

7. **a** $z = \frac{58 - 68.5}{15.4} = -0.68$. (While we use z-scores to describe scores in a standard Normal distribution, they can be used to describe relative standing in any distribution0.

8. **b** A blind study is one which the subjects do not know which treatment group they are in. This can control for the placebo effect if one treatment group is given a placebo.

9. **b** From the Venn diagram: $P(P \cap C) = 0.45$; $P(P \cap C^c) = 0.15$; $P(P^c \cap C) = 0.19$ $P(P^c \cap C^c) = 0.21$.

10. **d** This choice has the larger sample size, the larger significance level, and the alternative parameter value that is farther from the null value. Thus it will have the greatest power.

Part II

11. (a) $P(\text{Yes}|\text{Hispanic}) = \frac{91}{483} = 0.1884$ (b) Since the explanatory variable is ethnicity, row percentages are appropriate. See table below. (c) State: We are testing the hypothesis H_o: Ethnicity and whether or not the patient received pain medication are independent, against H_a: Ethnicity and whether or not the patient received pain medication are not independent. Plan: The procedure is a chi-square test for independence. Conditions: *Random*: the data come from a simple random sample of 2298 pediatric patients suffering from abdominal pain.. *10%*: It seems reasonable to assume that the population of pediatric patients at the hospitals used in this study exceeds 22,980. *Large counts*: Expected counts are given below. All are at least 5. Do: We are given $\chi^2 = 39.53, df = 3, P\text{-value} \approx 0$. Conclude: Since the *P*-value is smaller than any reasonable significance level, we can reject H_0. There is good evidence that ethnicity and whether or not pediatric patients suffering from abdominal pain at the hospital in this study are given pain medication are not independent.

Part (b): row percentages

	Given Medication?	
	Yes	No
White	**0.27**	**0.73**
Black	**0.16**	**0.84**
Hispanic	**0.19**	**0.81**
Other	**0.07**	**0.93**

Part (c): expected counts

	Given Medication?	
	Yes	No
White	**270.8**	**947.2**
Black	**122.8**	**429.3**
Hispanic	**107.4**	**375.6**
Other	**10.0**	**35.0**

12. (a) One possible correct answer: Assign a unique number from 0000 to 9999 to each student at the school. Using technology or a random digits table, chose 80 four-digit numbers, ignoring repeats. The students with these numbers form a simple random sample. (b) State: We are testing the hypothesis H_o: The population distribution of choices for goals in later life at the principal's school is the same as the given distribution for all U.S. middle schools, against H_a: The population distribution of choices at the principal's school is the not same as the given distribution for all U.S. middle schools. We will use a significance level of $\alpha = 0.05$. Plan: The procedure is a chi-square goodness-of-fit test. Conditions: *Random*: the data come from a simple random sample of 80 students at the school. *10%*: 80 students is less that 10% of the school's 1000 students. *Large counts*: Expected counts are: Happy=39.2, Healthy=12.8, Famous=12, Rich=16. All are at least 5.

Do: $\chi^2 = \frac{(48-39.2)^2}{39.2} + \frac{(15-12.8)^2}{12.8} + \frac{(9-12)^2}{12} + \frac{(8-16)^2}{16} = 7.104$, $df = 3$; P-value $= 0.0687$.

Conclude: Since the *P*-value is greater than $\alpha = 0.05$, we fail to reject H_0. We do not have sufficient evidence to conclude that the distribution of preferences for this school is different from the given distribution for all U.S. middle schools. (c) The order in which the options are presented may bias students' responses—they might, for example, be more inclined to choose the first choice that seems like a reasonable goal. This would overestimate the proportion of students who select "Happy."