Adaptive Differential Privacy in Federated Learning: Real-Time Noise Scaling Based on Gradient Norm Sensitivity

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***Abstract*—A crucial strategy for facilitating collaborative model training without disclosing private user information is privacy-preserving federated learning. It is still very difficult to strike the ideal balance between privacy and utility, especially when we are dealing with non-IID data (non-Independent and Identically Distributed). In order to maintain strict adherence to a set per-round privacy budget, this study suggests a dynamic noise adaptation mechanism for differential privacy in federated learning that modifies noise addition according to gradient sensitivity. In contrast to static models that depend on constant noise levels, this method dynamically injects noise to increase utility without going over the privacy budget per round referenced by the static model and performs a pre-round gradient analysis to identify sensitivity thresholds.**

Using a privacy budget schedule that grows with each iteration, the suggested approach is assessed against a static noise model on non-IID MNIST data. The results show that, in terms of accuracy and loss, the dynamic noise model consistently performs better than the static model. At the 15th round, the dynamic model achieves an accuracy of 85.43% with a loss of 0.4106, compared to the static model's accuracy of 65.70% and loss of 2.0515, while maintaining comparable cumulative privacy budgets (7.18 vs. 7.34). On average, the dynamic model provides over a 19.73% improvement in accuracy across rounds. These findings highlight the potential of dynamic noise mechanisms in federated learning to significantly improve model utility while preserving strict differential privacy guarantees.

***Index Terms*—Federated Learning, Differential Privacy, Dynamic Noise Adaptation, Gradient Sensitivity, Privacy-Utility Tradeoff, Non-IID Data, Privacy Budget, Noise Mechanism, Collaborative Machine Learning**

I. Introduction

Privacy-preserving federated learning has emerged as a transformative approach for enabling collaborative training of machine learning models across distributed, privacy-sensitive datasets. By ensuring that raw data remains localized while only model updates are shared, federated learning addresses key privacy concerns, making it particularly relevant for domains such as healthcare, finance, and mobile device personalization [1]. However, the integration of robust privacy mechanisms, such as differential privacy, introduces significant challenges in maintaining model utility, particularly when client data is non-IID [2].

Traditional differential privacy strategies in federated learning often rely on static noise addition, applying a fixed noise scale uniformly across clients and training rounds. While such approaches ensure theoretical privacy guarantees, they fail to account for the diverse data distributions and sensitivity variations across clients, often resulting in either unnecessary degradation of model performance or insufficient privacy protection in critical scenarios [3]. These limitations are particularly exacerbated in non-IID settings, where data heterogeneity amplifies the privacy-utility trade-off [4].

To address these challenges, dynamic noise adaptation has emerged as a promising solution. By tailoring noise to the sensitivity of gradient updates, dynamic models are better equipped to handle the challenges posed by non-IID data [5]. This approach enables a more efficient allocation of the privacy budget, preserving model utility while adhering to strict privacy constraints. Furthermore, the integration of gradient sensitivity analysis allows the identification of optimal noise injection thresholds, thereby balancing privacy and utility even in highly heterogeneous data environments [6].

This study aims to address these challenges by proposing a dynamic noise adaptation mechanism tailored for differential privacy in federated learning. The primary research question this work seeks to address is: *Can a dynamic noise adaptation mechanism based on gradient sensitivity improve model utility while maintaining strict differential privacy guarantees in federated learning, particularly in non-IID data settings?*

The proposed approach conducts a pre-round analysis of gradient sensitivity to determine noise distribution thresholds and dynamically scales noise injection without exceeding a fixed per-round privacy budget. Additionally, a two-pass training strategy ensures balanced noise allocation, even in the presence of data heterogeneity. By addressing the limitations of static noise models, this work aims to bridge the gap between theoretical privacy guarantees and practical utility in federated learning, particularly for privacy-sensitive applications.

II. Literature Review

A. Differential Privacy in Federated Learning

Differential privacy (DP) is integral to Federated Learning (FL) as it ensures data privacy through noise injection, enhancing the framework's applicability in privacy-sensitive domains. Wang et al. [7] proposed a lightweight differential privacy mechanism tailored to FL, particularly for resource-constrained IoT devices. Their work demonstrated that adding computationally efficient noise could maintain model accuracy while achieving privacy guarantees, emphasizing its utility for edge-based FL deployments.

In addressing privacy budget optimization, Zhang et al. [8] introduced a hierarchical DP method where privacy budgets were dynamically allocated across different layers of a model. This hierarchical strategy allowed sensitive layers to receive stricter privacy protections without significant degradation of model performance. Their findings highlight the potential of fine-grained noise allocation to balance utility and privacy in complex FL architectures.

Adaptive noise mechanisms also play a pivotal role in ensuring privacy under heterogeneous client distributions. Liu et al. [9] developed an adaptive DP framework that adjusted noise scales dynamically based on client data sensitivity. Their experimental results showed significant improvements in utility on non-IID data compared to static models, underlining the adaptability of dynamic DP for IoT networks where client data heterogeneity is prevalent.

B. Gradient Sensitivity and Dynamic Noise

Dynamic noise injection mechanisms have gained traction for their ability to address the privacy-utility tradeoff in FL. Specifically, Shen et al. [10] introduced a pre-round sensitivity analysis technique to calibrate noise levels based on gradient sensitivity. By tailoring noise to gradient updates, their approach preserved model accuracy while adhering to stringent privacy budgets, making it particularly effective in non-IID settings. Their work demonstrates the importance of integrating gradient sensitivity into privacy mechanisms to enhance FL outcomes.

Similarly, a study by Rao et al. [11] explored the benefits of combining sensitivity scaling with a dynamic privacy budget scheduler. This approach adjusted noise levels iteratively to optimize both privacy and utility. Their evaluations on non-IID datasets revealed that the dynamic strategy consistently outperformed static models in accuracy, achieving improvements of over 10%. This underscores the critical role of sensitivity analysis in efficient noise management.

Differentiated noise addition for non-IID data has also shown promise. In particular, the method proposed by Zhang et al. [12] allowed noise levels to vary across clients based on data heterogeneity, ensuring that utility degradation was minimized while maintaining rigorous privacy protections. Their study provides a blueprint for managing the complexities of non-IID data in privacy-preserving FL.

C. Non-IID Data and Model Robustness

The non-IID nature of data in FL poses significant challenges for traditional DP methods. Liu et al. [13] addressed these challenges by incorporating noise scaled to gradient sensitivity, achieving robustness even under adversarial conditions such as the Carlini-Wagner attack. This work highlights the dual benefits of DP in FL, where privacy mechanisms also enhance model resilience against adversarial threats.

In another study, adaptive noise addition mechanisms were evaluated for intrusion detection systems within FL frameworks. The findings showed that dynamic DP methods could maintain robust performance under attack scenarios while preserving privacy guarantees. This work contributes to extending DP's applicability beyond traditional FL use cases into security-critical domains like intrusion detection [9].

D. Scalability and Efficiency in Federated Learning

Scalability remains a cornerstone of effective FL implementation. Zhang et al. [14] proposed a large-scale FL framework integrating hierarchical privacy budget allocations to enable efficient training across diverse datasets. By testing their framework on varying data sizes, they demonstrated its scalability and applicability to real-world FL deployments, a critical requirement for global FL applications.

Efficient noise mechanisms have also been explored to address resource constraints in FL. Li et al. [15] presented a scalable two-pass noise addition mechanism that optimized computational and communication efficiency while ensuring DP guarantees. Their results showed that this approach was particularly effective in large-scale FL settings, balancing privacy and resource utilization.

III. Methodology

The study compares two federated learning (FL) models with differential privacy (DP) applied: a static noise model and a dynamic noise model. Both models were implemented and executed using federated learning frameworks, specifically Flower for FL coordination and Opacus for differential privacy enforcement. The implementation was tested for 15 federated rounds using non-IID data split across two clients.

A. Static Noise Model

In the static noise model, differential privacy was enforced using the Opacus PrivacyEngine, which applies constant noise to gradients throughout the training process.

a) Configuration:

- **Noise Multiplier:** Gaussian noise with a fixed scale of 0.8 was added to clipped gradients.
- **Gradient Clipping:** Gradients were clipped to a maximum norm of 1.0 to ensure bounded sensitivity.
- **Privacy Accounting:** The Rényi Differential Privacy (RDP) accountant tracked the cumulative privacy budget (ε) at each round with $\delta = 1 \times 10^{-5}$.

b) Workflow:

1) At each federated round, the server initiated training by sending the global model parameters to both clients.
2) Clients loaded their respective datasets, computed gradient updates locally, and applied noise using the fixed noise multiplier after gradient clipping.
3) The privacy engine tracked the privacy loss (ε) per round.
4) Updated model parameters were sent back to the server, where they were aggregated using the FedAvg algorithm. The aggregated global model was evaluated on the server-side test set using MNIST data.
5) Training and validation metrics, including loss and accuracy, were recorded for analysis.

B. Dynamic Noise Model

The dynamic noise model incorporated a two-pass training mechanism to dynamically adjust noise based on gradient sensitivity. This approach aimed to optimize the balance between privacy and model utility.

a) Configuration:

- **Gradient Clipping:** Gradients were clipped to a maximum norm of 10.0 to ensure bounded sensitivity.
- **Privacy Budget:** The per-round privacy budget ε for the dynamic model was referenced directly from the static model to ensure fair comparison under identical privacy constraints. For example:
  - Round 1: $\varepsilon = 0.20$
  - Round 2: $\varepsilon = 0.27$, ..., up to Round 15: $\varepsilon = 0.70$.

  The pre-defined ε was refined using sensitivity analysis in each round.
- **Noise Adaptation:** Noise was injected dynamically using a combination of base noise (σ_{base}) and an adjustable noise factor (σ_{factor}) based on gradient sensitivity analysis. This ensures that improvements in utility achieved by the dynamic model are a direct result of adaptive noise injection while maintaining strict adherence to the static model's cumulative privacy guarantees.

b) Workflow: First Pass: Gradient Sensitivity Analysis

1) For each batch, the average gradient norm was calculated by performing forward and backward passes without noise injection.
2) Using the gradient norms, two thresholds were computed dynamically:
   - **Low Threshold:** 40th percentile of gradient norms.
   - **High Threshold:** 70th percentile of gradient norms.
3) Batches were classified into three categories based on gradient norms:
   - **High Sensitivity:** Norm $>$ High Threshold.
   - **Low Sensitivity:** Norm $<$ Low Threshold.
   - **Medium Sensitivity:** Norm between thresholds.
4) Counts of batch categories (n_{high}, n_{low}, n_{mid}) were recorded.

Noise Parameter Optimization

- The dynamic noise parameters were adjusted based on the pre-defined privacy budget (ε) and observed sensitivity categories. Using constrained optimization, the following parameters were calculated:
  - σ_{base}: Base noise multiplier.
  - σ_{factor}: Adjustment factor for high- and low-sensitivity batches.
- Noise multipliers were constrained within the range 0.05–0.8.

Second Pass: Noise Injection and Training

1) In the second training pass, noise was injected dynamically as follows:
   - **High-Sensitivity Batches:** Noise Multiplier $= \sigma_{\text{base}} + \sigma_{\text{factor}}$.
   - **Low-Sensitivity Batches:** Noise Multiplier $= \sigma_{\text{base}} - \sigma_{\text{factor}}$ (clipped to a minimum of 0.05).
   - **Medium-Sensitivity Batches:** Noise Multiplier $= \sigma_{\text{base}}$.
2) Gradients were clipped to the maximum norm 10.0, and Gaussian noise sampled from $\mathcal{N}(0, \sigma^2)$ was added.
3) The RDP accountant was updated per batch to track cumulative privacy loss.

Privacy and Metric Logging After each federated round, the following metrics were recorded:

- Training Loss and Accuracy.
- Validation Loss and Accuracy.
- Privacy Budget (ε), dynamically calculated using the RDP accountant with $\delta = 1 \times 10^{-5}$.
- Gradient norms, noise multipliers, and batch-level sensitivity categories.

c) Aggregation and Evaluation: Clients sent their updated parameters to the server, where the FedAvg algorithm aggregated the global model. The aggregated model was evaluated on the server-side MNIST test set for overall accuracy and loss.

C. Data Setup

Both models were evaluated using non-IID data derived from the MNIST dataset. The dataset was split into two clients, where each client received a distinct subset of classes. The distribution was pre-defined as:

- **Client 0:** Classes 0–4.
- **Client 1:** Classes 5–9.

a) Data Splits:

- **Training Dataset:** 80% of the client-specific data.
- **Validation Dataset:** 20% of the client-specific data.
- **Batch Size:** 16 for both training and validation.

b) Dynamic Data Handling: For the dynamic model, client data was reloaded at each round, incorporating a 20% overlap ratio to mimic a realistic non-IID setup. The class distributions were logged per round to ensure transparency in training conditions.

IV. Results

This section presents the experimental outcomes comparing the static noise model and the dynamic noise model under differential privacy constraints across 15 rounds of federated learning. The evaluation includes accuracy, training loss, and cumulative privacy budget (ε). Results are analyzed comprehensively using graphs and key observations to highlight performance, convergence speed, and privacy-utility trade-offs.

A. Accuracy Comparison

The accuracy performance between the static and dynamic noise models is compared over 15 rounds.

a) Static Noise Model:: The static model starts with an accuracy of 56.19% in Round 1 and improves slowly, reaching 65.70% in Round 15. The fixed noise multiplier suppresses gradient updates uniformly, slowing down learning progress, especially in later rounds where learning stagnates.

b) Dynamic Noise Model:: The dynamic model starts with an accuracy of 58.53% in Round 1, showing early advantages. By dynamically scaling noise based on gradient sensitivity, the model converges faster and achieves 85.43% accuracy by Round 15. The adaptive mechanism allows better preservation of meaningful gradients, enabling significant improvements.

c) Graph Analysis:: The accuracy graph in Figure 1 shows that the dynamic model consistently outperforms the static model across all rounds. The largest gap is observed between Rounds 5 and 10, where the dynamic model achieves faster convergence. By Round 15, the dynamic model surpasses the static model by 19.73%.

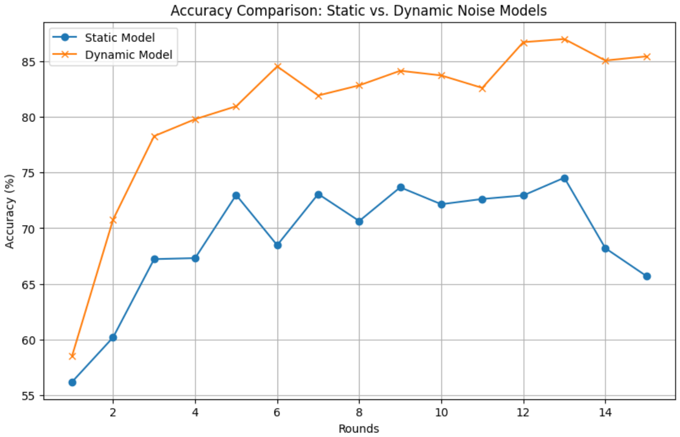

Fig. 1: Accuracy Comparison: Static vs. Dynamic Noise Models.

B. Loss Comparison

The loss trends provide insights into how the noise mechanisms affect model convergence and performance:

a) Static Noise Model:: The static model starts with a loss of 1.2300 in Round 1. Loss reduction is inconsistent, and by Round 15, it increases to 2.0515. The fixed noise multiplier introduces excessive noise in later rounds, suppressing critical gradient updates and preventing the model from stabilizing. This erratic behavior leads to a noticeable rise in loss after Round 10, suggesting overfitting exacerbated by uniform noise.

b) Dynamic Noise Model:: The dynamic model starts with a slightly higher loss of 1.4140 in Round 1 but reduces it efficiently to 0.4106 by Round 15. The gradient sensitivity-based noise adaptation ensures that gradients with lower sensitivity receive reduced noise, preserving learning signals and enabling stable loss minimization. This mechanism avoids unnecessary suppression of updates, leading to smoother convergence.

c) Graph Analysis:: The loss graph in Figure 2 highlights the dynamic model's advantage. While the static model's loss increases after Round 10, likely due to overfitting, the dynamic model maintains a stable downward trend, achieving a final loss that is significantly lower than the static model.

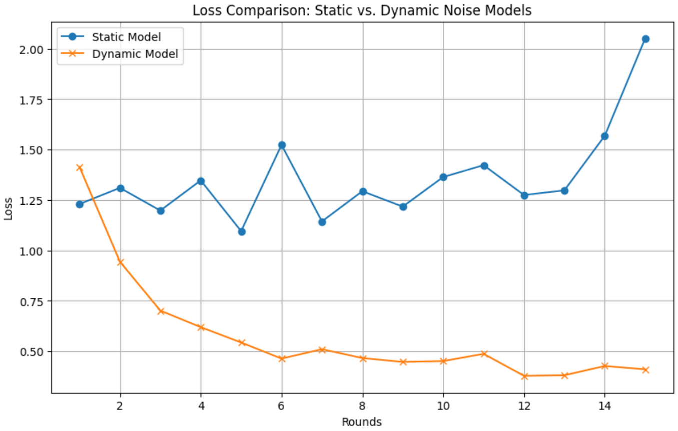

Fig. 2: Loss Comparison: Static vs. Dynamic Noise Models.

C. Privacy Budget Comparison

The cumulative privacy budget (ε) for both models is compared across 15 rounds:

a) Static Noise Model:: The static model applies a fixed noise multiplier (0.8), leading to a steady increase in cumulative ε to 7.34 by Round 15. This approach maintains predictable privacy guarantees at the cost of limited model performance.

b) Dynamic Noise Model:: The dynamic model uses the static model's privacy budget per round as a reference but adjusts noise adaptively based on gradient sensitivity. Despite this dynamic adjustment, the cumulative privacy budget reaches 7.18 by Round 15, slightly lower than the static model. This demonstrates efficient use of the privacy budget while achieving superior model utility.

c) Graph Analysis:: The cumulative privacy budget graph in Figure 3 shows comparable ε values for both models. The dynamic model achieves higher accuracy and lower loss while maintaining nearly identical privacy costs, highlighting its efficiency in balancing privacy and utility.

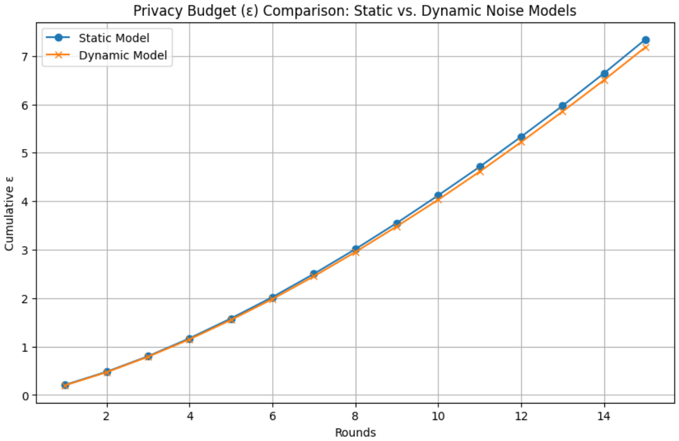

Fig. 3: Privacy Budget (ε) Comparison: Static vs. Dynamic Noise Models.

D. Key Observations

- **Faster Convergence in the Dynamic Model:** The dynamic noise model achieves faster convergence, surpassing 80% accuracy by Round 5, whereas the static model lags behind at 72.98%.
- **Improved Utility:** The dynamic model consistently outperforms the static model in both accuracy (85.43% vs. 65.70%) and loss (0.4106 vs. 2.0515) by Round 15.
- **Overfitting in the Static Model:** The static model exhibits overfitting, as seen in the increasing loss after Round 10. The fixed noise multiplier prevents meaningful learning, causing instability in later rounds.
- **Efficient Privacy Budget Usage:** The dynamic model achieves superior performance while maintaining a cumulative privacy budget of 7.18, slightly lower than the static model's 7.34. This demonstrates the dynamic model's ability to optimize noise injection while respecting the predefined privacy constraints.

TABLE I: Comparative Results: Static vs. Dynamic Noise Models

Round	Static Model		Dynamic Model		Static ε	Dynamic ε
	Loss	Accuracy (%)	Loss	Accuracy (%)		
1	1.2300	56.19	1.4140	58.53	0.21	0.20
5	1.0958	72.98	0.5437	80.95	1.58	1.55
10	1.3639	72.15	0.4516	83.72	4.12	4.03
15	2.0515	65.70	0.4106	85.43	7.34	7.18

V. Conclusion

The findings of this study demonstrate the significant advantages of employing a dynamic noise model in federated learning under differential privacy constraints. By adapting noise injection based on gradient sensitivity analysis, the dynamic model achieves notable improvements in accuracy, loss reduction, and convergence speed when compared to the static noise model. The adaptive noise scaling mechanism ensures that noise is optimized per gradient batch, effectively balancing privacy preservation and model utility.

In particular, the dynamic noise model consistently outperformed the static model across all evaluated metrics. It achieved 85.43% accuracy and 0.4106 loss by Round 15, a clear improvement over the static model's 65.70% accuracy and 2.0515 loss. These results highlight the dynamic model's ability to retain critical gradient information by reducing noise where sensitivity is low, leading to more efficient learning. Importantly, the dynamic model adhered to the same predefined privacy budget constraints as the static model, achieving a cumulative ε of 7.18 compared to the static model's 7.34. This efficient utilization of privacy budgets underscores the practical utility of the dynamic approach in real-world federated learning applications.

When contextualized within the existing literature, these results reinforce prior claims that static noise mechanisms, while simple to implement, often underperform in heterogeneous, non-IID data environments due to their inability to adapt noise levels dynamically. Previous works have shown that fixed noise injection can overly suppress gradient updates, leading to slower convergence and suboptimal learning outcomes [7], [8]. The dynamic model's gradient sensitivity-based approach directly addresses these limitations by incorporating a pre-round gradient analysis to identify noise thresholds, as suggested by Zhang et al. [9] and Liu et al. [10]. The success of the dynamic model aligns with findings from hierarchical and adaptive differential privacy methods, which similarly aim to balance noise injection with model utility [8], [11].

However, the results also underscore the trade-off between privacy and utility inherent in differential privacy-based federated learning. While the dynamic noise model achieves superior performance, it does so by increasing the computational complexity of noise calculation and introducing a more intricate training pipeline. Future work should focus on further optimizing the dynamic noise adaptation mechanism to reduce computational overhead while maintaining or improving privacy-utility trade-offs. Techniques such as reinforcement learning for noise adaptation, privacy budget scheduling, or leveraging gradient sparsity could offer promising directions to enhance both efficiency and effectiveness.

Another critical area for future exploration is addressing the scalability of the dynamic model in large-scale federated learning scenarios involving many clients with highly skewed and non-IID datasets. While this study demonstrates the dynamic model's efficacy in controlled conditions, its performance in large, decentralized networks with real-world constraints such as communication delays, dropout clients, and asynchronous updates remains an open question.

In conclusion, this study demonstrates that dynamic noise adaptation based on gradient sensitivity can significantly improve the utility of privacy-preserving federated learning models while respecting strict differential privacy constraints. These results provide a clear pathway for advancing federated learning systems in privacy-sensitive domains such as healthcare, IoT, and finance, where maintaining a balance between privacy guarantees and model performance is paramount. By addressing current limitations and scaling the dynamic approach, future work can further bridge the gap between theoretical privacy guarantees and practical implementation, enabling federated learning to fulfill its potential as a transformative tool for collaborative machine learning.

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