# Lindblad

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## 1 Lie Geometry of Density Matrix

Rather than parameterize $\rho$ through matrix entries, parameterize it via an exponential family. Use the known importance of generalized Gibbs ensembles. Also reveals the Poisson geometry underlying Fisher metric.

## 2 Lindblad

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{k=1}^{M} L_k \rho(t) L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho(t)\}$$

The action of $H$ on $V \otimes V^\dagger$ is given by coproduct and a sign flip for the $V^\dagger$ (More generally the antipode, but in this case just $-1$).

$$\Delta H = H \otimes 1 - 1 \otimes H$$

$$\begin{aligned}
\dot{\rho} &\in V \otimes V^\dagger \\
\dot{\rho} &= (-i\Delta H + \mathcal{G})\rho \\
\mathcal{G} &= \sum_{m=0}^{M} \bar{L}_m \otimes L_m - \frac{1}{2} Id \otimes L_m^\dagger L_m - \frac{1}{2} \bar{L}_m^\dagger \bar{L}_m \otimes Id
\end{aligned}$$

## 3 Effective Hamiltonian

### 3.1 Definition ($H_{eff}$)

$$H_{eff} = H - i\sum_k \frac{\gamma_k}{2} L_k^\dagger L_k$$

*The antiHermitian part is negative for $\gamma_k \geq 0$ in the sense of $L_k^\dagger L_k$ being a positive operator. In Lie algebraic terms $iH_{eff} \in \mathfrak{u}(n) \bigoplus \mathfrak{u}(n)^*_{\geq 0}$*

**3.2 Definition (Iwasawa Poisson-Lie Group)** *The one coming from $KAN$ decomposition. So for $GL(n, \mathbb{C})$ regarded as a real Lie group, we get Hermitian and antiHermitian split. TODO: Write down this parameterization.*

$$\frac{d\rho}{dt} = -i(H_{eff}\rho - \rho H_{eff}^\dagger) + \sum_k \gamma_k L_k \rho L_k^\dagger$$

# 4 Lie Parameterization

*Let* $\rho =$

# 5 Examples

**5.1 Example**

$$\begin{aligned} H &= 0 \\ \frac{\partial\rho}{\partial t} &= -i[0,\rho] + \frac{\gamma}{2}(\sigma_z\rho\sigma_z - \frac{1}{2}\{\sigma_z^\dagger\sigma_z, \rho\}) \end{aligned}$$

**5.2 Example**

$$\begin{aligned} H &= \hbar\omega a^\dagger a \\ L_1 &= a \\ \gamma_1 &= \frac{\gamma}{2}(\bar{n}+1) \\ L_2 &= a^\dagger \\ \gamma_2 &= \frac{\gamma}{2}(\bar{n}) \\ H_{eff} &= \hbar\omega a^\dagger a - i\frac{\gamma}{4}((\bar{n}+1)a^\dagger a + (\bar{n})aa^\dagger) \\ &= \hbar\omega a^\dagger a - i\frac{\gamma}{4}((2\bar{n}+1)a^\dagger a + \bar{n}) \end{aligned}$$

# 6 Hochschild Homology

https://books.google.com/books?id=_9UBCAAAQBAJ&pg=PA88&lpg=PA88&dq=lindblad+hochschild&
source=bl&ots=HI7IaXfgEe&sig=kOu64rDabBZnVO24_gnlgv0pcl4&hl=en&sa=X&ved=0ahUKEwictbSF7MTXAhUKj
v=onepage&q=lindblad%20hochschild&f=false

# 7 Quantum Chemistry

https://dash.harvard.edu/bitstream/handle/1/4646014/0807.0929v2.pdf?sequence=1

## 7.1 Unitary Evolution

https://arxiv.org/pdf/0905.0887.pdf

### 7.2 Open

For a particular small open system with known jump operators, can we come up with an auxiliary system to approximate the infinite dimensional bath Hilbert space in such a way that following the gates of "Towards quantum chemisty" on this auxiliary closed sytem problem implements the Lindblad evolution for the qubits $H_S = (\mathbb{C}^2)^d$?

Start with well studied $H_2$ molecule example. Decide what environment we should put it in...

Closest reference could find on this subject: `https://www.pks.mpg.de/~eisfeld/wwww/pdf/Mostame_et_al_Emulation_of_open_quant_scqb.pdf`