Lindblad

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1 Lie Geometry of Density Matrix

Rather than parameterize ρ through matrix entries, parameterize it via an exponential family. Use the known importance of generalized Gibbs ensembles. Also reveals the Poisson geometry underlying Fisher metric.

2 Lindblad

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{k=1}^{M} L_k \rho(t) L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho(t)\}$$

The action of H on $V \otimes V^\dagger$ is given by coproduct and a sign flip for the $V^\dagger$ (More generally the antipode, but in this case just -1).

$$\Delta H = H \otimes 1 - 1 \otimes H$$

$$\begin{aligned} \dot{\rho} &\in V \otimes V^\dagger \\ \dot{\rho} &= (-i\Delta H + \mathcal{G})\rho \\ \mathcal{G} &= \sum_{m=0}^{M} \bar{L}_m \otimes L_m - \frac{1}{2} Id \otimes L_m^\dagger L_m - \frac{1}{2} \bar{L}_m^\dagger \bar{L}_m \otimes Id \end{aligned}$$

3 Effective Hamiltonian

3.1 Definition (H_{eff})

$$H_{eff} = H - i \sum_k \frac{\gamma_k}{2} L_k^\dagger L_k$$

The antiHermitian part is negative for $\gamma_k \geq 0$ in the sense of $L_k^\dagger L_k$ being a positive operator. In Lie algebraic terms $iH_{eff} \in \mathfrak{u}(n) \bigoplus \mathfrak{u}(n)^*_{\geq 0}$

3.2 Definition (Iwasawa Poisson-Lie Group) *The one coming from KAN decomposition. So for $GL(n,\mathbb{C})$ regarded as a real Lie group, we get Hermitian and antiHermitian split. TODO: Write down this parameterization.*

$$\frac{d\rho}{dt} = -i(H_{eff}\rho - \rho H_{eff}^{\dagger}) + \sum_k \gamma_k L_k \rho L_k^{\dagger}$$

4 Lie Parameterization

Let $\rho =$

5 Examples

5.1 Example

$$\begin{aligned} H &= 0 \\ \frac{\partial\rho}{\partial t} &= -i[0,\rho] + \frac{\gamma}{2}(\sigma_z\rho\sigma_z - \frac{1}{2}\{\sigma_z^{\dagger}\sigma_z,\rho\}) \end{aligned}$$

5.2 Example

$$\begin{aligned} H &= \hbar\omega a^{\dagger}a \\ L_1 &= a \\ \gamma_1 &= \frac{\gamma}{2}(\bar{n}+1) \\ L_2 &= a^{\dagger} \\ \gamma_2 &= \frac{\gamma}{2}(\bar{n}) \\ H_{eff} &= \hbar\omega a^{\dagger}a - i\frac{\gamma}{4}((\bar{n}+1)a^{\dagger}a + (\bar{n})aa^{\dagger}) \\ &= \hbar\omega a^{\dagger}a - i\frac{\gamma}{4}((2\bar{n}+1)a^{\dagger}a + \bar{n}) \end{aligned}$$

6 Fisher Information Norm of Lindblad Vector Field

The ordinary differential equation described above determines a vector field on the space of density matrices (M,g) where g is the Fisher information metric tensor. So one can evaluate $v(\rho)$ which is the direction of flow when starting at ρ. One can use g to give $g(v(\rho),v(\rho))$ which measures the length thereof. The actual dynamics is a trajectory for this vector field by solving for the flow ϕ_t. We want to calculate the length of the curve.

$$L = \int_0^T \sqrt{g(v(\rho(t)),v(\rho(t)))}dt$$

where $\rho(t) = \phi_t\rho(0)$ is the solution to the ordinary differential equation on M.

7 Hochschild Homology

https://books.google.com/books?id=_9UBCAAAQBAJ&pg=PA88&lpg=PA88&dq=lindblad+hochschild&
source=bl&ots=HI7IaXfgEe&sig=kOu64rDabBZnVO24_gnlgv0pcl4&hl=en&sa=X&ved=0ahUKEwictbSF7MTXAhUKj
v=onepage&q=lindblad%20hochschild&f=false

7.1 Theorem *Let α be a completely positive unital map $\mathcal{N} \to \mathcal{M}$ and ρ a faithful normal state on $\mathcal{M}$. Then there exists a unique $\mathcal{N} - \mathcal{M}$ bimodule $\mathcal{H}_\alpha$ defined by having a cyclic vector ξ_α and*

$$\begin{aligned} \langle \xi_\alpha \mid\mid \ell_\alpha(n)\xi_\alpha \rangle &= \rho(\alpha(n)) \\ \langle \xi_\alpha \mid\mid r_\alpha(m)\xi_\alpha \rangle &= \rho(m) \end{aligned}$$

to define the left and right actions.

Proof https://indico.oist.jp/indico/event/5/picture/116.pdf □

7.2 Theorem *The entropy of this channel is* $\log Ind(\mathcal{H}_\alpha)$ *where taking Vaughan Jones' index.*

7.3 Theorem (Holevo) *How this is interpreted if assume finite dimensions.*

7.4 Theorem (Choi)

7.5 Theorem (Stinespring Factorization)

7.6 Corollary (GNS Reconstruction) *Let H be 1 dimensional. Apply Stinespring to the state Φ on the Von Neumann algebra A. V^* $H \to K$ is specified by $V^*1 = \xi$ where ξ is a normalized vector in K.*

$$\begin{aligned} \Phi(a) &= V\pi(a)V^* = \langle V\pi(a)V^* \mid 1 \rangle_H \\ &= \langle \pi(a)\xi \mid \xi \rangle_K \end{aligned}$$

So K is the reconstructed Hilbert space and π is how anything in A acts as a concrete operator instead of abstractly as an element of an algebra.

7.7 Corollary (Lindblad)

As the channel changes, the bimodule changes. That is why this goes with the Hochschild section. Hochschild cohomology is about deformations of algebras and bimodules.

7.8 Definition (Modular Operator) *Let $\mathcal{H}$ be a $\mathcal{N} - \mathcal{M}$ bimodule with finite Jones' index. Give faithful normal states ρ_1 and ρ_2 on each side.*

$$\Delta_{\mathcal{H}}(\rho_1 \mid \rho_2) \equiv \frac{d(\rho_1 \cdot \ell^{-1})}{d(\rho_2 \cdot r^{-1} \cdot \epsilon)}$$

Proof https://indico.oist.jp/indico/event/5/picture/116.pdf □

Each channel or basic operation gives a bimodule, a program then tensors all these together in some order. Further fitting with the philosophy espoused in Coxeter compiler and Hamiltonian-Learning (where Lagrangian correspondences replace bimodules because classical).

8 Quantum Chemistry

https://dash.harvard.edu/bitstream/handle/1/4646014/0807.0929v2.pdf?sequence=1

8.1 Unitary Evolution

https://arxiv.org/pdf/0905.0887.pdf

8.2 Open

For a particular small open system with known jump operators, can we come up with an auxiliary system to approximate the infinite dimensional bath Hilbert space in such a way that following the gates of "Towards quantum chemisty" on this auxiliary closed sytem problem implements the Lindblad evolution for the qubits $H_S = (\mathbb{C}^2)^d$?

Start with well studied H_2 molecule example. Decide what environment we should put it in...

Closest reference could find on this subject: https://www.pks.mpg.de/~eisfeld/wwww/pdf/Mostame_et_al_Emulation_of_open_quant_scqb.pdf