Graph States

Ammar Husain

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1 Graph States

1.1 Definition (Graph State) *Let G be an undirected graph with V vertices.*

$$| \psi_G \rangle \equiv \prod_{(a,b) \in E} CZ_{a,b} \mid + \rangle^{\otimes V}$$

This is well founded because $CZ_{a,b} = CZ_{b,a}$ so the choice of orientation for $(a,b) \in E$ does not matter. Also they commute so the order of the product does not have to be specified.

1.2 Lemma *For $K_i = X_i \otimes_{j \in N(i)} Z_j$, $K_i \mid \psi_G \rangle = (+1) \mid \psi_G \rangle$. $K_i^2 = Id$, and they commute. So they form an independent set in the sense of the Coxeter graph as the general program described in the Coxeter compiler paper.*

2 Hypergraph States

2.1 Definition (k-Uniform Hypergraph) *A hypergraph such that each hyperedge contains exactly k vertices. If $k = 2$, that is a usual graph.*

2.2 Definition (Uniform Hypergraph State) *Let G be a k-uniform hypergraph on V vertices.*

$$| \psi_G \rangle \equiv \prod_{(a_1, a_2 \cdots a_k) \in E} CZ_{a_1, a_2 \cdots a_k} \mid + \rangle^{\otimes V}$$

where $CZ_{a_1, a_2 \cdots a_k}$ acts as -1 only on $\mid 1 \rangle^{\otimes k}$ and 1 otherwise. Again the symmetry means there is no impact of the choice of ordering $a_1 \cdots a_k$ necessary in the hyperedge.

2.3 Lemma *For $K_i = X_i \otimes \prod_{(i, i_2 \cdots i_k) \in E} CZ_{i_2 \cdots i_k}$, $K_i \mid \psi_G \rangle = (+1) \mid \psi_G \rangle$. Again $K_i^2 = Id$, and they commute. Again they form an independent set in the sense of the Coxeter graph as the general program described in the Coxeter compiler paper.*

2.4 Definition (Hypergraph state)

Proof `https://arxiv.org/pdf/1211.5554.pdf` `https://arxiv.org/pdf/1612.06418.pdf` □

3 Bayesian DAG States

3.1 Definition *Let G be a DAG where the vertices v_i represent random variables on d_i possibilities. So the entire Hilbert space is $\bigotimes_{i=0}^{V-1} \mathbb{C}^{d_i}$ The edges indicate causation.*

Let b be vertex and $\{a_1 \cdots a_n\}$ be it's possibly empty set of immediate predecessors $U_{\{a\},b}$ is then chosen to be a unitary satisfying

$$\begin{aligned} U_{\{a\},b} \mid e_{i_1}\rangle \otimes \cdots \mid e_{i_n}\rangle \otimes \mid 0\rangle &= \sqrt{P_j} \mid e_{i_1}\rangle \otimes \cdots \mid e_{i_n}\rangle \otimes \mid j\rangle \\ P_j &= P(b = j \mid a_1 = i_1 \cdots a_n = i_n) \end{aligned}$$

Initialize in the state $\mid 0\rangle^{\otimes V}$. Do a topological sort on G. Then apply the operators $U_{\varnothing,b}$ for the bottom of the poset. Then build up from there with $U_{\{a\},b}$

If you wish to ask a question conditioned on some variables like the random variable for vertex v_j is in state i_j, perform amplitude amplification first. The ratios between the terms with $v_j = i_j$ remain the same amongst each other. That is what one is probing anyways. Relative likelihoods.