Lindblad

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1 Lie Geometry of Density Matrix

Rather than parameterize ρ through matrix entries, parameterize it via an exponential family. Use the known importance of generalized Gibbs ensembles. Also reveals the Poisson geometry underlying Fisher metric.

2 Lindblad

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{k=1}^{M} L_k \rho(t) L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho(t)\}$$

The action of H on $V \otimes V^\dagger$ is given by coproduct and a sign flip for the $V^\dagger$ (More generally the antipode, but in this case just -1).

$$\Delta H = H \otimes 1 - 1 \otimes H$$

$$\begin{aligned} \dot{\rho} &\in V \otimes V^\dagger \\ \dot{\rho} &= (-i\Delta H + \mathcal{G})\rho \\ \mathcal{G} &= \sum_{m=0}^{M} \bar{L}_m \otimes L_m - \frac{1}{2} Id \otimes L_m^\dagger L_m - \frac{1}{2} \bar{L}_m^\dagger \bar{L}_m \otimes Id \end{aligned}$$

3 Effective Hamiltonian

3.1 Definition (H_{eff})

$$H_{eff} = H - i \sum_k \frac{\gamma_k}{2} L_k^\dagger L_k$$

The antiHermitian part is negative for $\gamma_k \geq 0$ in the sense of $L_k^\dagger L_k$ being a positive operator. In Lie algebraic terms $iH_{eff} \in \mathfrak{u}(n) \bigoplus \mathfrak{u}(n)^*_{\geq 0}$

3.2 Definition (Iwasawa Poisson-Lie Group) *The one coming from KAN decomposition. So for $GL(n,\mathbb{C})$ regarded as a real Lie group, we get Hermitian and antiHermitian split. TODO: Write down this parameterization.*

$$\frac{d\rho}{dt} = -i(H_{eff}\rho - \rho H_{eff}^{\dagger}) + \sum_k \gamma_k L_k \rho L_k^{\dagger}$$

4 Lie Parameterization

Let $\rho =$

5 Examples

5.1 Example

$$\begin{aligned} H &= 0 \\ \frac{\partial\rho}{\partial t} &= -i[0,\rho] + \frac{\gamma}{2}(\sigma_z\rho\sigma_z - \frac{1}{2}\{\sigma_z^{\dagger}\sigma_z,\rho\}) \end{aligned}$$

5.2 Example

$$\begin{aligned} H &= \hbar\omega a^{\dagger}a \\ L_1 &= a \\ \gamma_1 &= \frac{\gamma}{2}(\bar{n}+1) \\ L_2 &= a^{\dagger} \\ \gamma_2 &= \frac{\gamma}{2}(\bar{n}) \\ H_{eff} &= \hbar\omega a^{\dagger}a - i\frac{\gamma}{4}((\bar{n}+1)a^{\dagger}a + (\bar{n})aa^{\dagger}) \\ &= \hbar\omega a^{\dagger}a - i\frac{\gamma}{4}((2\bar{n}+1)a^{\dagger}a + \bar{n}) \end{aligned}$$

6 Fisher Information Norm of Lindblad Vector Field

The ordinary differential equation described above determines a vector field on the space of density matrices (M,g) where g is the Fisher information metric tensor. So one can evaluate $v(\rho)$ which is the direction of flow when starting at ρ. One can use g to give $g(v(\rho),v(\rho))$ which measures the length thereof. The actual dynamics is a trajectory for this vector field by solving for the flow ϕ_t. We want to calculate the length of the curve.

$$L = \int_0^T \sqrt{g(v(\rho(t)),v(\rho(t)))}dt$$

where $\rho(t) = \phi_t\rho(0)$ is the solution to the ordinary differential equation on M.

7 Hochschild Homology

https://books.google.com/books?id=_9UBCAAAQBAJ&pg=PA88&lpg=PA88&dq=lindblad+hochschild&source=bl&ots=HI7IaXfgEe&sig=kOu64rDabBZnV024_gnlgv0pcl4&hl=en&sa=X&ved=0ahUKEwictbSF7MTXAhUKj
v=onepage&q=lindblad%20hochschild&f=false

8 Quantum Chemistry

https://dash.harvard.edu/bitstream/handle/1/4646014/0807.0929v2.pdf?sequence=1

8.1 Unitary Evolution

https://arxiv.org/pdf/0905.0887.pdf

8.2 Open

For a particular small open system with known jump operators, can we come up with an auxiliary system to approximate the infinite dimensional bath Hilbert space in such a way that following the gates of "Towards quantum chemisty" on this auxiliary closed sytem problem implements the Lindblad evolution for the qubits $H_S = (\mathbb{C}^2)^d$?

Start with well studied H_2 molecule example. Decide what environment we should put it in...

Closest reference could find on this subject: https://www.pks.mpg.de/~eisfeld/wwww/pdf/Mostame_et_al_Emulation_of_open_quant_scqb.pdf