

# Group Rewriting

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## 1 Clifford Group

### 1.1 Pauli Group

**1.1 Definition ( $P_n$ )** *The Pauli Group on  $n$  qubits is the group generated by  $\sigma_x$   $\sigma_y$   $\sigma_z$  and  $i$  operating on each of  $n$  qubits. As a group it is isomorphic to  $(D_4 \rtimes C_2)^n$*

### 1.2 Clifford Group

**1.2 Definition ( $C_n$ )** *The normalizer of  $P_n$  in  $U(2^n)$ .*

**1.3 Lemma** *It is generated by the generators of  $P_n$  along with the following ?.*

### 1.3 Cliff+T

### 1.4 Theorem (Universality)

### 1.5 Theorem (Solovay-Kitaev)

As a corollary of universality we know that we can take an arbitrary rotation of one qubit  $SU(2)$  to an approximation using only Cliff+T gates. A desired error bound must be given. An algorithm for this is given by

<https://arxiv.org/pdf/1403.2975.pdf>

but it is faster than Solovay-Kitaev which is more general.

## 2 Coxeter Groups

2,3 qubit relations with weyl exceptional other fun groups

<https://hal.inria.fr/file/index/docid/420456/filename/E8Weyl.pdf>

<https://arxiv.org/pdf/0807.3650.pdf>

Rewriting system for coxeter group

[https://ac.els-cdn.com/0022404994900191/1-s2.0-0022404994900191-main.pdf?\\_tid=1f07609c-04b2-11acdnat=1517202550\\_f41a15336d6c1eb1d880008e0214ec41](https://ac.els-cdn.com/0022404994900191/1-s2.0-0022404994900191-main.pdf?_tid=1f07609c-04b2-11acdnat=1517202550_f41a15336d6c1eb1d880008e0214ec41)

Words with intervening property for a quick check of reducedness. Outputs a witness of nonreducedness. But no witness found does not imply reduced.

Given an expression and a coxeter graph, algorithm to find a segment of the word to show it is not reduced/null if this is inconclusive

Then feed that into the reducing algorithm. Repeat until intervening property says the expression might be reduced.

[http://emis.ams.org/journals/EJC/Volume\\_17/PDF/v17i1n9.pdf](http://emis.ams.org/journals/EJC/Volume_17/PDF/v17i1n9.pdf)

Algorithm for reducing words in Coxeter group

<https://mathoverflow.net/questions/109071/algorithm-for-reducing-words-in-a-coxeter-group>

### 3 Finite Complete Rewriting Systems

Surface groups but more importantly for here theorem about short exact sequences

<https://arxiv.org/pdf/math/9611205.pdf>

Rewriting system for coxeter group

[https://ac.els-cdn.com/0022404994900191/1-s2.0-0022404994900191-main.pdf?\\_tid=1f07609c-04b2-11acdnat=1517202550\\_f41a15336d6c1eb1d880008e0214ec41](https://ac.els-cdn.com/0022404994900191/1-s2.0-0022404994900191-main.pdf?_tid=1f07609c-04b2-11acdnat=1517202550_f41a15336d6c1eb1d880008e0214ec41)

Gap for all finitely presented groups

[http://doc.sagemath.org/html/en/reference/groups/sage/groups/finitely\\_presented.html](http://doc.sagemath.org/html/en/reference/groups/sage/groups/finitely_presented.html)

### 4 Unsorted

<https://arxiv.org/pdf/1509.02004.pdf>

<https://arxiv.org/pdf/1701.05200.pdf>