Analysis of continuous beam by force method

Input data

Span lengths - $\vec{l} = [4; 7; 3; 5] \cdot \text{ m} = [4\,\text{m} \;\; 7\,\text{m} \;\; 3\,\text{m} \;\; 5\,\text{m}]$

Number of spans - $n = \text{len}(\vec{l}) = 4$

Coordinates of supports - $\vec{x} = [4\,\text{m} \;\; 11\,\text{m} \;\; 14\,\text{m} \;\; 19\,\text{m}]$

Total beam length - $L = \vec{x}_4 = 19\,\text{m}$

Load - $q = 10\,\frac{\text{kN}}{\text{m}}$

Elastic modulus of the material - $E = 30\,\text{GPa}$

Cross section

Rectangular section with dimensions: $b = 250\,\text{mm}$, $h = 500\,\text{mm}$

Area - $A = b \cdot h = 125000\,\text{mm}^2$

Moment of inertia - $I = \frac{b \cdot h^3}{12} = 2604166667\,\text{mm}^4$

Shear area - $A_Q = \frac{5}{6} \cdot b \cdot h = 104167\,\text{mm}^2$

Solution

The solution will be performed by the force method with a primary system - simply supported beam with internal supports removed and replaced by unknown forces X_i

Bending moments

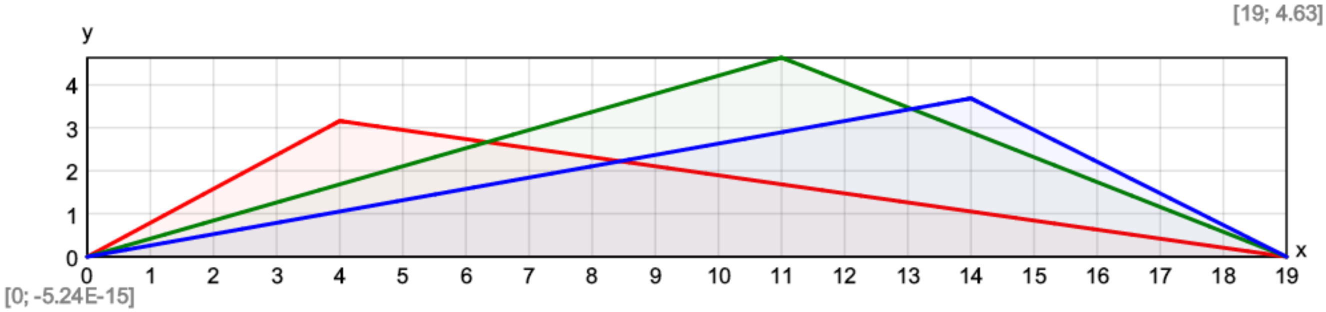
- in section *a* due to unit force at distance *x* from the beginning of the beam:

$$M_{1,max}(x) = \left(\frac{x}{L} - 1\right) \cdot x$$

$$M_{1,a}(a; x) = M_{1,max}(x) \cdot \begin{cases} if\ a < x: & \dfrac{a}{x} \\ else: & \dfrac{L - a}{L - x} \end{cases}$$

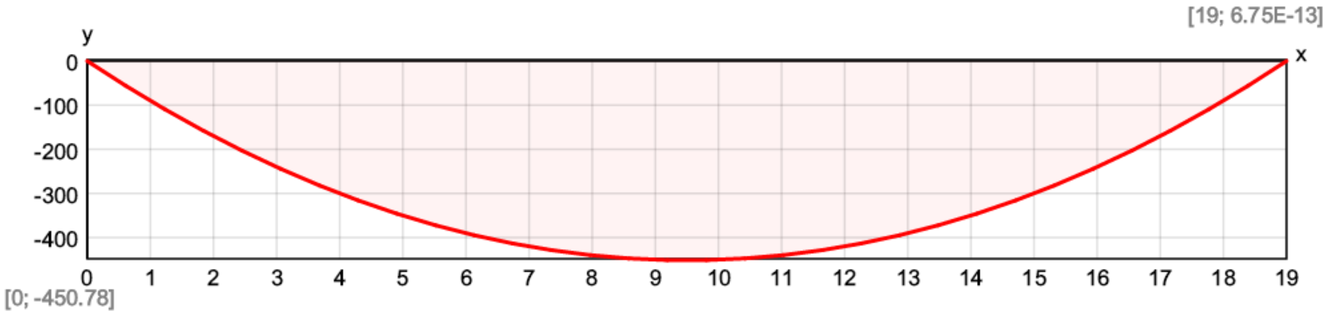
- in section *a*, due to unit force X_i:

$$M_1(a; i) = M_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$M_F(x) = \frac{q \cdot x}{2} \cdot (L - x)$$



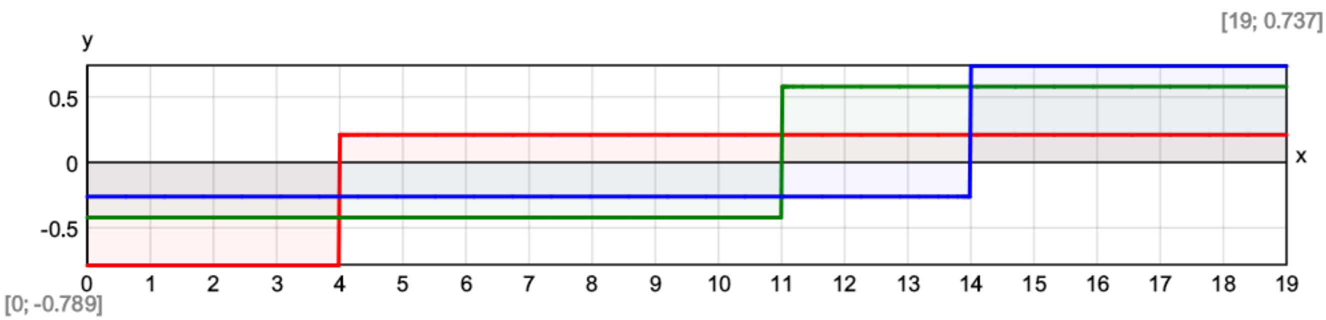
Shear forces

- in section *a* due to unit force at distance *x* from the beginning of the beam:

$$V_{1,a}(a; x) = \begin{cases} if\ a < x: & \frac{x}{L} - 1 \\ else: & \frac{x}{L} \end{cases}$$

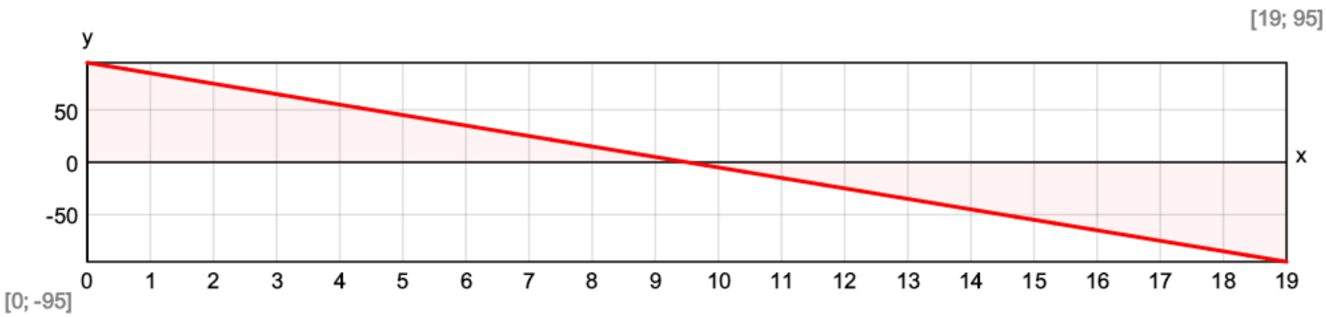
- in section *a*, due to unit force X_i:

$$V_1(a; i) = V_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$V_F(x) = q \cdot \left(\frac{L}{2} - x\right)$$



Number of unknowns by force method - $n_1 = n - 1 = 3$

Flexibility coefficients

$$\delta(i;j) = \int_{0\,\mathrm{m}}^{L} \frac{M_1(x;i) \cdot M_1(x;j)}{E \cdot I}\, dx + \int_{0\,\mathrm{m}}^{L} \frac{V_1(x;i) \cdot V_1(x;j)}{E \cdot A_Q}\, dx$$

$$\Delta_F(i) = \int_{0\,\mathrm{m}}^{L} \frac{M_1(x;i) \cdot M_F(x)}{E \cdot I}\, dx + \int_{0\,\mathrm{m}}^{L} \frac{V_1(x;i) \cdot V_F(x)}{E \cdot A_Q}\, dx$$

$$\delta = \mathrm{symmetric}(n_1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \vec{\Delta}_F = \mathrm{vector}(n_1) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\$Repeat\{\$Repeat\{\delta_{i,j} = \delta(i;j); i = 1...n_1\}; j = 1...n_1\} = 0.0011\,\mathrm{m/\,kN}$$

$$\$Repeat\{\vec{\Delta}_{F.i} = \Delta_F(i); i = 1...n_1\} = -0.161\,\mathrm{m}$$

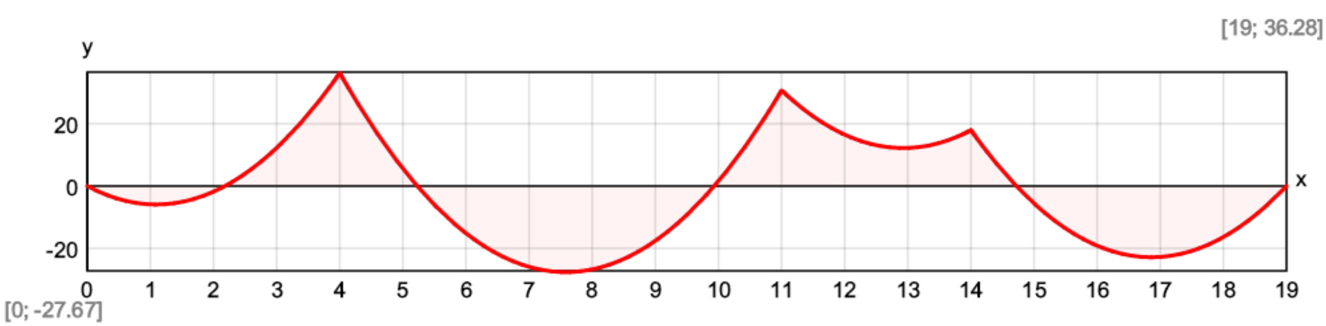
$$\delta = \begin{bmatrix} 0.000809\,\mathrm{m/\,kN} & 0.00101\,\mathrm{m/\,kN} & 0.000719\,\mathrm{m/\,kN} \\ 0.00101\,\mathrm{m/\,kN} & 0.00174\,\mathrm{m/\,kN} & 0.00133\,\mathrm{m/\,kN} \\ 0.000719\,\mathrm{m/\,kN} & 0.00133\,\mathrm{m/\,kN} & 0.0011\,\mathrm{m/\,kN} \end{bmatrix}, \vec{\Delta}_F = \begin{bmatrix} -0.135\,\mathrm{m} & -0.211\,\mathrm{m} & -0.161\,\mathrm{m} \end{bmatrix}$$

Calculation of the unknown forces X_i

$$\vec{X} = -\mathrm{clsolve}(\delta; \vec{\Delta}_F) = \begin{bmatrix} 64.9\,\mathrm{kN} & 53.47\,\mathrm{kN} & 39.33\,\mathrm{kN} \end{bmatrix}$$

Results

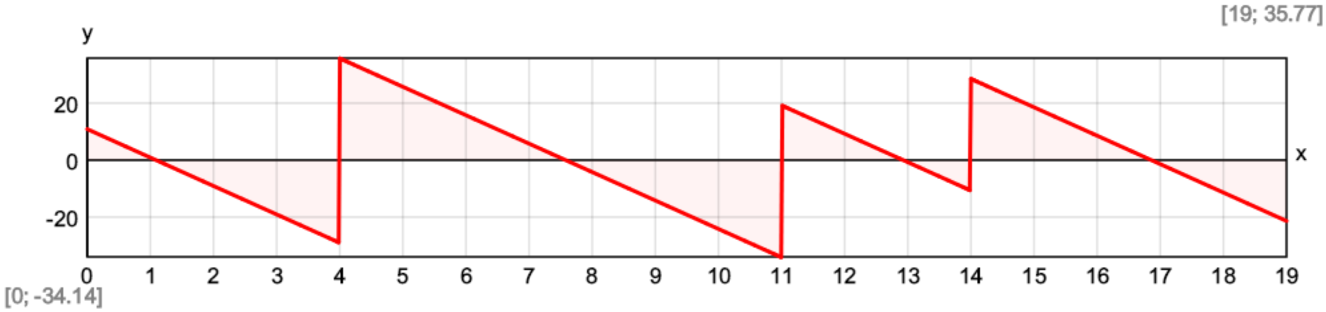
Bending moment diagram - $M(x) = M_F(x) + \sum_{i=1}^{n_1} M_1(x;i) \cdot X_i$



$$\vec{M}_{max} = \begin{bmatrix} 5.94\,\mathrm{kNm} & 27.69\,\mathrm{kNm} & -12.23\,\mathrm{kNm} & 22.91\,\mathrm{kNm} \end{bmatrix}$$

$\vec{M}_{min} = [-36.39\,\mathrm{kNm} \quad -30.79\,\mathrm{kNm} \quad -17.98\,\mathrm{kNm}]$

Shear force diagram - $V(x) = V_F(x) + \sum_{i=1}^{n_1} V_1(x;i) \cdot \vec{X}_i$



$\vec{V}_{max} = [10.9\,\mathrm{kN} \quad 35.8\,\mathrm{kN} \quad 19.27\,\mathrm{kN} \quad 28.6\,\mathrm{kN}]$

$\vec{V}_{min} = [-29.1\,\mathrm{kN} \quad -34.2\,\mathrm{kN} \quad -10.73\,\mathrm{kN} \quad -21.4\,\mathrm{kN}]$

Deflections

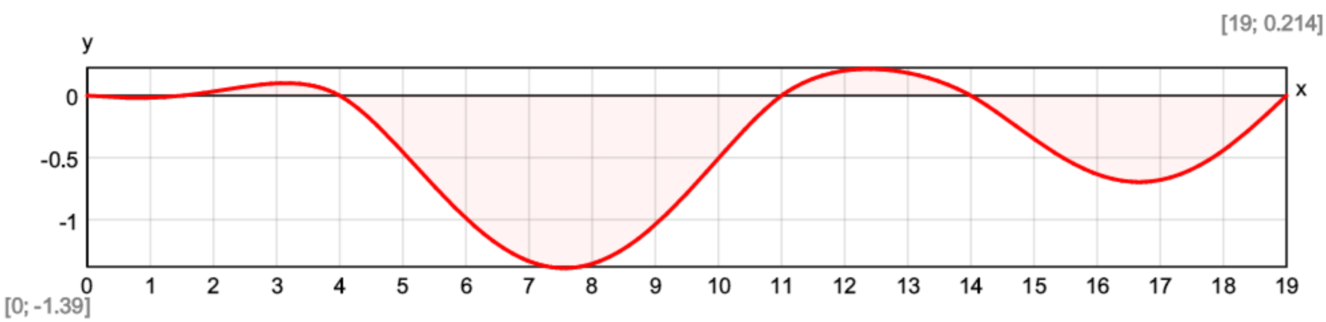
- in section *a*, due to unit force X_i:

$$d_1(a;i) = \int_{0\,\mathrm{m}}^{L} \frac{M_{1,a}(x;a) \cdot M_1(x;i)}{E \cdot I}\,dx + \int_{0\,\mathrm{m}}^{L} \frac{V_{1,a}(x;a) \cdot V_1(x;i)}{E \cdot A_Q}\,dx$$

- due to external loads in primary system:

$$d_F(a) = \int_{0\,\mathrm{m}}^{L} \frac{M_{1,a}(x;a) \cdot M_F(x)}{E \cdot I}\,dx + \int_{0\,\mathrm{m}}^{L} \frac{V_{1,a}(x;a) \cdot V_F(x)}{E \cdot A_Q}\,dx$$

$$d(x) = d_F(x) + \sum_{i=1}^{n_1} d_1(x;i) \cdot \vec{X}_i$$

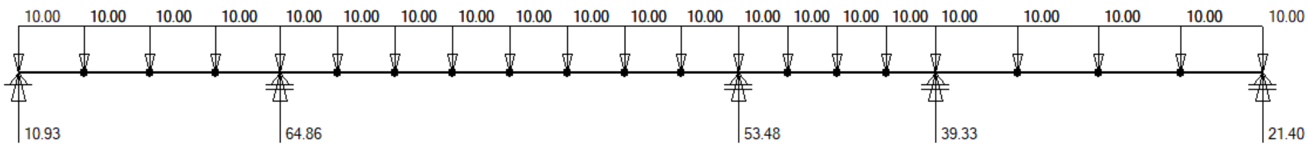


Maximum deflection - $d_{max} = \$Inf\{d(x); x \in [0\,\mathrm{m}; L]\} = -1.39\,\mathrm{mm}$

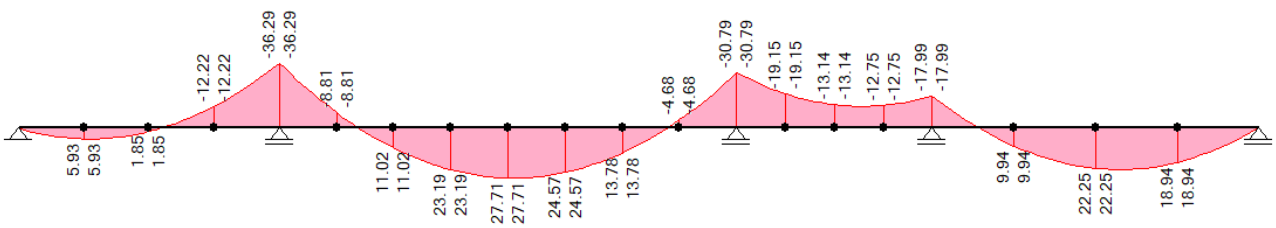
At a distance from the origin - $x_{inf} = 7.56\,\mathrm{m}$

Comparison with Stadyps 6.0 structural analysis software

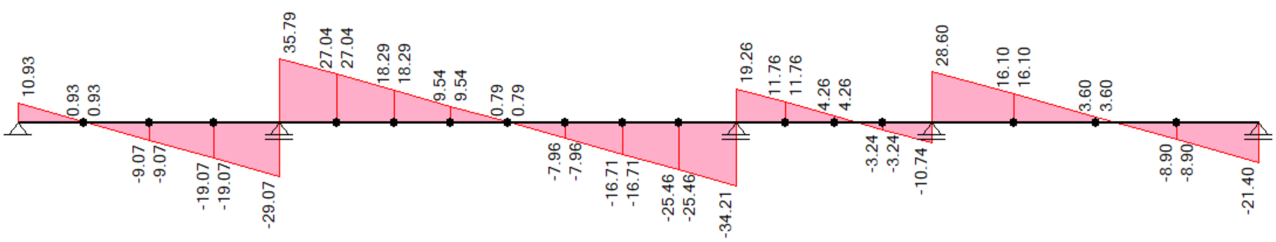
Loads and support reactions, kN



Bending moments, kNm



Shear forces, kN



Deflections, mm