

# Modal analysis of steel pole with arbitrary, variable cross-section along height

## Static scheme

Cantilever with length -  $L = 110$  m

## Material

### Steel

Elastic modulus -  $E = 206$  GPa      Poisson's ratio -  $\nu = 0.3$

Shear modulus -  $G = \frac{E}{2 \cdot (1 + \nu)} = \frac{206 \text{ GPa}}{2 \cdot (1 + 0.3)} = 79.23$  GPa

Mass density -  $\gamma_s = 7.85 \frac{\text{t}}{\text{m}^3}$

## Cross-section

Section height:

- at bottom -  $h_b = 3000$  mm
- at top -  $h_t = 750$  mm
- difference -  $\Delta h = h_t - h_b = 750 \text{ mm} - 3000 \text{ mm} = -2250$  mm
- as a function of distance to base -  $h(x) = h_b + \frac{\Delta h \cdot x}{L}$

Section width:

- at bottom -  $w_b = 3000$  mm
- at top -  $w_t = 750$  mm
- difference -  $\Delta w = w_t - w_b = 750 \text{ mm} - 3000 \text{ mm} = -2250$  mm
- as a function of distance to base -  $w(x) = w_b + \frac{\Delta w \cdot x}{L}$

Cross-section shape - **Elliptical tube**

Thickness -  $t_b = 20$  mm ,  $t_t = 9$  mm,  $t(x) = t_b + \frac{(t_t - t_b) \cdot x}{L}$

Function of cross-section outline

$$b_i(x; z) = w(x) - 2 \cdot t(x) \quad h_i(x; z) = h(x) - 2 \cdot t(x)$$

$$b_1(x; z) = \frac{2 \cdot w(x)}{h(x)} \cdot \sqrt{\left(\frac{h(x)}{2}\right)^2 - \left(z - \frac{h(x)}{2}\right)^2}$$

$$b_2(x; z) = \begin{cases} \text{if } (z > t(x)) \cdot (z < h(x) - t(x)): & \frac{2 \cdot b_i(x; z)}{h_i(x; z)} \cdot \sqrt{\left(\frac{h_i(x; z)}{2}\right)^2 - \left(z - \frac{h(x)}{2}\right)^2} \\ \text{else:} & 0 \text{ m} \end{cases}$$

$$b(x; z) = b_1(x; z) - b_2(x; z)$$

## Cross section properties

$$\text{Area} - A(x) = \int_{0 \text{ mm}}^{h(x)} b_1(x; z) dz - \left( \int_{t(x)}^{h(x)-t(x)} b_2(x; z) dz \right)$$

$$\text{First moment of area} - S(x) = \int_{0 \text{ mm}}^{h(x)} b_1(x; z) \cdot z dz - \left( \int_{t(x)}^{h(x)-t(x)} b_2(x; z) \cdot z dz \right)$$

$$\text{Geometrical center} - z_c(x) = \frac{S(x)}{A(x)}$$

Second moment of area

$$I_1(x) = \int_{0 \text{ mm}}^{h(x)} b_1(x; z) \cdot (z - z_c(x))^2 dz \quad I_2(x) = \int_{t(x)}^{h(x)-t(x)} b_2(x; z) \cdot (z - z_c(x))^2 dz$$

$$I(x) = I_1(x) - I_2(x)$$

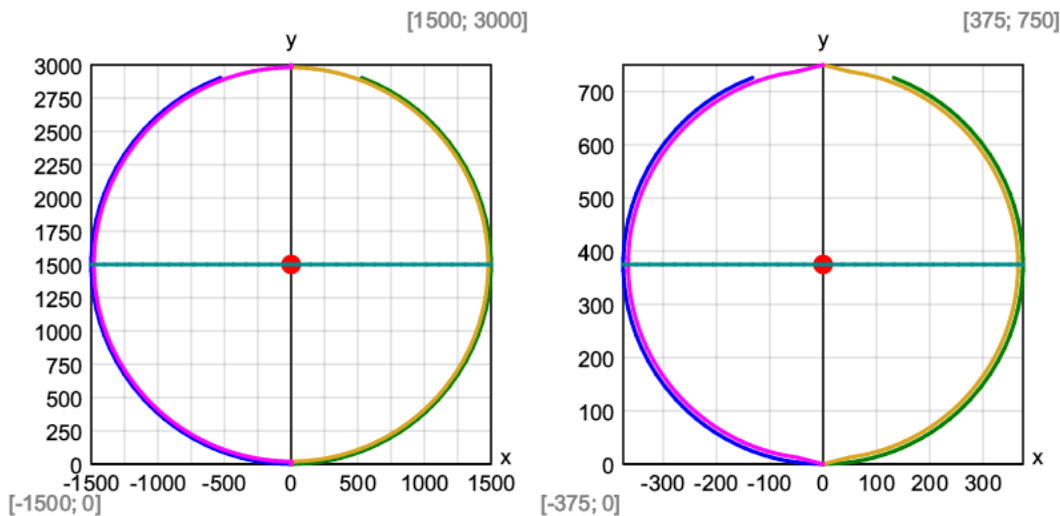
First moment of area under z

$$S_1(x; z) = \int_{\delta}^z b_1(x; \zeta) \cdot (z_c(x) - \zeta) d\zeta \quad S_2(x; z) = \int_{t(x)+\delta}^z b_2(x; \zeta) \cdot (z_c(x) - \zeta) d\zeta$$

Shear area

$$A_{Q1}(x) = \frac{I_1(x)^2}{\int_{\delta}^{h(x)-\delta} \frac{S_1(x; z)^2}{b_1(x; z)} dz} \quad A_{Q2}(x) = \frac{I_2(x)^2}{\int_{t(x)+\delta}^{h(x)-t(x)-\delta} \frac{S_2(x; z)^2}{b_2(x; z)} dz}$$

$$A_Q(x) = A_{Q1}(x) - A_{Q2}(x)$$



## Mass

Distributed mass along height -  $m(x) = A(x) \cdot \gamma_s$

## Solution

Number of nodes -  $n_j = 11$

Length of one segment -  $\Delta x = \frac{L}{n_j} = \frac{110 \text{ m}}{11} = 10 \text{ m}$

Elevation of node  $j$  -  $x(j) = \Delta x \cdot j$

Bending due to horizontal force  $F_j = 1$  at node  $j$  -  $M(j; x) = \max(x(j) - x; 0 \text{ m})$

Flexibility matrix

$$D(i; j) = \left( \int_{0 \text{ m}}^L \frac{M(i; x) \cdot M(j; x)}{I(x)} dx \right) \cdot \frac{1}{E} + \left( \int_{0 \text{ m}}^L \frac{1}{A_Q(x)} dx \right) \cdot \frac{1}{G}$$

$$D = \begin{bmatrix} 0.00911 & 0.0219 & 0.0346 & 0.0474 & 0.0601 & 0.0729 & 0.0856 & 0.0984 & 0.11 & 0.12 & 0.14 \\ 0.0219 & 0.0722 & 0.13 & 0.18 & 0.24 & 0.3 & 0.35 & 0.41 & 0.47 & 0.52 & 0.58 \\ 0.0346 & 0.13 & 0.26 & 0.4 & 0.54 & 0.68 & 0.82 & 0.96 & 1.1 & 1.24 & 1.38 \\ 0.0474 & 0.18 & 0.4 & 0.67 & 0.95 & 1.23 & 1.51 & 1.79 & 2.07 & 2.35 & 2.63 \\ 0.0601 & 0.24 & 0.54 & 0.95 & 1.43 & 1.93 & 2.42 & 2.92 & 3.41 & 3.91 & 4.41 \\ 0.0729 & 0.3 & 0.68 & 1.23 & 1.93 & 2.73 & 3.55 & 4.37 & 5.2 & 6.02 & 6.84 \\ 0.0856 & 0.35 & 0.82 & 1.51 & 2.42 & 3.55 & 4.83 & 6.15 & 7.46 & 8.78 & 10.09 \\ 0.0984 & 0.41 & 0.96 & 1.79 & 2.92 & 4.37 & 6.15 & 8.16 & 10.22 & 12.29 & 14.36 \\ 0.11 & 0.47 & 1.1 & 2.07 & 3.41 & 5.2 & 7.46 & 10.22 & 13.38 & 16.63 & 19.87 \\ 0.12 & 0.52 & 1.24 & 2.35 & 3.91 & 6.02 & 8.78 & 12.29 & 16.63 & 21.66 & 26.86 \\ 0.14 & 0.58 & 1.38 & 2.63 & 4.41 & 6.84 & 10.09 & 14.36 & 19.87 & 26.86 & 35.2 \end{bmatrix} \frac{\text{mm}}{\text{kN}}$$

Mass matrix

$$d_{M,j} = \int_{x(j) - \frac{\Delta x}{2}}^{x(j) + \frac{\Delta x}{2}} m(x) dx = 1.06 \text{ t}$$

$$M = \begin{bmatrix} 13.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 11.43 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.94 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8.55 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.26 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.08 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Total mass of structure -  $M_{tot} = \text{sum}(\vec{d}_M) = 71.62 \text{ t}$

Eigenvalues

$$M_{sq} = \sqrt{M} = \begin{bmatrix} 3.61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.38 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.53 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.95 \end{bmatrix}$$

$$C = \text{copy}(M_{sq} \cdot D \cdot M_{sq}; \text{symmetric}(n_J); 1; 1) = \text{copy}(M_{sq} \cdot D \cdot M_{sq}; \text{symmetric}(11); 1; 1)$$

$$= \begin{bmatrix} 0.12 & 0.27 & 0.39 & 0.5 & 0.58 & 0.65 & 0.69 & 0.71 & 0.71 & 0.68 & 0.47 \\ 0.27 & 0.82 & 1.37 & 1.82 & 2.19 & 2.47 & 2.66 & 2.77 & 2.78 & 2.69 & 1.86 \\ 0.39 & 1.37 & 2.59 & 3.7 & 4.6 & 5.29 & 5.79 & 6.07 & 6.14 & 5.98 & 4.14 \\ 0.5 & 1.82 & 3.7 & 5.73 & 7.48 & 8.86 & 9.85 & 10.46 & 10.68 & 10.48 & 7.3 \\ 0.58 & 2.19 & 4.6 & 7.48 & 10.39 & 12.79 & 14.58 & 15.73 & 16.25 & 16.09 & 11.29 \\ 0.65 & 2.47 & 5.29 & 8.86 & 12.79 & 16.57 & 19.54 & 21.57 & 22.62 & 22.66 & 16.03 \\ 0.69 & 2.66 & 5.79 & 9.85 & 14.58 & 19.54 & 24.11 & 27.47 & 29.43 & 29.95 & 21.43 \\ 0.71 & 2.77 & 6.07 & 10.46 & 15.73 & 21.57 & 27.47 & 32.67 & 36.13 & 37.57 & 27.31 \\ 0.71 & 2.78 & 6.14 & 10.68 & 16.25 & 22.62 & 29.43 & 36.13 & 41.73 & 44.85 & 33.37 \\ 0.68 & 2.69 & 5.98 & 10.48 & 16.09 & 22.66 & 29.95 & 37.57 & 44.85 & 50.54 & 39 \\ 0.47 & 1.86 & 4.14 & 7.3 & 11.29 & 16.03 & 21.43 & 27.31 & 33.37 & 39 & 31.82 \end{bmatrix}$$

$$\vec{\lambda} = \text{reverse}(\text{last}(\text{eigenvals}(C \cdot 10^{-3}); 3)) = [0.19 \ 0.0172 \ 0.00339]$$

Natural frequencies

$$\vec{\omega} = \sqrt{\frac{1}{\vec{\lambda}}} = [2.27 \ 7.61 \ 17.18]$$

Vibration frequencies

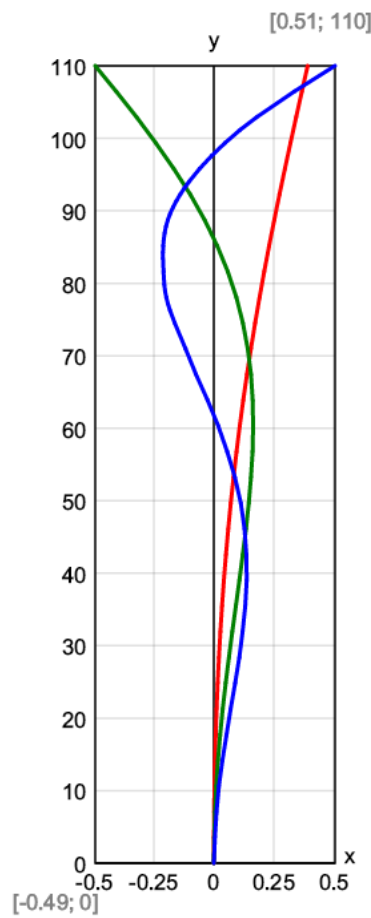
$$\vec{f} = \frac{\vec{\omega}}{2 \cdot \pi} \cdot \text{Hz} = \frac{\vec{\omega}}{2 \cdot 3.14} \cdot \text{Hz} = [0.36 \text{ Hz} \ 1.21 \text{ Hz} \ 2.73 \text{ Hz}]$$

Vibration periods

$$\vec{T} = \frac{1}{\vec{f}} = [2.77 \text{ s} \ 0.83 \text{ s} \ 0.37 \text{ s}]$$

Mode shapes

$$\begin{aligned}
 \Phi &= \text{inverse}(M_{sq}) \cdot \text{extract}_{\text{cols}}(\text{eigenvecs}(C \cdot 10^{-3}); \text{range}(n_j; n_j - 2; -1)) \\
 &= \text{inverse}(M_{sq}) \cdot \text{extract}_{\text{cols}}(\text{eigenvecs}(C \cdot 10^{-3}); \text{range}(11; 11 - 2; -1)) \\
 &= \begin{bmatrix} 0.00249 & 0.00878 & 0.0192 \\ 0.0103 & 0.0331 & 0.0641 \\ 0.024 & 0.0693 & 0.11 \\ 0.0443 & 0.11 & 0.14 \\ 0.0718 & 0.15 & 0.11 \\ 0.11 & 0.16 & 0.0221 \\ 0.15 & 0.15 & -0.11 \\ 0.2 & 0.0757 & -0.21 \\ 0.26 & -0.0587 & -0.18 \\ 0.32 & -0.26 & 0.0684 \\ 0.39 & -0.49 & 0.51 \end{bmatrix}
 \end{aligned}$$



Comparison with ASCE SEI 7/22

(C26.11-13)

$$\lambda_1 = 1.9 \cdot \left( \exp \left( -4 \cdot \frac{h_t}{h_b} \right) \right) + \frac{6.65}{0.9 + \left( \frac{t_t}{t_b} \right)^{0.67}} = 1.9 \cdot \left( \exp \left( -4 \cdot \frac{750 \text{ mm}}{3000 \text{ mm}} \right) \right) + \frac{6.65}{0.9 + \left( \frac{9 \text{ mm}}{20 \text{ mm}} \right)^{0.67}} = 5.17$$

Fundamental natural frequency

(C26.11-12)

$$n_1 = \frac{\lambda_1}{2 \cdot \pi \cdot L^2} \cdot \sqrt{\frac{E \cdot I(0\text{ m})}{m(0\text{ m})}} = \frac{5.17}{2 \cdot 3.14 \cdot (110\text{ m})^2} \cdot \sqrt{\frac{206\text{ GPa} \cdot I(0\text{ m})}{m(0\text{ m})}} = 0.37\text{ Hz}$$

Fundamental period of vibrations

$$T_1 = \frac{1}{n_1} = \frac{1}{0.37\text{ Hz}} = 2.72\text{ s}$$

## Comparison with SAP 2000 structural analysis software

Input data:

### STATIC LOAD CASES

STATIC CASE	CASE TYPE	SELF WT FACTOR
LOAD1	DEAD	0.0000

### JOINT DATA

JOINT	GLOBAL-X	GLOBAL-Y	GLOBAL-Z	RESTRAINTS	ANGLE-A	ANGLE-B	ANGLE-C
1	0.00000	0.00000	0.00000	1 1 1 1 1 1	0.000	0.000	0.000
2	0.00000	0.00000	10.00000	0 0 0 0 0 0	0.000	0.000	0.000
3	0.00000	0.00000	20.00000	0 0 0 0 0 0	0.000	0.000	0.000
4	0.00000	0.00000	30.00000	0 0 0 0 0 0	0.000	0.000	0.000
5	0.00000	0.00000	40.00000	0 0 0 0 0 0	0.000	0.000	0.000
6	0.00000	0.00000	50.00000	0 0 0 0 0 0	0.000	0.000	0.000
7	0.00000	0.00000	60.00000	0 0 0 0 0 0	0.000	0.000	0.000
8	0.00000	0.00000	70.00000	0 0 0 0 0 0	0.000	0.000	0.000
9	0.00000	0.00000	80.00000	0 0 0 0 0 0	0.000	0.000	0.000
10	0.00000	0.00000	90.00000	0 0 0 0 0 0	0.000	0.000	0.000
11	0.00000	0.00000	100.00000	0 0 0 0 0 0	0.000	0.000	0.000
12	0.00000	0.00000	110.00000	0 0 0 0 0 0	0.000	0.000	0.000

### FRAME ELEMENT DATA

FRAME	JNT-1	JNT-2	SECTION	ANGLE	RELEASES	SEGMENTS	R1	R2	FACTOR	LENGTH
13	1	2	VAR01	0.000	000000	2	0.000	0.000	1.000	10.000
14	2	3	VAR02	0.000	000000	2	0.000	0.000	1.000	10.000
15	3	4	VAR03	0.000	000000	2	0.000	0.000	1.000	10.000
16	4	5	VAR04	0.000	000000	2	0.000	0.000	1.000	10.000
17	5	6	VAR05	0.000	000000	2	0.000	0.000	1.000	10.000
18	6	7	VAR06	0.000	000000	2	0.000	0.000	1.000	10.000
19	7	8	VAR07	0.000	000000	2	0.000	0.000	1.000	10.000
20	8	9	VAR08	0.000	000000	2	0.000	0.000	1.000	10.000
21	9	10	VAR09	0.000	000000	2	0.000	0.000	1.000	10.000
22	10	11	VAR10	0.000	000000	2	0.000	0.000	1.000	10.000
23	11	12	VAR11	0.000	000000	2	0.000	0.000	1.000	10.000

### MATERIAL PROPERTY DATA

MAT LABEL	MODULUS OF ELASTICITY	POISSON'S RATIO	THERMAL COEFF	WEIGHT PER UNIT VOL	MASS PER UNIT VOL
STEEL	206000000	0.300	1.120E-05	78.500	7.850
CONC	24821128.4	0.200	9.900E-06	23.562	2.401
OTHER	24821128.4	0.200	9.900E-06	23.562	2.401

# FRAME SECTION PROPERTY DATA

SECTION LABEL	MAT LABEL	SECTION TYPE	DEPTH	FLANGE WIDTH TOP	FLANGE THICK TOP	WEB THICK	FLANGE WIDTH BOTTOM	FLANGE THICK BOTTOM
P00	STEEL		3.000	0.000	0.000	2.000E-02	0.000	0.000
P01	STEEL		2.795	0.000	0.000	1.900E-02	0.000	0.000
P02	STEEL		2.591	0.000	0.000	1.800E-02	0.000	0.000
P03	STEEL		2.386	0.000	0.000	1.700E-02	0.000	0.000
P04	STEEL		2.182	0.000	0.000	1.600E-02	0.000	0.000
P05	STEEL		1.977	0.000	0.000	1.500E-02	0.000	0.000
P06	STEEL		1.773	0.000	0.000	1.400E-02	0.000	0.000
P07	STEEL		1.568	0.000	0.000	1.300E-02	0.000	0.000
P08	STEEL		1.364	0.000	0.000	1.200E-02	0.000	0.000
P09	STEEL		1.159	0.000	0.000	1.100E-02	0.000	0.000
P10	STEEL		0.955	0.000	0.000	1.000E-02	0.000	0.000
P11	STEEL		0.750	0.000	0.000	9.000E-03	0.000	0.000

# FRAME SECTION PROPERTY DATA

SECTION LABEL	AREA	TORSIONAL INERTIA	MOMENTS OF INERTIA I33	MOMENTS OF INERTIA I22	SHEAR AREAS A2	SHEAR AREAS A3
P00	0.187	0.416	0.208	0.208	9.462E-02	9.462E-02
P01	0.166	0.319	0.160	0.160	8.375E-02	8.375E-02
P02	0.145	0.241	0.120	0.120	7.356E-02	7.356E-02
P03	0.127	0.178	8.876E-02	8.876E-02	6.398E-02	6.398E-02
P04	0.109	0.128	6.385E-02	6.385E-02	5.508E-02	5.508E-02
P05	9.246E-02	8.898E-02	4.449E-02	4.449E-02	4.679E-02	4.679E-02
P06	7.736E-02	5.985E-02	2.992E-02	2.992E-02	3.917E-02	3.917E-02
P07	6.351E-02	3.839E-02	1.920E-02	1.920E-02	3.217E-02	3.217E-02
P08	5.097E-02	2.329E-02	1.165E-02	1.165E-02	2.584E-02	2.584E-02
P09	3.967E-02	1.307E-02	6.536E-03	6.536E-03	2.014E-02	2.014E-02
P10	2.969E-02	6.629E-03	3.314E-03	3.314E-03	1.509E-02	1.509E-02
P11	2.095E-02	2.876E-03	1.438E-03	1.438E-03	1.068E-02	1.068E-02

# FRAME SECTION PROPERTY DATA

SECTION LABEL	SECTION MODULII S33	SECTION MODULII S22	PLASTIC MODULII Z33	PLASTIC MODULII Z22	RADII OF GYRATION R33	RADII OF GYRATION R22
P00	0.139	0.139	0.178	0.178	1.054	1.054
P01	0.114	0.114	0.146	0.146	0.981	0.981
P02	9.295E-02	9.295E-02	0.119	0.119	0.910	0.910
P03	7.440E-02	7.440E-02	9.541E-02	9.541E-02	0.838	0.838
P04	5.853E-02	5.853E-02	7.507E-02	7.507E-02	0.766	0.766
P05	4.501E-02	4.501E-02	5.774E-02	5.774E-02	0.694	0.694
P06	3.375E-02	3.375E-02	4.332E-02	4.332E-02	0.622	0.622
P07	2.449E-02	2.449E-02	3.144E-02	3.144E-02	0.550	0.550
P08	1.708E-02	1.708E-02	2.194E-02	2.194E-02	0.478	0.478
P09	1.128E-02	1.128E-02	1.450E-02	1.450E-02	0.406	0.406
P10	6.941E-03	6.941E-03	8.931E-03	8.931E-03	0.334	0.334
P11	3.835E-03	3.835E-03	4.942E-03	4.942E-03	0.262	0.262

## Results:

### ASSEMBLED JOINT MASSES IN GLOBAL COORDINATES

JOINT	UX	UY	UZ	RX	RY	RZ
1	7.067329	7.067329	7.067329	.000000	.000000	.000000
2	13.024974	13.024974	13.024974	.000000	.000000	.000000
3	11.437717	11.437717	11.437717	.000000	.000000	.000000
4	9.949356	9.949356	9.949356	.000000	.000000	.000000
5	8.562760	8.562760	8.562760	.000000	.000000	.000000
6	7.275225	7.275225	7.275225	.000000	.000000	.000000
7	6.089295	6.089295	6.089295	.000000	.000000	.000000
8	5.002586	5.002586	5.002586	.000000	.000000	.000000
9	4.017318	4.017318	4.017318	.000000	.000000	.000000
10	3.131435	3.131435	3.131435	.000000	.000000	.000000
11	2.346830	2.346830	2.346830	.000000	.000000	.000000
12	0.936644	0.936644	0.936644	.000000	.000000	.000000

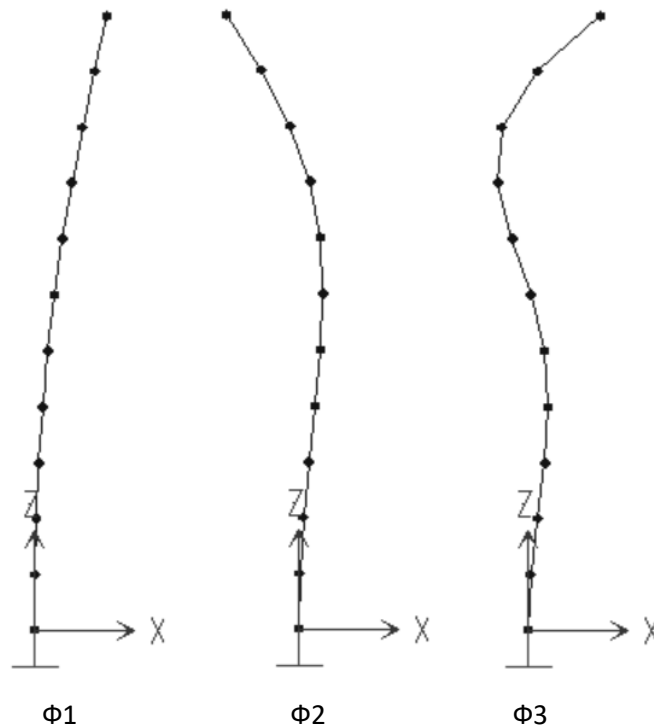
### TOTAL ASSEMBLED JOINT MASSES IN GLOBAL COORDINATES

	UX	UY	UZ	RX	RY	RZ
TOTAL	78.841467	78.841467	78.841467	.000000	.000000	.000000

### MODAL PERIODS AND FREQUENCIES

MODE	PERIOD (TIME)	FREQUENCY (CYC/TIME)	FREQUENCY (RAD/TIME)	EIGENVALUE (RAD/TIME)**2
1	2.784527	0.359127	2.256464	5.091629
2	0.831336	1.202883	7.557935	57.122388
3	0.369511	2.706277	17.004037	289.137267

## Modal shapes





Differences:

First mode -  $\delta_1 = (2.773 - 2.785)/2.773 = 0.43\%$

Second mode -  $\delta_2 = (0.826 - 0.831)/0.826 = 0.60\%$

Third mode -  $\delta_3 = (0.367 - 0.370)/0.367 = 0.81\%$

Convergence of solution in dependance of mesh density:

Number of nodes	Period, s			Difference, %		
	$\Phi 1$	$\Phi 2$	$\Phi 3$	$\Phi 1$	$\Phi 2$	$\Phi 3$
3	3,02	1,06	0,440	9,42%	30,86%	24,65%
6	2,82	0,870	0,401	2,17%	7,41%	13,60%
12	2,77	0,824	0,365	0,36%	1,73%	3,40%
24	2,76	0,813	0,356	0,00%	0,37%	0,85%
48	2,76	0,810	0,353	0,00%	0,00%	0,00%
96	2,76	0,810	0,353	0,00%	0,00%	0,00%
SAP (11)	2,79	0,831	0,370	0,91%	2,59%	4,82%

Plot of convergence vs number of nodes:

