The Landé g-factor
Experimentally Determined for Atomic Mercury

Jacob Thompson

May 19, 2020

I Introduction

When an atom with a well defined quantum number and energy level is placed in a strong external magnetic field, the energy level will split into a larger number of levels [3, 2, 1]. This also splits the spectral lines associated with the specific energy level in an effect known as the Zeeman effect [2]. Unless an atom is in the ${}^1S_0$ state it will have some net magnetic dipole moment [1]. This interacts with the magnetic field and has some potential energy associated with its orientation creating the discrete energy levels. These energy levels are quantized meaning angular momentum is also quantized, as theorized by the solution to the Schrodinger equation. Therefore, observing the spectral lines due to the Zeeman efffect provides experimental support for the quantization of angular momentum.

The potential energy due to the orientation can also be related to the total angular momentum of the atom. The geometric factor known as the Landé g factor g_J directly relates the eigenvalues $\hbar M_\mathrm{J}$ of the angular momentum operator to the change in energy of the atom [3]. Therefore, the Landé g factor can be determined experimentally through the measurement of the difference in energy between the now non-degenerate energy states. I will be determining a value for the Landé g factor for neutral mercury undergoing a transition from the ${}^3P_1$ excited state to the ${}^3P_0$ ground state.

The magnetic field B places a torque on the total atomic magnetic moment μ. This creates the change in the potential energy ΔE observed in the Zeeman effect. This change is given by [1, 2, 3]

$$\Delta E = -\vec{\mu} \cdot \vec{B}. \tag{1}$$

Both the spin angular momentum and the orbital angular momentum contribute to the total angular momentum and therefore to the net magnetic moment of an atom. This net magnetic moment can be related to the total angular momentum $\vec{J}$ via [1]

$$\mu_\mathrm{J} = -g_\mathrm{J} \frac{e}{2m_\mathrm{e}} \vec{J}, \tag{2}$$

where e is the electron charge and m_e is the electron mass. $\vec{J}$ is the total angular momentum vector and has eigenvalues $\hbar M_\mathrm{J}$. A neutral mercury atom in its ground states has its energy levels $n = 1, 2, 3, 4, 5$ completely filled, and although there is no net magnetic moment to the atom, these angular momenta must still be accounted for. This gives rise to the Landé g factor g_{J} seen in Eq. (2). Relating the total angular momentum vector to its eigenvalues in Eq. (2) and combining it with Eq. (1) yields [1]

$$\Delta E = g_{\mathrm{J}} \mu_{\mathrm{B}} B M_{\mathrm{J}}, \tag{3}$$

where $\mu_{\mathrm{B}} = e\hbar/2m_{\mathrm{e}} = 9.271024$ J/T and is referred to as the Bohr magneton. This can be rearranged to yield

$$g_{\mathrm{J}} = \frac{\Delta E}{\mu_{\mathrm{B}} B M_{\mathrm{J}}}. \tag{4}$$

The wavelength λ of the photon emitted as an electron drops from an excited state will be measured. Since the energy of this photon is the same as the energy difference between the initial and ground state of the atom, the energy of the initial state is $E = hc/\lambda$. Therefore the energy difference between two distinct energy states ΔE is given by [3]

$$\Delta E = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_2}. \tag{5}$$

Using Eq. (5), Eq. (4) and spectroscopy data for atomic mercury transitions from the ${}^3P_1$ excited state to the ${}^3P_0$ ground state, I will determine a value for the Landé g factor.

II Instruments and Data Source

To collect spectral data for atomic mercury undergoing transitions between the ${}^3P_1$ excited state and the ${}^3P_0$ ground state a mercury lamp was used. It was shone into a spectrometer, likely an older 1250M model. Within the spectrometer the light from the mercury lamp reflects of curved mirror M1 which directs it to a diffraction grating G. The light from the diffraction grating is then reflected off two more mirrors M2 and M3 before being detected by a photodetector. The output from this detector is sent to a computer. The mercury lamp is surrounded by an electromagnet whose strength can be adjusted based on the strength of the current running through it. A detailed schematic of a setup generally used for this type of data collection is shown in Fig. 1. The data sent to the computer are stored as ordered pairs of a specific wavelength of light, and the ratemeter voltage associated with it. The ratemeter voltage corresponds to the intensity of that particular wavelength. Therefore, if the ordered pairs were plotted, the spectral lines from the mercury deexcitation would show up as peaks on the plot.

Spectral data were collected for mercury transitions from the ${}^3P_1$ excited state to the ${}^3P_0$ ground state, under varying magnetic field strengths. The strength of the field is dependent on the current running through the electromagnet, and is measured to a precision of 0.1 T. The measurements of field strength, however are reported to four decimal places. Spectroscopy data were taken for 6 different field strengths varying from $B = 0$ T to $B = 2.9399 \pm .1$ T. These data were saved as .CSV files and made available online. Table I shows the six electromagnet currents used, and the corresponding magnetic field strength due to the current.

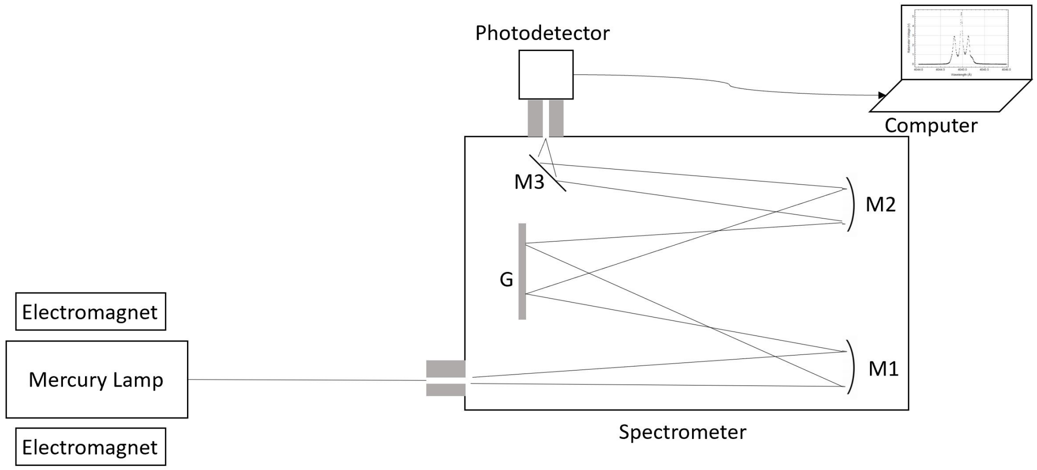


Fig. 1. Schematic for experiments involving the use of a spectrometer to measure the wavelength of light released due to mercury deexcitation. A mercury lamp is shone at the entrance of a spectrometer. An electromagnet surrounds the lamp and generates a magnetic field. The field strength can be adjusted by adjusting the current flowing through the electromagnet. The schematic shows an older 1250M spectrometer. The light from the lamp enters the spectrometer through an adjustable entrance slit where it is reflected off a curved mirror M1 toward a diffraction grating G. The light reflects off the grating and back to another curved mirror M2. The light is then reflected off mirror M3 which directs it to a lateral adjustable exit slit. A photo-multiplier tube is mounted as a photodetector to this exit slit. The output from this photodetector is sent to a computer to record the necessary data.

Table I. The magnetic field strength generated by an electromagnet. The first column shows the current in amps running through the electromagnet. The second column shows the corresponding magnetic field strength B given in teslas. The field strength is not linearly related to the current. It is also a measured value reported to four decimal places of precision, however the uncertainty is given as .1 T.

Current A	B T
0	0 $\pm$.1
5	1.1079 $\pm$.1
10	1.8475 $\pm$.1
15	2.3160 $\pm$.1
25	2.7531 $\pm$.1
40	2.9399 $\pm$.1

III Data Presentation and Analysis

The six data files for mercury spectra were downloaded. These data were taken for six different magnetic field strengths corresponding to six different electric currents powering the electromagnet. There is a small offset in the absolute wavelength reported. This is likely due to the equipment not being calibrated accurately. For this reason I corrected the data to display the true wavelength. This was done by plotting all six data sets on separate plots with wavelength in angstroms on the x-axis, and ratemeter voltage in volts on the y-axis. I then magnified the central peak of each of the plots. This central peak corresponds to the $M_{\mathrm{J}} = 0$ transition and exists even in the absence of a magnetic field. I determined the wavelength corresponding to the point where the peak crests to the nearest thousandth of an angstrom. The wavelengths of these peaks were taken to be exact as the uncertainty is on the order of one hundredth of an angstrom (if not smaller) and will not contribute to the uncertainty of g_{J} in a statistically significant way. This will become even more apparent when I discuss the method of calculating the uncertainty of g_{J}. The wavelengths of these six peaks were averaged. The NIST value for the wavelengths of a photon released from this transition is 4046.5650 Å[4]. Subtracting the average wavelength of the peaks from this value yields the offset of the data. The data were then corrected to account for this offset.

I next plotted each of the six data sets with corrected wavelength in angstroms on the x-axis and ratemeter voltage in volts on the y-axis. One of these data sets was taken with no magnetic field present, and therefore only showed the $M_{\mathrm{J}} = 0$ transition. It was therefore eliminated from consideration. The remaining five plots showed three peaks corresponding to the $M_{\mathrm{J}} = \pm 1, 0$ transitions. The plot for an electromagnet current of 40 A is shown in Fig. 2. The peaks are labeled with their respective quantum number M_{J}.

The wavelength of the maximum of three peaks were determined in the same way I determined the wavelength for the central peaks earlier. This was done for each of the five remaining data sets. These wavelengths will eventually be used to calculate the Landé g factor. In doing this I will obtain fifteen distinct values for g_{J}. I will average these values to obtain a final value for g_{J}, and take a population standard deviation to determine the uncertainty. Any uncertainty corresponding to the wavelengths of the peak will be statistically insignificant compared to this standard deviation. Therefore I took these wavelengths to be exact. For this same reason I will also take the measured magnetic field strength to be exact. While the fractional uncertainty of the field strength is larger it will still be statistically insignificant after it is propagated through the calculations. Once the wavelength of each peak was obtained, I calculated the difference in energy ΔE between the $M_{\mathrm{J}} = -1$ and the $M_{\mathrm{J}} = 0$ peak, the $M_{\mathrm{J}} = 0$ and the $M_{\mathrm{J}} = 1$ peak and finally the $M_{\mathrm{J}} = -1$ and the $M_{\mathrm{J}} = 1$ peak using Eq. (5). I defined ΔE in my calculations such that it was always positive. I have also calculated an energy difference between the $M_{\mathrm{J}} = -1$ and the $M_{\mathrm{J}} = 1$ peak in order to obtain more data to work with. For these reasons the M_{J} in Eq. (4) should be redefined to be the difference in the M_{J} quantum number corresponding to the peaks ΔM_{J}. Here ΔM_{J} must always be a positive quantity.

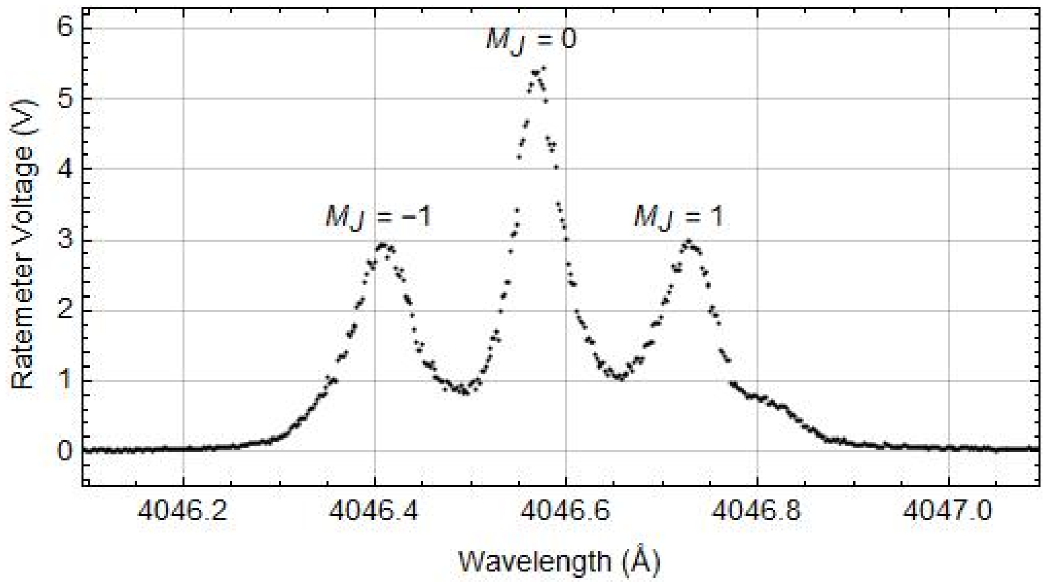


Fig. 2. A plot of the corrected spectral data from mercury undergoing a transition from the ${}^3P_1$ excited state to the ${}^3P_0$ ground state in the presence of a magnetic field. This feild was generated with a 40 A current flowing through an electromagnet. The plot shows wavelength on the x-axis in angstroms and ratemeter voltage in volts on the y-axis. The ratemeter voltage corresponds to the intensity of the specific wavelength of light. It shows three distinct peaks located at wavelengths of $\lambda \approx 4046.4$ Å, 4046.55 Å, and 4046.75 Å. The two outer peaks have an amplitude of approximately 3 V, while the central peak has an amplitude of approximately 5.5 V. The three peaks are labeled $M_\mathrm{J} = -1$, $M_\mathrm{J} = 0$ and $M_\mathrm{J} = 1$ from lower to higher wavelength.

With this correction I can use Eq. (4) to calculate 15 distinct values for g_J corresponding to each difference in energy between the excited quantum states. Averaging these I obtain a value of $g_\mathrm{J} = 1.96502$. Taking a population standard deviation σ_g of these values returns $\sigma_\mathrm{g} = 0.0768684$. Therefore I am confident in stating an experimentally determined value for the Landé g factor in the case of atomic mercury transitioning from the ${}^3P_1$ excited state to the ${}^3P_0$ ground state to be $g_\mathrm{J} = 1.97(8)$. Quantum Mechanical predictions state the Landé g factor in this case is $g_\mathrm{J} = 2$ which is well within my calculated uncertainty.

IV Future Directions

These data were very interesting. I had remembered learning about the Zeeman effect previously, however I had not been fully exposed to it. I knew that analyzing these data would force me to research more on the Zeeman effect, and the Landé g factor. I prefer to know the derivation of the equations I am using, so I researched both the practical analysis of the Zeeman effect and the theory behind it. This was very enlightening. When analyzing the data I noticed the plots all had a similar plateau after the peak with the highest wavelength. All six data sets showed this plateau. It can be seen in Fig. 2, and was even more distinct in some others. My first thought was this is a Compton plateau but I believe it is on the wrong side of the peak. Therefore, I am unsure what this is due to and am interested in learning if it has any significance.

To continue this analysis I think it would be interesting to back track and determine the spin g factor using our determination of the Landé g factor. Our calculation of the Landé g factor depended on the spin g factor equaling exactly 2. Using the Landé g factor I could solve for the spin g factor. I am interested to see what I would obtain. This could be trivial if I simply backtracked my calculations, but if I solve for the spin g factor first by relating the net magnetic moment to the magnetic moment from the orbital and spin angular momenta it could yield a different result. This still might be trivial and only give an result other than 2 due to precision errors, but I am not sure that would be the case. I feel it is worth doing some preliminary calculations.

I would also like to research further into the polarization effects of the Zeeman effect. I am interested in what causes the the light to polarize. Also how does this polarization change with a magnetic field of different strength? Is there a relationship between the intensity of polarized light and magnetic field strength? Will the angle of polarization change? There are a number of questions I have about this effect and I am looking forward to researching it more.

References

[1] R. Eisberg and R. Resnick, Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles. John Wiley Sons, 1985.

[2] R. A. Serway, C. J. Moses, and C. A. Moyer, Modern Physics. Belmont, CA: Thomson Brooks/Cole, 2005.

[3] J. S. Townsend, Quantum Physics. Sausalito, CA: University Science Books, 2010.

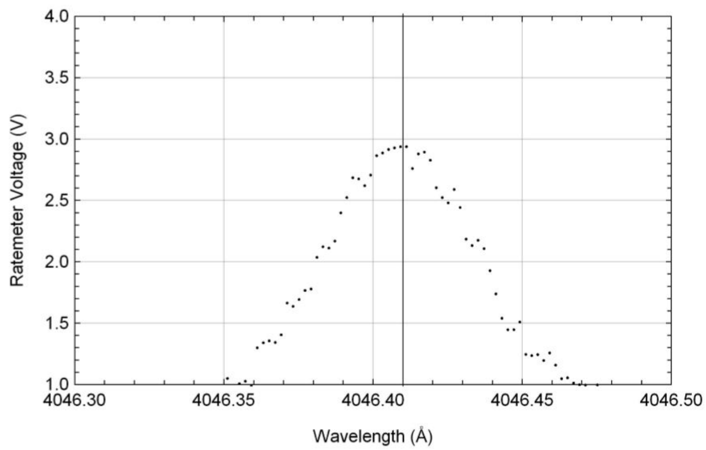
[4] NIST, A Kramida, Yu. Ralchenko, and J. Reader. NIST Atomic Spectra Database. (ver. 5.3)[Online.]. Available: `http://physics.nist.gov/asd` (accessed Mar. 2020). National Institute of Standards and Technology, Gaithersburg, MD., 2015.

Appendix

The following is some excerpts of code to better illustrate the analysis process.

The central peak of the 5 A data set (after it has been corrected) is marked with a black line vertical which can be adjusted to ensure it is at the maximum of the peak.

```
ListPlot[cor2, PlotRange → {{4046.3, 4046.5}, {1, 4}}, Frame → True, FrameStyle → Black,
 PlotStyle → Black, FrameLabel → {"Wavelength (Å)", "Ratemeter Voltage (V)"},
 GridLines → Automatic, Epilog → {Black, Line[{{4046.410, 0}, {4046.410, 6}}]}]
```



The same was done for the other two peaks and their wavelengths are put in a list called L2. The difference in energy levels from the Zeeman effect are calculated for L2.

```
ΔE2 = {En[L2[[1]], L2[[3]]], En[L2[[1]], L2[[2]]], En[L2[[2]], L2[[3]]]};
(* Units jouls *) (* En[λ1,λ2] is defined as the positive
 energy difference between photons of wavelengths λ1 and λ2 *)
```

The same was done for the other five data sets and a nested list was created called ΔE.

```
g1 = {g[2, FSC[[1, 2]], ΔE[[1, 1]]],
   g[1, FSC[[1, 2]], ΔE[[1, 2]]], g[1, FSC[[1, 2]], ΔE[[1, 3]]]};
(* g[MJ,B,ΔE] yeids the Landé g factor. Three values of gJ are calculated for
 the 5 A data set *)(* FSC[[All,2]] is the magnetic feild strength in teslas *)
```

This is repeated for the other data sets and these values are entered into a new list. The mean of this list determines g_J, the population standard deviation determines the uncertainty.