Index of Refraction of Air

Experimentally Determined For HeNe and GreNe Laser Light

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Abstract

We determine the index of refraction for both HeNe and GreNe laser light in air using a Michelson interferometer with a gas tube on one leg. We use a 10-cm-long gas tube for preliminary exploration and confirmation of our methods for HeNe light. We also use a 1-m-long gas tube for both HeNe and GreNe light. For HeNe light we determine an index of refraction of $n = 1.0002707(3)$ in air using a 10-cm-long gas tube and $n = 1.00026264(15)$ using a 1-m-long gas tube. The predicted value $n = 1.000262625(31)$ is within our uncertainty for the value from the 1-m-long tube result but not that from the 10-cm-long tube. For GreNe light we determine an index of refraction of $n = 1.00026405(3)$ in air. The predicted value $n = 1.000263997(31)$ is not within our uncertainty, however, our value is within the uncertainty of the predicted value.

I Introduction and Theory

A Michelson interferometer has the ability to measure changes in the optical path length of a laser beam. It consists of light which is split along two legs, then recombined at a later time and place. If the optical path length of one leg changes, these changes are represented as interference fringes when the light recombines. By placing a gas tube in one leg of the interferometer we can measure the change in wavelength of light in a vacuum compared to in air at atmospheric pressure. The wavelength of light in air and in a vacuum are related by

$$\lambda_{\text{air}} = \lambda_{\text{vac}} n, \tag{1}$$

where n is the index of refraction for that wavelength of light in air. When the gas tube is completely evacuated the number of wavelengths within the gas tube is given by

$$N_{\text{vac}} = \frac{2L}{\lambda_{\text{vac}}}, \tag{2}$$

where L is the length of the tube. In our interferometer the light travels through the gas tube, reflects off a mirror, and travels back through the tube, hence the factor of $2L$ in Eq. (2). Similarly the number of wavelengths within the pressurized tube is $N_{\text{vac}} = 2L/\lambda_{\text{vac}}$ which when related to Eq. (1) yields

$$N_{\text{air}} = \frac{2L}{\lambda_{\text{vac}} n}. \tag{3}$$

Due to the gas tube only being present in one leg, this decrease in waves present in the gas tube will be observed as interference fringes. Subtracting Eq. (4) from Eq. (2) and solving for the index of refraction n we obtain

$$n = \frac{2L}{2L - \lambda_{\text{air}} N}, \tag{4}$$

where the number of fringes $N \equiv N_{\text{vac}} - N_{\text{air}}$. By counting the number of interference fringes present as the gas tube pressurizes to 1 atm, we can determine the index of refraction of air for a specific wavelength of light. To determine an uncertainty in this value we should use propagation of uncertainty described in Taylor[2]. This yields

$$\frac{\delta(n-1)}{n-1} = \sqrt{(\frac{\delta N}{N})^2 + (\frac{\delta L}{L})^2 + (\frac{\delta \lambda_{\text{air}}}{\lambda_{\text{air}}})^2}. \tag{5}$$

II Experimental Procedure

We collected data using a Michelson interferometer. We set up this experiment such that laser light left HeNe and GreNe lasers and was split at a beam splitter. One path brings the light to mirror 1 M1 which is located on a motorized stage. It reflects off M1 and returns to the beam splitter. The other beam travels through a gas tube, reflects off mirror 2 M2 and back to the beam splitter where it is recombined with the first beam. The combined beams then travel to a prism to separate the HeNe and GreNe light. These separate beams travel to two separate photo-detectors whose output connects to a pre-amplifier then to our oscilloscope. The gas tube can be depressurized using a pump, and pressurized to the pressure of the lab room using a leak valve. The pressure of the gas tube is measured by a pressure gauge, and its output is also sent to the oscilloscope. The full schematic for our experiment involving the Michelson interferometer is shown in Fig. 1.

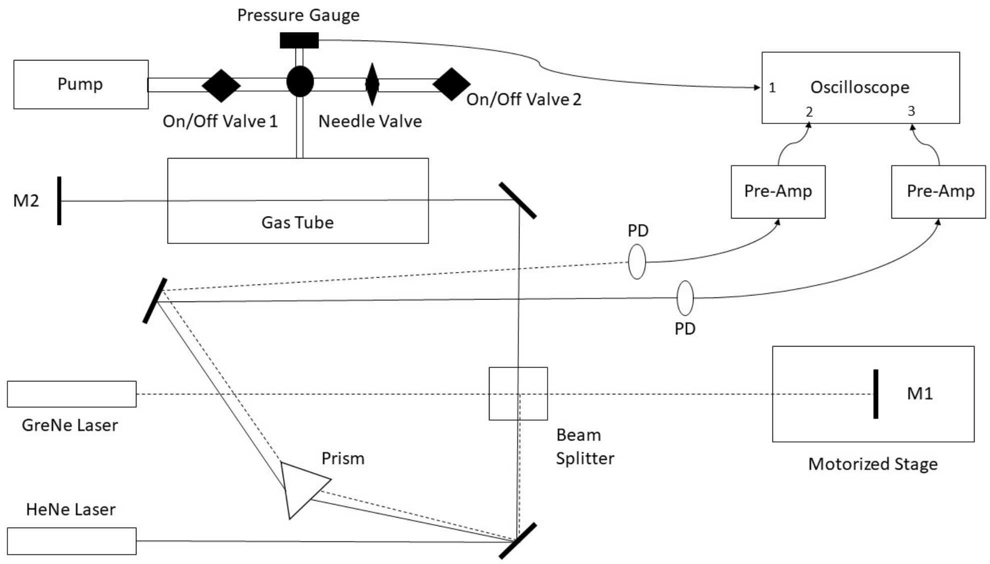
Pressure Gauge
Pump
On/Off Valve 1
Needle Valve
On/Off Valve 2
Oscilloscope
1
2
3
M2
Gas Tube
Pre-Amp
Pre-Amp
PD
PD
GreNe Laser
Beam
Splitter
M1
Motorized Stage
Prism
HeNe Laser

Fig. 1

Fig. 1 *(previous page)*. Schematic of the Michelson interferometer to measure the index of refraction of air for red and green wavelengths at atmospheric pressure. The laser light leaves the HeNe and GreNe lasers and is split at the beam splitter. One beam travels to mirror 1 M1 which is located on a motorized stage. The beam then reflects back to the beam splitter. The other beam travels through a gas tube, reflects off mirror 2 M2 and returns to the beam splitter through the gas tube. The beams then recombine and travel through a prism where the HeNe and GreNe light are separated. These two beams then are detected by two different photo-detectors PD. The output from these photo-detectors is sent to separate pre-amplifiers, and then the pre-amplifier output is sent to the oscilloscope. The gas tube is connected to a pressure gauge which is connected to the oscilloscope. There is a pump connected to the gas tube via valve 1. The gas tube can also be pressurized by opening valve 2 which is open to the pressure of the lab room. Between valve 2 and the gas tube is a needle valve which when adjusted changes how fast the gas tube pressurizes.

Initially we left the gas tube pressurized and translated M1 across the motorized stage to obtain an interference pattern on our oscilloscope for both the HeNe and GreNe lasers. We used this to adjust the gain on the pre-amplifiers for both signals and ensure that the photo-detectors were aligned correctly. Next we depressurized the gas tube to between zero and one millibar and adjusted the leak valve such that the gas tube took approximately 100 s to pressurize to the pressure of the lab room. To collect our data we first evacuated the gas tube to approximately zero millibars. We began our data collection on the oscilloscope. This collects both the oscillations in photo-detector signal corresponding to interference fringes in the GreNe and HeNe beams and the pressure in the gas tube. We then pressurized the gas tube using the needle valve and collected data until the pressure in the tube had equalized. We repeated this process to collect five distinct data sets using a 10-cm-long gas tube and only the HeNe laser. We then exchanged the gas tube for a 1-m-long tube and collected five distinct data sets for both the GreNe and HeNe laser.

III Analysis

The fringe data gathered using the 10-cm-long gas tube was used for preliminary exploration and proof of the methodology. From the five data files taken for this gas tube, I chose the one with the most time before and after the data was taken. This ensures that we can clearly see where the data begins and ends. The data shows a section of approximately constant value, followed by a series of oscillations, before returning to a constant value. I then normalized this data such that the midpoint of the oscillations is at zero volts. These normalized data is shown in Fig. 2. I counted the total number N_{initial} of whole and half fringes shown and determined $N_{\text{initial}} = 85$ oscillations. I next needed to take into account partial oscillations. First I determined where the relevant data began and ended. This was done by greatly magnifying a plot of the data set and finding the location where the oscillations either started or stopped. I then averaged the voltage of each of the peaks to determine the average amplitude of an oscillation in this data set. This in conjunction with the location of the beginning and end of our data set will be used to determine the fraction of an oscillation completed when the gas tube first started pressurizing N_{beg}, or when the gas tube finished pressurizing N_{end}.

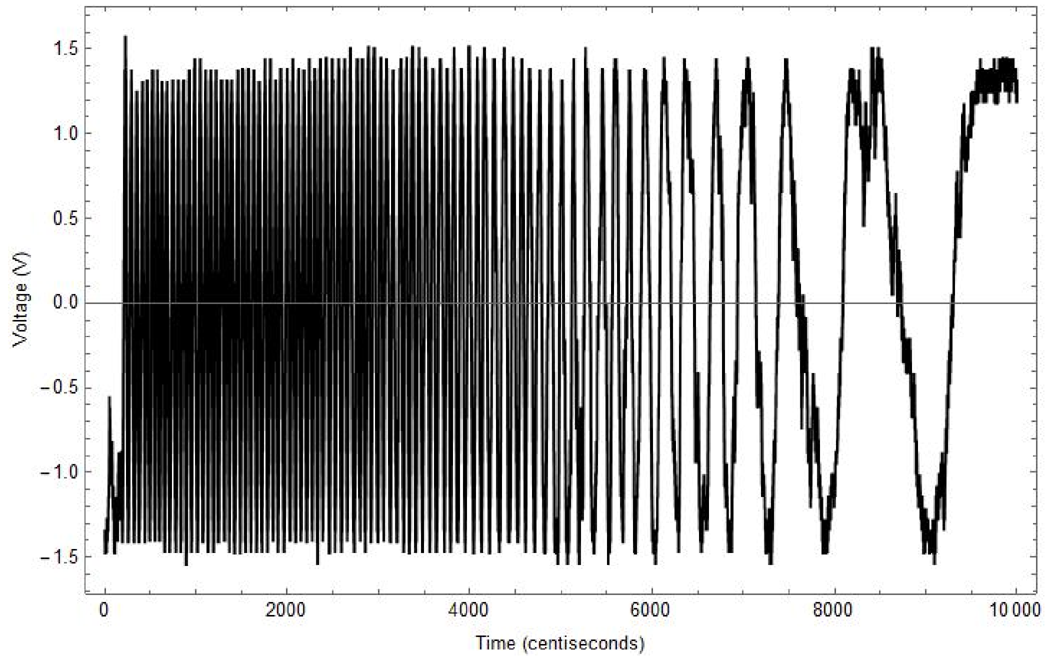

Fig. 2. A plot of the normalized data from the photo-detector in the interferometer. The plot shows the interference pattern present as the 10-cm-long gas tube pressurizes. It shows time in centiseconds on the x-axis and the voltage in volts sent from the photo-detector on the y-axis. The oscillations are normalized such that their midpoint is located at zero volts. There is some background noise before the fringe pattern begins at $Time \approx 100$ cs. The data plateaus beginning at $Time \approx 9500$ cs signifying the gas tube has pressurized to the pressure of the room. The oscillations have an amplitude A of $A \approx 1.5$ V.

Using the average amplitude A of each oscillation and the voltage V of a partial oscillation at the beginning or end of the data, we can compute the height difference d from the end of the partial oscillation to the nearest maximum or minimum. This is illustrated in Fig. 3 for the beginning of the data set. Computing the fraction F of an oscillation completed is then given by $F = d/(4A)$. I determined that $N_{\text{beg}} = .06$ and $N_{\text{end}} = .48$. Summing $N_{\text{beg}} + N_{\text{end}} + N_{\text{initial}}$ yields the total number N of oscillations in the data set. I determined $N = 85.54$ oscillations.

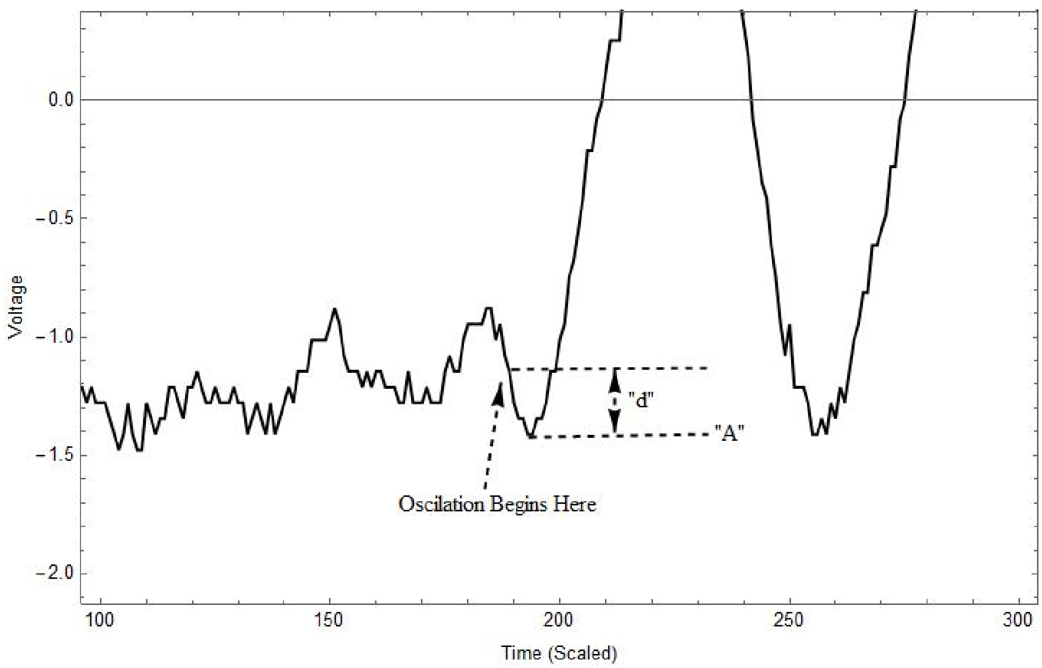

Fig. 3. A plot of the normalized data from the photo-detector in the interferometer magnified to see where the interference pattern begins. This location is labeled. It shows time in centiseconds on the x-axis and the voltage in volts sent from the photo-detector on the y-axis. The oscillations are normalized such that their midpoint is located at zero volts. The data begins at $Time = 189$ cs and $Voltage = 1.14$ V. The average amplitude of the peaks A is also labeled and is located at $A = 1.45$ V. The distance between the nearest minimum and where the data begins is also shown.

This number of oscillations determined for the HeNe laser and a 10-cm-long gas tube does have some uncertainty associated with it. The methodology for determining this uncertainty was found to be too equivocal, and was therefore abandoned when analyzing the data from the 1-m-long gas tube.

Since this data set is merely being used as a proof of the methodology, I will not go into detail about the exact methodology used. I will however give an overview and quote the result.

After determining where the data began I determined two locations which encompassed this beginning point and I felt could plausibly encompass all the points in which the data could have begun. I then calculated the fraction of a fringe that these two points encompassed. This value would vary greatly depending on if there was any elevation change between the points I chose. The same approach was taken at the end of the data, and the fringe fractions were summed. This yielded $N = 85.54(6)$. While this value is highly precise I am not confident in my determination of N to that level of precision.

We will now determine a value for the number of fringes shown as the 1-m-long gas tube pressurizes for both HeNe and GreNe light. We have five data sets for both the HeNe and GreNe light interference patterns. Of these five, two did not show adequate time before collection began and therefore needed to be eliminated from consideration. For each of the remaining data files a value for the total number of fringes shown for both the HeNe and GreNe light was determined. I used the same methodology as given above for the 10-cm-long gas tube and HeNe laser. This yielded three results for the number of fringes from the HeNe light, and three for the number of fringes from the GreNe light. These results were averaged to obtain a mean value for the number of fringes from HeNe light and a mean value for the number of fringes from the GreNe light. Half the range of each of these data determined the uncertainty. This yielded the total number of fringes for the HeNe light in a 1-m-long gas tube $N = 829.8(5)$ and the total number of fringes for the

GreNe light in a 1-m-long gas tube $N = 971.66(8)$. These values will be used to determine the index of refraction of air for HeNe and GreNe light.

Using the values for N we can determine an index of refraction of air for each of the laser wavelengths. Eq. (4) establishes this value where each variable is taken to be exact. We will determine uncertainty through propagation of uncertainty later. The NIST values for the wavelengths of HeNe and GreNe light are used where $\lambda_{\text{air HeNe}} = 6328.1646(4)$ and $\lambda_{\text{air GreNe}} = 5433.6513(4)$ (in air at STP[1]). We can determine the uncertainty in this value using Eq. (5). Where $\delta L = .0001$ m. This yields a value for the index of refraction of air at STP for both HeNe and GreNe light. These results are given in conjunction with the laser used, its wavelength at STP, and the length of the gas tube used in Table I. The total number of fringes counted is also given. The index of refraction found using the 10-cm-long gas tube is much less precise than that found using the 1-m-long gas tube, due to the higher relative uncertainty in L and N.

Table I. The index of refraction of air for different wavelengths of light. The index of refraction of air n is established for both HeNe and GreNe light using a Michelson interferometer. The length L of the gas tube used and the number N of fringes observed are also given. The NIST values for the wavelengths of HeNe and GreNe light are shown(in air at STP[1]).

Laser	λ (Å)	L (m)	N	n
HeNe	6328.1646(4)	0.0100(1)	85.54(6)	1.0002707(3)
HeNe	6328.1646(4)	1.0000(1)	829.8(5)	1.00026264(15)
GreNe	5433.6513(4)	1.0000(1)	971.66(8)	1.00026405(3)

We can predict the theoretical value for the index of refraction using Reference[3]. We predict $n = 1.000262625(31)$ for HeNe light. I chose to use a temperature of 23 degrees Celsius, a relative humidity of 50% and a pressure of 991 mbar in this calculations. These values seem reasonable for temperature and humidity. They are also not meant to be exact, but instead a ballpark for us to check our value against. The value for pressure was obtained from our pressure gauge when the 1-m-long gas tube pressure was equalized with the room. This predicted value falls within the uncertainty of our value for HeNe light using the 1-m-long gas tube $n = 1.00026264(15)$. It does not however fall within the uncertainty of our value found using the 10-cm-long gas tube $n = 1.0002707(3)$. This could be due to the data being taken on different days, under different atmospheric conditions, however I believe it is due to the rougher analysis. We only used one data set instead of three due to it being a proof of the methodology and was not intended to be a final result.

For GreNe light we predicted $n = 1.000263997(31)$ using the same atmospheric conditions. This value does not fall within the uncertainty of our value $n = 1.00026405(3)$. However, our value falls within the uncertainty of the predicted value.

IV Conclusion

We determined the index of refraction of air at STP for HeNe light to be $n = 1.00026264(15)$ experimentally. The predicted value is $n = 1.000262625(31)$ which falls within our uncertainty. We also determine the index of refraction of GreNe light at STP to be $n = 1.00026405(3)$. The predicted value is $n = 1.000263997(31)$. This value does not fall within the uncertainty of our value, however our value falls within the uncertainty of this predicted value. This confirms the theoretical prediction that the index of refraction of air does change depending on the wavelength of light. It also shows that light does not slow very much in air when compared to its speed in a vacuum.

References

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