Nuclear Decay

For Multiple Half-Lives of Ba-137m

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I Abstract

We use a scintillator to take count rate data for the decay of isolated Ba-137m, with a dwell time of $\Delta t = 4$s. We analyze these data to determine a half-life of $\tau_{1/2} = 153 \pm 4$s and confirm the exponential decay model. When compared to the documented value $\tau_{1/2} = 153.12$s [1] we find it is well within our uncertainty. Calculating the percent difference yields just 0.08%.

II Introduction and Theory

When the nucleus of an isotope decays from an excited state to a less excited state it is most commonly via gamma decay. The number of protons and neutrons remain unchanged, however the nucleus is more tightly bound. This means there is less binding energy needed. The excess binding energy is released in the form of a gamma ray. One particular isotope which decays via gamma decay is Ba-137 in the excited state $I = 11/2$. This is called a meta-stable isotope and is denoted Ba-137m. This particular isotope has a relatively long half-life when compared to others which decay via gamma radiation. This makes it useful for study. A relatively pure sample of Ba-137m can be created in our laboratory which we will use to measure its half-life.

The half-life of an isotope is defined as the time it takes for the number of, in this case, excited nuclei to decay by half. If the initial number of excited isotopes is N_0 then the number at time t is given by

$$N(t) = N_0 e^{-\lambda t}. \tag{1}$$

Differentiating this and taking the natural logarithm yields

$$lnR(t) = lnR_0 - \lambda t. \tag{2}$$

This is a linear relationship where λ is the rate constant. Setting $N(t) = 1/2N_0$ and solving Eq. (1) for t returns

$$\tau_{1/2} = \frac{-ln(2)}{\lambda}. \tag{3}$$

Therefore we are able to calculate the half-life of an isotope if we know the rate constant λ.

III Experimental Procedure

We set up our experiment with an NaI scintillator to generate photons which enter a photo-multiplier tube. The output from this tube is eventually brought to a multi-channel analyzer (MCA) which has 256 distinct channels for binning data. The full schematic is shown in Fig. 1. In order to narrow the range of energy values for which we obtain counts, we first analyzed the decay of Cs-137. Cs-137 decays to Ba-137m 100% of the time, via β decay then γ decay. Cs-137, however, has a much greater half life than Ba-137m, so we use it to isolate the peak energy from the decay of Ba-137m [1].

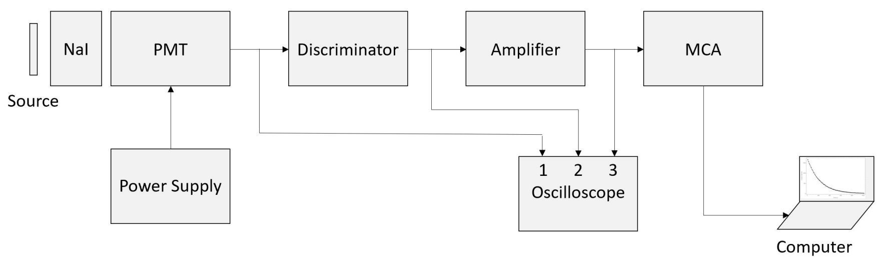

Fig. 1. Schematic of the setup for experiments involving a NaI scintillator to measure the gamma radiation due to decay of certain isotopes. When photons are ejected from the scintillator, they enter the photo-multiplier tube (PMT), where the electrical signal is generated. That signal is adjusted by the discriminator and amplifier then sent to the oscilloscope and multi-channel analyzer (MCA).

We then used the MCA to collect spectroscopy data on the computer. We used a pulse height analysis (PHA) mode to collect our data. This bins pulses according to their amplitude. We noticed a distinct peak in the spectroscopic data. At energies higher than the value at which this peak was located there were practically no counts, so we only needed to adjust the lower level discriminator on the MCA to remove the Compton plateau and Compton edge located at a lower energy than the peak.

After adjusting the discriminator to isolate the primary peak for Cs-137, we configured the MCA for multi-channel scaling (MCS) mode. This counts the number of pulses within a certain dwell time across the 256 channels of our MCA. We adjusted the dwell time such that we could acquire data for at least five half-lives of Ba-137m. Because the documented half-life of Ba-137m is $\tau_{1/2} = 2.552$min [1], we chose a dwell time of $\Delta t = 4$s. This along with our 256 channels gives us a total collection time of 17 minutes which is well over 5 half lives of Ba-137m. With no sample near the scintillator (and all our samples contained in a lead box) we obtained count data for the background radiation.

We prepared a liquid radioactive source containing Ba-137m (with 10 drops of eluted solution). We placed the Ba-137m solution under the scintillator and promptly recorded data using the same dwell time $\Delta t = 4$s.

IV Results and Analysis

Using the MCS mode and the scintillator, we obtained both background count data and Ba-137m decay count data for a dwell time of $\Delta t = 4$s. I omitted the first two data points from all of our MCS data. These I assumed contained artificial counts due to their anomalous nature. The remaining 254 elements are plotted in Fig. 2, in which we can see that the raw Ba-137m count data follows a trend of decay, as we would expect for radioactive decay. It also shows that our background counts could conceivably skew our data after around 600 seconds. This is confirmed in Table I where we see that our background counts make up less than 1% of the raw Ba-137m count data originally, but by the end of our data the background counts make up 16% of our signal. For this reason we must remove the background radiation from our raw Ba-137m count data. Because the background count data and the raw Ba-137m count data were obtained at different times, it would be illegitimate to subtract the number of background counts for any arbitrary time t from the number of Ba-137m counts at said time t. However, the background and raw Ba-137m count data were obtained under similar conditions. Therefore, it is legitimate to subtract the mean number of background counts per second from the number of raw Ba-137m counts at every time t. Calculating the mean number of counts per second we obtain $c_b = 7.9 \pm 1.4$ counts per second. We will only use the mean in the following calculation. We subtract this from our raw Ba-137m count data to obtain a new data set which I will refer to as corrected Ba-137m count data.

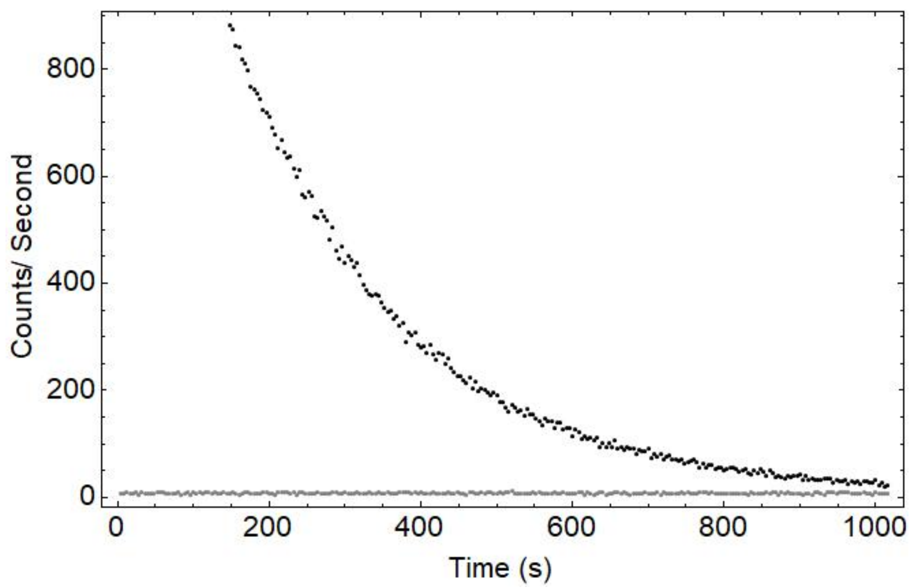

Fig. 2. The raw number of gamma particle counts counted by the MCA each second for both Ba-137m decay (black), and the background data with no source (gray). We can see a decay pattern in the gamma particle counts and a somewhat static number of background counts per second.

Table I. The contribution of the background count rate. The mean background count rate is reported as a percentage of the mean decay signal per half-life. We see this percentage is large enough to skew our data even after just a few half-lives. The mean count rate is also reported for each half-life. The uncertainty was calculated by taking the population standard deviation of the counts in any singular half-life.

Time	Count Rate (cps)	Percent
$\tau_{1/2}$	1200 ± 200	0.6%
$2\tau_{1/2}$	630 ± 120	1%
$3\tau_{1/2}$	320 ± 60	2%
$4\tau_{1/2}$	160 ± 30	5%
$5\tau_{1/2}$	87 14	9%
$6\tau_{1/2}$	48 ± 8	16%

In Fig. 3 we plot the natural log of the corrected Ba-137m count data. A natural log function was chosen so that we could use the slope of the best fit line to determine the half-life of Ba-137m. On this plot I calculated a line of best fit,

$$y_{\text{fit}}(t) = -0.00452s^{-1}t + 8.835. \tag{4}$$

This was calculated using a *least-squares* fit algorithm [2]. After determining this fit line I plotted it with the natural log of our corrected Ba-137m count data to qualitatively ensure the best fit function fit my data. I also located two lines which I felt represented the minimum, and maximum slope of the data,

$$y_{\text{min slope}}(t) = -0.00439s^{-1}t + 8.835 \tag{5}$$

and

$$y_{\text{max slope}}(t) = -0.00465s^{-1}t + 8.835 \tag{6}$$

respectively. These were decided such that they encompass approximately 68% (or one standard deviation) of the data and can best be seen in Fig. 4. The increased detail also shows how well our fit line bisects our decay data. I chose to have the same y intercept for all three of these lines as I assumed the uncertainty in counts at time $t = 0$ to be zero. This y intercept was determined when finding the line of best fit.

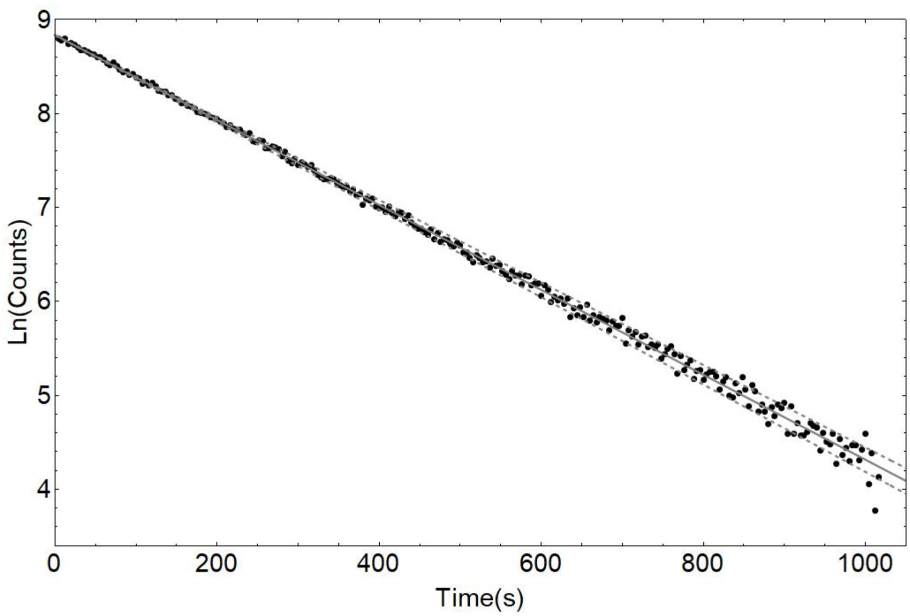

Fig. 3. The natural log of our corrected Ba-137m decay count data plotted with respect to time. On the same plot is the line of best fit (solid), and lines (dashed) with the minimum and maximum slope to encompass one standard deviation of the data, while maintaining the y intercept from the fit line. See Fig. 4 for a more detailed view of these data, and the corresponding lines of best fit, minimum slope, and maximum slope.

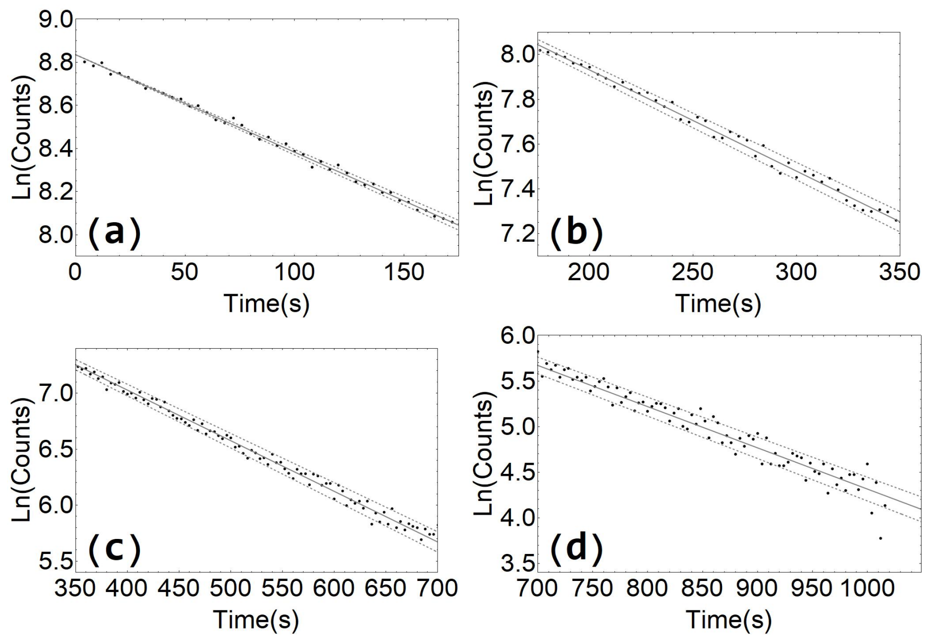

Fig. 4. Decay data partitioned into four different contiguous time intervals (a-d) to show increased detail in the data, the fit line (solid) and the lines of minimum and maximum slope (dashed). With the increased detail we see the fit line bisects our data set. Also we see our lines of minimum and maximum slope enclose the majority of our data.

We can use the slopes of the lines given by Eq. (4), Eq. (5), and Eq. (6) to find the half life of Ba-137m. From Eq. (2) we can see that the slope of Eq. (4) yields the rate constant λ. Therefore we can solve for the half-life using Eq. (3). We evaluate this equation with the slope of the fit line to find $\tau_{Fit} = 153$s. Next we evaluate Eq. (3) with the slopes of Eq. (5) and Eq. (6) to determine the uncertainty of our measured half-life. We obtain $\tau_{Min} = 149$s and $\tau_{Max} = 157$s. Finding the magnitude of the difference between τ_{Min}

and τ_{Fit}, and the magnitude of the difference between τ_{Max} and τ_{Fit}, yields an uncertainty of $\delta\tau_{\tau+}$ = 4.54115s and $\delta\tau_{\tau-}$ = 4.28724s. Calculating the mean of $\delta\tau_{\tau+}$ and $\delta\tau_{\tau-}$ yields our uncertainty $\delta\tau_{1/2}$ = 4.41420s, and therefore $\tau_{1/2}$ = 153 ± 4s.

The documented half life of Ba-137m is $\tau_{1/2}$ = 2.552 min = 153.12s [1]. The documented value is well within the uncertainty of our experimental value $\tau_{1/2}$ = 153 ± 4s. In fact the percent difference is only 0.08%. Therefore, our experimental value for $\tau_{1/2}$ is extremely close to the documented value meaning our decay data follows the trend of exponential decay we expect from radioactive decay.

V Conclusion

We determined the half-life of Ba-137m to be $\tau_{1/2}$ = 153 ± 4s. The documented value $\tau_{1/2}$ = 153.12s is within the uncertainty of our value, and we find a percent difference of only 0.08%. We did have a rather large uncertainty. This is likely due to the method by which it was calculated. Estimating the lines which encompassed approximately 68% of our data leaves a lot to be desired in the way of precision. Perhaps the precision could be improved through calculating the R-squared value of our fit line and using that to determine uncertainty in some way.

References

[1] NNDC *National Nuclear Data Center (NNDC) Chart of the Nuclides (Interactive Online Database)*. Available at `http://www.nndc.bnl.gov/chart` (Accessed March 2019)

[2] Wolfram Language System Documentation Center *Curve Fitting*. Available at `https://reference.wolfram.com/language/tutorial/CurveFitting.html` (Accessed March 2019)