Week 02 Problem Set Data Structures and Algorithms

Elementary Data and Control Structures in

 \Box

1. (Arithmetic)

There is a 5-digit number that satisfies $4 \cdot abcde = edcba$, that is, when multiplied by 4 yields the same number read backwards. Write a C-program to find this number.

Answer:

Solution: 21978

2. (Arrays)

a. Write a C-function that returns the *inner product* of two *n*-dimensional vectors **a** and **b**, encoded as 1-dimensional arrays of *n* floating point numbers.

Use the function prototype float innerProduct(float a[], float b[], int n).

Hint: The inner product of two vectors is calculated as $\sum_{i=1..n} \mathbf{a}_i \cdot \mathbf{b}_i$

b. Write a C-function to compute C as the matrix product of matrices A and B.

Use the function prototype **void matrixProduct(float a[M][N], float b[N][P], float c[M][P])**. You can assume that *M*, *N*, *P* are given as symbolic constants, e.g.

```
#define M 3
#define N 4
#define P 4
```

Hint: The product of an $m \times n$ matrix \mathbf{A} and an $n \times p$ matrix \mathbf{B} is the $m \times p$ matrix \mathbf{C} such that $\mathbf{C}_{ij} = \sum_{k=1...n} \mathbf{A}_{ik} \cdot \mathbf{B}_{kj}$

for all $i \in \{1..m\}$ and $j \in \{1..p\}$.

Answer:

```
float innerProduct(float a[], float b[], int n) {
   int i;
   float product = 0.0;

for (i = 0; i < n; i++)
     product += a[i] * b[i];

return product;
}

void matrixProduct(float a[M][N], float b[N][P], float c[M][P]) {
   int i, j, k;</pre>
```

```
for (i = 0; i < M; i++) {
      for (j = 0; j < P; j++) {
         c[i][j] = 0.0;
         for (k = 0; k < N; k++) {
            c[i][j] += a[i][k] * b[k][j];
      }
  }
}
```

3. (Characters)

Write a C-program that outputs, in alphabetical order, all strings that use each of the characters 'c', 'a', 't', 'd', 'o', 'g' exactly once.

How many strings does your program generate?

Answer:

There are 6! = 720 permutations of "catdog".

A straightforward solution is to use six nested loops and a conditional statement to filter out all strings with duplicate characters. The following program includes a counter to check how many strings have been generated.

```
#include <stdio.h>
int main(void) {
  char catdog[] = { 'a','c','d','g','o','t' };
  int count = 0;
   int i, j, k, l, m, n;
   for (i=0; i<6; i++)
      for (j=0; j<6; j++)
         for (k=0; k<6; k++)
            for (1=0; 1<6; 1++)
               for (m=0; m<6; m++)
                  for (n=0; n<6; n++)
                     if (i!=j && i!=k && i!=l && i!=m && i!=n &&
                          j!=k && j!=l && j!=m && j!=n &&
                         k!=1 && k!=m && k!=n &&
                         l!=m && l!=n && m!=n) {
                           printf("%c%c%c%c%c%c\n", catdog[i], catdog[j],
                                                     catdog[k], catdog[l],
                                                     catdog[m], catdog[n]);
                           count++;
  printf("%d\n", count);
  return 0;
}
```

4. (Elementary control structures)

- a. Write a C-function that takes a positive integer n as argument and outputs a series of numbers according to the following process, until 1 is reached:
 - if *n* is even, then $n \leftarrow n/2$
 - if *n* is odd, then $n \leftarrow 3^*n+1$
- b. The Fibonacci numbers are defined as follows:
 - Fib(1) = 1
 - Fib(2) = 1
 - Fib(n) = Fib(n-1)+Fib(n-2) for n≥3

Write a C program fibonacci.c that applies the process described in Part a. to the first 10 Fibonacci numbers.

The output of the program should begin with

```
Fib[1] = 1
Fib[2] = 1
Fib[3] = 2
```

```
Fib[4] = 3
3
10
5
16
8
4
2
1
```

We have created a script that can automatically test your program. To run this test you can execute the dryrun program that corresponds to the problem set and week, i.e. prob02 for this week. It expects to find a program named fibonacci.c in the current directory. You can use dryrun as follows:

```
prompt$ -cs9024/bin/dryrun prob02
```

Note: Please ensure that your output follows exactly the format shown above.

Answer:

```
#include <stdio.h>
#define MAX 10
void collatz(int n) { // named after the German mathematician who invented this problem
  printf("%d\n", n);
  while (n != 1) {
      if (n % 2 == 0) {
        n = n / 2;
      } else {
        n = 3*n + 1;
     printf("%d\n", n);
}
int main(void) {
   int fib[MAX] = { 1, 1 };  // initialise the first two numbers
   int i;
   for (i = 2; i < MAX; i++) { // compute the first 10 Fibonacci numbers
      fib[i] = fib[i-1] + fib[i-2];
   for (i = 0; i < MAX; i++) { // apply Collatz's process to each number
     printf("Fib[%d] = %d\n", i+1, fib[i]);
     collatz(fib[i]);
  return 0;
}
```

5. (Elementary data structures)

Define a data structure to store all information of a single ride with the Opal card. Here are three sample records:

| Transaction number | Date/time | Mode | Details | Journey number | Fare Applied | Fare | Discount | Amount |
|--------------------|----------------------------|----------|--|-------------------|-----------------|--------|----------|---------|
| 2013 | Mon 30/07/2018 10:16 | B | Flinders St af Ox- ford St to Anzac Pde D opp UNSW | 1 | | \$1.46 | \$0.00 | -\$1.46 |
| 2011 | Mon 30/07/2018 10:05 | B | Victoria St at Liv- erpool to Oxford St op Palmer St | 1 | | \$2.20 | \$0.00 | -\$2.20 |
| 2009 | Sun 29/07/2018 17:35 | • | Bondi Junction to Kings Cross | | Day Cap | \$3.54 | \$3.54 | \$0.00 |

You may assume that individual stops (such as "Anzac Pde D opp UNSW") require no more than 31 characters.

Determine the memory requirements of your data structure, assuming that each integer and floating point number takes 4 bytes.

If you want to store millions of records, how would you improve your data structure?

Answer:

There are of course many possible ways in which this data can be structured; the following is just one example:

```
typedef struct {
   int day, month, year;
} DateT;
typedef struct {
  int hour, minute;
} TimeT;
typedef struct {
  int transaction;
  char weekday[4];
                            // 3 chars + terminating '\0'
  DateT date:
  TimeT time;
                            // 'B', 'F' or 'T'
  char mode;
  char from[32], to[32];
  int
         journey;
  char faretext[12];
  float fare, discount, amount;
} JourneyT;
```

Memory requirement for one element of type JourneyT: 4 + 4 + 12 + 8 + 1 (+ 3 padding) + $2 \cdot 32 + 4 + 12 + 3 \cdot 4 = 124$ bytes.

The data structure can be improved in various ways: encode both origin and destination (from and to) using Sydney Transport's unique stop IDs along with a lookup table that links e.g. 203311 to "Anzac Pde Stand D at UNSW"; use a single integer to encode the possible "Fare Applied" entries; avoid storing redundant information like the weekday, which can be derived from the date itself.

6. Challenge Exercise

Write a C-function that takes 3 integers as arguments and returns the largest of them. The following restrictions apply:

- You are not permitted to use if statements.
- You are not permitted to use loops (e.g. while).
- You are not permitted to call any function.
- You are only permitted to use data and control structures introduced in Week 2's lecture.

Answer:

The following makes use of the fact that a true condition has value 1 and a false condition has value 0:

```
int max(int a, int b, int c) {
  int d = a * (a >= b) + b * (a < b);  // d is max of a and b
  return c * (c >= d) + d * (c < d);  // return max of c and d
}</pre>
```