

Linear MPC

$$x_{k+1} = Ax_k + Bu_k$$

$k \rightarrow$ time instant

$x_k \in X \subseteq \mathbb{R}^n$: state vector

$u_k \in U \subseteq \mathbb{R}^m$: Control Input

$A \in \mathbb{R}^{n \times n}$: System Matrix

$B \in \mathbb{R}^{n \times m}$: Input Matrix

$$X = \{x \in \mathbb{R}^n : F_x x \leq g_x\}$$

$$U = \{u \in \mathbb{R}^m : F_u u \leq g_u\}$$

Cost function,

$$J = x_{N_T}^T Q_{N_T} x_{N_T} + \sum_{k=0}^{N_T-1} x_k^T Q x_k + u_k^T R u_k$$

$$x_{i+1|k} = A x_{i|k} + B u_{i|k}$$

$$\begin{bmatrix} x_{k|k} \\ x_{k+1|k} \\ \vdots \\ x_{N+1|k} \end{bmatrix} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 & 0 \dots & \vdots \\ B & 0 & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix} \begin{bmatrix} v_{k|k} \\ v_{k+1|k} \\ \vdots \\ v_{k+N-1|k} \end{bmatrix}$$

$$A_x = \begin{bmatrix} I \\ A \\ \vdots \\ A^N \end{bmatrix} \quad B_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dots \\ B & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix}$$

we define,

$$Q_x = \begin{bmatrix} Q & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & Q & 0 \\ 0 & \dots & 0 & Q_N \end{bmatrix}$$

$$R_U = \begin{bmatrix} R & 0 & \dots & 0 \\ 0 & R & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R \end{bmatrix}$$

The cost function can be represented in terms of x_k & u_k

$$Y = x_k^T Q x_k + u_k^T R_U u_k$$

Constraints can be as :-

$$F_x = \begin{bmatrix} F_x & 0 & \dots & 0 \\ 0 & F_x & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & F_x \end{bmatrix}$$

$$g_x = \begin{bmatrix} g_x \\ g_x \\ \vdots \\ g_x \end{bmatrix}$$

$$F_U = \begin{bmatrix} F_U & 0 & \dots & 0 \\ \vdots & F_U & \dots & \vdots \\ 0 & 0 & \dots & F_U \end{bmatrix} \quad g_U = \begin{bmatrix} g_U \\ g_U \\ \vdots \\ g_U \end{bmatrix}$$

Constraint Equations now are :-

$$F_X X_k \leq g_X$$

$$F_U U_k \leq g_U$$

$$Z = \begin{bmatrix} X_k \\ U_k \end{bmatrix} \quad H = \begin{bmatrix} R_X & 0 \\ 0 & R_U \end{bmatrix}$$

$$g = \begin{bmatrix} g_X \\ g_U \end{bmatrix} \quad F_{eq} = \begin{bmatrix} \text{optimal-current} \\ I - B_U \end{bmatrix} \quad \begin{matrix} \hookrightarrow \text{loss function} \end{matrix}$$

$$F = \begin{bmatrix} F_x & 0 \\ 0 & F_v \end{bmatrix} \quad g_{eq} = A_x x_k$$

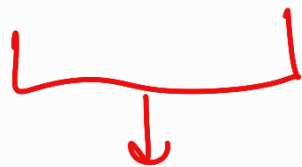
MPC is given as

$$z^T H z, \quad Fz \leq g$$

$$f_{eq} z \leq g_{eq}$$

Control input :

$$V_k = [v_k^*]_1 = v_{k|k}^d$$



first element will be
the control input.

$$1 \rightarrow \text{find } x_k = [x]_{k+1}$$

$$\text{find } F_{eq}, g_{eq}$$

$$z^* = \begin{bmatrix} x_k^* \\ u_k^* \end{bmatrix}$$

$$u_k = [u_k^*],$$

$$z_0 = z^*$$

→ Constraint optimised using fmincon:

$$fmincon(f, z_0, F, g, F_{eq}, g_{eq}, lb, ub)$$

↓
lower bound

upper bound.

→ In the first code stimulation we are using for loop which really puts us in a bad spot and takes a lot of time to compute.

→ Value 0.9 gives us the initial rocket landing with a heavy thrust motion causing it to have a jerky landing.

→ Initially has a high thrust which reduces and stabilises in our optimal MPC model.