Linear MPC

$$x_{k+1} = A x_k + B u_k$$

$k \Rightarrow$ time instant

$x_k \in X \subseteq \mathbb{R}^n$: state vector

$u_k \in U \subseteq \mathbb{R}^n$: Control Input

$A \in \mathbb{R}^{n \times n}$: System Matrix

$B \in \mathbb{R}^{n \times m}$: Input Matrix

$$X = \{ x \in \mathbb{R}^n : F_x x \leq g_x \}$$

$$U = \{ u \in \mathbb{R}^n : F_u u \leq g_u \}$$

Cost function,

$$J = x_{N_T}^T Q_{N_T} x_{N_T} + \sum_{k=0}^{N_T-1} x^T Q x_k + u_k^T R u_k$$

$$\boxed{x_{i+1|k} = A x_{i|k} + B u_{i|k}}$$

$$\begin{bmatrix} x_{k|k} \\ x_{k+1|k} \\ \vdots \\ x_{N+1|k} \end{bmatrix} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 & 0 & \cdots & \vdots \\ B & 0 & 0 & \cdots & \vdots \\ \vdots & & & & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & & B \end{bmatrix} \begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \vdots \\ u_{k+N-1|k} \end{bmatrix}$$

$$A_x = \begin{bmatrix} I \\ A \\ \vdots \\ A^N \end{bmatrix} \qquad B_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ B & 0 & 0 & 0 & 0 & \cdots \\ \vdots & & & & & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & & & B \end{bmatrix}$$

---

we define,

$$Q_x = \begin{bmatrix} Q & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & Q & 0 \\ 0 & \cdots & 0 & Q_N \end{bmatrix}$$

$$R_U = \begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & R & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & R \end{bmatrix}$$

The cost function can be represented in forms of X_k & U_k

$$Y = X_k^T Q_x X_k + U_k^T R_U U_k$$

Constraints can be as :-

$$F_x = \begin{bmatrix} F_x & 0 & & 0 \\ 0 & F_x & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ & & & F_x \end{bmatrix} \qquad g_x = \begin{bmatrix} g_x \\ g_x \\ \vdots \\ g_x \end{bmatrix}$$

$$F_U = \begin{bmatrix} F_u & 0 & \cdots & 0 \\ \vdots & F_u & \cdots & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & F_u \end{bmatrix} \qquad g_U = \begin{bmatrix} g_u \\ g_u \\ \vdots \\ g_u \end{bmatrix}$$

<u>Constraint Equations now are :-</u>

$$F_x X_k \leq g_x$$

$$F_U U_k \leq g_U$$

$$Z = \begin{bmatrix} X_k \\ U_k \end{bmatrix} \qquad H = \begin{bmatrix} Q_x & 0 \\ 0 & R_x \end{bmatrix}$$

$$g = \begin{bmatrix} g_x \\ g_u \end{bmatrix} \qquad F_{eq} = \begin{bmatrix} I - B_U \end{bmatrix}$$

optimal - current

loss function

$$F = \begin{bmatrix} F_x & 0 \\ 0 & F_u \end{bmatrix} \qquad g_{eq} = A_x x_k$$

<u>MPC is given as</u>

$$z^T H z \quad , \quad \begin{matrix} Fz \leq g \\ F_{eq} z \leq g_{eq} \end{matrix}$$

Control input :

$$u_k = \underbrace{[U_k^*]_1}_{\text{first element will be the control input.}} = u^*_{k|k}$$

1 → find $x_k = [x]_{k+1}$

find F_{eq}, g_{eq}

$$z^* = \begin{bmatrix} x_k^* \\ u_k^* \end{bmatrix}$$

$$u_k = [u_k^*],$$

$$z_0 = z^*$$

→ Constraint optimised using fmincon:

$$\text{fmincon}(f, z_0, F, g, F_{eq}, g_{eq}, lb, ub)$$

lower bound

upper bound.

→ In the first code simulation we are using for loop which really puts us in a bad spot and takes a lot of time to compute.

→ Value 0.4 gives us the initial rocket landing with a heavy thrust motion causing it to have a jerky landing.

→ Initialy has a high thrust which reduces and stabalises in our optimal MPC model.