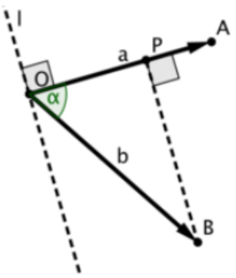
void special cases when denominator is equal to 0:

$$\tan\alpha = \frac{y_a}{x_a}, \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{|a||b|\sin\alpha}{|a||b|\cos\alpha} = \frac{a \wedge b}{a * b}.$$

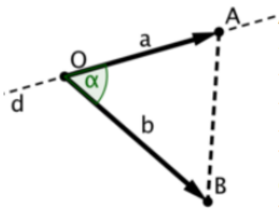
ter-clockwise direction using the well-known formula

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Dot product: $a * b = x_a x_b + y_a y_b = |a||b| cos\ \alpha$.
To explain its meaning, lets draw dashed line l perpendicular to the vector a.
Cosine properties apply that:
- if $a * b > 0$, then b points to the same side of l as a (as in the picture);
- if $a * b = 0$, then b is collinear with l, so it's perpendicular to a;
- if $a * b < 0$, then b points to the opposite side of l from a.

To sum it up, **dot product sign indicates whether two vectors point in about the same direction.**
Now let's look at the absolute value. $|b||cos\ \alpha| = |OP|$ — the projection of b onto a.
So, the dot product's absolute value is the projection length of b onto a times the length of a.
Nothing special, just remember that we can find projection length and angle cosine from it.



2D cross product: $a \wedge b = x_a y_b - y_a x_b = |a||b| sin\ \alpha$.
This name is not canon, but I didn't find a better one.
Again, for clarity we draw a line, but this time dashed line d through the vector a.
Sine properties apply that:
- if $a \wedge b > 0$, then b points to the left side of d if we're looking in the direction of a;
- if $a \wedge b = 0$, then b lies on d, so it's collinear with a;
- if $a \wedge b < 0$, then b points to the right side of d (as in the picture).

To sum it up, **2D cross product indicates whether the shortest turn from *a* to *b* is in the counter-clockwise direction.**
The following function ccw emphasizes this meaning. It acts like a comparator returning negative value if OA goes before OB in the counter-clockwise ordering (the commonly used ordering) and positive value if after. Origin O is $(0, 0)$ by default. If you're having trouble remembering which sign to use, draw vectors $a = (1, 0)$ and $b = (0, 1)$, then it's easy to compute that $a \wedge b = 1$.