~~eigenvalues $0 \le \mu_1 \le \dots \le \mu_{d_2}$ of C_{YY}.~~

~~Now, let $(w_x, w_y) = (\sum_{i=1}^{d_1} \alpha_i e_i, \sum_{j=1}^{d_2} \beta_j \tilde{e}_j)$ be a solution~~

2) (c) The first row reads

$$A \cdot B \alpha_y = \rho A^2 \alpha_x$$

$$\Leftrightarrow X^T X Y^T Y \alpha_y = \rho X^T X X^T X \alpha_x$$

$$\Rightarrow \alpha_x^T X^T \cdot \frac{1}{N} X Y^T Y \alpha_y = \rho \cdot \alpha_x^T X^T X X^T X \alpha_x$$

$$\Leftrightarrow w_x^T C_{xy} w_y = \rho w_x^T C_{xx} w_x = \rho$$

Hence, $w_x^T C_{xy} w_y$ will be maximal for the greatest value of ρ.

~~For $d_2 = 1$ the scalar $w_y \in \mathbb{R}^{d_1} = \mathbb{R}$ is already determined by the second constraint since (reformulating the problem~~

~~a factor, it will simply determine $w_x^T C_{xy}$ to be nonnegative.~~

3) For $d_2 = 1$, ~~we can simply~~ the scalar $w_y \in \mathbb{R}^1 = \mathbb{R}$ is already determined up to sign by the second constraint:

$$1 \stackrel{!}{=} w_y \cdot \frac{1}{N} Y^T Y \, w_y = w_y^2 \cdot \frac{\|y\|_2^2}{N} \Rightarrow w_y = \pm \frac{\sqrt{N}}{\|y\|_2},$$

where we reformulated the constraint for $y \in \mathbb{R}^{N \times 1}$ instead of $Y \in \mathbb{R}^{1 \times N}$.

$$\arg \min_{v \in \mathbb{R}^D} v^T X X^T v - 2 v^T X y + \underbrace{\ldots}_{\text{const.}}$$

---

(2) (d) Simply calculate $w_x = X\alpha_x$, $w_y = Y\alpha_y$?

The term we want to optimize can now be written as, setting $w = w_x$:

$$w^T C_{xy} w_y = \underbrace{\frac{\sqrt{N}}{\|y\|_2}}_{>0,\, const.} \cdot (\pm w)^T C_{xy}$$

Since the constraint $w^T C_{xx} w = 1$ is invariant to changing the sign of w, the problem boils down to the simpler version:

$$\arg\max_{w \in \mathbb{R}^D} w^T C_{xy} \quad \text{subject to} \quad w^T C_{xx} w = 1 \text{ . or}$$

$$\arg\max_{w \in \mathbb{R}^D} w^T X y \quad \text{subject to} \quad w^T X X^T w = 1 \text{ .}$$

The LSR problem ~~[illegible]~~ can be rewritten as

$$\|X^T v - y\|^2 = v^T X X^T v - 2 v^T X y + \underbrace{\|y\|_2^2}_{const.} \text{ , hence}$$

$$\arg\min_{v \in \mathbb{R}^D} v^T X X^T v - 2 v^T X y \text{ .}$$

· Notation: $\mathbb{E}[X$

Bem: $\mathbb{E}[X$

$\mathbb{E}[$

(Dazu Beweis)

$P[\max_{k \in [0,}$

$\cdot (X_n)_t$

Sheet 02

(1)(a) The ~~second~~ first row of the generalized eigenvalue problem reads:

$$\cancel{C_{xy} w_y = C_{xy} w_y}$$

$$C_{xy} w_y = \alpha\, C_{xx} w_x$$

Multiplication by w_x^T yields that the term we want to maximize can be written as

$$w_x^T C_{xy} w_y = \alpha \underbrace{w_x^T C_{xx} w_x}_{=1} = \alpha ,$$

showing the claim.

(b) $(-w_x)^T C_{xy} (-w_y) = w_x^T C_{xy} w_y$,

$(-w_x)^T C_{xx} (-w_x) = w_x^T C_{xx} w_x = 1$ and by the same argumentation the second restriction is fulfilled.

(2)(b) primal: $\frac{1}{N} X Y^T w_y = \alpha \frac{1}{N} X X^T w_x$, the second restriction is fulfilled.

(2)(b) primal: $\frac{1}{N} X Y^T w_y = \lambda \frac{1}{N} X X^T w_x$,

$$\frac{1}{N} Y X^T w_x = \lambda \cdot \frac{1}{N} Y Y^T w_y$$

Multiplication by $N X^T$ respectively $N Y^T$ yields

$$X^T X Y^T Y \alpha_y = \lambda X^T X X^T X \alpha_x ,$$

$$Y^T Y X^T X \alpha_x = \lambda Y^T Y Y^T Y \alpha_y .$$

This can be rewritten as

$$A \cdot B \alpha_y = \lambda A \cdot A \alpha_x$$

$$B \cdot A \alpha_x = \lambda B \cdot B \alpha_y$$

or equivalently

$$\begin{bmatrix} 0 & AB \\ BA & 0 \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix} = \lambda \begin{bmatrix} A^2 & 0 \\ 0 & B^2 \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix} .$$

3