

# On the performance of volatility-managed portfolios

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The **volatility-managed portfolios** is characterized by conservative positions in the underlying factors when volatility was recently high and more aggressively levered positions following periods of low volatility.

$$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$$

## 1. Introduction

### Background:

Recent studies document strong empirical performance for volatility-managed versions of a wide range of popular trading strategies. The findings have important implications for investors.

### Motivation:

1. Moreira and Muir's (2017) spanning regression tests suggest that volatility-scaled portfolios are potentially **more valuable when used in combination** with their original counterparts.
2. Although a volatility-managed portfolio constructed directly is straightforward to construct in real time, the **combined investment strategy (scaled and unscaled portfolios)** is not.

**Whether real-time investors are able to capture the economic gains implied by the spanning regressions?**

### Main Work and Conclusions:

We assess whether volatility management is systematically advantageous for investors and place specific emphasis on real-time implementation, and the same time.

1. We find **no statistical or economic evidence that volatility-managed portfolios systematically earn higher Sharpe ratios.**
2. **The trading strategies implied by the spanning regressions are not implementable in real time.**
3. We provide evidence that this “underperform” result is driven by **substantial structural instability in the underlying spanning regressions for these strategies.**

### Innovations and Meaning:

Our findings suggest that the **in-sample alphas and utility gains do not readily translate into enhanced portfolio outcomes for investors**, offering a complementary viewpoint.

## 2. Data

### 2.1. Data description

**Database:** CRSP, Compustat and IBES

### Factors:

- a. **9 equity factors:** market (MKT), size (SMB), value (HML) factors, momentum factor (MOM), profitability (RMW) and investment (CMA) factors, profitability (ROE) and investment (IA) factors and betting-against-beta factor (BAB)
- b. **94 anomaly variables** reported in Hou et al. (2015) and McLean and Ponti (2016)

**Note:** For each anomaly, we construct a **value-weighted** portfolio that takes a **long (short) position** in the decile of stock.

### 2.2. Construction of volatility-managed portfolios

For an anomaly portfolio, we construct the corresponding **volatility-managed portfolio return** as:

$$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$$

where the **realized variance in month t** is

$$\hat{\sigma}_t^2 = \frac{22}{J_t} \sum_{j=1}^{J_t} (f_t^j)^2$$

$f_t$ : the buy-and-hold excess portfolio return in month t

$\sigma_{t-1}^2$ : the realized variance of daily returns during the month preceding the portfolio formation date

$c^*$ : select the scaling parameter such that  $f_t$  and  $f_{\sigma,t}$  have **the same unconditional volatility**.

$c^*/\sigma_{t-1}^2$ : a measure of the **leverage required to invest in the volatility-managed portfolio**

**Note:** The managed portfolio is a scaled version of the original strategy, with investment positions proportional to the inverse of lagged variance.

### 3. Direct comparisons

**Table 1** Volatility-managed and original factors

	Factor								
	<i>MKT</i> (1)	<i>SMB</i> (2)	<i>HML</i> (3)	<i>MOM</i> (4)	<i>RMW</i> (5)	<i>CMA</i> (6)	<i>ROE</i> (7)	<i>IA</i> (8)	<i>BAB</i> (9)
Panel A: Performance measures for original factors									
Mean	7.80	2.57	4.84	7.94	2.92	3.72	6.52	4.99	8.23
Standard deviation	18.61	11.12	12.14	16.39	7.71	6.97	8.83	6.48	10.71
Sharpe ratio	0.42	0.23	0.40	0.48	0.38	0.53	0.74	0.77	0.77
Panel B: Performance measures for volatility-managed factors									
Mean	9.55	0.86	4.64	16.17	3.94	2.79	9.39	4.69	10.81
Standard deviation	18.61	11.12	12.14	16.39	7.71	6.97	8.83	6.48	10.71
Sharpe ratio	0.51	0.08	0.38	0.99	0.51	0.40	1.06	0.72	1.01
Panel C: Performance comparisons									
Sharpe ratio difference	0.09 [0.30]	−0.15 [0.09]	−0.02 [0.86]	0.50 [0.00]	0.13 [0.29]	−0.13 [0.23]	0.32 [0.01]	−0.05 [0.68]	0.24 [0.01]
Panel D: Properties of volatility-managed factors									
Correlation with original factor	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
$P_{01}(c^*/\hat{\sigma}_{t-1}^2)$	0.04	0.03	0.04	0.04	0.04	0.06	0.06	0.06	0.04
$P_{50}(c^*/\hat{\sigma}_{t-1}^2)$	0.96	0.81	1.02	1.01	1.11	0.97	1.08	0.96	1.00
$P_{99}(c^*/\hat{\sigma}_{t-1}^2)$	6.47	5.07	5.89	8.64	5.02	4.56	4.73	4.45	5.09

#### Observations:

In **five cases** the volatility-managed factor earns a higher average return and Sharpe ratio than the original strategy does. **Three of the nine differences are significantly positive**

**Table 2** Summary of volatility-managed and original portfolios: Broad sample

		Sharpe ratio difference	
Sample (1)	Total (2)	$\Delta SR > 0$ [Signif.] (3)	$\Delta SR < 0$ [Signif.] (4)
Panel A: Combined sample			
All trading strategies	103	53 [8]	50 [4]
Panel B: By category			
Factors	9	5 [3]	4 [0]
Anomaly portfolios	94	48 [5]	46 [4]
Panel C: By trading strategy type			
Accruals	10	4 [0]	6 [0]
Intangibles	10	3 [0]	7 [0]
Investment	11	3 [0]	8 [1]
Market	1	1 [0]	0 [0]
Momentum	9	9 [5]	0 [0]
Profitability	22	15 [1]	7 [1]
Trading	21	11 [1]	10 [1]
Value	19	7 [1]	12 [1]

### Observations:

Volatility management leads to improved and worsened performance at **roughly the same frequency**, and few of the differences are statistically significant. The majority of the significantly positive Sharpe ratio differences are attributable to the **nine momentum** strategies.

**Null hypothesis: the expected Sharpe ratios for the volatility-managed and original versions of each strategy are equal**

—The p-value of **0.84** indicates that a null hypothesis of equal performance is not rejected.

**How to interpret the broad-based results?** —We find volatility management **is likely to be successful if volatility is persistent and the risk-return relation is flat**.

Implement:

- (i) Generate 100,000 bootstrap samples under the null hypothesis (exhibits persistence in conditional volatility, but zero correlation between lagged volatility and future expected return).
- (ii) Run 103 performance comparisons and count the number of positive performance differences.
- (iii) Compare the number of positive differences in the data.

### Observations:

1. Across the bootstrap samples, **the average number of positive Sharpe ratio differences is 66**. The two-tailed bootstrap p-value of **0.01** indicates that **the null hypothesis is rejected**.
2. A positive risk-return relation for a given strategy works to degrade the performance of a given volatility-managed portfolio.

### 4. Combination strategies

Moreira and Muir's (2017) spanning regression tests suggest that volatility-scaled portfolios are potentially **more valuable when used in combination** with their original counterparts.

#### 4.1. Spanning regressions

Moreira and Muir's (2017) **evaluate** volatility-managed factors by estimating time-series regressions of the form

$$f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t.$$

### How to link spanning test results to appraisal ratios and utility gains for investors?

A mean-variance investor who has access to a risk-free security, while allocates between excess returns  $f_t$  and  $f_{\sigma,t}$  aims to:

$$\max_a U(a) = a^\top \hat{\mu} - \frac{\gamma}{2} a^\top \hat{\Sigma} a$$

The vector of optimal portfolio weights is given by

$$a = \begin{bmatrix} x_\sigma^* \\ x^* \end{bmatrix} = \frac{1}{\gamma} \hat{\Sigma}^{-1} \hat{\mu}$$

where  $\hat{\mu} = [\bar{f}_{\sigma,t} \quad \bar{f}_t]^\top$  and the vector of optimal relative weights in the two risky assets is

$$\begin{bmatrix} w_\sigma^* \\ w^* \end{bmatrix} = \frac{\hat{\Sigma}^{-1} \hat{\mu}}{1_2^\top \hat{\Sigma}^{-1} \hat{\mu}}$$

the covariance matrix is

$$\hat{\Sigma} = \hat{\sigma}_f^2 \begin{bmatrix} 1 & \hat{\rho} \\ \hat{\rho} & 1 \end{bmatrix}$$

**The ex post optimal allocation to the volatility-managed portfolio is then proportional to the spanning regression alpha**

$$x_{\sigma}^* = \frac{\hat{\alpha}}{\gamma \hat{\sigma}_f^2 (1 - \hat{\rho}^2)}.$$

We combine the optimal investment policy with the definition of the volatility-managed portfolio to generate the dynamic investment rule

$$y_t^* = x_{\sigma}^* \left( \frac{c^*}{\hat{\sigma}_{t-1}^2} \right) + x^*.$$

**Note:** However, it is not straightforward to construct in real time, because the optimal weighting of scaled and unscaled portfolios depends on in-sample return moments.

#### Evaluation:

$SR(y_t^*)$ : the Sharpe ratio earned by this combination strategy (and the riskless asset)

$SR(z_t^*)$ : the Sharpe ratio earned by a mean-variance investor who does not have access to the volatility-managed portfolio (original portfolio and the riskless asset)

$AR$ : the appraisal ratio for a given scaled strategy:

$$AR = \frac{\hat{\alpha}}{\hat{\sigma}_{\epsilon}}.$$

$AR^2$ : the squared appraisal ratio (reflects the extent to which volatility management can be used to increase the slope of the mean-variance frontier)

$$AR^2 = SR(y_t^*)^2 - SR(z^*)^2,$$

$CER$ : the certainty equivalent return, quantify the in-sample utility gains from volatility management

$$\Delta CER = \frac{SR(y_t^*)^2 - SR(z^*)^2}{2\gamma}$$

**Note:** Positive alphas indicate that volatility management Sharpe ratios increase and utility gain.

## 4.2. In-sample tests

**Table 3 Spanning regressions.**

	Factor								
	<i>MKT</i>	<i>SMB</i>	<i>HML</i>	<i>MOM</i>	<i>RMW</i>	<i>CMA</i>	<i>ROE</i>	<i>IA</i>	<i>BAB</i>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A: Univariate regressions									
Panel A.1: Regression results									
Alpha, $\alpha$ (%)	4.63 (3.08)	−0.76 (−0.87)	1.87 (1.88)	12.39 (7.31)	2.23 (2.57)	0.26 (0.39)	4.97 (5.10)	1.18 (1.83)	5.74 (5.97)
Beta, $\beta$	0.63 (11.32)	0.63 (7.75)	0.57 (7.65)	0.48 (7.13)	0.59 (7.10)	0.68 (13.82)	0.68 (11.12)	0.70 (13.59)	0.62 (12.97)
$R^2$	0.40	0.40	0.33	0.23	0.34	0.46	0.46	0.50	0.38
Appraisal ratio, $AR$	0.32	−0.09	0.19	0.86	0.36	0.05	0.77	0.26	0.68
Panel A.2: Ex post optimization parameters									
Scaling parameter, $c^*$	10.33	2.63	2.95	4.60	1.48	1.53	2.06	1.64	3.20
Risky allocation, $x_{\sigma}^* + x^*$	0.61	0.34	0.82	1.22	1.45	1.60	2.44	0.70	2.05
Relative factor weights									
Vol-managed factor, $w_{\sigma}^*$	0.72	−0.60	0.46	0.98	0.79	0.12	0.97	0.41	0.78
Original factor, $w^*$	0.28	1.60	0.54	0.02	0.21	0.88	0.03	0.59	0.22
Panel A.3: Portfolio performance measures									
Sharpe ratio									
Original factor	0.42	0.23	0.40	0.48	0.38	0.53	0.74	0.77	0.77
Combination strategy	0.53	0.25	0.44	0.99	0.52	0.54	1.06	0.81	1.03
Difference	0.11	0.02	0.04	0.50	0.14	0.00	0.32	0.04	0.26
CER (%)									
Original factor	1.76	0.53	1.59	2.35	1.44	2.85	5.46	5.92	5.90
Combination strategy	2.79	0.61	1.94	9.74	2.71	2.88	11.32	6.57	10.52
Difference	1.03	0.08	0.35	7.39	1.27	0.03	5.86	0.65	4.63

**Observations:**

Almost all combination strategies exhibit strong in-sample performance gains relative to the original factors. The general conclusions are robust.

**Table 4 Summary of spanning regressions: Broad sample.**

Sample (1)	Total (2)	Univariate regressions		Additional controls for Fama-French (1993) factors	
		$\alpha > 0$ [Signif.] (3)	$\alpha < 0$ [Signif.] (4)	$\alpha > 0$ [Signif.] (5)	$\alpha < 0$ [Signif.] (6)
Panel A: Combined sample					
All trading strategies	103	77 [23]	26 [3]	70 [21]	33 [3]
Panel B: By category					
Factors	9	8 [5]	1 [0]	7 [6]	2 [0]
Anomaly portfolios	94	69 [18]	25 [3]	63 [15]	31 [3]

**Observations:**

We find that 77 of the 103 scaled portfolios earn positive alphas in univariate spanning tests and, accordingly, are assigned positive weights in the ex post optimal combination portfolios. Twenty-three of the positive estimates are statistically significant at the 5% level.

**4.3. Out-of-sample tests**

We examine whether investors would have benefitted from volatility management based on information available in real time.

**4.3.1. Out-of-sample strategy design**

**Base case:** We start with a sample of  $T$  monthly excess return observations for a given factor or anomaly portfolio. We use the first  $K = 120$  months as the initial training period and employ an expanding-window approach to estimate the relevant portfolio parameters, construct portfolio weights and evaluate portfolio performance over the subsequent out-of-sample period.

**Optimal portfolio weights:**

$$\begin{bmatrix} x_{\sigma,t} \\ x_t \end{bmatrix} = \frac{1}{\gamma} \hat{\Sigma}_t^{-1} \hat{\mu}_t$$

**The investment position in the original factor:**

$$y_t = x_{\sigma,t} \left( \frac{c_t}{\hat{\sigma}_{t-1}^2} \right) + x_t$$

We impose a leverage constraint of  $|y_t| \leq 5$  against our results being driven by extreme outliers.

**Evaluation:**

$\mu_j, \sigma_j$  are the mean and standard deviation of excess returns for the real-time strategy that invests in the original portfolio and risk-free asset

Sharpe ratio difference:

$$\Delta SR = \frac{\hat{\mu}_i}{\hat{\sigma}_i} - \frac{\hat{\mu}_j}{\hat{\sigma}_j}$$

CER gains:

$$\Delta CER = \left( \hat{\mu}_i - \frac{\gamma}{2} \hat{\sigma}_i^2 \right) - \left( \hat{\mu}_j - \frac{\gamma}{2} \hat{\sigma}_j^2 \right)$$

**4.3.2. Out-of-sample results**

**Table 5 Real-time combination strategies**

	Factor								
	<i>MKT</i> (1)	<i>SMB</i> (2)	<i>HML</i> (3)	<i>MOM</i> (4)	<i>RMW</i> (5)	<i>CMA</i> (6)	<i>ROE</i> (7)	<i>IA</i> (8)	<i>BAB</i> (9)
Panel A: Real-time combination strategies									
Sharpe ratio									
[S1] Combination strategy (real time)	0.42	0.14	0.38	0.92	0.44	0.52	1.13	0.70	1.09
[S2] Original factor (real time)	0.46	0.19	0.43	0.49	0.31	0.56	0.78	0.68	0.79
Difference, [S1]–[S2]	−0.04	−0.06	−0.06	0.44	0.13	−0.03	0.36	0.02	0.30
	[0.64]	[0.37]	[0.41]	[0.00]	[0.53]	[0.20]	[0.00]	[0.74]	[0.00]
[S3] Combination strategy (ex post optimal)	0.53	0.26	0.50	0.99	0.58	0.64	1.21	0.73	1.11
Difference, [S1]–[S3]	−0.11	−0.12	−0.12	−0.07	−0.14	−0.11	−0.07	−0.03	−0.02
	[0.01]	[0.14]	[0.08]	[0.07]	[0.37]	[0.00]	[0.20]	[0.41]	[0.78]
CER (%)									
[S1] Combination strategy (real time)	1.56	0.00	1.41	8.47	1.96	2.74	12.25	4.19	10.88
[S2] Original factor (real time)	1.75	0.38	1.61	2.29	0.91	3.09	5.44	3.68	6.23
Difference, [S1]–[S2]	−0.19	−0.37	−0.20	6.18	1.04	−0.35	6.81	0.51	4.65
	[0.83]	[0.27]	[0.73]	[0.00]	[0.57]	[0.21]	[0.00]	[0.60]	[0.00]
[S3] Combination strategy (ex post optimal)	2.79	0.67	2.47	9.87	3.42	4.04	14.55	5.36	12.34
Difference, [S1]–[S3]	−1.23	−0.66	−1.06	−1.40	−1.46	−1.30	−2.30	−1.17	−1.46
	[0.01]	[0.13]	[0.10]	[0.07]	[0.39]	[0.03]	[0.15]	[0.25]	[0.30]

**Observations:**

Volatility management has potential benefits for real-time investors in some factors, but the gains are less impressive, and are less useful to investors with access to market, size, and value strategies.

**Table 6 Summary of real-time combination strategies: Broad sample**

Panel A: Real-time combination strategies						
Sample (1)	Total (2)	Sharpe ratio difference: Combination strategy (real time) versus		CER difference: Combination strategy (real time) versus		
		Original factor (real time)	Combination strategy (ex post optimal)	Original factor (real time)	Combination strategy (ex post optimal)	
		$\Delta SR$ + / - (3)	$\Delta SR$ + / - (4)	$\Delta CER$ + / - (5)	$\Delta CER$ + / - (6)	
Panel A.1: Combined sample						
All trading strategies	103	45 [8] / 58 [2]	1 [0] / 102 [39]	31 [7] / 72 [7]	0 [0] / 103 [41]	
Panel A.2: By category						
Factors	9	5 [3] / 4 [0]	0 [0] / 9 [2]	5 [3] / 4 [0]	0 [0] / 9 [2]	
Anomaly portfolios	94	40 [5] / 54 [2]	1 [0] / 93 [37]	26 [4] / 68 [7]	0 [0] / 94 [39]	
Panel A.3: By trading strategy type						
Accruals	10	3 [0] / 7 [1]	0 [0] / 10 [5]	3 [0] / 7 [2]	0 [0] / 10 [4]	
Intangibles	10	4 [0] / 6 [0]	0 [0] / 10 [0]	1 [0] / 9 [1]	0 [0] / 10 [4]	
Investment	11	5 [0] / 6 [0]	0 [0] / 11 [6]	5 [0] / 6 [0]	0 [0] / 11 [5]	
Market	1	0 [0] / 1 [0]	0 [0] / 1 [1]	0 [0] / 1 [0]	0 [0] / 1 [1]	
Momentum	9	8 [4] / 1 [0]	0 [0] / 9 [5]	8 [5] / 1 [0]	0 [0] / 9 [5]	
Profitability	22	10 [1] / 12 [0]	1 [0] / 21 [7]	6 [1] / 16 [1]	0 [0] / 22 [5]	
Trading	21	10 [1] / 11 [1]	0 [0] / 21 [6]	6 [1] / 15 [1]	0 [0] / 21 [8]	
Value	19	5 [2] / 14 [0]	0 [0] / 19 [9]	2 [0] / 17 [2]	0 [0] / 19 [9]	

**Observations:**

The real-time combination strategies tend to underperform the real-time strategies that exclude the volatility-managed portfolios and their ex post optimal versions, **no matter whether the Fama-French (1993) factors are included or not.**

**Robustness test:**

We display robustness results pertaining to **the training sample type, risk aversion parameter, training sample length, and leverage constraint.** None of the alternative specifications meaningfully improves performance relative to the base case.

**Table 7 Summary of real-time combination strategies: Robustness tests.**

Description (1)	Sharpe ratio difference: ( $N = 103$ )		CER difference: ( $N = 103$ )	
	$\Delta SR$	Binomial	$\Delta CER$	Binomial
	+ / - (2)	$p$ -value (3)	+ / - (4)	$p$ -value (5)
Panel A: Real-time combination strategies				
Base case design	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Rolling-window training sample	49 [2] / 54 [1]	0.694	17 [1] / 86 [19]	0.000
Risk aversion, $\gamma = 2$	48 [9] / 55 [2]	0.555	35 [8] / 68 [7]	0.001
Risk aversion, $\gamma = 10$	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Initial training sample length, $K = 240$	45 [9] / 58 [10]	0.237	36 [8] / 67 [11]	0.003
Initial training sample length, $K = 360$	40 [9] / 63 [6]	0.030	31 [8] / 72 [8]	0.000
Leverage constraint, $L \leq 1.0$	49 [10] / 54 [2]	0.694	38 [4] / 65 [5]	0.010
Leverage constraint, $L \leq 1.5$	47 [10] / 56 [3]	0.431	38 [7] / 65 [7]	0.010
Leverage constraint, $L \leq \infty$	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Panel B: Real-time combination strategies including Fama-French (1993) three factors				
Base case design	32 [2] / 71 [13]	0.000	32 [3] / 71 [10]	0.000
Rolling-window training sample	32 [0] / 71 [8]	0.000	20 [0] / 83 [16]	0.000
Risk aversion, $\gamma = 2$	22 [3] / 81 [13]	0.000	24 [3] / 79 [10]	0.000
Risk aversion, $\gamma = 10$	31 [3] / 72 [12]	0.000	32 [3] / 71 [10]	0.000
Initial training sample length, $K = 240$	31 [6] / 72 [11]	0.000	35 [9] / 68 [10]	0.001
Initial training sample length, $K = 360$	30 [6] / 73 [9]	0.000	28 [7] / 75 [9]	0.000
Leverage constraint, $L \leq 1.0$	22 [2] / 81 [11]	0.000	27 [3] / 76 [12]	0.000
Leverage constraint, $L \leq 1.5$	21 [3] / 82 [13]	0.000	26 [2] / 77 [9]	0.000
Leverage constraint, $L \leq \infty$	31 [3] / 72 [12]	0.000	32 [3] / 71 [11]	0.000

**Observations:**

Incorporating volatility-managed portfolios into the real-time portfolio decision tends to **harm performance**.

**4.3.3. Explanations for poor out-of-sample performance**

It is natural to explore the economic drivers of these results.

**Table 8 Comparison of VM strategies with traditional anomaly strategies: Broad sample.**

Description (1)	Total (2)	Alpha:		Sharpe ratio difference:		CER difference:	
		$\alpha$	Binomial	$\Delta SR$	Binomial	$\Delta CER$	Binomial
		+ / - (3)	$p$ -value (4)	+ / - (5)	$p$ -value (6)	+ / - (7)	$p$ -value (8)
Panel A: Spanning regressions							
Spanning regressions	103	77 [23] / 26 [3]	0.000	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Spanning regressions with FF3 controls	103	70 [21] / 33 [3]	0.000	32 [2] / 71 [13]	0.000	32 [3] / 71 [10]	0.000
Panel B: Anomaly regressions							
CAPM regressions	102	93 [73] / 9 [3]	0.000	75 [19] / 27 [1]	0.000	68 [18] / 34 [1]	0.001
FF3 regressions	100	81 [60] / 19 [5]	0.000	66 [18] / 34 [1]	0.002	55 [19] / 45 [1]	0.368

**Note:** Panel B Column 2 shows in sample results. Column 5 and 7 show the **real-time** results.

**Observations:** Real-time anomaly strategy **fare better** compared with volatility-managed strategy.

**Why the statistical support for out-of-sample combination strategies is particularly weak in the volatility-managed portfolios setting?**

We consider three potential explanations:

- (i) **Estimation risk** in the out-of-sample portfolio choice exercise (DeMiguel et al. (2009b).
- (ii) **Low power in the out-of-sample tests** because the evaluation period is shorter (e.g., Inoue and Kilian, 2004).



(iii) **Structural instability** in the conditional risk-return tradeoff for the various factors and anomaly portfolios.—We investigate this issue using Bai and Perron's (1998, 2003) **structural break tests**.

**Table 9 Summary of multiple structural break tests for spanning regressions and anomaly regressions: Broad sample**

Description (1)	Total (2)	Frequency distribution for breaks					$\bar{N}_b$ (8)
		$N_b = 0$ (3)	$N_b = 1$ (4)	$N_b = 2$ (5)	$N_b = 3$ (6)	$N_b \geq 4$ (7)	
Panel A: Spanning regressions							
Spanning regressions	103	0	10	52	34	7	2.37
Spanning regressions with FF3 controls	103	1	8	53	35	6	2.37
Panel B: Anomaly regressions							
CAPM regressions	102	15	38	39	9	1	1.44
FF3 regressions	100	10	25	36	21	8	1.92

#### Observations:

In the volatility-managed portfolios setting, the prevalence of breaks often works to the detriment of real-time investors who rely on past data in portfolio construction.

#### 5. Conclusion

- We find **no a pervasive link between volatility management and improved performance for real-time investors** (direct investments in volatility-managed portfolios or combination portfolios).
- We show that combination strategies tend to exhibit positive alphas **in sample**, however, **structural instability in the spanning regression parameters** limits the appeal of this approach to investors conditioning their portfolios on real-time information in the view of the Sharpe ratios and CERs.