On the stability of portfolio selection models

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Outline

- Introduction
- Research design
  - Portfolio selection models
- Empirical study
  - Monte Carlo method
  - Resampling method
- Conclusion

1. Introduction-- Motivation

- Portfolio selection: assessing the effect of the estimation errors of the parameters
  - **Minimum risk models**.
- No sensitivity analysis for the **Risk Parity diversification** approach.
- Nor for other portfolio selection models requiring **maximum gain–risk ratios.**

1. Introduction-- Questions

- What is the **amount of data** required to significantly reduce the effects of the estimation errors?

  Smaller n/T

- Given n tradable assets and T observations, what is **the portfolio selection model that is less sensitive** to the information deficit?

  Risk Parity

1. Introduction-- Contribution

- We examine and compare the noise sensitivity of financial portfolios obtained using various risk and gain measures, and different selection approaches.

2. Portfolio selection models

List of portfolio strategies.

♯	Model	Abbreviation
Risk Diversification strategy		
1	Risk Parity portfolio	RP
2	Most Diversified portfolio	MDP
Optimization strategy		
Minimum Risk portfolios		
3	Min Variance	MinV
4	Min SMAD	MinSMAD
5	Min CVaR ($\varepsilon = 0.3$)	MinCVaR
6	Min MaxLoss	MinMaxLoss
Maximum gain–risk ratio portfolios		
7	Max Sharpe Ratio	MaxSR
8	Max MAD Ratio	MaxMADR
9	Max STARR Ratio ($\varepsilon = 0.3$)	MaxSTARR
10	Max MaxLoss Ratio	MaxMaxLossR
11	Max Omega Ratio	MaxOmegaR

2.1 Risk Diversification strategy

- **The Risk Parity (RP) approach(1996)**

The risk is measured by the volatility

$$\sigma(x) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}}$$

The total risk contribution of asset i is described by

$$TRC_i^{\sigma}(x) = x_i \frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i\,(\Sigma x)_i}{\sigma(x)}$$

The RP approach requires equality of all total risk contributions

$$TRC_i^{\sigma}(x) = TRC_j^{\sigma}(x) \Leftrightarrow x_i\,(\Sigma x)_i = x_j\,(\Sigma x)_j \quad \forall i, j\,.$$

$$\Rightarrow \begin{cases} x_i\,(\Sigma x)_i = \lambda & i = 1, \ldots, n \\ \sum_{i=1}^{n} x_i = 1 & \\ x_i \geq 0 & i = 1, \ldots, n \end{cases}$$

2.1 Risk Diversification strategy

- **The Most Diversified portfolio(2008)**

Maximize a measure of diversification

$$DR(\boldsymbol{x}) = \frac{\boldsymbol{x}^T \boldsymbol{\sigma}}{\sqrt{\boldsymbol{x}^T \Sigma \boldsymbol{x}}}, \qquad \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T$$

Choueifaty et al. (2013)

$$\begin{aligned} \min_{\boldsymbol{y}} \quad & \frac{1}{2} \boldsymbol{y}^T \Sigma \boldsymbol{y} \\ s.t. \quad & \\ & \sum_{i=1}^{n} y_i \sigma_i = 1 \\ & y_i \geq 0 \qquad i = 1, \ldots, n \end{aligned}$$

$$x_i = \frac{y_i}{\sum_i^n y_i} \text{ for } i = 1, \ldots, n.$$

2.2 Minimum risk portfolios

$$\min_{\boldsymbol{x}} \quad Risk(\boldsymbol{x})$$
$$s.t. \quad \sum_{i=1}^{n} x_i = 1, \; x_i \geq 0 \quad i = 1, \ldots, n.$$

- Variance: $\sigma_P^2(\boldsymbol{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$
- Semi-Mean Absolute Deviation (SMAD): $MAD(\boldsymbol{x}) = \frac{1}{T}\sum_{t=1}^{T}|\sum_{i=1}^{n}(\mu_i - r_{it})x_i|$

$$SMAD(\boldsymbol{x}) = \frac{1}{T}\sum_{t=1}^{T}\max(0, \sum_{i=1}^{n}(\mu_i - r_{it})x_i).$$

- CVaR: $CVaR_\varepsilon(\boldsymbol{x}) = -\frac{1}{\varepsilon}\int_0^\varepsilon Q_{R_P(\boldsymbol{x})}(\alpha)d\alpha$ (ε =0.30)
- MaxLoss: $l_P^{max}(x) = -r_P^{min}(x) = -\min_{1\leq t\leq T}\sum_{i=1}^{n}x_i r_{i,t}$

2.3 Maximum gain–risk ratio portfolios

Model	Gain measure	Risk measure
max Sharpe ratio	$\mu_P(\boldsymbol{x}) - r_f$	Variance (6)
max MAD ratio	$\mu_P(\boldsymbol{x}) - r_f$	MAD (7)
max STARR ratio	$\mu_P(\boldsymbol{x}) - r_f$	CVaR (8)
max MaxLoss ratio	$\mu_P(\boldsymbol{x}) - r_f$	MaxLoss (9)
max Omega ratio	$E[\max(0, R_P(\boldsymbol{x}) - r_f)]$	$E[\min(0, R_P(\boldsymbol{x}) - r_f)]$

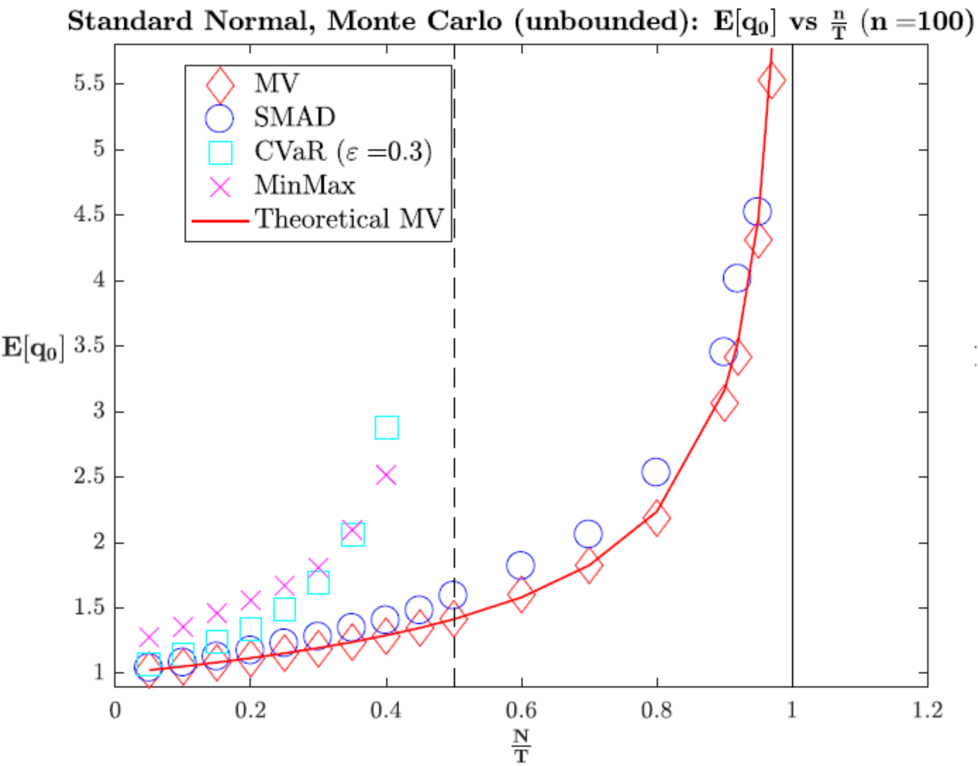
$$\max_{\boldsymbol{x}} \quad \frac{Gain(\boldsymbol{x})}{Risk(\boldsymbol{x})}$$

$$s.t.$$

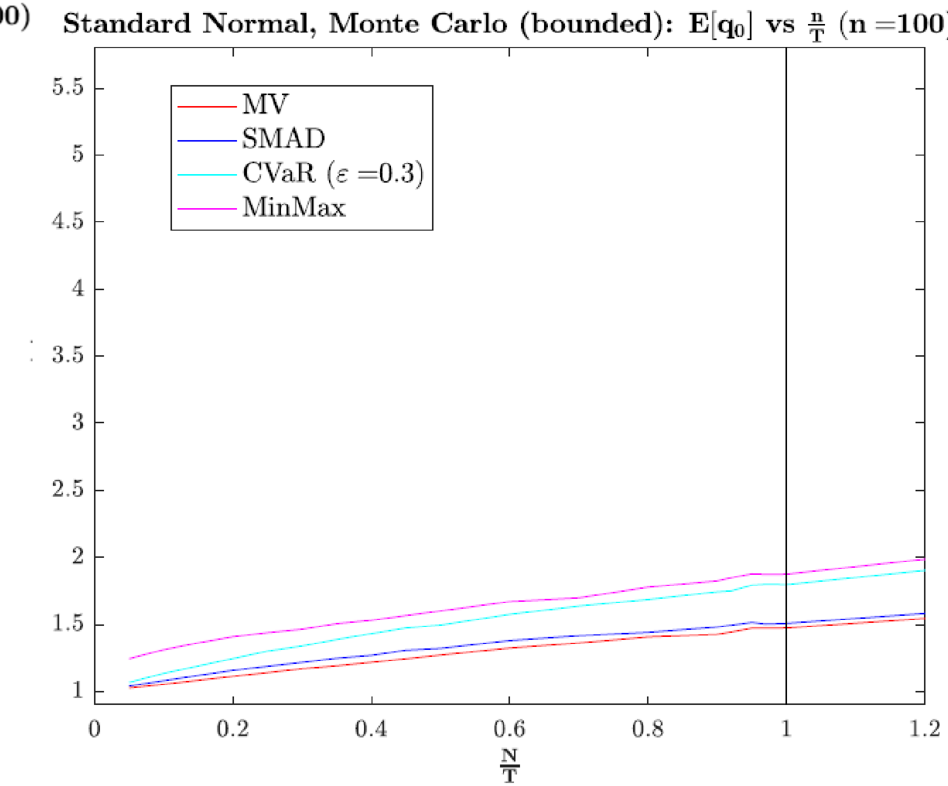
$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0 \qquad i = 1, \ldots, n .$$

3. Stability measures



(a) Long-short minimum risk portfolios



(b) Long-only minimum risk portfolios

$$q_0^2 = \frac{x^{*\prime}\,\Sigma^{(0)}\,x^{*}}{x^{(0)\prime}\,\Sigma^{(0)}\,x^{(0)}}\,. \xrightarrow{\text{Papp et al. (2005)}} E[q_0] = \frac{1}{\sqrt{1-\frac{n}{T}}}$$

- $x^{(0)}$:the "true" optimal portfolio x^{*} :the "perturbed" optimal portfolio

3. Stability measures

- $\boldsymbol{x}^{(\mathbf{0})}$:the “true” optimal portfolio
- $\boldsymbol{x}^{*}$:the “perturbed” optimal portfolio

- **The Euclidean norm**: $d_2 = \|\mathbf{x}^{(0)} - \mathbf{x}^*\|$
- The Root Mean Squared Error (RMSE): $d_{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i^{(0)} - x_i^*)^2}$
- The L_1 norm : $d_1 = \sum_{i=1}^{n} |x_i^{(0)} - x_i^*|$
- The L_∞ norm: $d_\infty = \max_{1 \le i \le n} |x_i^{(0)} - x_i^*|$

4. Data

- Artificial market:

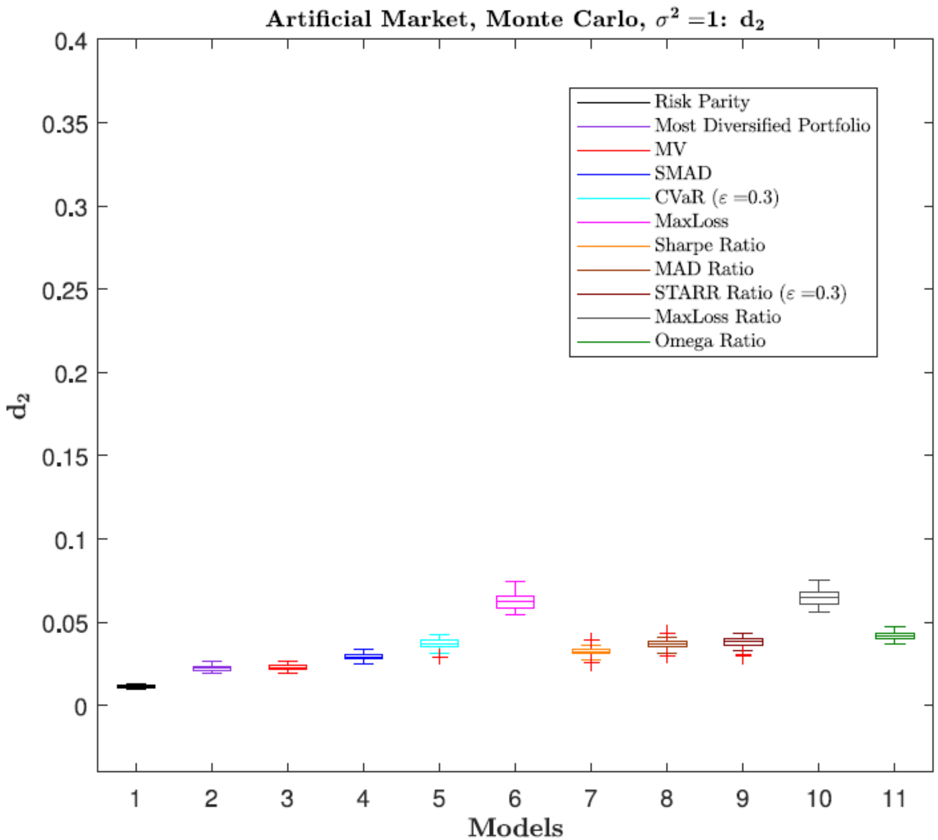
$$\boldsymbol{R} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}),\ \boldsymbol{\mu} = (0.1, 0.1, \ldots, 0.1),\ \text{and}\ \boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}$$

  - The "true" optimal portfolio: $\boldsymbol{x}^{(\mathbf{0})} = \frac{\mathbf{1}}{\mathbf{n}}$
- **Artificial data**: the Monte Carlo technique with n = 100, $\sigma^2 = 1$, portfolio: M=50

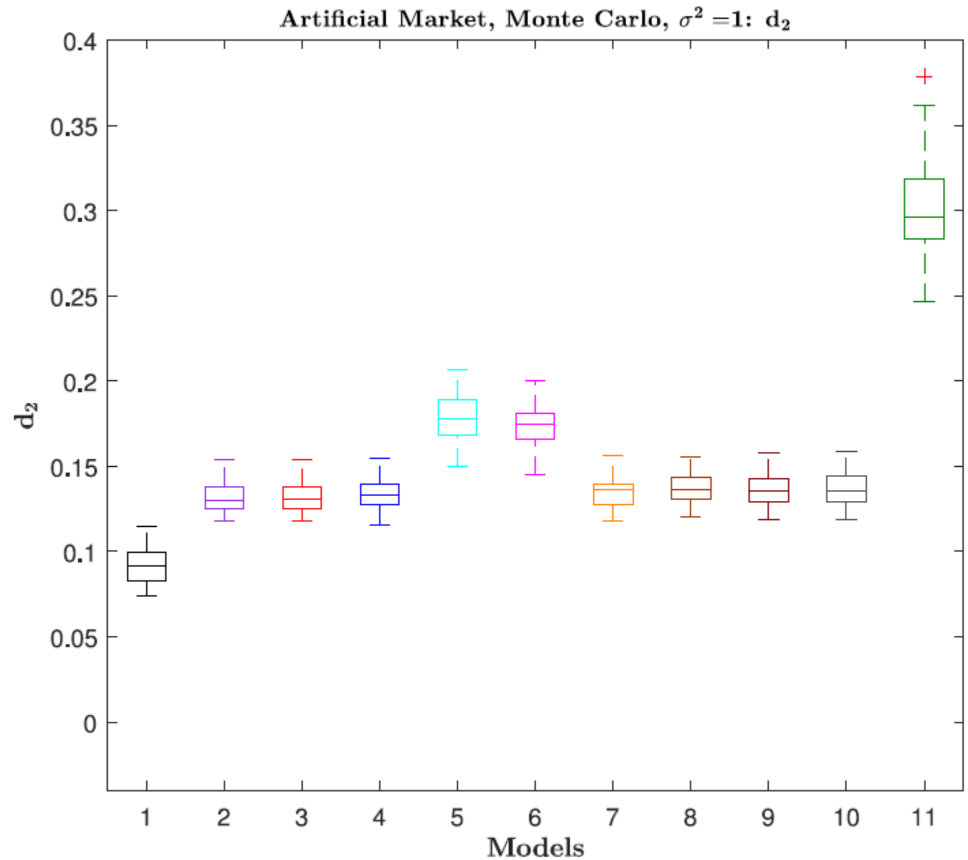
- Five real-world datasets
  - DJIA: 28, 16/02/1990 to 07/04/2016
  - HSI, 43, 25/11/2005 to 11/04/2016
  - STOXX50 :49, 22/05/2001 to 11/04/2016
  - NDX :82, 03/11/2004 to 11/04/2016
  - FTSE :83, 11/07/2002 to 11/04/2016
- The Monte Carlo :$\boldsymbol{R} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ estimated from the real-world.
- Resampling: the historical scenarios represented by the returns

5.1 Stability analysis with the Monte Carlo method

- **Artificial market**



(a) Boxplot for $\frac{n}{T} = 0.05$

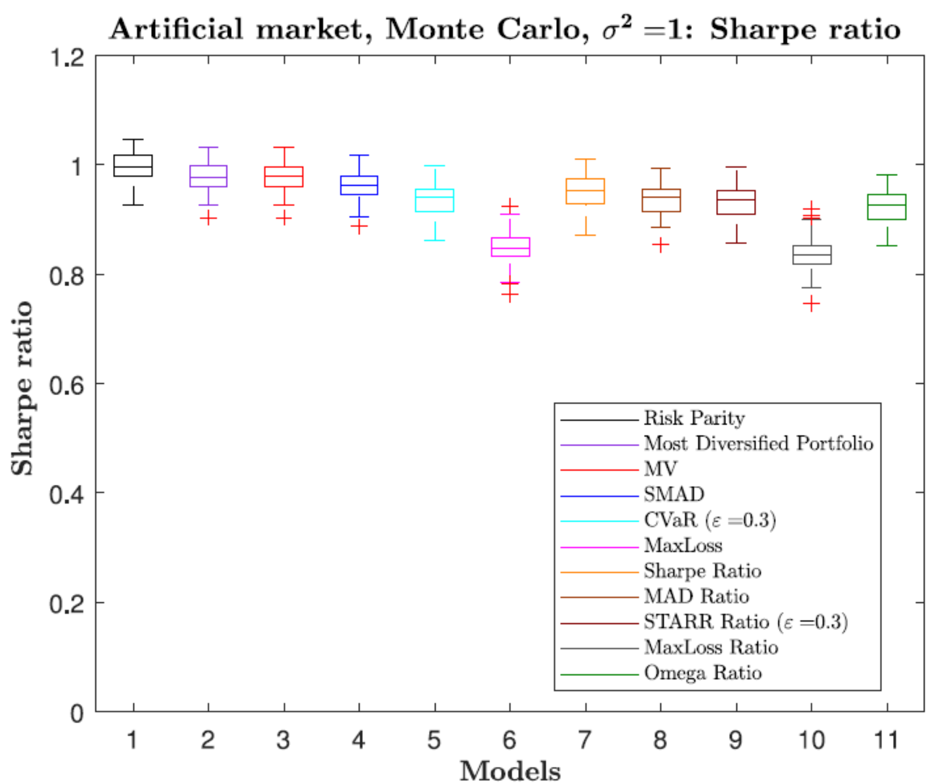


(d) Boxplot for $\frac{n}{T} = 1.5$

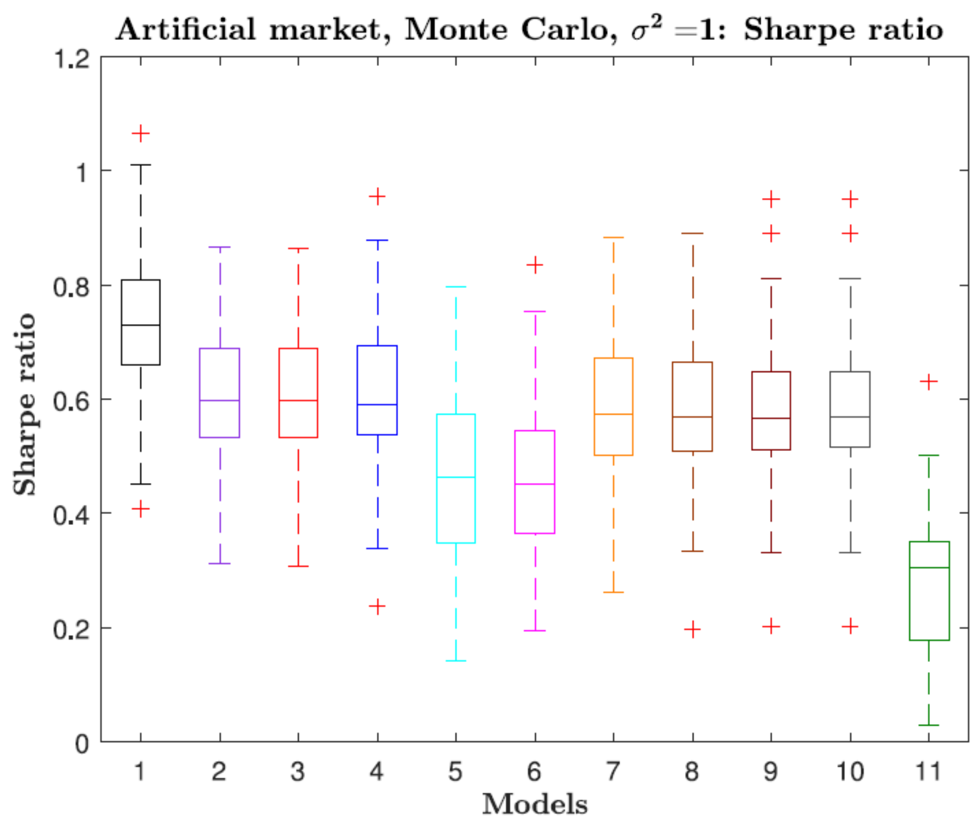
- The Risk Parity portfolio is always the most stable strategy

5.1 Stability analysis with the Monte Carlo method

- **Artificial market**



(a) Boxplot for $\frac{n}{T}$ = 0.05

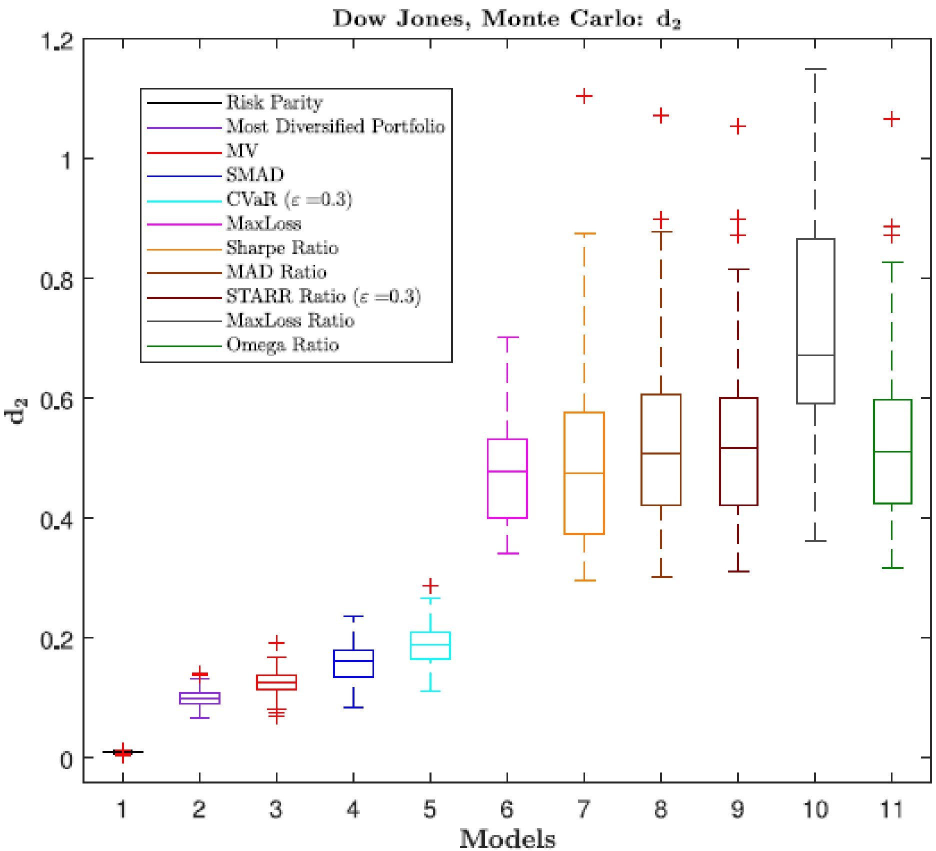


(d) Boxplot for $\frac{n}{T}$ = 1.5

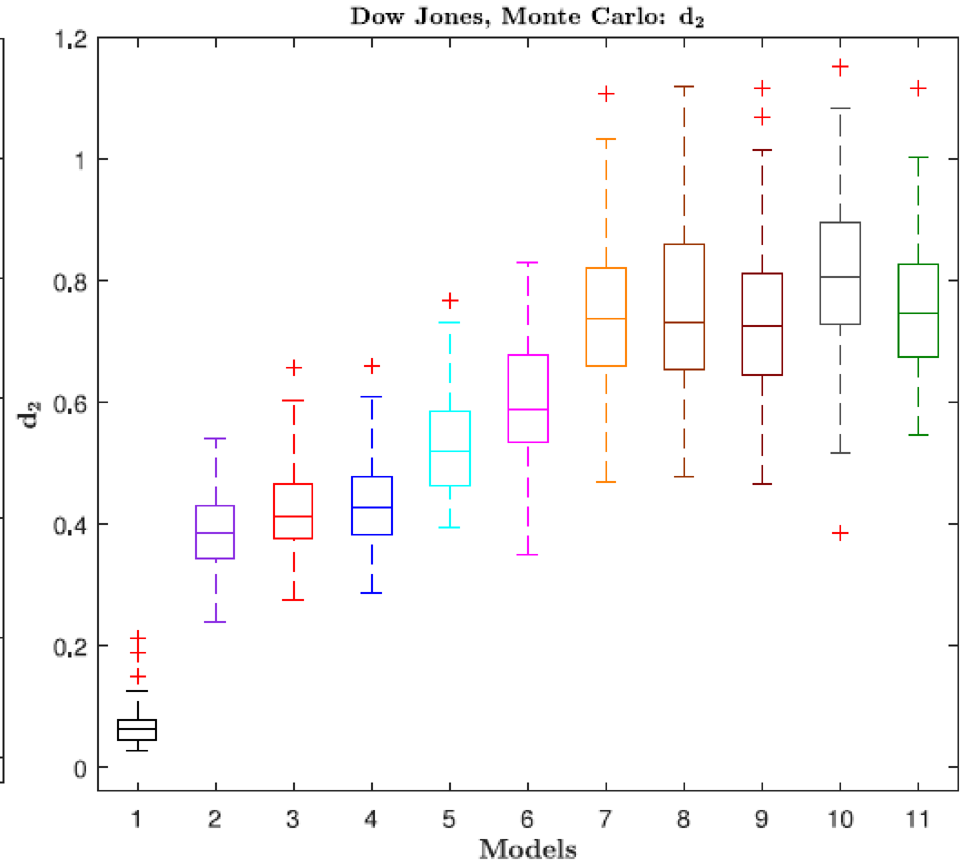
- The Risk Parity portfolio is also the most stable strategy

5.1 Stability analysis with the Monte Carlo method

- **Normal market with inputs estimated from real world datasets**



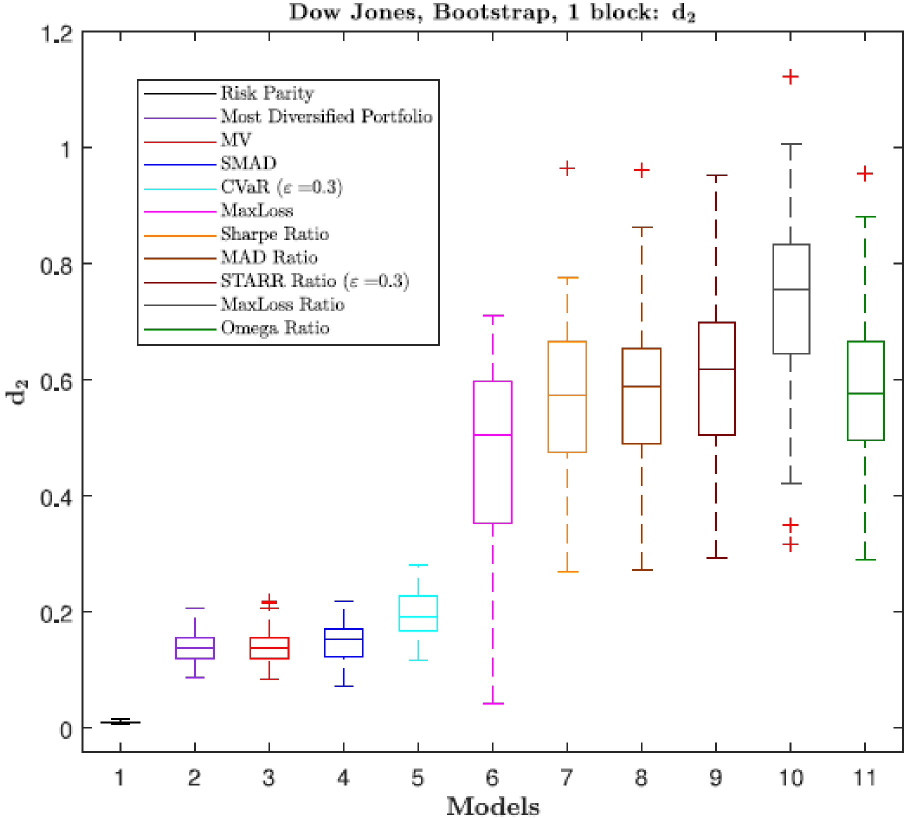
(a) Boxplot for $\frac{n}{T} = 0.05$



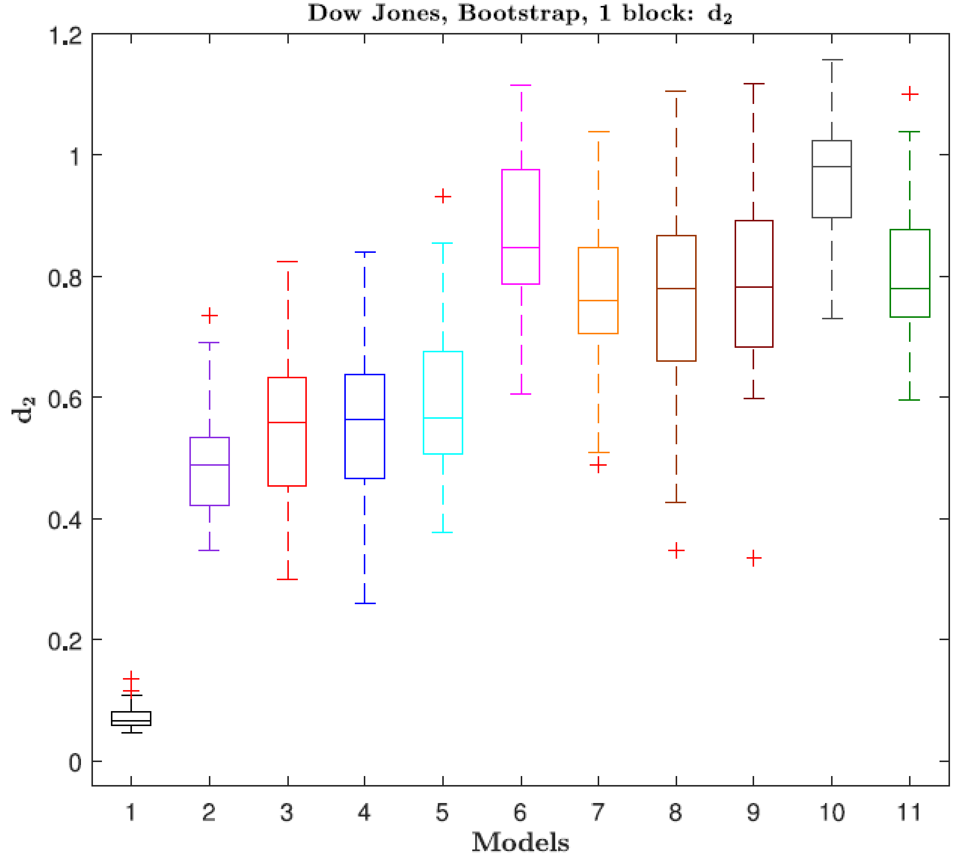
(d) Boxplot for $\frac{n}{T} = 1.5$

As expected, the dispersion around the ''true'' optimal portfolio increases when $\frac{n}{T}$

5.2 Resampling method



(a) Boxplot for $\frac{n}{T}$ = 0.05

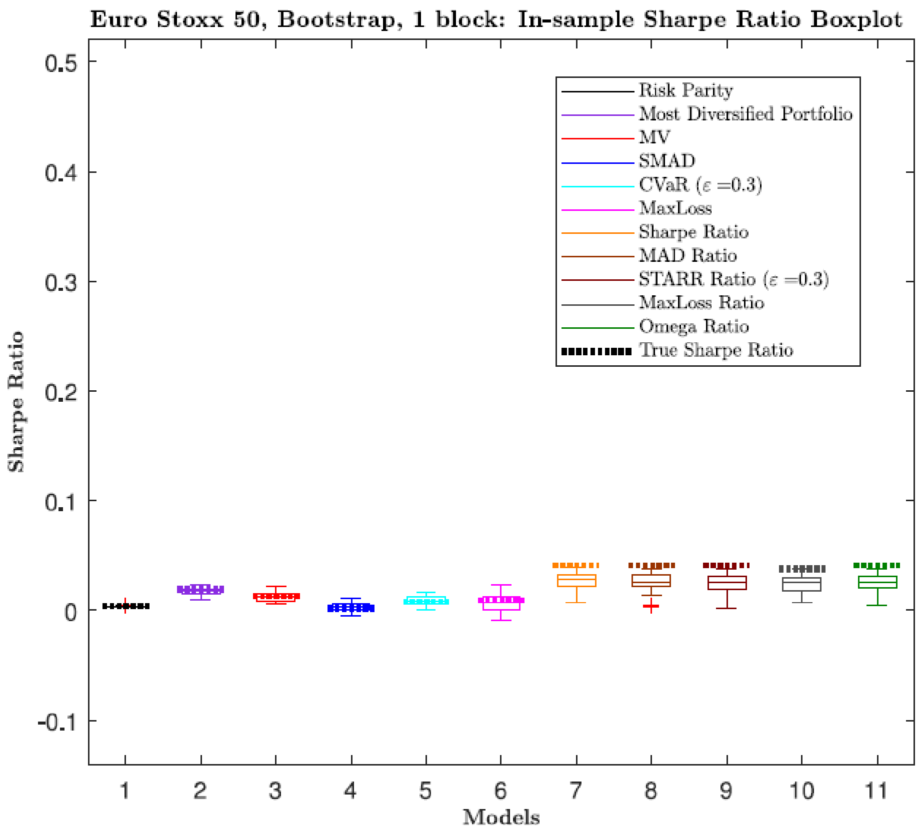


(d) Boxplot for $\frac{n}{T}$ = 1.5

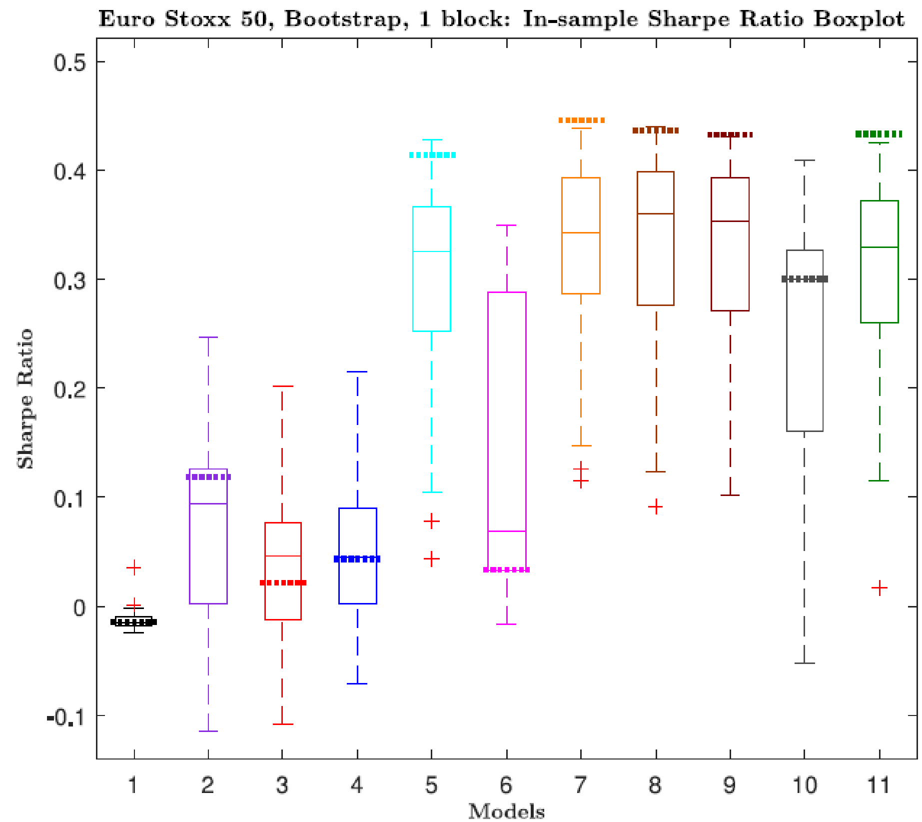
- The Risk Parity portfolio is always the most stable strategy

5.3. Profitability analysis

- **In-sample analysis**



(a) Boxplot for $\frac{n}{T} = 0.05$

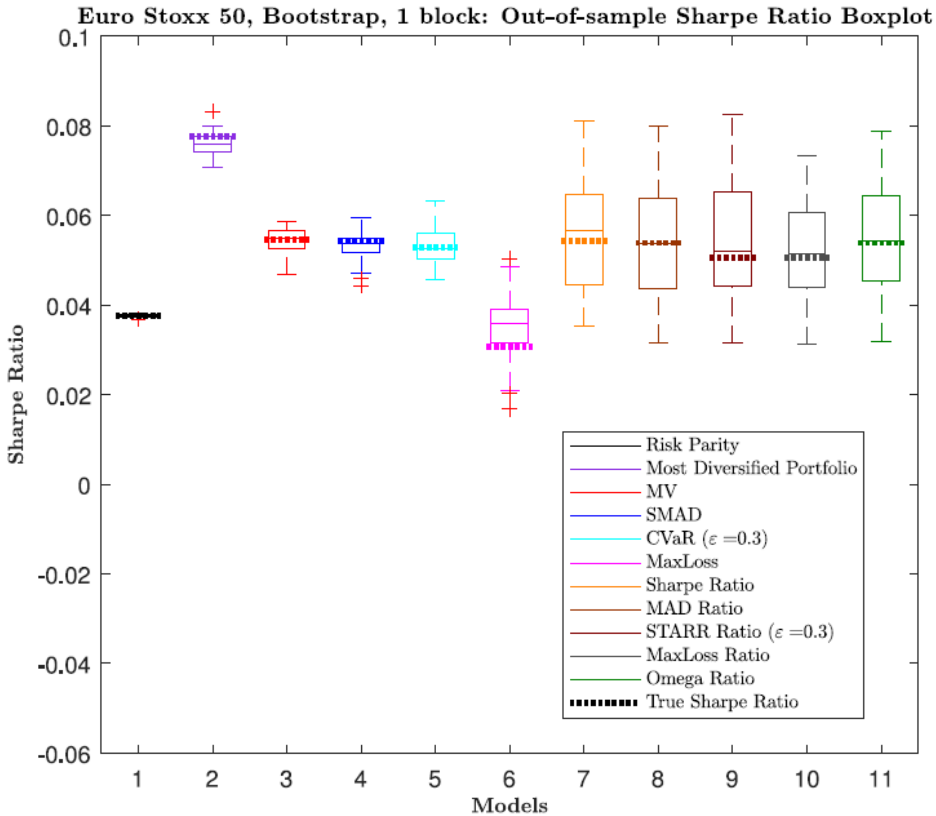


(d) Boxplot for $\frac{n}{T} = 1.5$

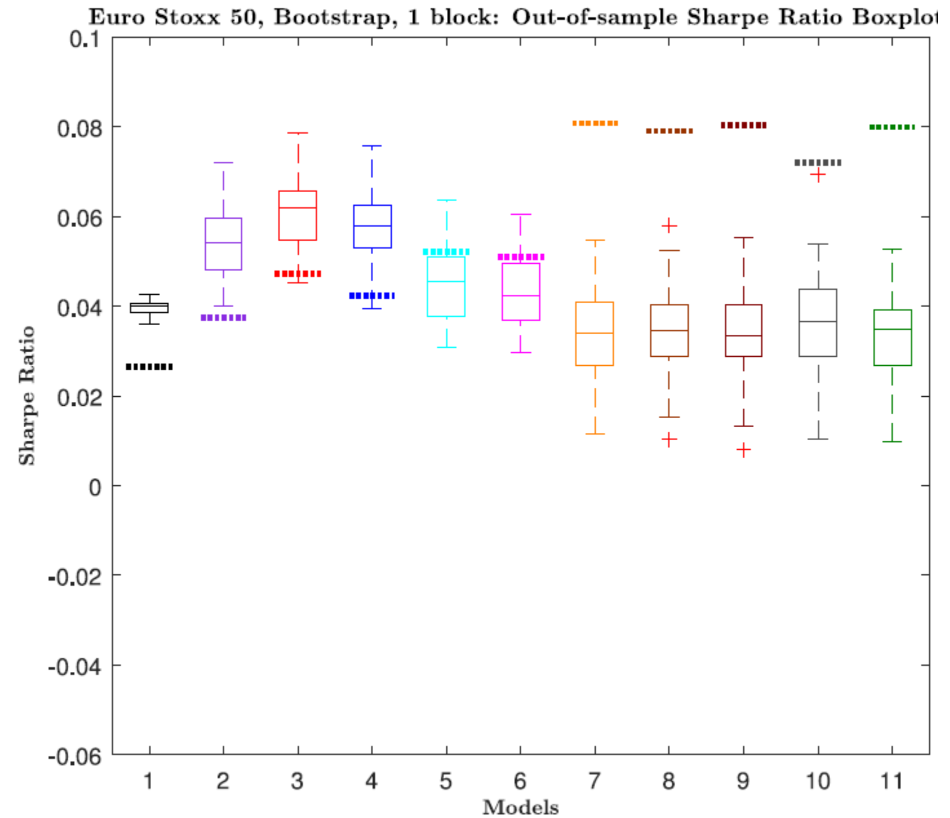
- The SRs obtained by MaxSR are, as expected, always the highest in-sample values.

5.3. Profitability analysis

- **out-of-sample**



(a) Boxplot for $\frac{n}{T} = 0.05$



(d) Boxplot for $\frac{n}{T} = 1.5$

- The SR of the Risk Parity portfolio is the most stable one, even though its value is not generally satisfactory

6. Conclusion

- The maximum gain–risk portfolios are the most sensitive to noise, while risk diversification strategies are generally the most stable.
- The Risk Parity portfolios are the least dispersed, have the worse Sharpe ratios

7. Future research

- Obtain theoretical justifications
- Extend the stability analysis to other classes of portfolio selection models such as those based on stochastic dominance