The Cross-Section of Volatility and Expected Returns

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Definition

Volatility

Aggregate Volatility

• Idiosyncratic Volatility relative to FF3 model

Background and Motivation

- Considerable research has examined the time-series relation between the volatility of the market and the expected return.
- As a systematic risk factor, aggregate volatility should be priced.
- Option studies reveal a negative price for market volatility
- Earlier researchers find positive relation between idiosyncratic volatility and average return

Literature

Pricing volatility

Campbell and Hentschel(1992): framework

Chernov and Ghysels (2000), Benzoni (2002), and Jones(2003): option studies have estimated a negative price of risk for market volatility using options on an aggregate market index or options on individual stocks

Lintner(1965), Lehmann(1990): positive coefficient on IVOL and average return

Contribution

• A systematic investigation of whether the volatility of the market is a priced risk factor and estimation of the price of aggregate volatility risk.

• Examine the idiosyncratic volatility relative to standard Fama and French model at firm level

Research Design

Theory and Empirical Framework

• Multifactor model concludes that:

$$a_t^i = E_tig(r_{t+1}^iig) = eta_{m,t}^i\lambda_{m,t} + eta_{v,t}^i\lambda_{v,t} + \sum_{k=1}^Keta_{k,t}^i\lambda_{k,t}$$

• β is the coefficient for risk and λ is the price of risk

Research Design

Theory and Empirical Framework

- Proxy for aggregate volatility risk
 VIX: index represent the implied volatility of an option contract on S&P100 index that has a maturity of 1 month
- Two-factor model

$$r_t^i = \beta_0 + \beta_{MKT}^i MKT_t + \beta_{\Delta VIX}^i \Delta VIX_t + \varepsilon_t^i,$$

 \bullet β measures the sensitive to aggregate volatility

Research Design

Theory and Empirical Framework

• Define idiosyncratic volatility by ff3:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i.$$

• IVOL is the standard deviation of ε : $\sqrt{\operatorname{var}(\varepsilon_t^i)}$

Portfolio-sorted Regressions

Portfolios sorted by exposure to aggregate volatility risk

Rank	Mean	Std. Dev.	% Mkt Share	Size	B/M	ÇAPM Alpha	FF-3 Alpha	Factor Loadings				
								Pre-Formation $\beta_{\Delta VIX}$	Pre-Formation eta_{FVIX}	Next Month Post-Formation $eta_{\Delta VIX}$	Full Sample Post-Formation β_{FVIX}	
1	1.64	5.53	9.4%	3.70	0.89	0.27	0.30	-2.09	-2.00	-0.033	-5.06	
						[1.66]	[1.77]				[-4.06]	
2	1.39	4.43	28.7%	4.77	0.73	0.18	0.09	-0.46	-0.42	-0.014	-2.72	
						[1.82]	[1.18]				[-2.64]	
3	1.36	4.40	30.4%	4.77	0.76	0.13	0.08	0.03	0.08	0.005	-1.55	
						[1.32]	[1.00]				[-2.86]	
4	1.21	4.79	24.0%	4.76	0.73	-0.08	-0.06	0.54	0.62	0.015	3.62	
						[-0.87]	[-0.65]	0.000			[4.53]	
5	0.60	6.55	7.4%	3.73	0.89	-0.88	-0.53	2.18	2.31	0.018	8.07	
						[-3.42]	[-2.88]				[5.32]	
5-1	-1.04					-1.15	-0.83					
	[-3.90]					[-3.54]	[-2.93]					
Joint to	st p-value					0.01	0.03				0.00	

Portfolio-sorted Regressions

• Fama-Macbeth factor premiums on 25 portfolios sorted by β_{MKT} and $\beta_{\Delta VIX}$

Panel A: Fama-MacBeth (1973) Factor Premiums							
	I	II	III	IV			
Constant	-0.145	-0.527	-0.202	-0.247			
	[-0.23]	[-0.88]	[-0.31]	[-0.36]			
MKT	0.977	1.276	1.034	1.042			
	[1.11]	[1.47]	[1.13]	[1.13]			
FVIX	-0.080		-0.082	-0.071			
	[-2.49]		[-2.39]	[-2.02]			
STR		-0.194					
		[-2.32]					
SMB	-0.638	-0.246	-0.608	-0.699			
	[-1.24]	[-0.59]	[-1.13]	[-1.25]			
HML	-0.590	-0.247	-0.533	-0.232			
	[-0.95]	[-0.40]	[-0.82]	[-0.34]			
UMD			0.827	0.612			
			[0.83]	[0.59]			
LIQ				-0.021			
12				[-1.00]			
$\mathrm{Adj}R^2$	0.67	0.56	0.65	0.79			

Portfolio-sorted Regressions

Portfolios sorted by VOL and IVOL

Rank	Mean	Std. Dev.	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha
		Panel A:	Portfolios Sor	ted by Tota	l Volatility		
1	1.06	3.71	41.7%	4.66	0.88	0.14	0.03
						[1.84]	[0.53]
2	1.15	4.48	33.7%	4.70	0.81	0.13	0.08
						[2.14]	[1.41]
3	1.22	5.63	15.5%	4.10	0.82	0.07	0.12
						[0.72]	[1.55]
4	0.99	7.15	6.7%	3.47	0.86	-0.28	-0.17
						[-1.73]	[-1.42]
5	0.09	8.30	2.4%	2.57	1.08	-1.21	-1.16
V-1	periodes.	2000-00-0	1390000	5-6520457	HP\$45.75	[-5.07]	[-6.85]
5-1	-0.97					-1.35	-1.19
	[-2.86]					[-4.62]	[-5.92]
	Panel B:	Portfolios S	orted by Idiosy	vncratic Vol	atility Rela	tive to FF-3	
1	1.04	3.83	53.5%	4.86	0.85	0.11	0.04
						[1.57]	[0.99]
2	1.16	4.74	27.4%	4.72	0.80	0.11	0.09
						[1.98]	[1.51]
3	1.20	5.85	11.9%	4.07	0.82	0.04	0.08
						[0.37]	[1.04]
4	0.87	7.13	5.2%	3.42	0.87	-0.38	-0.32
						[-2.32]	[-3.15]
5	-0.02	8.16	1.9%	2.52	1.10	-1.27	-1.27
						[-5.09]	[-7.68]
5-1	-1.06					-1.38	-1.31
	[-3.10]					[-4.56]	[-7.00]

Portfolio-sorted Regressions

 Portfolios sorted by IVOL controlling for other factors

		1 Low	2	3	4	5 High	5-1
NYSE Stocks Only		0.06	0.04	0.02	-0.04	-0.60	-0.66
	93	[1.20]	[0.75]	[0.30]	[-0.40]	[-5.14]	[-4.85]
Size Quintiles	Small 1	0.11	0.26	0.31	0.06	-0.43	-0.55
		[0.72]	[1.56]	[1.76]	[0.29]	[-1.54]	[-1.84]
	2	0.19	0.20	-0.07	-0.65	-1.73	-1.91
		[1.49]	[1.74]	[-0.67]	[-5.19]	[-8.14]	[-7.69]
	3	0.12	0.21	0.03	-0.27	-1.49	-1.61
		[1.23]	[2.40]	[0.38]	[-3.36]	[-10.1]	[-7.65]
	4	0.03	0.22	0.17	-0.03	-0.82	-0.86
		[0.37]	[2.57]	[2.47]	[-0.45]	[-6.61]	[-4.63]
	Large 5	0.09	0.04	0.03	0.14	-0.17	-0.26
	ANTENNES (1.24)	[1.62]	[0.72]	[0.51]	[1.84]	[-1.40]	[-1.74]
Controlling for Siz	ze	0.11	0.18	0.09	-0.15	-0.93	-1.04
		[1.30]	[2.49]	[1.35]	[-1.99]	[-6.81]	[-5.69]
Controlling for Bo	ok-to-Market	0.61	0.69	0.71	0.50	-0.19	-0.80
Š.		[3.02]	[2.80]	[2.49]	[1.47]	[-0.48]	[-2.90]
Controlling for Le	verage	0.11	0.11	0.08	-0.24	-1.12	-1.23
	A THE SEC	[2.48]	[2.20]	[1.19]	[-2.45]	[-7.81]	[-7.61]
Controlling for Lie	quidity	0.08	0.09	-0.01	-0.16	-1.01	-1.08
74 1000 to \$44	06	[1.71]	[1.53]	[-0.09]	[-1.62]	[-8.61]	[-7.98]
Controlling for Volume		-0.03	0.02	-0.01	-0.39	-1.25	-1.22
125		[-0.49]	[0.39]	[-0.32]	[-7.11]	[-10.9]	[-8.04]
Controlling for Tu	rnover	0.11	0.03	-0.11	-0.49	-1.34	-1.46
		[2.49]	[0.58]	[-1.79]	[-6.27]	[-11.0]	[-10.7]
Controlling for Bi	d-Ask Spreads	-0.07	-0.01	-0.09	-0.49	-1.26	-1.19
		[-1.21]	[-0.18]	[-1.14]	[-5.36]	[-9.13]	[-6.95]
Controlling for Co	skewness	-0.02	-0.00	0.01	-0.37	-1.40	-1.38
		[-0.32]	[-0.02]	[0.08]	[-2.30]	[-6.07]	[-5.02]
Controlling for Di	spersion	0.12	-0.07	0.11	0.01	-0.27	-0.39
in Analysts' For	ecasts	[1.57]	[-0.76]	[1.12]	[0.09]	[-1.76]	[-2.09]

Conclusions

- Estimate a cross-sectional price of volatility risk of approximately -1% per annum, and this estimate is robust to controlling for size, value, momentum, and liquidity effects
- Stocks with high idiosyncratic volatility have abysmally low average returns.

THANKS!