Hedge Fund Crowds and Mispricing

Richard Sias, H. J. Turtle, Blerina Zykaj .

Mangement science,2016

Crowded Trades and Tail Risk

Gregory W. Brown, Philip Howard, Christian T. Lundblad.

The Review of Financial Studies,2022

Crowding and Tail Risk in Momentum Returns

Pedro Barroso, Roger M. Edelen, Paul Karehnke.

Journal of Financial and Quantitative Analysis,2022

解读者：Tu Xueyong

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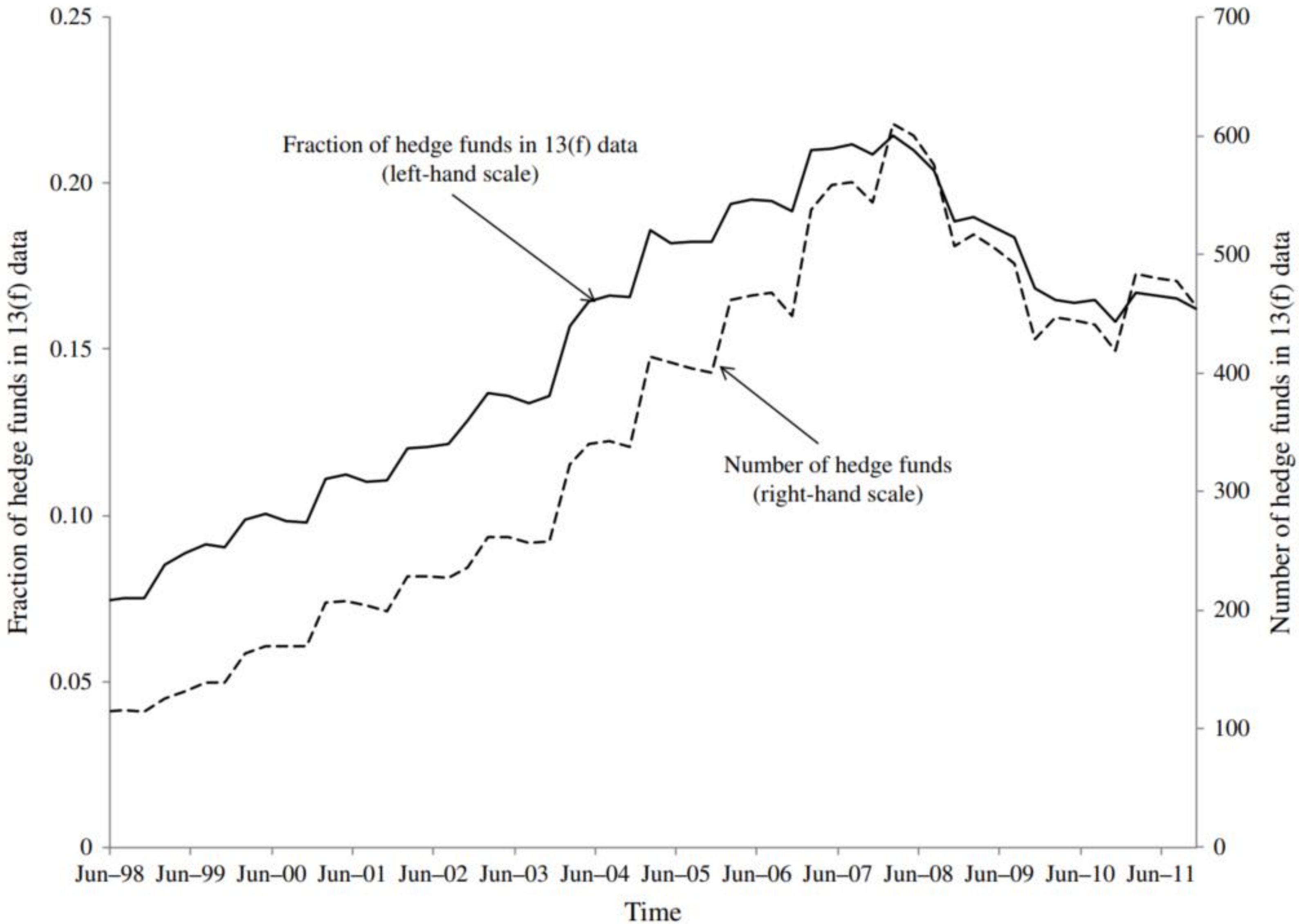
1. Introduction-- Motivation

Hedge Funds in the 13(f) Data Over Time (1998–2011)



1. Introduction-- Motivation

- Hedge fund assets under management grew more than 1,420% between the end of 1997 and 2012

- Recent models suggest hedge funds follow similar strategies resulting in crowded equity positions that **destabilize** markets

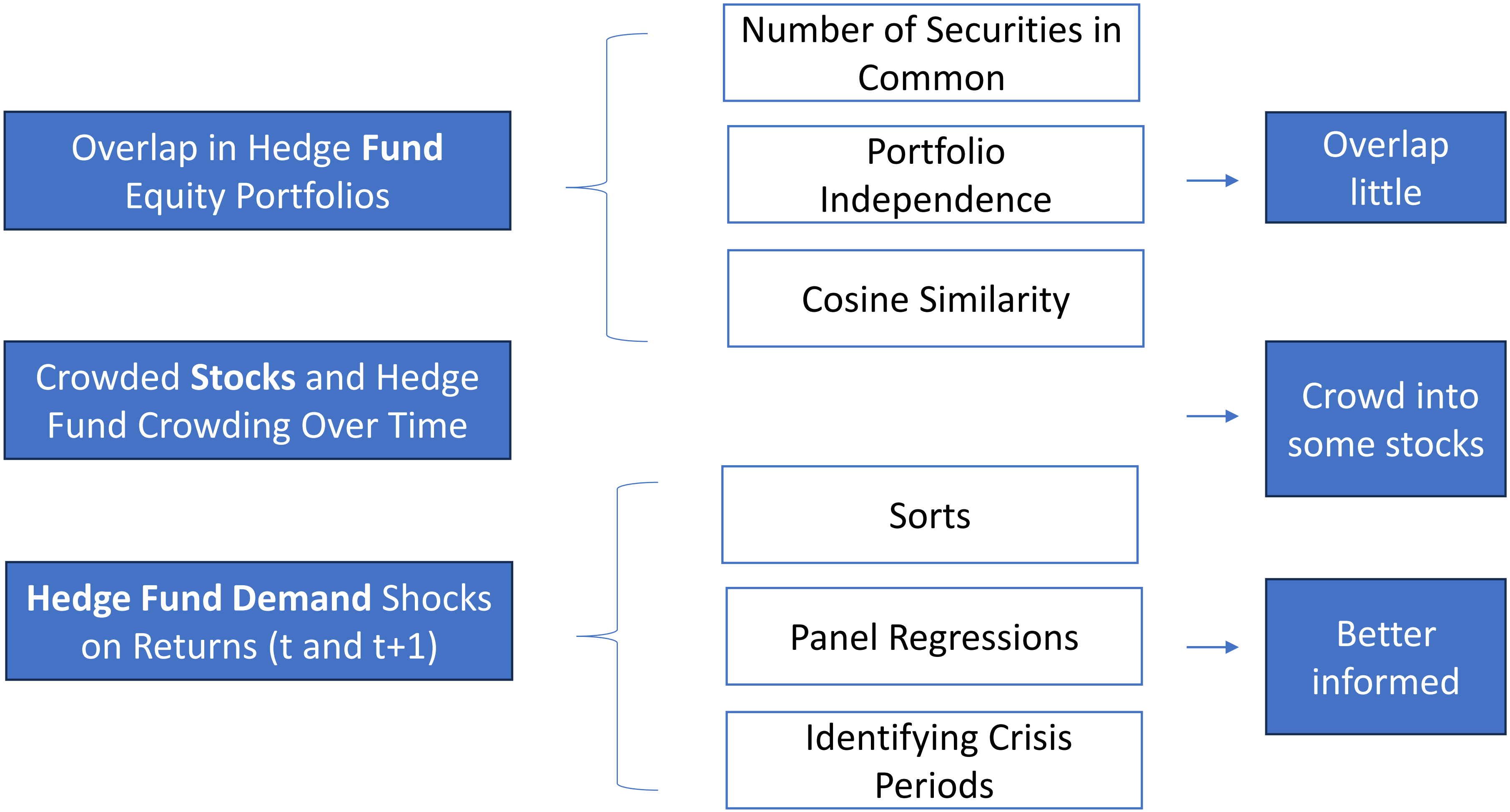
- Stein (2009, p. 1517) points out two negative externalities:
  - Hedge funds can drive prices from value
  - The negative externalities are exacerbated in a funding crisis

1. Introduction– Literature review

- Hedge fund crowds play a meaningful role in destabilizing markets.
  - the 1998 financial crisis (e.g., Kyle and Xiong 2001, Gromb and Vayanos 2002)
  - the 2007 “quant crisis” (e.g., Khandani and Lo 2011, Brunnermeier 2009)
  - the 2008–2009 financial crisis (e.g., Acharya et al. 2009, Pedersen 2009)

- These concerns are echoed by the popular press, regulators, and hedge fund managers themselves.

1. Introduction-- Framework



1. Introduction-- Contribution

- Inconsistent with the existing assertion, we find that hedge fund equity portfolios are **remarkably independent**.
- Hedge Fund demand shocks are, on average, positively related to subsequent raw and risk-adjusted returns.
- Our results have important implications for the ongoing debate regarding hedge fund regulation.

2. Data

- Quarterly 13(f) reports between 1998 and 2011
- 13(f) data have two primary limitations:
  - do **not** capture **all hedge funds** or **all hedge fund positions**.
  - **no** information about equity derivatives or **short positions** of hedge funds
- Thomson Reuters database: the parent management companies
- Securities must have returns for each month in quarter Center for Research in Security Prices (CRSP) share code of 10 or 11

3. Overlap in Hedge Fund Equity Portfolios

1. Number of Securities in Common
   - the number of securities that each pair of hedge funds hold in common
2. Portfolio Independence

$$PI(h_t, j_t) = \frac{1}{2}\sum_{k=1}^{K} |w_{h,k,t} - w_{j,k,t}|,$$

3. Cosine Similarity

$$s(h_t, j_t) = \sum_{k=1}^{K} w_{h,k,t} w_{j,k,t} \Bigg/ \left(\sqrt{\sum_{k=1}^{K} w_{h,k,t}^2}\sqrt{\sum_{k=1}^{K} w_{j,k,t}^2}\right).$$

4. Cosine Similarity in positive active weights

$$w_{h,k,t} - w_{mkt,k,t}$$

3. Overlap in Hedge Fund Equity Portfolios

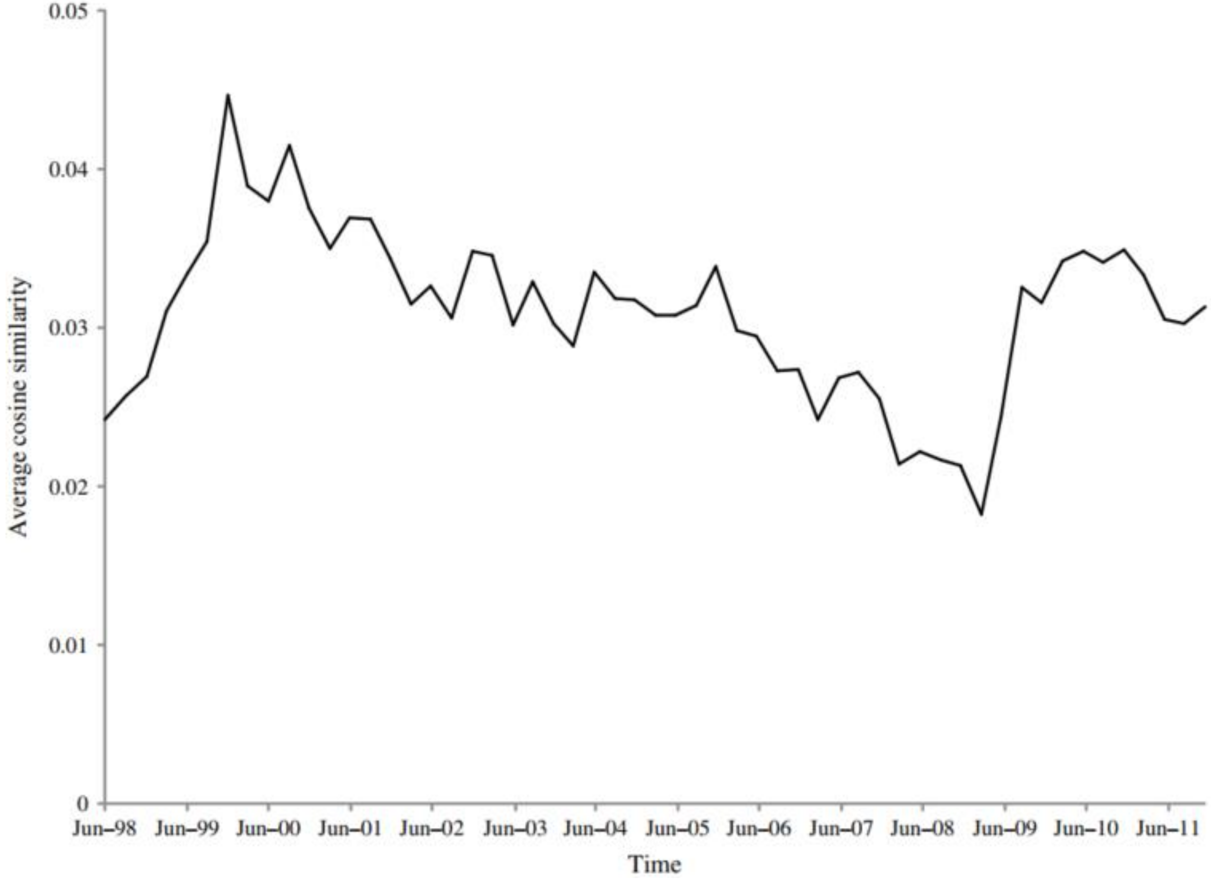
Table 2 Overlap in Hedge Fund Portfolios and Similar Size Non-Hedge Fund Institution Portfolios

	95th percentile	Median	5th percentile	Average
Panel A: Number of common securities				
Hedge funds	16.000	0.800	0.000	4.131
(No. of securities held)	(278)	(51)	(36)	
Non-hedge funds	74.982	9.000	0.000	20.865
(No. of securities held)	(319)	(103)	(54)	
Number positive	0			0
[Number significant]	[0]			[0]
Number negative	55			55
[Number significant]	[55]			[55]
Panel B: Portfolio independence				
Hedge funds	1.000	0.997	0.904	0.978
Non-hedge funds	1.000	0.946	0.645	0.895
Number positive			55	55
[Number significant]			[55]	[55]
Number negative			0	0
[Number significant]			[0]	[0]

	95th percentile	Median	5th percentile	Average
Panel C: Cosine similarity in portfolio weights				
Hedge funds	0.153	0.002	0.000	0.031
Non-hedge funds	0.509	0.061	0.000	0.139
Number positive	0			0
[Number significant]	[0]			[0]
Number negative	55			55
[Number significant]	[55]			[55]
Panel D: Cosine similarity in positive active weights				
Hedge funds	0.123	0.001	0.000	0.024
Non-hedge funds	0.271	0.022	0.000	0.066
Number positive	0			0
[Number significant]	[0]			[0]
Number negative	55			55
[Number significant]	[55]			[55]

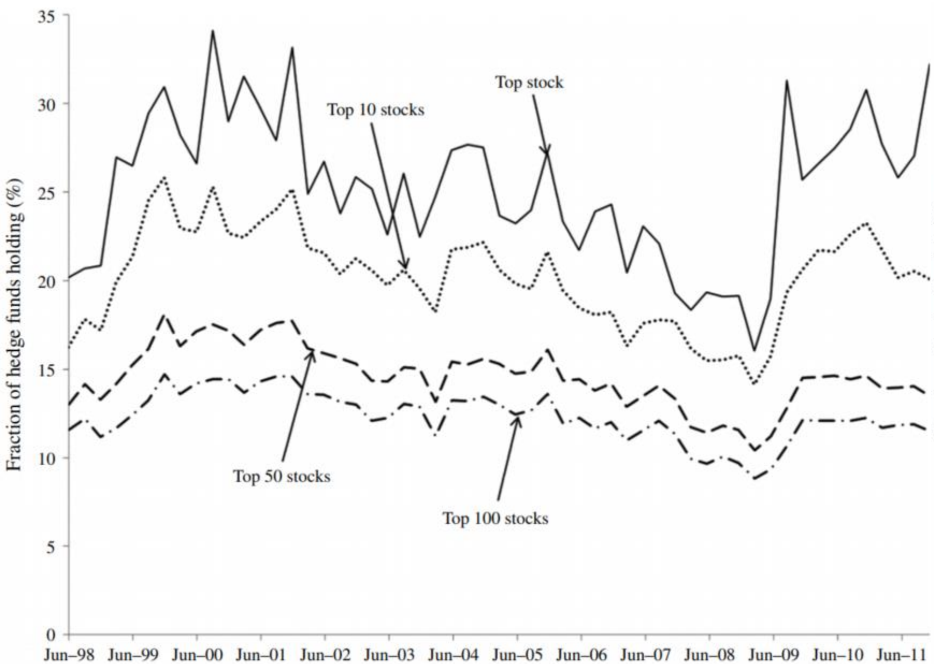
- Hedge funds have lower levels of portfolio overlap than similar size non-hedge fund institutions.

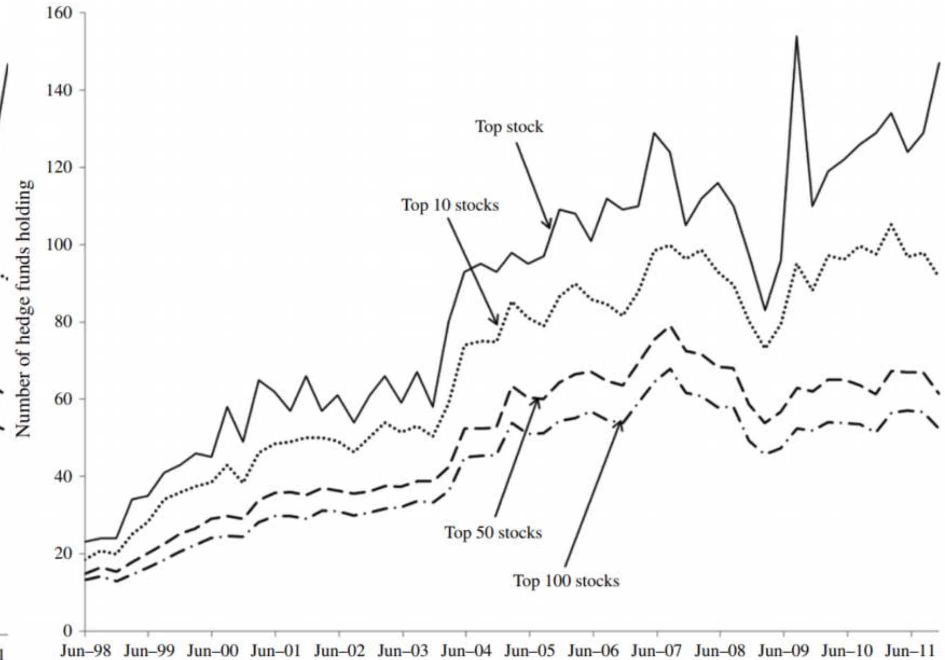
4. Crowded Stocks and Hedge Fund Crowding Over Time



- **No evidence** that portfolio overlap in the average hedge fund pair has systematically **increased over time**.

4. Crowded Stocks and Hedge Fund Crowding Over Time





- Hedge fund crowds in individual securities have grown over time as a result of the **growth in the number of hedge funds**
- and **not** because individual hedge funds increasingly **focus on the same securities**.

5. Hedge Fund Demand Shocks and Returns

Table 3 The Relation Between Hedge Fund Demand Shocks and Contemporaneous and Subsequent Raw Returns

			Mean quarterly return (%)			Benchmark adjusted returns (%)			Earnings surprises (%)	
Portfolio	No. of stocks	Hedge fund demand$_{t=0}$ [# buy: # sell]	Return$_{t=0}$	Return$_{t=1}$	Return$_{t=1\text{ to }4}$	Adjusted return$_{t=0}$	Adjusted return$_{t=1}$	Adjusted return$_{t=1\text{ to }4}$	CAR$_{t=1}$	I/B/E/S$_{t=1}$
Heavy sell	637	−5.158 [4.359: 9.517]	1.873 (0.92)	1.672 (0.93)	2.473 (1.46)	0.114 (0.26)	−0.391 (−0.80)	0.449 (1.21)	−0.202 (−3.68)***	0.046 (8.00)***
Sell	768	−1.500 [3.729: 5.229]	1.118 (0.65)	2.094 (1.28)	2.613 (1.70)*	−0.813 (−2.31)**	−0.279 (−0.91)	0.434 (1.50)	−0.138 (−2.61)**	0.045 (8.29)***
No change	1,404	0.000 [1.306: 1.306]	1.040 (0.61)	2.673 (1.63)	2.626 (1.77)*	−0.746 (−1.33)	0.407 (0.80)	0.436 (0.93)	−0.181 (−3.22)***	0.043 (5.67)***
Buy	820	1.510 [5.127: 3.617]	3.347 (1.84)*	3.442 (2.09)**	3.060 (2.01)**	1.241 (2.24)**	1.085 (2.59)**	0.818 (2.56)**	−0.009 (−0.12)	0.048 (8.12)***
Heavy buy	713	5.372 [9.528: 4.156]	8.125 (3.32)***	4.510 (2.58)**	3.133 (1.99)*	5.721 (3.77)***	2.232 (4.03)***	0.990 (2.56)**	0.042 (0.69)	0.057 (8.35)***
Heavy buy – Heavy sell			6.252 (3.15)***	2.838 (5.14)***	0.660 (2.80)***	5.607 (3.22)***	2.623 (5.28)***	0.542 (3.13)***	0.245 (5.15)***	0.011 (3.63)***

- No evidence that hedge fund demand shocks, in general, drive prices from value
- Superior hedge fund information regarding fundamental values

5. Hedge Fund Demand Shocks and Returns

Table 4 Panel Regressions of Returns on Hedge Fund and Non-Hedge Fund Demand

	(1) $Return_{t=0}$	(2) $Return_{t=1}$	(3) $Return_{t=2}$	(4) $Return_{t=3}$	(5) $Return_{t=4}$
ln(*Capitalization*)	−0.733	−0.689	−1.063	−1.056	−0.889
	(−6.89)***	(−9.27)***	(−13.55)***	(−13.95)***	(−11.30)***
Turnover	5.542	−0.577	−0.473	−0.523	−0.426
	(14.67)***	(−6.21)***	(−4.07)***	(−6.10)***	(−5.16)***
Idiosyncratic σ^2	−0.340	−0.066	−0.041	−0.134	−0.456
	(−1.76)*	(−0.92)	(−0.54)	(−1.70)*	(−2.47)**
Beta	−2.203	0.110	0.502	0.435	0.443
	(−15.94)***	(1.20)	(5.38)***	(4.82)***	(4.82)***
$Return_{t=(0,-1)}$		0.013	0.007	−0.007	−0.017
		(6.74)***	(3.65)***	(−4.09)***	(−8.91)***
HF demand	0.253	0.547	0.192	0.134	0.033
	(3.81)***	(12.06)***	(4.19)***	(2.69)**	(0.66)
NHF demand	3.349	−0.539	−0.479	−0.620	−0.471
	(16.46)***	(−11.14)***	(−9.00)***	(−10.87)***	(−8.72)***
Firms (clusters)	10,636	10,636	10,375	10,091	9,800
Observations	248,836	248,836	243,272	237,516	231,730
R^2 (%)	17.21	13.88	12.86	13.24	13.30

$$\begin{aligned} Return_{k,t+j} &= \sum_{t=1}^{55} \gamma_{0,t} + \gamma_1 \ln(Capitalization)_{k,t} \\ &+ \gamma_2 Turnover_{k,t} + \gamma_3 Idiosyncratic\ \sigma^2_{k,t} + \gamma_4 Beta_{k,t} \\ &+ \gamma_5 Return_{k,t=\{0,-1\}} + \gamma_6 NHF\ demand_{k,t} \\ &+ \gamma_7 HF\ demand_{k,t} + \varepsilon_{k,t}. \end{aligned}$$

➢ No evidence that hedge fund demand shocks result in subsequent return reversals

5. Hedge Fund Demand Shocks and Returns

Table 6 Panel Regressions of Returns on Hedge Fund and Non-Hedge Fund Demand

	(1) $Return_{t=0}$	(2) $Return_{t=1}$	(3) $Return_{t=2}$	(4) $Return_{t=3}$	(5) $Return_{t=4}$
ln(*Capitalization*)	−0.792	−0.700	−1.060	−1.059	−0.898
	(−7.40)***	(−9.38)***	(−13.46)***	(−13.91)***	(−11.39)***
Turnover	5.514	−0.583	−0.471	−0.525	−0.430
	(14.64)***	(−6.27)***	(−4.06)***	(−6.12)***	(−5.21)***
Idiosyncratic σ^2	−0.342	−0.067	−0.041	−0.134	−0.457
	(−1.76)*	(−0.93)	(−0.54)	(−1.70)*	(−2.47)**
Beta	−2.197	0.111	0.502	0.435	0.444
	(−15.92)***	(1.20)	(5.38)***	(4.82)***	(4.83)***
$Return_{t=(0,-1)}$		0.013	0.007	−0.007	−0.017
		(6.73)***	(3.65)***	(−4.09)***	(−8.92)***
HF demand noncrises	0.378	0.618	0.167	0.163	0.068
	(5.24)***	(12.86)***	(3.56)***	(3.31)***	(1.27)
NHF demand noncrises	3.566	−0.541	−0.472	−0.628	−0.452
	(16.45)***	(−10.43)***	(−8.81)***	(−10.95)***	(−7.96)***
HF demand crises	−0.405	0.073	0.364	−0.063	−0.177
	(−2.88)***	(0.55)	(2.33)**	(−0.33)	(−1.31)
NHF demand crises	1.992	−0.477	−0.546	−0.541	−0.564
	(9.85)***	(−3.64)***	(−3.28)***	(−3.36)***	(−4.93)***
R^2 (%)	17.25	13.88	12.86	13.24	13.30

$$
\begin{aligned}
&Return_{k,\,t+j}\\
&=\sum_{t=1}^{55}\gamma_{0,\,t}+\gamma_1\ln(Capitalization)_{k,\,t}\\
&\quad+\gamma_2 Turnover_{k,\,t}+\gamma_3 Idiosyncratic\ \sigma^2_{k,\,t}\\
&\quad+\gamma_4 Beta_{k,\,t}+\gamma_5 Return_{k,\,t=\{0,\,-1\}}\\
&\quad+\gamma_6 NHF\ demand_{k,\,t}*Noncrisis\ dummy_t\\
&\quad+\gamma_7 HF\ demand_{k,\,t}*Noncrisis\ dummy_t\\
&\quad+\gamma_8 NHF\ demand_{k,\,t}*Crisis\ dummy_t\\
&\quad+\gamma_9 HF\ demand_{k,t}*Crisis\ dummy_t+\varepsilon_{k,t}.
\end{aligned}
$$

- ➢ No evidence that hedge fund demand shocks result in subsequent return reversals in crisis

5. Conclusion

- Inconsistent with the existing assertion, we find that hedge fund equity portfolios are **remarkably independent**.
- Hedge Fund demand shocks are, on average, positively related to subsequent raw and risk-adjusted returns.
- Our results have important implications for the ongoing debate regarding hedge fund regulation.

Crowded Trades and Tail Risk

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1. Introduction-- Motivation

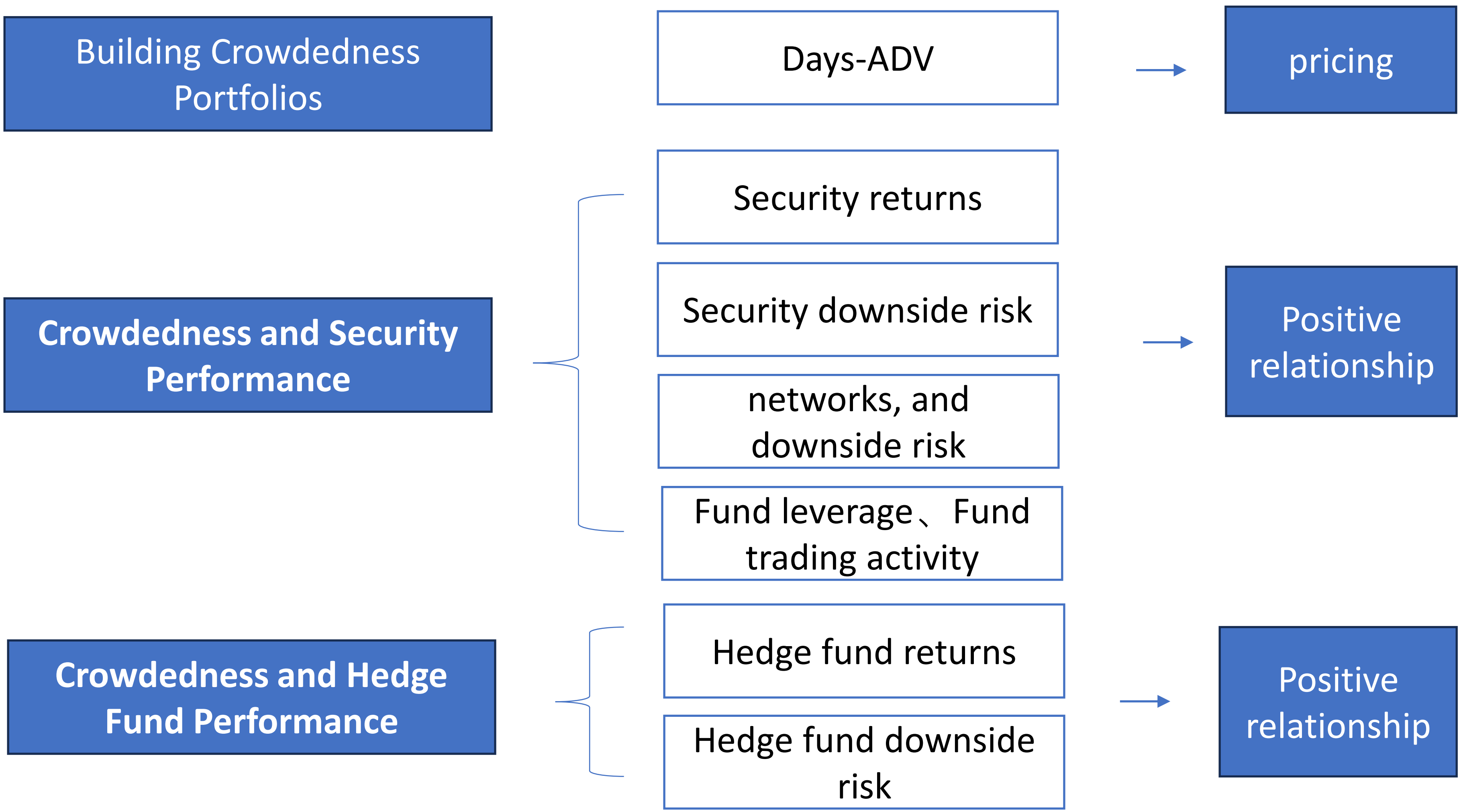
- The increasing interest in **concentrated positions** in investment assets, by investors,regulators, and researchers
- Referred to as “**crowded trades**”
- **Concern on downward price pressure** resulting from the liquidation of concentrated positions
- **Far less** is known about the impact of hedge fund ownership on equity downside risk

1. Introduction– Literature review

- Ben-David, Franzoni, and Moussawi (2012)
  - document evidence of **forced sales** by hedge funds during the financial crisis stemming from **margin calls and redemptions**

- Sias, Turtle, and Zykaj (2016)
  - **no evidence** of adverse return shocks related to common positions

1. Introduction-- Framework



1. Introduction-- Contribution

- First, we extend the literature related to explaining **cross-sectional hedge fund returns**
- Second, our measure extends work on the importance of **portfolio networks**
- Third, we also extend work documenting the role played by **liquidity** in hedge fund portfolios and returns
- Finally, our measure extends work on **downside risk**

2. Data

- Fund: Quarterly 13(f) reports between 2004 and 2017
- Novus—data with the parent management companies
- Security: CRSP common shares (share codes 10 and 11)
- We remove the smallest 20% of securities
- Monthly returns, average daily trading volume (hereafter, ADV), and market capitalization

2. Data- Three measures of crowdedness

- The number of invested funds (NHF)
  - Invested in an individual security at a point in time
- Share ownership (PSO)
  - PSO=$\frac{\text{Total value invested by hedge funds}}{\text{market capitalization}}$
- **Required liquidity (Days-ADV)**

$$ILLIQ = \frac{\text{market capitalization}}{ADV}$$

  - Days-ADV= $\frac{\text{Total value invested by hedge funds}}{\text{ADV}}$ =PSO $\times$ $ILLIQ$
  - Average daily volume (ADV) for each security using the daily dollar volume over the previous 90 trading days

2. Data- Three measures of crowdedness

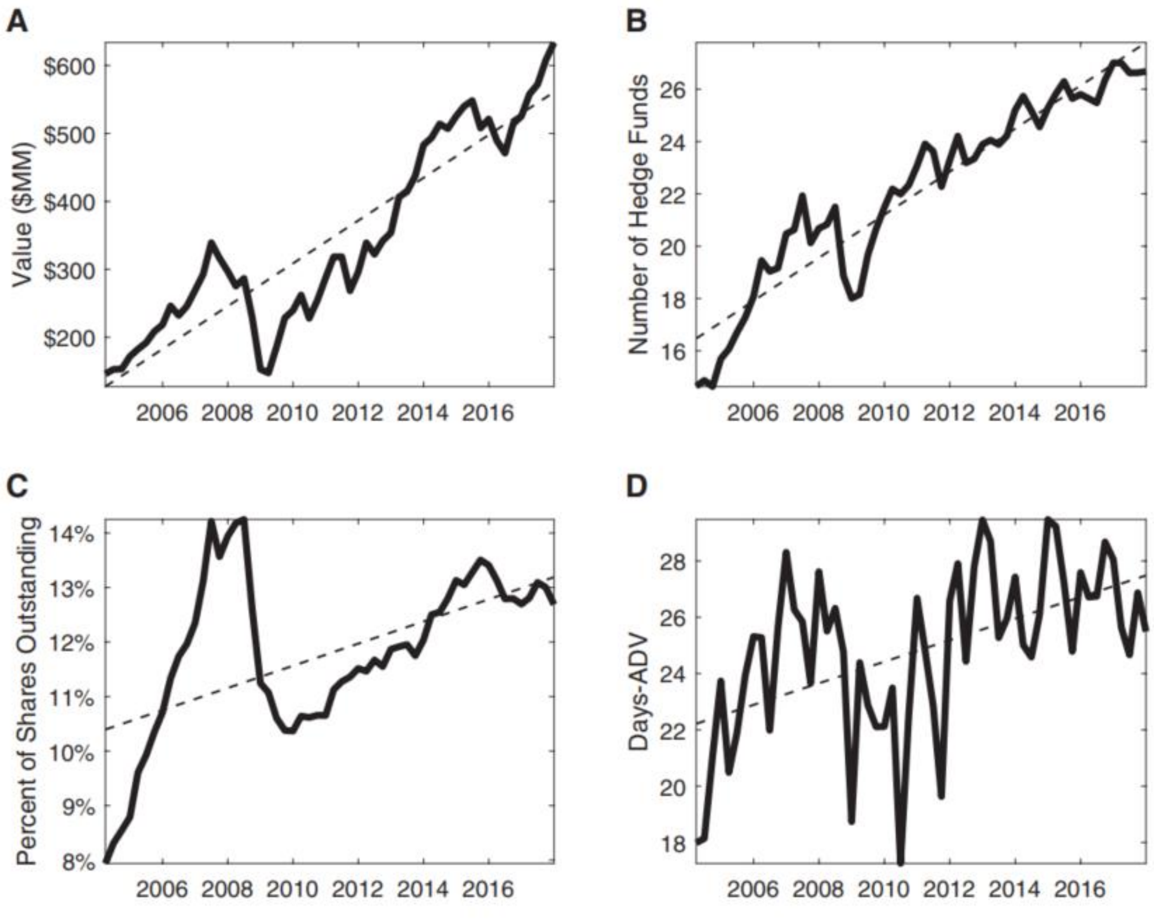


Figure 1
Hedge fund universe's position in average security

- ➢ The average security's crowdedness has increased

3. Building Crowdedness Portfolios

- Sort firms into quintile portfolios by crowdedness

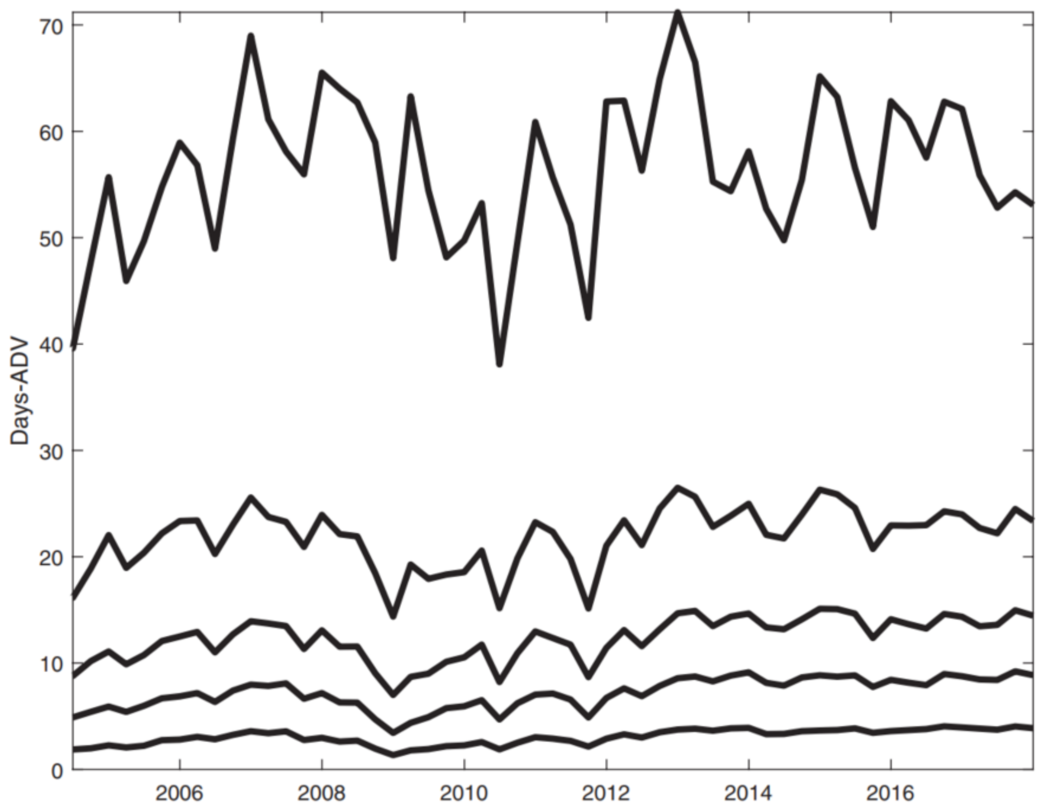


Figure 2
Median days-ADV quantiles

➢ Days-ADV: **How many days** it would take the hedge fund industry to exit its collective position.

3.1 Crowdedness spread portfolios

Table 2
Portfolio return statistics

A. Equal-weighted portfolios

	Low	Q2	Q3	Q4	High	HML
Days-ADV	6.5%	11.0%	13.1%	14.4%	15.6%	9.1%***
	(21.3%)	(19.6%)	(19.3%)	(19.4%)	(17.9%)	(8.0%)
PSO	10.0%	11.6%	11.7%	13.5%	13.9%	3.9%**
	(17.1%)	(18.7%)	(19.3%)	(20.4%)	(21.5%)	(7.2%)
ILLIQ	8.7%	10.4%	12.9%	13.9%	14.8%	6.1%*
	(25.5%)	(20.0%)	(18.4%)	(18.0%)	(15.9%)	(12.6%)

B. Value-weighted portfolios

	Low	Q2	Q3	Q4	High	HML
Days-ADV	8.5%	10.1%	10.4%	12.1%	11.3%	2.8%*
	(14.9%)	(13.4%)	(13.6%)	(14.0%)	(13.8%)	(6.2%)
PSO	7.8%	10.8%	9.5%	11.1%	13.3%	5.5%**
	(12.1%)	(14.6%)	(15.7%)	(16.0%)	(18.1%)	(9.8%)
ILLIQ	9.8%	9.0%	9.6%	9.8%	7.5%	−2.3%
	(21.0%)	(16.6%)	(13.2%)	(10.7%)	(12.2%)	(14.6%)

- Firms associated with relatively crowded positions exhibit larger average returns

3.2 Crowdedness: Mutual funds versus hedge funds

Table 3
Days-ADV factor regressions: Hedge funds versus mutual funds

	Hedge funds		Mutual funds	
	Equal weighted	Value weighted	Equal weighted	Value weighted
Average	9.08%	2.79%	6.71%	0.61%
SD	8.04%	6.25%	13.29%	14.44%
Sharpe ratio	1.13	0.45	0.51	0.04
Market β	−0.25	−0.13	−0.46	−0.58
CAPM	11.22%	3.87%	10.27%	5.08%
	(0.00)	(0.02)	(0.00)	(0.13)
FF3	10.97%	4.18%	10.22%	5.04%
	(0.00)	(0.01)	(0.00)	(0.13)
+Mom	10.62%	4.36%	9.08%	4.41%
	(0.00)	(0.01)	(0.00)	(0.12)
+Reversal	9.76%	4.04%	7.92%	4.51%
	(0.00)	(0.02)	(0.00)	(0.12)
+Pastor-Stambaugh	9.78%	4.05%	8.02%	4.59%
	(0.00)	(0.02)	(0.00)	(0.12)
+Amihud	7.42%	3.01%	6.80%	1.45%
	(0.00)	(0.06)	(0.01)	(0.64)
+ILLIQ	3.22%	2.88%	1.16%	0.75%
	(0.03)	(0.05)	(0.64)	(0.73)
+Fung-Hsieh	3.02%	2.26%	0.56%	0.61%
	(0.04)	(0.16)	(0.84)	(0.78)
+BAB+DVL+QMJ	4.10%	3.70%	−3.72%	−1.37%
	(0.00)	(0.01)	(0.09)	(0.56)
Stepwise	4.62%	4.67%	−3.15%	−1.88%
	(0.00)	(0.00)	(0.12)	(0.37)

- Hedge funds' positions deliver outperformance, while mutual funds do not.

3.3 Crowdedness and downside risk

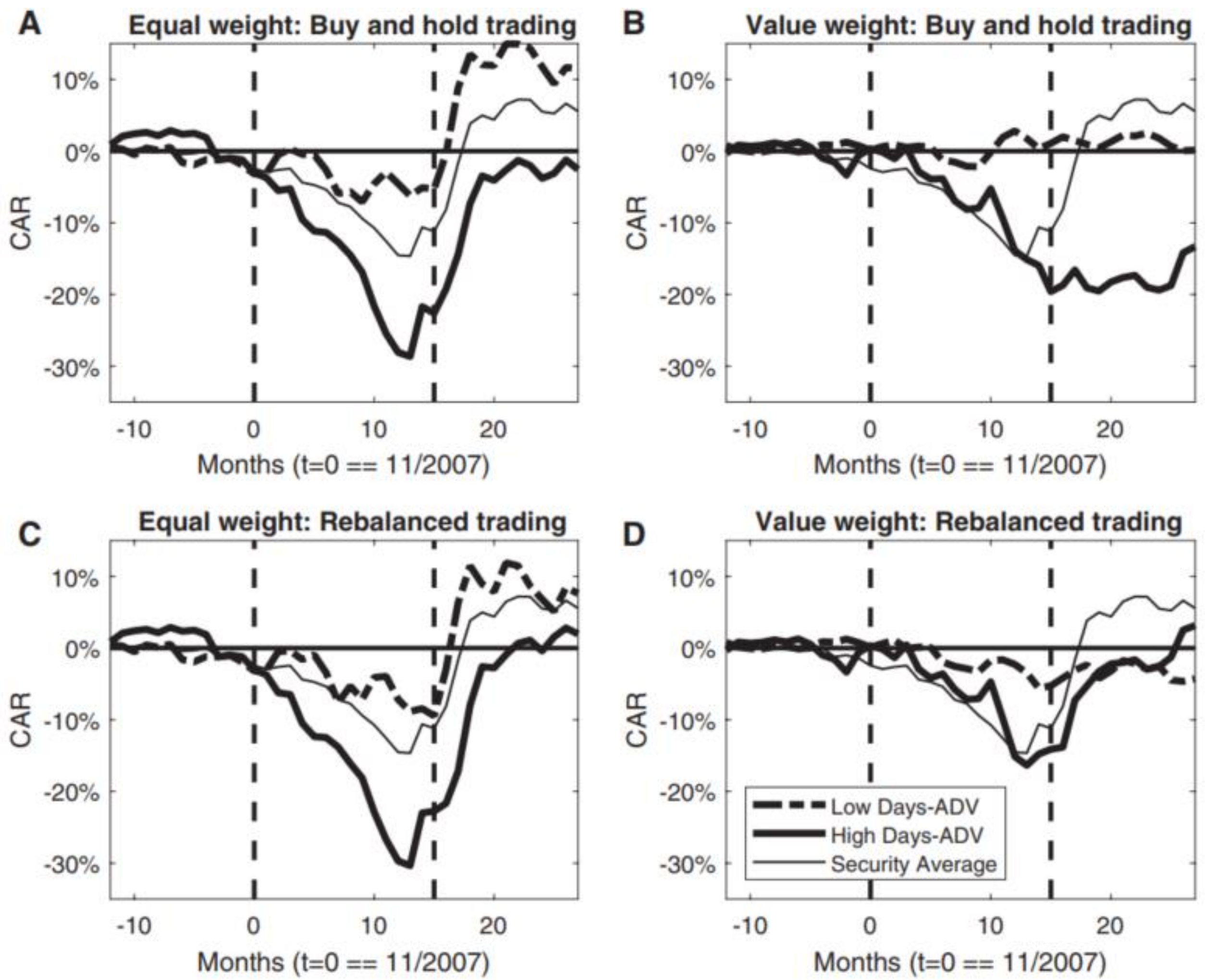


Figure 4
Days-ADV drawdown CAR over November 2007–February 2009

- Investor crowdedness correlate with downside risk

3.3 Crowdedness, networks, and downside risk

- Weighted average of Days-ADV for each hedge fund holding
    - **A hedge fund** has its capital 25% in Apple and 75% in Microsoft
    - Apple and Microsoft have Days-ADV of 10 and 30
    - Hedge fund’s portfolio Days-ADV=25%*10+75%*30=25
- **Peer-ADV**： weighted average of the hedge-fund level Days-ADV
    - **Apple** is held with AQR holding 20% and Point72 holding 80%
    - AQR and Point72 have hedge fund Days-ADV of 5 and 30
    - Apple’s Peer-ADV=20%*5+80%*30=25

3.3 Crowdedness, networks, and downside risk

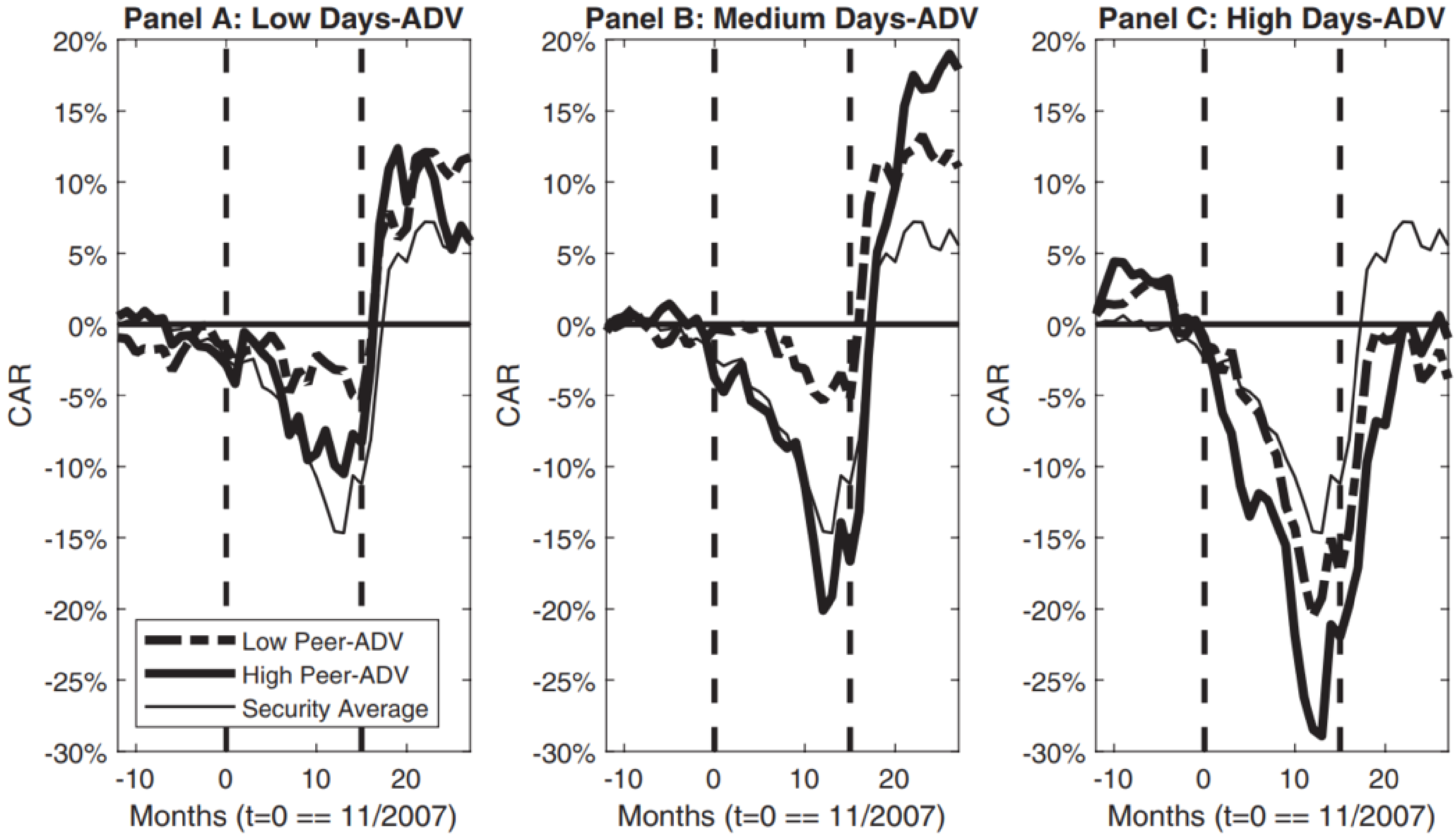


Figure 5
Buy and hold peer days-ADV drawdown CAR over November 2007–February 2009

- The price declines associated with **two-dimensional** crowdedness fully **reverse** in the quarters

3.3 Fund leverage | Fund trading activity

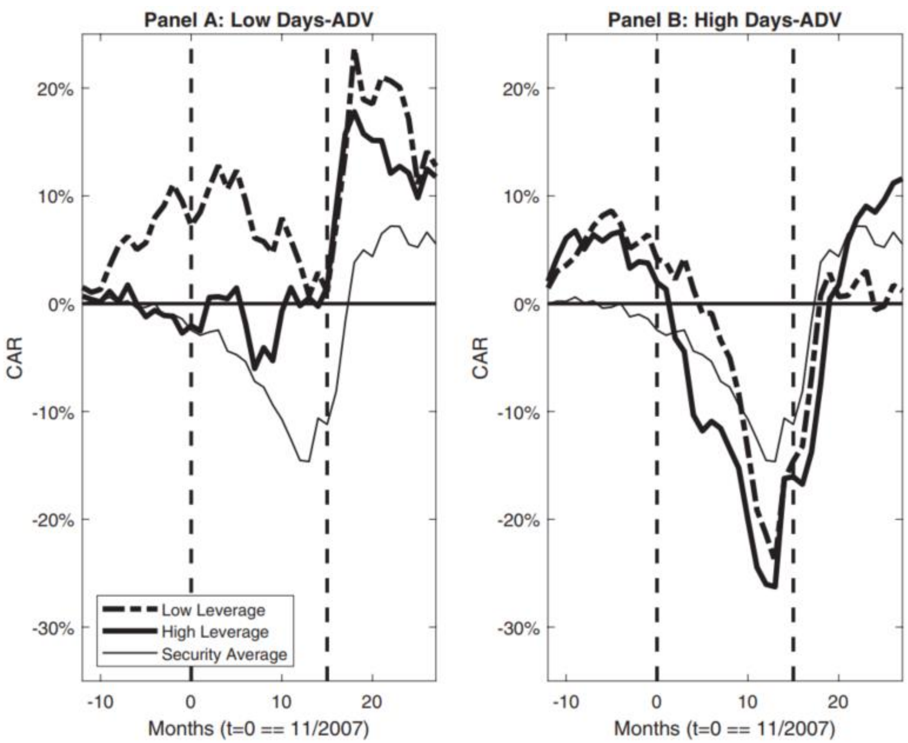


Figure 6
Buy and hold days-ADV × leverage drawdown CAR over November 2007– February 2009

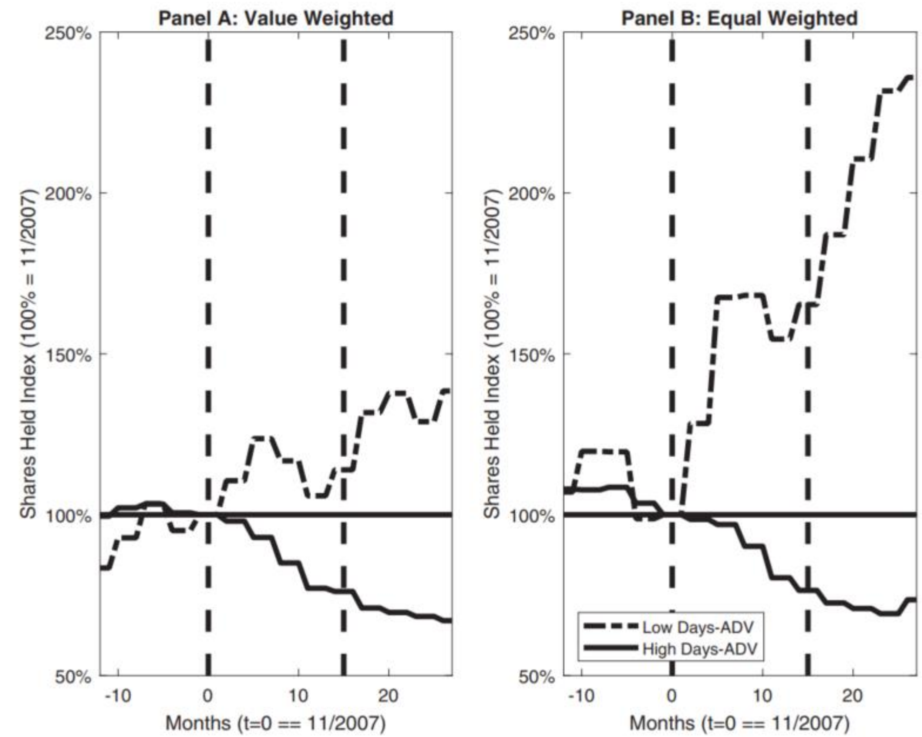


Figure 7
Net flows during the financial crisis

- Fund leverage and Fund trading activity are related to downside risk

4 Crowdedness and Hedge Fund Performance

$$R_{it}^{e} = \alpha_i + \beta_{iM} \times R_{Mt}^{e} + \beta_{iC} \times R_{ADVt} + \varepsilon_{it}.$$

Table 4
Percentage of hedge funds with significant Days-ADV crowdedness exposure

Strategy	Funds	25th percentile	Median	75th percentile	Significant
Full - All Funds	10,383	−0.08	0.11	0.28	35%
HF Bear Market Equity	33	−0.14	0.04	0.19	27%
HF Convertible Arbitrage	167	0.10	0.27	0.50	64%
HF Distressed Securities	159	0.08	0.26	0.43	59%
HF Diversified Arbitrage	57	−0.09	0.00	0.16	40%
HF Equity Market Neutral	296	−0.06	0.07	0.27	42%
HF Event Driven	249	0.05	0.24	0.37	58%
HF Fund of Funds - Equity	908	0.13	0.25	0.36	62%
HF Fund of Funds - Event	165	0.21	0.30	0.39	73%
HF Fund of Funds - Macro/Systematic	245	−0.07	0.08	0.20	27%
HF Fund of Funds - Multistrategy	1,533	0.10	0.23	0.37	56%
HF Fund of Funds - Other	243	0.05	0.22	0.36	43%
HF Fund of Funds - Relative Value	128	0.09	0.21	0.32	63%
HF Global Long/Short Equity	472	0.00	0.18	0.38	43%
HF Global Macro	411	−0.17	0.00	0.18	21%

- Exposures to crowded stocks are an important component of hedge fund returns

4 .1 Hedge fund downside risk

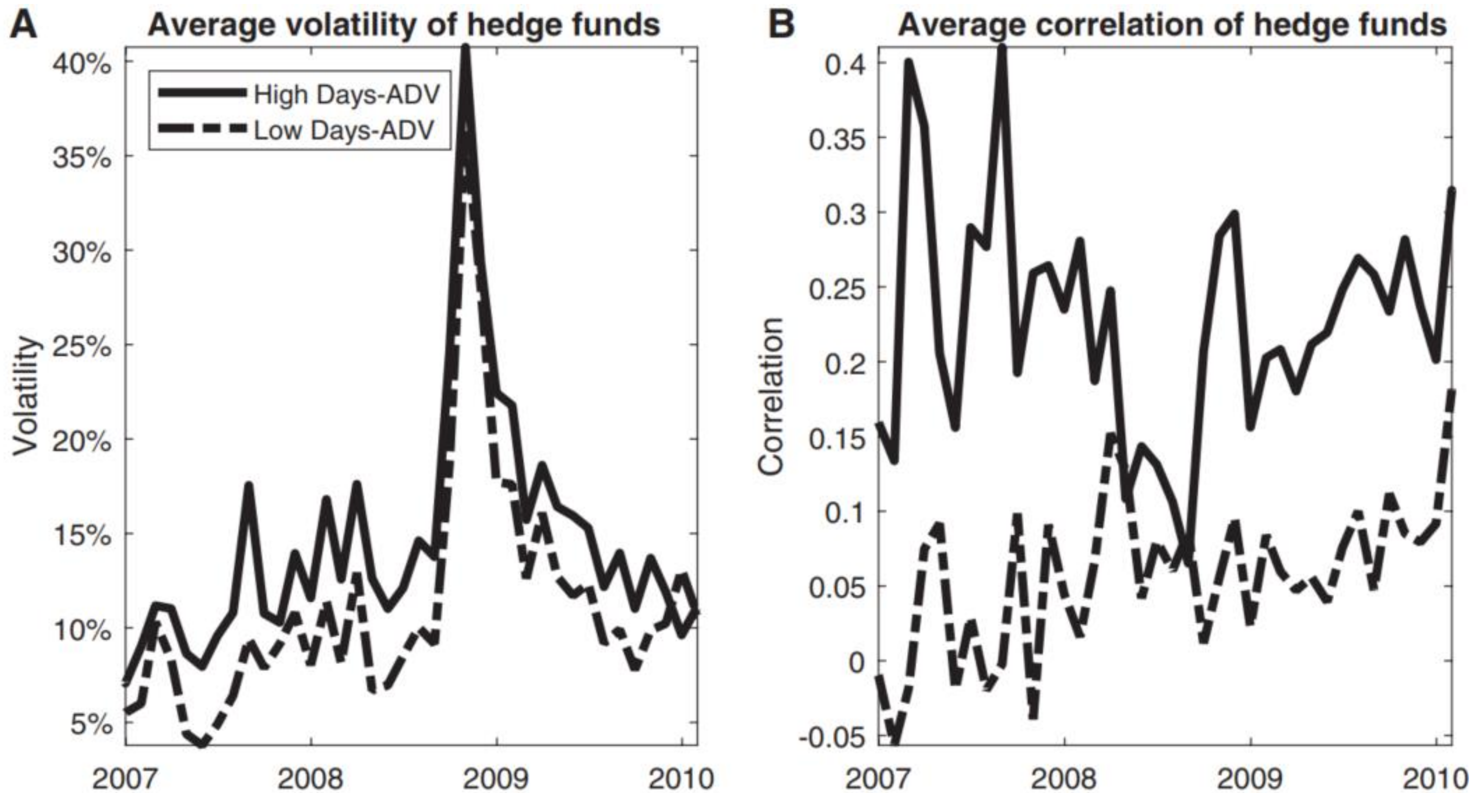


Figure 8
Hedge fund volatility and correlation

- "High Days-ADV" is the quantile for the largest Days-ADV exposures
- The exposure to our crowdedness measure helps to explain the magnitude of hedge fund **drawdowns**

5. Conclusion

- This paper examines how **crowded equity positions** are linked to the performance of individual securities and the hedge funds

- Hedge funds in especially crowded trades earn excess returns on average and have more downside risk in crisis

Crowding and Tail Risk in Momentum Returns

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1. Introduction-- Motivation

- What is the role of crowding in generating tail risk in investment strategies?
- **Financially constrained arbitrageurs** can generate tail risk
  - segmented margin accounts (Gromb and Vayanos (2002))
  - self-imposed loss limits (Morris and Shin (2004))
  - illiquidity in funding markets (Brunnermeier and Pedersen (2009)).
- Brown, Howard, and Lundblad (2019) find that concentrated positions amplify tail risk **in times of market distress**.

➢ We explore **whether crowding per se generates tail risk, without conditioning on market distress**.

1. Introduction-- Motivation

- The momentum strategy fits that setting well
  - Arbitrageurs follow **a strategy** based on prices
  - **Incomplete information** about peer actions
  - A setting where **crowding induced tail risk** is plausible

- We developing a theory of crowding and momentum tail risk

1. Introduction-- Framework

- Model
- Simulations
  - Base Case (Known Crowding)
  - Rational Beliefs → No risk
  - Myopic Beliefs → Risk
  - Optimal Linear Beliefs

- **Empirical Section**
  - Crowding and Momentum Conditional **Expected Returns**
  - Crowding and **Tail Risk** in Momentum Returns → Negative relation
  - Crowding and the **Volatility** of Momentum Returns

1. Introduction-- Contribution

- Theoretically, we develop a **crowding model for momentum returns** and tail risk

- Our analysis makes an important contribution to the literature on **risk management** in investment strategies

2. Model

- A single call auction as in Stein (2009):
- 2-period setting:
  - formation-period return: informed investors observe a signal
  - evaluation period return: the fundamental value is revealed
- 3 types of stocks: **good, bad, and neutral**, at time 0, no investor knows
- 3 types of investors: **informed, momentum, and counterparty**.

- Formation period (time 0–1) informed investors observe the type of each stock, and **a valuation signal $\boldsymbol{\delta}$**
- The fundamental value $P_{j,2}$ of each stock *j* is revealed to all investors at time 2 to be

$$\ln P_{j,2} = \ln P_{j,0} + \chi + \iota_j \frac{\delta + \varepsilon}{2}$$

2. Model

- ➢ Demand Schedules
- All investors have power utility preferences, choosing a time 1 **demand schedule** for the momentum portfolio to maximize
-
$$E[u(K_2)] = E\left[\frac{K_2^{1-\gamma}}{1-\gamma}\right],$$
- We use the second-order approximation approach of Campbell and Viceira (2002), yielding
$$Demand = \frac{E_{\text{type}}[m+\varepsilon]}{\gamma Var_{\text{type}}[m+\varepsilon]} K_{\text{type},0},$$
- Let f denote the formation-period log return on the momentum portfolio
- The evaluation period log return is then $\delta + \varepsilon - f$
- Let $m = E(\delta + \varepsilon - f \mid \delta, f) = \delta - f$ denote the expected momentum return that informed investors

2. Model

- Equilibrium

- Summing demands across the 3 investor types and equating to supply (zero) gives the market clearing condition

$$f = \frac{1}{D}\left(\delta k_{\mathrm{I}} + \frac{\delta^{E}}{1+\frac{\delta^{V}}{\sigma_{\varepsilon}^{2}}} k_{\mathrm{M}}\right)$$

where $D = \left(1 - \frac{\delta^{V}}{\sigma_{\varepsilon}^{2}+\delta^{V}} k_{\mathrm{M}}\right)$ and $k_{\text{type}} = K_{\text{type}}/(K_{\mathrm{C}} + K_{\mathrm{I}} + K_{\mathrm{M}})$ indicates the fraction of capital from each investor type. The random variable k_{M} is the momentum-investor relative crowd size.[8] The initial capital allocations k_{I}, k_{C}, and k_{M} are randomly drawn from a symmetric Dirichlet distribution (discussed in more detail in Section III).

2. Model

- Equilibrium with a **perfect-information** base case and various assumptions on momentum investor rationality

- 1. Known Crowding: **capital ratio $\boldsymbol{k_M}$ and $\boldsymbol{k_I}$** are observed prior to trading

$$f = \lambda\delta,\ with\ \ \lambda = k_I + k_M.$$

  - Beliefs $\boldsymbol{\delta^E} = \boldsymbol{\delta} = \lambda^{-1} f$ and $\boldsymbol{\delta^V} = \mathbf{0}$

- 2. Myopic Beliefs: momentum investors **know the mean of the distribution of capital**, doesn't infer realizations from market prices

$$f = \lambda\delta,\ with\ \ \lambda = Ek_I + Ek_M.$$

$$f = \lambda\left(\frac{k_{\mathrm{I}}}{\lambda - k_{\mathrm{M}}}\right)\delta = \lambda\left(\frac{k_{\mathrm{I}}}{Ek_{\mathrm{I}} - (k_{\mathrm{M}} - Ek_{\mathrm{M}})}\right)\delta$$

  - negative value for f when $k_M > \lambda$

2. Model

- 3. Optimal Linear Beliefs: momentum investors restrict their strategy space to linear beliefs, $\delta^E = \lambda^{-1} f$
  - **prevents catastrophic losses** from crowding-induced feedback effects.

- 4. Rational Beliefs: momentum investors account for the asymmetric signal that f provides **regarding the presence of crowding-induced feedback effects.**
  - Momentum investors compute δ^E and δ^V given this joint density, observation of f, and the definition of conditional expectations

$$\delta^E = \int_0^\infty \delta p(\delta|f) d\delta, \qquad \text{and} \qquad \delta^V = \int_0^\infty \left(\delta - \delta^E\right)^2 p(\delta|f) d\delta,$$

$$p(\delta|f) = \frac{\frac{g(\delta)}{\delta} \int_0^1 h\left(k_{\mathrm{M}}, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\left(\delta^V/\sigma_\varepsilon^2\right)} k_{\mathrm{M}}\right)\right) D dk_{\mathrm{M}}}{\int_0^\infty \frac{g(\delta)}{\delta} \int_0^1 h\left(k_{\mathrm{M}}, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\left(\delta^V/\sigma_\varepsilon^2\right)} k_{\mathrm{M}}\right)\right) D dk_{\mathrm{M}} d\delta}$$

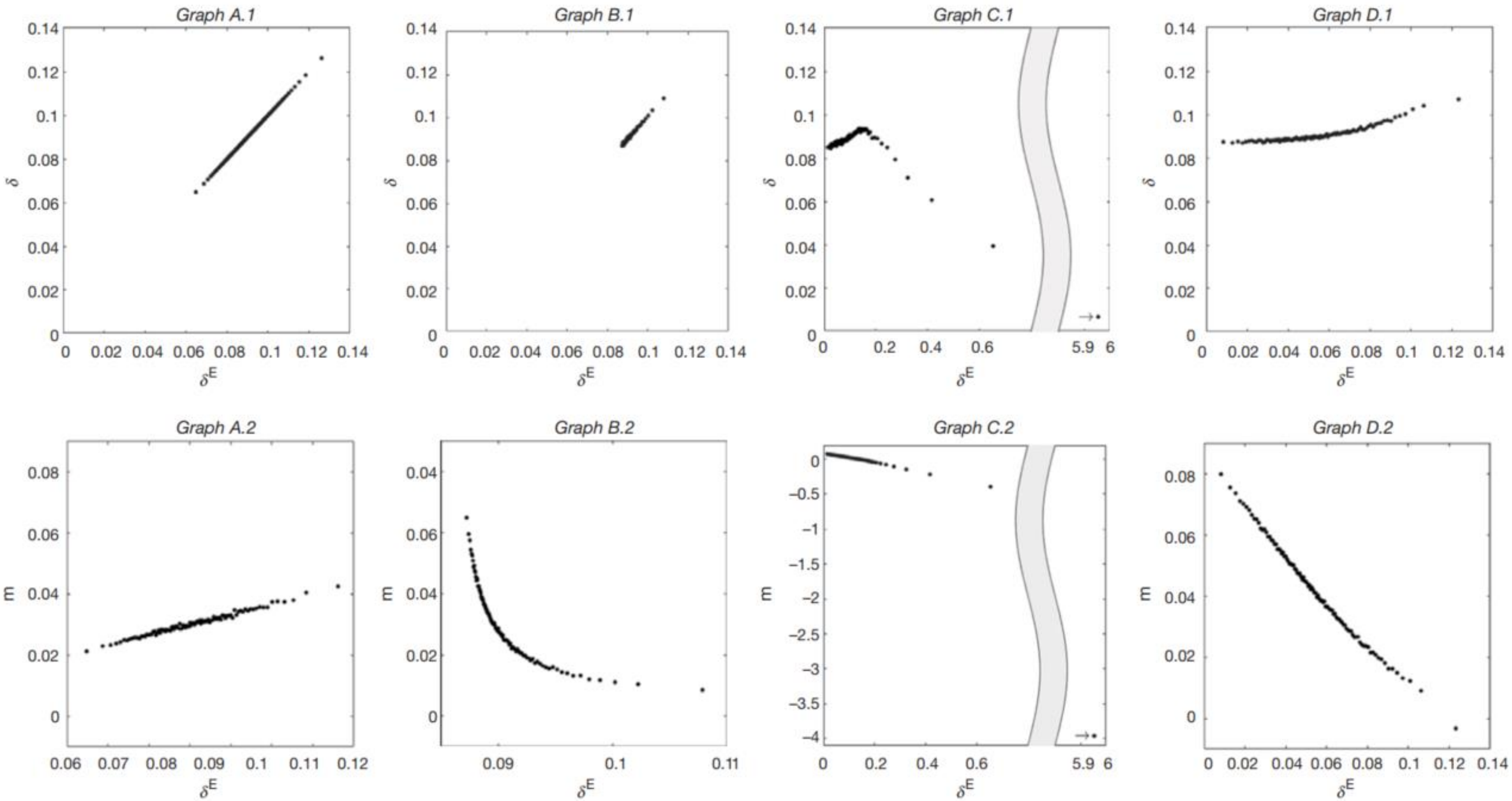
3. Simulations

- 100,000 random draws of the market conditions (i.e., δ, k_M, and k_I) in each of the 4 belief cases

known crowding rational beliefs myopic beliefs optimal linear beliefs

Value

Mom return



4. Empirical Section

- Thomson Reuters Institutional 13f database 1980-2015
- Stock data are from the CRSP database
- The **momentum return** at time t is defined as the **return of winners** (stocks in the top 10%, sorting on returns from months t 12 to t 2) **minus the return of losers** (stocks in the bottom 10% similarly constructed)

4.1 Crowding Proxies

- We base our designation of a momentum investor on the following score (Grinblatt, Titman, and Wermers, 1995)

$$\text{SCORE}_{i,q} = \sum_{j=1}^{J} \left(\omega_{i,j,q} - \omega_{i,j,q-1}\right) r_{j,q-1},$$

- where $r_{j,q}$ is the quarter q return on stock j and ω is a portfolio weight with

$$\omega_{i,j,q} - \omega_{i,j,q-1} = \frac{w_{i,j,q} P_{j,q-1}}{\sum_{j=1}^{J} w_{i,j,q} P_{j,q-1}} - \frac{w_{i,j,q-1} P_{j,q-1}}{\sum_{j=1}^{J} w_{i,j,q-1} P_{j,q-1}},$$

- $w_{i,j,q}$ indicates shares held in stock j by institution i at the end of quarter q and $P_{j,q-1}$ is the price of stock j at the end of quarter $q-1$.

- A **positive SCORE** implies trading aligned with a **momentum strategy.**

$$\mathbb{1}_{\text{MOM}_{i,q}} = \mathbb{1}_{\sum_{l=0}^{3} \mathbb{1}_{\text{SCORE}_{i,q-l} > 0} = 4}$$

4.1 Crowding Proxies

- 3 measures:
  - CNT: the count of institutions following a momentum strategy
  - AUM: their assets under management
  - TRD: quarterly change in holdings

- 2 sets:
  - Factor level
  - Security level

4.1 Crowding Proxies i为基金，j 为股票

每个季度动量基金数量占比

$$\mathrm{CNT\_F}_q = \frac{1}{N_q}\sum_{i=1}^{N_q} \mathbb{1}_{\mathrm{MOM}_{i,q}}$$

被动量组合持有的多空头寸*被动量基金持有占比

$$\mathrm{CNT\_S}_q = \sum_{j=1}^{J}\left(\overline{\omega}_{j,q} - \underline{\omega}_{j,q}\right)\frac{\sum_{i=1}^{N_q} \mathbb{1}_{w_{i,j,q}>0}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} \mathbb{1}_{w_{i,j,q}>0}},$$

$$\mathrm{AUM\_F}_q = \frac{\sum_{i=1}^{N_q} \mathrm{HOLD}_{i,q}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} \mathrm{HOLD}_{i,q}}$$

$$\mathrm{AUM\_S}_q = \sum_{j=1}^{J}\left(\overline{\omega}_{j,q} - \underline{\omega}_{j,q}\right)\frac{\sum_{i=1}^{N_q} w_{i,j,q}P_{j,q}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} w_{i,j,q}P_{j,q}},$$

$$\mathrm{TRD\_F}_q = \frac{\sum_{i=1}^{N_q} \mathrm{WTRD}_{i,q}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} \mathrm{WHOLD}_{i,q}} - \frac{\sum_{i=1}^{N_q} \mathrm{LTRD}_{i,q}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} \mathrm{LHOLD}_{i,q}}$$

$$\mathrm{TRD\_S}_q = \sum_{j=1}^{J}\left(\overline{\omega}_{j,q} - \underline{\omega}_{j,q}\right)\frac{\sum_{i=1}^{N_q}\left(w_{i,j,q} - w_{i,j,q-1}\right)P_{j,q-1}\mathbb{1}_{\mathrm{MOM}_{i,q}}}{\sum_{i=1}^{N_q} w_{i,j,q}P_{j,q}}$$

$$\mathrm{HOLD}_{i,q} = \sum_{j=1}^{J} w_{i,j,q}P_{j,q},\quad \mathrm{WHOLD}_{i,q} = \sum_{j=1}^{J} w_{i,j,q}P_{j,q}\mathbb{1}_{\iota_{j,q}=1},\quad \mathrm{LHOLD}_{i,q} = \sum_{j=1}^{J} w_{i,j,q}P_{j,q}\mathbb{1}_{\iota_{j,q}=-1},$$

$$\mathrm{WTRD}_{i,q} = \sum_{j=1}^{J}\left(w_{i,j,q} - w_{i,j,q-1}\right)P_{j,q-1}\mathbb{1}_{\iota_{j,q}=1},\quad \mathrm{LTRD}_{i,q} = \sum_{j=1}^{J}\left(w_{i,j,q} - w_{i,j,q-1}\right)P_{j,q-1}\mathbb{1}_{\iota_{j,q}=-1},$$

4.2 Crowding and Momentum Conditional Expected Returns

TABLE 3

Factor Model

Table 3 contains the factor exposures of quarterly momentum returns on the Fama–French 3-factor model (FF3) and a dynamic extension in which the 3 factors are interacted with dummies for positive past annual factor returns (DFF3). Alphas are monthly and *t*-statistics (in parentheses) use White (1980) standard errors. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	Alpha	Mkt	SMB	HML	Dmkt	DSMB	DHML	Adj. R^2
FF3	0.016***	−0.35**	−0.48**	−0.59**				12%
	(4.39)	(−1.98)	(−2.06)	(−2.06)				
DFF3	0.014***	−0.85***	−0.68***	−0.95***	0.81***	0.48	1.02**	25%
	(4.26)	(−2.93)	(−2.71)	(−2.68)	(2.77)	(1.22)	(2.14)	

- Momentum return can not be priced by risk factors.

4.2 Crowding and Momentum Conditional Expected Returns

Level of Analysis	Factor			Security		
Measure of Crowding	AUM	TRD	CNT	AUM	TRD	CNT
Panel A. Dynamic FF3 Model						
$\Delta \mathrm{CROWD}_q$	−0.21***	−0.34*	−0.33*	−0.07	−0.41**	−0.04
	(−2.64)	(−1.94)	(−1.84)	(−0.95)	(−2.05)	(−0.31)
CROWD_{q-1}	−0.24***	−0.34*	−0.58***	−0.18***	−0.36**	−0.26**
	(−4.32)	(−1.87)	(−4.35)	(−3.00)	(−2.39)	(−2.52)
$\mathrm{CROWD\_EVOL}_q$	3.06**	2.30	6.60***	0.19	1.58	0.20
	(2.47)	(1.31)	(3.75)	(0.75)	(1.08)	(0.56)
Realized vol. of Mom rets.	−0.27***	−0.28**	−0.25**	−0.29**	−0.28***	−0.28**
	(−2.62)	(−2.54)	(−2.21)	(−2.45)	(−2.63)	(−2.30)
Adj. R^2	40.7%	31.8%	37.7%	32.6%	34.0%	31.7%

- The first two measure consistently and significantly negatively predict momentum returns.

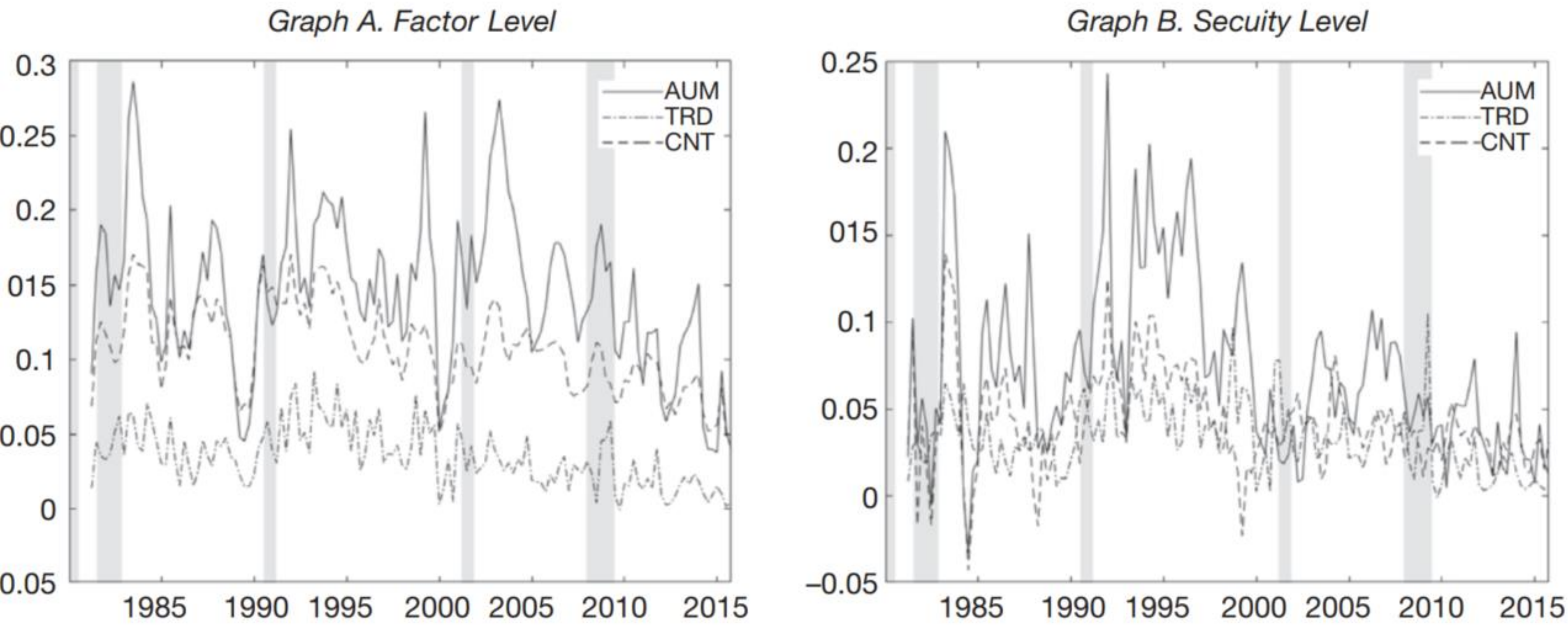
4.3 Crowding and Tail Risk in Momentum Returns

Level of Analysis	Factor			Security		
Measure of Crowding	AUM	TRD	CNT	AUM	TRD	CNT
Panel A. Predicting the 5% Left Tail						
$\Delta\mathrm{CROWD}_q$	8.4	12.3	19.2	−1.4	11.8	3.8
	(1.49)	(1.15)	(1.17)	(−0.21)	(1.03)	(0.28)
	[0.35]	[0.92]	[0.60]	[0.22]	[0.20]	[0.90]
$\mathrm{CROWD}_{q\text{-}1}$	7.2	14.4	24.5*	4.9	26.4	22.1
	(1.55)	(1.00)	(1.79)	(0.77)	(1.39)	(1.33)
	[0.86]	[0.81]	[0.98]	[0.98]	[0.53]	[0.46]
$\mathrm{CROWD\_EVOL}_q$	−1.4	43.2	−213.3	−11.8	−221.3	−91.4
	(−0.02)	(0.41)	(−1.09)	(−0.42)	(−1.30)	(−1.15)
	[0.29]	[0.43]	[0.48]	[0.17]	[0.05]*	[0.11]
Realized vol. of Mom rets.	10.1***	9.9***	10.8***	9.9***	12.3***	9.9***
	(3.22)	(3.18)	(3.10)	(2.92)	(2.87)	(2.94)
	[0.00]***	[0.00]***	[0.00]***	[0.00]***	[0.00]***	[0.00]***

- No evidence that crowding increases tail risk.

4.3 Crowding and Tail Risk in Momentum Returns

➢ Measures of Crowding



➢ No evidence that crowding increases tail risk.

4.3 Crowding and Tail Risk in Momentum Returns

	Factor			Security			
	AUM	TRD	CNT	AUM	TRD	CNT	Realized Vol. of Mom Rets.
Panel A. CROWD							
Volatility							
T1	23.8	25.7	32.3	23.1	23.3	31.3	15.3
T2	28.0	22.3	26.8	33.2	30.3	27.7	17.4
T3	25.8	29.5	16.5	20.1	23.8	17.2	38.3
	(0.40)	(0.74)	(−3.63)***	(−1.28)	(0.13)	(−3.23)***	(5.62)***
Skewness							
T1	−1.8	−1.5	−1.7	−0.3	−1.9	−1.7	−0.3
T2	−1.6	−0.7	−1.1	−2.0	−1.9	−1.4	−0.3
T3	−1.1	−1.8	−0.6	−0.1	0.0	0.0	−1.2
	(0.57)	(−0.26)	(1.83)*	(0.51)	(1.59)	(2.66)***	(−2.06)**
Kurtosis							
T1	14.6	10.8	10.6	4.3	15.5	10.9	4.0
T2	11.5	4.9	8.1	11.1	10.7	8.7	4.0
T3	8.9	12.6	4.7	4.3	5.9	3.4	6.7
	(−1.11)	(0.37)	(−2.77)***	(0.06)	(−2.08)**	(−3.40)***	(2.29)**

- Feedback effects from crowding do not explain the tendency for high lag volatility to predict crashes

4.4 Crowding and the Volatility of Momentum Returns

- realized volatility of momentum returns computed from raw and risk-adjusted (dynamic FF3) daily returns over the quarter.

Level of Analysis	Factor			Security		
Measure of Crowding	AUM	TRD	CNT	AUM	TRD	CNT
Panel A. Dynamic FF3 Model						
$\Delta CROWD_q$	0.03	−0.10	−0.05	0.04	−0.03	0.01
	(0.65)	(−0.72)	(−0.51)	(0.67)	(−0.24)	(0.11)
$CROWD_{q-1}$	−0.06**	−0.18*	−0.10	−0.03	−0.16**	−0.08*
	(−2.24)	(−1.82)	(−1.21)	(−1.15)	(−2.03)	(−1.80)
$CROWD\_EVOL_q$	0.12	−0.13	−0.75	−0.31**	−0.83	−0.42**
	(0.14)	(−0.12)	(−0.56)	(−2.53)	(−0.86)	(−2.27)
Realized vol. of Mom rets.	0.77***	0.77***	0.74***	0.72***	0.81***	0.73***
	(8.83)	(8.42)	(9.10)	(8.70)	(10.02)	(8.59)
Adj. R^2	59.4%	59.2%	59.5%	60.2%	60.1%	59.6%

- Momentum volatility is not positively predicted by measures of crowding

4.5 Determinants of Crowding

- realized volatility of momentum returns computed from raw and risk-adjusted (dynamic FF3) daily returns over the quarter.

Level of Analysis	Factor			Security		
Measure of Crowding	AUM	TRD	CNT	AUM	TRD	CNT
1YR_RET$_{q-1}$	0.72**	0.25***	0.41***	0.29	0.24***	0.17
	(2.57)	(3.40)	(2.69)	(1.32)	(2.62)	(1.34)
1YR_RET$_{q-5}$	0.92***	0.40***	0.52***	0.35	0.27***	0.21
	(3.22)	(3.93)	(2.94)	(1.29)	(3.08)	(1.34)
1YR_VOL$_{q-1}$	−0.28*	−0.20***	−0.36***	−0.39**	−0.08	−0.18**
	(−1.94)	(−3.31)	(−4.50)	(−2.46)	(−1.34)	(−2.09)
1YR_VOL$_{q-5}$	0.41*	0.07	0.18**	−0.10	0.02	0.08
	(1.90)	(1.34)	(2.33)	(−0.68)	(0.41)	(1.22)
Adj. R^2	10.9%	20.1%	19.1%	12.9%	10.6%	2.2%

- 1-year volatilities indeed predict negatively crowding in momentum

5. Conclusion

- We provide a model of crowding with momentum investors who attempt to infer informed investors' private signals from prices.

- Examine proxies for momentum investing by institutional investors, in contrast to crowding from return covariances or volatility.

- A generally inverse relation between momentum investing and future tail risk in momentum returns.