Time series momentum: Is it there?

Huang, Dashan, Jiangyuan Li, Liyao Wang, and Guofu Zhou.
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解读者：屠雪永

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Outline

- Introduction
- Research design
  - Univariate time series regression
  - Pooled regression
  - Bootstrap tests
- Empirical results
- Conclusion

1. Introduction-- Motivation

- Time series momentum (TSM) refers to the predictability of the past 12-month return on the next one-month return.
- Moskowitz et al. (MOP, 2012) conclude that time series momentum (TSM) is everywhere: The past 12-month return positively predicts the next one- to 12-month return
- Whether time series predictability is present at the 12-month frequency remains an open question.
- In this paper, using the same data set as MOP (Moskowitz et al., 2012), we reexamine the evidence of TSM, we find that the evidence on TSM is weak.

1. Introduction-- Research question

- 1.How to prove that the evidence on TSM is weak ?

- 2. Why is the TSM strategy profitable even though the statistical evidence on time series predictability is weak?

1. Introduction-- Framework

Answers to the first question

We run a **time series regression** of monthly return for each asset on its past 12-month return.

We follow MOP's approach and run a **pooled regression** by stacking all asset returns together.

To assess the degree of over-rejection, we use **two bootstrap methods.**

Answers to the second question

We propose a **times series history (TSH)** strategy that buys assets if their historical mean returns are positive and sells them otherwise.

1. Introduction-- Contribution

- 1. This paper proves the evidence on TSM is weak,and explain the phenomenon why the TSM strategy is profitable.

- 2. The predictability in the asset classes, if it exists, is not as simple as a constant 12-month return rule.

2. Data

- Futures prices: 24 commodities, 9 developed country equity indexes, 13 developed government bonds, and 9 currency forwards from the same data sources as MOP(55).
- Sample period : 1985.01 ~2015.12.
- Futures returns：For each day, we calculate the daily excess return of each futures contract with the nearest- or next-nearest-to-delivery contract and compound the daily returns to a cumulative **month return index**.

3. Univariate time series regression

- We run univariate time series regressions to explore the predictability of the past 12-month return for individual assets.

$$r_{t+1}^{i} = \alpha + \beta r_{t-12,t}^{i} + \varepsilon_{t+1}^{i}, \qquad (1)$$

- Where r_{t+1}^{i} is the return of asset i in month $t+1$ and $r_{t-12,t}^{i}$ is its past 12-month return (i.e., the return between months t - 12 and t).

$$R_{OS}^{2} = 1 - \frac{\sum_{t=K}^{T-1}(r_{t+1}^{i} - \hat{r}_{t+1}^{i})^{2}}{\sum_{t=K}^{T-1}(r_{t+1}^{i} - \bar{r}_{t+1}^{i})^{2}}, \qquad (2)$$

$$\hat{r}_{t+1}^{i} = \hat{\alpha}_{t} + \hat{\beta}_{t} r_{t-12,t}^{i}$$

- Where K is the initial sample size for parameters training.

$\bar{r}_{t+1}^{i}$ is the sample mean of asset i with data up to month t.

Data spilt:15+16 years

3. Univariate time series regression--results

- Table 2 In- and out-of-sample performance of time series momentum (TSM) with time series regression.

Asset	β_i	t-stat	R^2	R^2_{OS}
Panel A: Commodity futures				
Aluminum	0.30	0.88	0.28	−1.42
Brent oil	0.34	0.69	0.14	−1.29
Cattle	0.38**	2.23	0.94	−0.62
Cocoa	−0.14	-0.28	0.03	−1.51
Coffee	0.21	0.40	0.04	0.23
Copper	0.77*	1.69	0.97	0.21
Average across asset classes			0.39	−0.67
#(10% significance)	$\boxed{8}$			$\boxed{3}$

The evidence of TSM across all the assets is very weak. Of the 55 assets, only eight display significant regression slopes at the 10% level ; the significance is not concentrated but disperse among the four asset classes; R^2_{OS} only three are significant at the 10% level.

4. Pooled regression

We first replicate the Pooled regression in MOP (overstate the presence of TSM):

$$r^i_{t+1}/\sigma^i_t = \alpha + \beta r^i_{t-h+1}/\sigma^i_{t-h} + \varepsilon^i_{t+1}, \tag{3}$$

$$(\sigma^i_t)^2 = 261 \sum_{j=0}^{\infty} (1-\delta)\delta^j (r^i_{t-1-j} - \bar{r}^i_t)^2, \tag{4}$$

$$r^i_{t+1}/\sigma^i_t = \alpha + \beta sign(r^i_{t-h+1}) + \varepsilon^i_{t+1}, \tag{5}$$

where *sign* is the sign function that equals +1 when $r^i_{t-h+1} \geq 0$ and -1 when $r^i_{t-h+1} < 0$. To highlight fixed effects, a possible specification is

$$r^i_{t+1}/\sigma^i_t = \alpha + \beta r^i_{t-h+1}/\sigma^i_{t-h} + \mu_i/\sigma_i + \varepsilon^i_{t+1}, \tag{6}$$

$$\hat{\beta} = \beta + \frac{\mathrm{Cov}(r^i_{t-h+1}/\sigma^i_{t-h}, \mu_i/\sigma_i)}{\mathrm{Var}(r^i_{t-h+1}/\sigma^i_{t-h})}. \tag{7}$$

the slope estimate of Eq. (3) is biased upward

4. Pooled regression--Bootstrap tests

Table 3 p-value from the test that all assets have the same mean or Sharpe ratio.

	ANOVA	Welch's ANOVA	Kruskal-Wallis	Bootstrap
Mean	0.08	$< 10^{-3}$	$< 10^{-10}$	0
Sharpe ratio	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-15}$	0

Two standard bootstrap approaches:

The first is a more restrictive parametric wild bootstrap

The second is a more general nonparametric **pairs bootstrap** that resamples the predictor and the dependent variable simultaneously.

$$\hat{\varepsilon}^i_{t+1} = r^i_{t+1}/\sigma^i_t - \hat{\alpha} - \hat{\beta} r^i_{t-h+1}/\sigma^i_{t-h}. \tag{8}$$

wild bootstrap $$r^{i*}_{t+1}/\sigma^{i*}_t = \hat{\alpha} + \hat{\beta} r^i_{t-h+1}/\sigma^i_{t-h} + \hat{\varepsilon}^i_{t+1} v^i_{t+1}, \tag{9}$$

$$v^i_t = \begin{cases} 1 & \text{with probability } 1/2, \\ -1 & \text{with probability } 1/2. \end{cases} \tag{10}$$

4. Pooled regression--results

- **Table 4** *t*-statistic of pooled regression **without controlling for fixed effects.**

h	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs
Panel A: Forecast with return lagged h months						
	$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h r^i_{t-h+1}/\sigma^i_{t-h} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h sign(r^i_{t-h+1}) + \varepsilon^i_{t+1}$		
1	3.11	9.26	3.63	2.90	8.18	3.41
2	1.31	4.98	1.98	1.62	4.44	2.31
3	2.89	8.61	3.45	2.83	6.84	3.45
4	0.24	2.46	1.06	1.20	2.12	1.99
5	-0.17	1.88	0.60	-0.34	1.83	0.54
6	0.97	4.18	1.71	1.58	3.62	2.28
Panel B: Forecast with past h-month returns						
	$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h r^i_{t-h,t}/\sigma^i_{t-1} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h sign(r^i_{t-h,t}) + \varepsilon^i_{t+1}$		
1	3.11	9.26	3.63	2.90	8.18	3.41
2	2.92	9.46	3.46	3.07	8.32	3.61
3	3.74	11.45	4.22	4.15	10.20	4.61
4	3.49	10.71	3.97	4.57	9.49	4.96
5	3.11	9.58	3.63	4.24	8.85	4.72
6	3.29	9.65	3.80	3.88	8.88	4.39

At the 5% level ,define the distribution of the *t*-statistic's 97.5% quantile as the simulated *t*-statistic for significance.

The t-statistic from the real data is smaller than the simulated t-statistics respectively, suggesting that the evidence is weak in support of TSM.

4. Pooled regression--results

- **Table 6** t-statistic of pooled regression **without volatility scaling** and **without controlling for fixed effects**

h	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs
Panel A: Forecast with return lagged h months						
	$r^i_{t+1} = \alpha_h + \beta_h r^i_{t-h+1} + \varepsilon^i_{t+1}$			$r^i_{t+1} = \alpha_h + \beta_h sign(r^i_{t-h+1}) + \varepsilon^i_{t+1}$		
1	1.80	5.49	2.51	2.20	6.13	2.85
2	0.52	2.58	1.47	1.65	2.67	2.45
3	1.43	4.57	2.19	1.84	4.58	2.58
4	0.67	3.21	1.58	1.47	3.21	2.26
5	−1.33	−0.10	−0.14	−0.89	−0.08	0.28
6	1.03	3.37	1.92	1.77	3.45	2.48
Panel B: Forecast with past h-month returns						
	$r^i_{t+1} = \alpha_h + \beta_h r^i_{t-h,t} + \varepsilon^i_{t+1}$			$r^i_{t+1} = \alpha_h + \beta_h sign(r^i_{t-h,t}) + \varepsilon^i_{t+1}$		
1	1.80	5.49	2.51	2.20	6.13	2.85
2	1.39	4.56	2.21	2.57	5.10	3.18
3	1.71	5.26	2.45	3.06	5.81	3.62
4	1.82	5.30	2.59	3.75	5.94	4.25
5	1.27	4.27	2.09	3.23	4.75	3.77
6	1.55	4.85	2.39	2.71	5.38	3.32

The t-statistics without volatility scaling are much smaller than those with volatility scaling. Volatility scaling seems at least partially responsible for the performance of the TSM trading strategy.

4. Pooled regression--results

- **Table 7** t-statistic of pooled regression without controlling for fixed effects over **1985:01–2009:12.**

h	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs
Panel A: Forecast with return lagged h months						
	$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h r^i_{t-h+1}/\sigma^i_{t-h} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h sign(r^i_{t-h+1}) + \varepsilon^i_{t+1}$		
1	3.71	10.68	4.20	3.75	9.31	4.19
2	0.97	4.07	1.68	1.34	3.65	2.02
3	2.48	7.43	3.11	2.44	6.09	3.01
4	0.22	2.40	1.14	0.65	2.28	1.59
5	-0.15	1.53	0.67	-0.38	1.56	0.66
6	0.52	3.08	1.30	1.35	2.78	2.15
Panel B: Forecast with past h-month returns						
	$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h r^i_{t-h,t}/\sigma^i_{t-1} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha_h + \beta_h sign(r^i_{t-h,t}) + \varepsilon^i_{t+1}$		
1	3.71	10.68	4.20	3.75	9.31	4.19
2	3.09	9.53	3.54	3.19	8.39	3.70
3	3.74	11.53	4.27	4.43	9.96	4.94
4	3.45	10.37	3.98	4.78	9.19	5.19
5	3.06	9.27	3.63	4.36	8.39	4.85
6	3.17	9.31	3.73	4.03	8.52	4.46

For the 1985 to 2009 sample period, the t-statistics from real data are still smaller than the simulated t-statistics.

4. Pooled regression $r^i_{t+1}/\sigma^i_t - \overline{r^i/\sigma^i} = \beta(r^i_{t-h+1}/\sigma^i_{t-h} - \overline{r^i_{-h+1}/\sigma^i_{-h}}) + \varepsilon^i_{t+1},$ (12)

- **Table 8** t-statistic of pooled regression **controlling for fixed effects.**

h	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs	t-statistic	Bootstrapped t-statistic Wild	Bootstrapped t-statistic Pairs
Panel A: Forecast with return lagged h months						
	$r^i_{t+1}/\sigma^i_t = \alpha^i_h + \beta_h r^i_{t-h+1}/\sigma^i_{t-h} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha^i_h + \beta_h sign(r^i_{t-h+1}) + \varepsilon^i_{t+1}$		
1	2.80	8.51	3.39	2.66	7.60	3.19
2	0.96	4.17	1.66	0.94	3.85	1.67
3	2.53	7.77	3.12	2.17	6.36	2.83
4	−0.19	1.56	0.70	0.36	1.56	1.27
5	−0.56	1.00	0.25	−0.94	1.03	0.02
6	0.58	3.26	1.36	1.07	2.90	1.79
Panel B: Forecast with past h-month returns						
	$r^i_{t+1}/\sigma^i_t = \alpha^i_h + \beta_h r^i_{t-h,t}/\sigma^i_{t-1} + \varepsilon^i_{t+1}$			$r^i_{t+1}/\sigma^i_t = \alpha^i_h + \beta_h sign(r^i_{t-h,t}) + \varepsilon^i_{t+1}$		
1	2.80	8.51	3.39	2.66	7.60	3.19
2	2.51	8.43	3.07	2.62	7.41	3.19
3	3.23	10.17	3.74	3.56	9.08	4.17
4	2.89	9.24	3.46	3.60	8.36	4.11
5	2.44	7.89	2.99	3.17	7.45	3.66
6	2.53	7.97	3.12	3.15	7.27	3.70

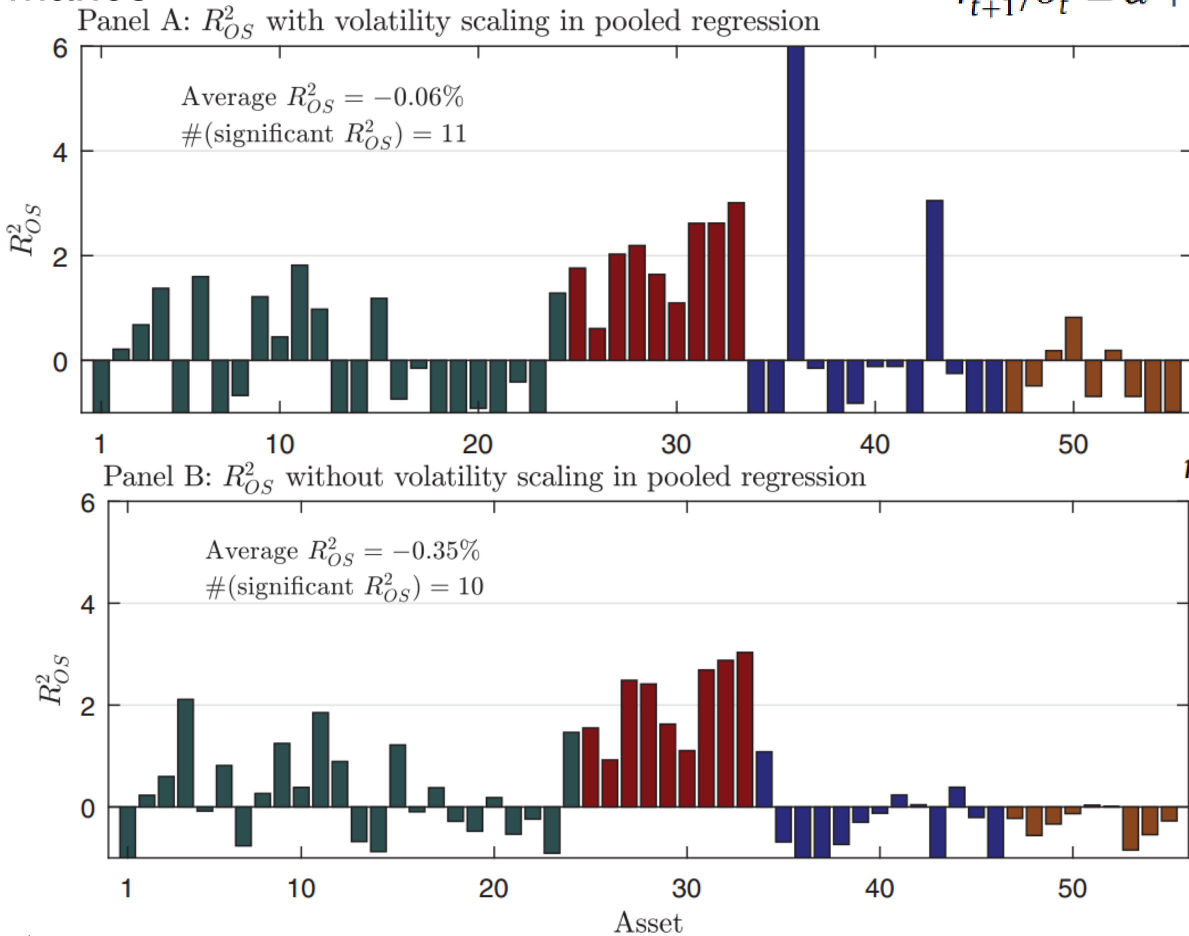
Compared with Table 4, after controlling for the fixed effects, the t-statistic is smaller than that without controlling for fixed effects. Insufficient evidence exists in support of TSM.

4. Pooled regression-- Out-of-sample performance

- **Fig. 4.** Time series momentum (TSM) with pooled regression: out-of-sample performance

$$r^i_{t+1}/\sigma^i_t = \alpha + \beta r^i_{t-12,t}/\sigma^i_{t-1} + \varepsilon^i_{t+1},$$

$$r^i_{t+1} = \alpha + \beta r^i_{t-12,t} + \varepsilon^i_{t+1}$$



A pooled regression can improve the out-of-sample forecasting power, but such improvement is restricted to international equity markets. it cannot provide significant support for TSM either.

5. Trading strategy--TSM versus TSH at asset level

- asset i ~ iid ~ N(μ^i,σ^i), the probability of the past 12-month return being positive is

$$\Pr(r^i_{t-12,t} > 0) = 1 - \Pr\left(\frac{r^i_{t-12,t} - 12\mu^i}{\sqrt{12}\sigma^i} \leq -\sqrt{12}\frac{\mu^i}{\sigma^i}\right)$$
$$= \Phi(\sqrt{12}\mu^i/\sigma^i), \quad (16)$$

$$r^{\mathrm{TSH},i}_{t+1} = sign(r^i_{1,t})r^i_{t+1}, \quad (17)$$

$$r^{\mathrm{TSM},i}_{t+1} = sign(r^i_{t-12,t})r^i_{t+1}. \quad (18)$$

TSH :buys the futures contract if its **historical sample mean** is non-negative and sells it if its historical sample mean is negative.
TSM :buys the future contract if its **past 12-month return** is non-negative and sells it if its past 12-month return is negative.

5. Trading strategy-- TSM versus TSH **at asset level**

- **Table 9** Time series momentum (TSM) versus time series history (TSH) at the asset level(55).

Asset	TSM return	TSH return	TSM Sharpe ratio	TSH Sharpe ratio	Return difference	*p*-value of return difference	Sharpe ratio difference	*p*-value of Sharpe ratio difference
Aluminum	0.27	−0.47	0.05	−0.08	0.74**	0.04	0.13**	0.04
Brent oil	0.80	0.32	0.09	0.04	0.48	0.44	0.05	0.44
Cattle	0.28	0.08	0.07	0.02	0.20	0.48	0.05	0.48
Cocoa	−0.46	0.13	−0.06	0.02	−0.59	0.32	−0.08	0.31
Coffee	0.18	−0.55	0.02	−0.05	0.73	0.30	0.07	0.30
Copper	0.77	0.94	0.10	0.12	−0.17	0.74	−0.02	0.73
JPY/USD	0.46	0.09	0.14	0.03	0.37	0.11	0.11	0.11
NOK/USD	0.06	−0.06	0.02	−0.02	0.12	0.58	0.04	0.58
NZD/USD	0.24	0.02	0.07	0.01	0.22	0.38	0.06	0.38
SEK/USD	0.04	−0.05	0.01	−0.02	0.09	0.70	0.03	0.70
CHF/USD	0.18	0.19	0.05	0.06	−0.01	0.96	−0.01	0.97
GBP/USD	0.00	−0.03	0.00	−0.01	0.03	0.87	0.01	0.87
#(significance)					7		7	

Of the 55 assets, only five(+) show that the TSM strategy generates a higher average return than the TSH strategy.
The TSM strategy does not significantly outperform at the asset level the TSH strategy that does not require predictability.

5. Trading strategy-- TSM versus TSH **at portfolio level**

- **Table 10** Time series momentum (TSM) versus time series history (TSH) at the portfolio level.

		Fama–French four-factor model						Asness-Moskowitz-Pedersen three-factor model				
	Mean	Alpha	MSCI World Index	SMB	HML	UMD	R^2	Alpha	MSCI World Index	Value everywhere	Momentum everywhere	R^2
Panel A: Equal weighting, i.e., portfolio weight $= \frac{1}{N}$												
TSM strategy												
Long leg	0.34***	0.12**	0.16***	0.05**	0.09***	0.32***	42.57%	0.09	0.16***	0.14***	0.38***	41.92%
	(4.92)	(2.29)	(7.73)	(2.24)	(2.62)	(7.19)		(1.60)	(7.21)	(2.99)	(7.33)	
Short leg	−0.05	−0.03	0.14***	0.11***	0.03	−0.28***	48.31%	0.02	0.13***	-0.09	−0.32***	45.85%
	(−0.72)	(−0.46)	(5.30)	(4.01)	(0.96)	(−8.34)		(0.23)	(5.08)	(−1.42)	(−6.89)	
Long - short	0.39***	0.15*	0.02	−0.06*	0.06	0.60***	46.03%	0.07	0.03	0.23**	0.70***	47.39%
	(4.73)	(1.94)	(0.61)	(−1.83)	(1.01)	(9.99)		(1.01)	(0.93)	(2.50)	(8.79)	
TSH strategy												
Long leg	0.27***	0.07	0.28***	0.14***	0.10***	0.08*	49.07%	0.10	0.26***	0.01	0.08*	45.30%
	(2.56)	(0.95)	(8.79)	(4.41)	(3.90)	(1.93)		(1.13)	(7.73)	(0.24)	(1.65)	
Short leg	0.02	0.02	0.03***	0.03*	0.02	−0.04**	8.73%	0.01	0.03***	0.03	−0.03	8.04%
	(0.61)	(0.52)	(3.51)	(1.83)	(1.47)	(−2.01)		(0.25)	(3.17)	(1.15)	(−1.04)	
Long - short	0.25***	0.05	0.25***	0.11***	0.08***	0.13***	44.83%	0.09	0.23***	−0.02	0.11*	42.01%
	(2.70)	(0.80)	(8.54)	(3.70)	(2.96)	(2.76)		(1.14)	(7.59)	(−0.29)	(1.94)	
TSM versus TSH												
Mean difference	0.14 [0.19]											
Alpha difference		0.10 [0.26]						−0.02 [0.84]				

$$r_{t+1}^{TSM} = \frac{1}{N_t}\sum_{i=1}^{N_t} sign(r_{t-12,t}^{i}) r_{t+1}^{i}$$

$$r_{t+1}^{TSH} = \frac{1}{N_t}\sum_{i=1}^{N_t} sign(r_{1,t}^{i}) r_{t+1}^{i},$$

1.The performance of the two strategies mainly stems from the long legs;
2.The alpha differential between the TSM and TSH strategies is always indifferent from zero.

5. TSM and TSH forecast comparison: predictive slope

- **Table 11** Time series momentum (TSM) and time series history (TSH) forecast comparison.

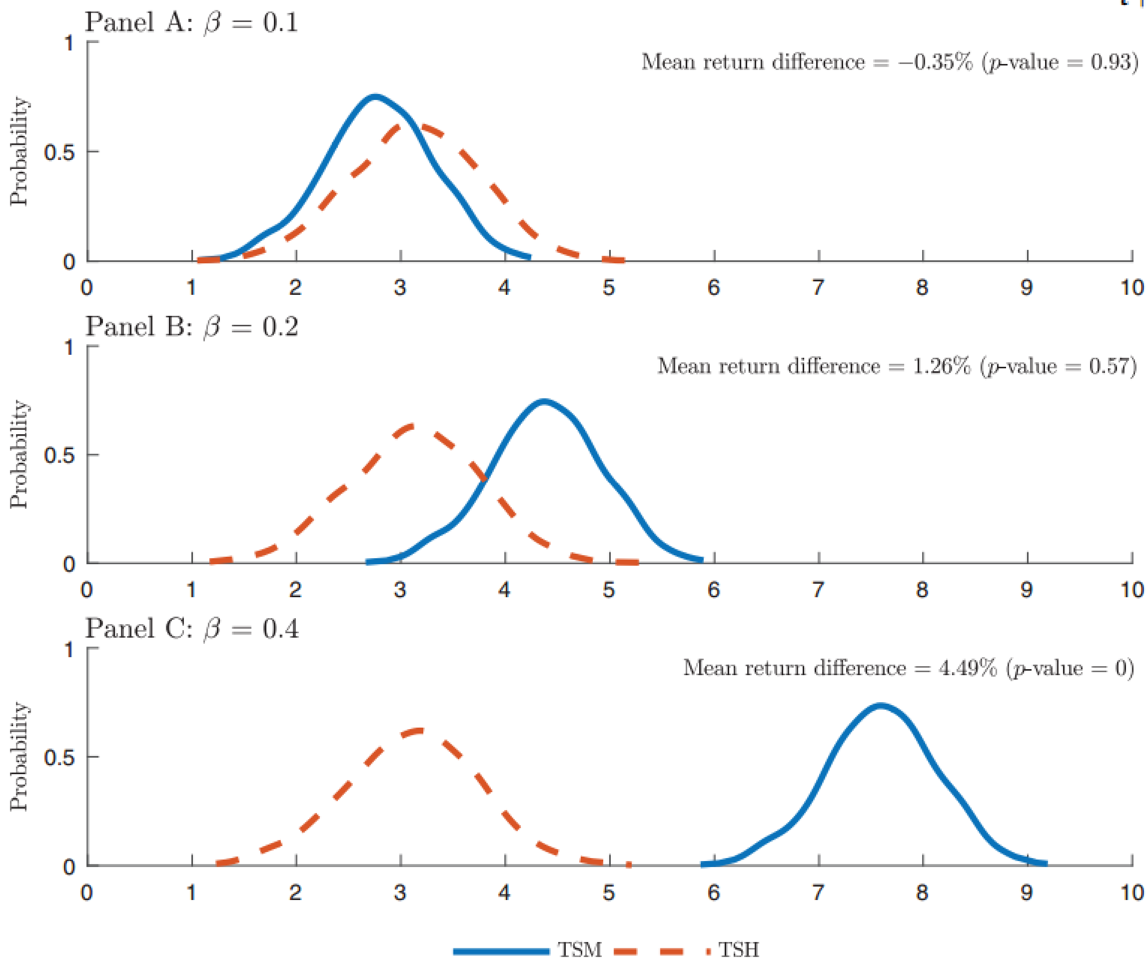
	$r^i_{t+1} = \alpha + \beta \hat{r}^{\mathrm{TSM},i}_{t+1} + \varepsilon^i_{t+1}$			$\hat{r}^{\mathrm{TSM},i}_{t+1} = d\hat{r}^{\mathrm{TSH},i}_{t+1} + u^i_t$		
Asset class	β	t-statistic	R^2	d	t-statistic	R^2
Panel A: $\hat{r}^{\mathrm{TSM},i}_{t+1}$ is estimated with volatility scaling						
Overall	0.19	0.61	0.04	1.09***	18.56	40.33
Commodity	0.15	0.42	0.02	1.24***	11.62	23.53
Equity	0.07	0.10	0.01	0.84***	14.90	45.06
Bond	0.23	0.60	0.08	0.99***	68.75	92.27
Currency	−0.08	−0.12	0.01	1.01***	14.95	4.45
Panel B: $\hat{r}^{\mathrm{TSM},i}_{t+1}$ is estimated without volatility scaling						
Overall	0.30	0.45	0.03	1.04***	41.89	54.96
Commodity	0.09	0.11	0.00	1.01***	26.53	37.65
Equity	−0.37	−0.32	0.07	0.93***	35.34	77.93
Bond	−0.49	−0.52	0.07	1.00***	72.40	91.38
Currency	0.03	0.03	0.00	1.64***	19.78	14.32

The TSM strategy has little predictive power and behaves in a very similar manner to the TSH strategy.

5. When does the TSM outperform the TSH?

- **Fig. 5.** Annualized mean return difference between time series momentum (TSM) and time series history (TSH)

$$r^i_{t+1} = \alpha^i + \beta \frac{r^i_{t-12,t}}{12} + \varepsilon^i_{t+1},$$



When the slope is 0.4, the TSM dominates the TSH in the sense that it does better in almost all the simulated data sets.

6. Conclusion

- Firstly, The predictability of the TSM is weak .
- Secondly, The performance of the TSM strategy is likely driven by differences in mean returns, not predictability.