Variance Risk in Aggregate Stock Returns and Time-Varying Return Predictability

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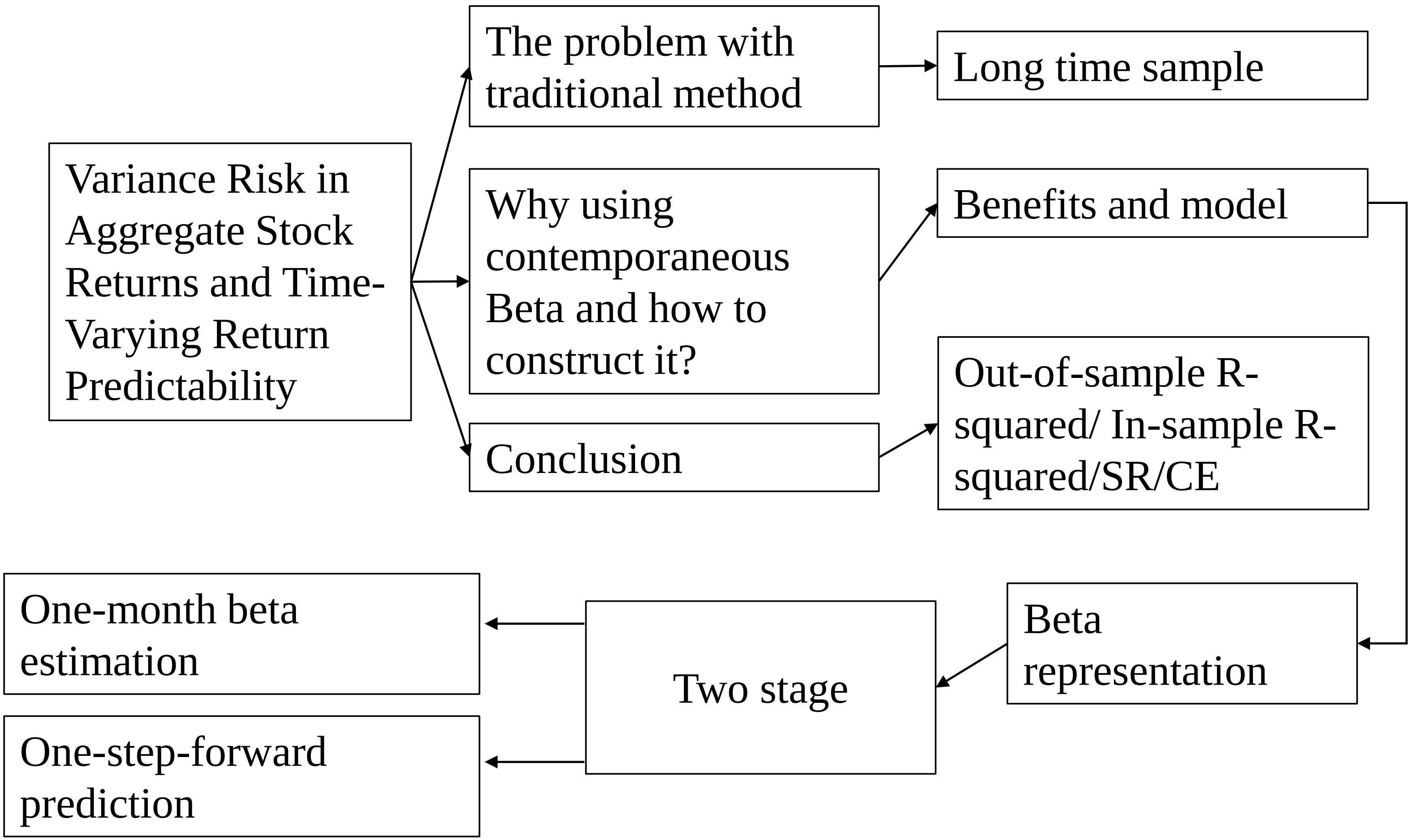
Outline


Variance Risk in Aggregate Stock Returns and Time-Varying Return Predictability
The problem with traditional method
Why using contemporaneous Beta and how to construct it?
Conclusion
Long time sample
Benefits and model
Out-of-sample R-squared/ In-sample R-squared/SR/CE
Beta representation
Two stage
One-month beta estimation
One-step-forward prediction

Outline

- Introduction
- Research design
- Empirical result
- Robust Test
- Conclusion

Introduction: Background

- Various approaches have been studied for market returns prediction
   A. Existing Problem

      First, predictive relationships appear to change over time, with some variables being successful in certain periods (Fama and French, 1988a) or at specific periods of the business cycle (Dangl and Halling, 2012).

      Second, predictors that perform well in sample often fail out of sample (Goyal and Welch,2008; Campbell and Thompson, 2008). Lastly, return predictions typically perform worse for shorter horizons (Fama and French, 1988a), with many well-known predictors failing to forecast returns at the horizons below six months.

Introduction: Background

- Various approaches have been studied for market returns prediction

  B. Variance risk premium (VRP)

  A recent study by Bollerslev, Tauchen, and Zhou (2009) suggests that even monthly or quarterly market returns are predictable by the one-month variance risk premium (VRP),measured as the difference between option-implied variance and realized variance. They report a positive and statistically significant slope coefficient for the regression.

Introduction: VRP

$$\frac{dS_t}{S_t} = \mu_t dt + \sqrt{V_t}(\rho_t dW_t^v + \sqrt{1-\rho_t^2} dW_t^o)$$

$$dV_t = \theta_t dt + \sigma_v dW_t^v.$$

$$\frac{dS_t}{S_t} = \mu_t dt + \rho_t \frac{\sqrt{V_t}}{\sigma_v}(dV_t - \theta_t dt) + \sqrt{(1-\rho_t^2)V_t} dW_t^o.$$

$$\begin{aligned}\mathrm{Cov}_T\left(-SDF_{T,T+1}, \int_T^{T+1} \frac{dS_t}{S_t}\right) &= -\rho_T \frac{V_T}{\sigma_v} \mathrm{Cov}_T\left(SDF_{T,T+1}, \int_T^{T+1} dV_t\right) \\ &\quad - \sqrt{(1-\rho_T^2)V_T}\, \mathrm{Cov}_T\left(SDF_{T,T+1}, \int_T^{T+1} dW_t^o\right)\end{aligned}$$

Introduction: VRP

$$VRP_T = Cov_T\left(SDF_{T,T+1}, \int_T^{T+1} dV_t\right)$$

$$\approx E_T^Q\left[\int_T^{T+1} dV_t\right] - E_T\left[\int_T^{T+1} dV_t\right]$$

Introduction: New Method

- Contemporaneous Beta Regression

A. Contemporaneous Beta Estimation

the exposure (β) can be estimated by the slope of the contemporaneous regression of market returns on the unexpected changes in realized variance (RV):

$$R_{m,t} = \beta_{v,0} + \beta_v(RV_t - \mathrm{E}_{t-1}[RV_t]) + \epsilon_{o,t}.$$

Introduction: New Method

- Contemporaneous Beta Regression

B. Beta Substitution

The estimated coefficients of the predictive regression are then used to form a one-step-ahead out-of-sample forecast.

The new approach directly uses the **contemporaneous variance beta ($\beta v,t$) in place of the predictive beta (βp).**

The size of the slope can then be multiplied by the VRP to form a return forecast for the following month.

Introduction: Motivation

- First, the one-month market risk premium should be related to the VRP by the market's exposure to variance risk. This logic follows intuitively from what is known as the "beta representation," i.e., that the risk premium of an asset is related to the price of risk by the size of risk exposure.

Introduction: Motivation

- The second observation is that when variance risk is responsible for a larger fraction of market risk, the VRP should explain a greater share of the market risk premium. When market risk is decomposed into two parts – a variance-related component and an unrelated component – the combination of the beta and the VRP should exactly explain the market risk premium due to the variance-related component.

Introduction: Contribution

- The empirical section of this paper shows that the new approach strictly outperforms the traditional way of return forecasting at the monthly horizon.
- In particular, the new approach predicts one-month market returns in a statistically and economically significant manner. Specifically, the traditional approach, which requires running a series of rolling predictive regressions , are unable to produce accurate forecasts of one-month returns.

Introduction: Contribution

- Across multiple VRP measures considered, some of the out-of-sample R2s are positive (-0.8% – 5.2%), but they are all far from being statistically significant. However, when we combine the VRP with the contemporaneous variance beta of the market, the R2s are always much higher (6.1% – 8.4%) and the corresponding Wald statistics are always statistically significant.
- These results are robust regardless of whether a constant or zero premium on the orthogonal component is assumed. Finally, there is a gain of more than 0.13 (21% increase) in the annual Sharpe ratio and 4% (100% increase) in the certainty equivalent when forming a trading strategy based on the new approach.

Research Design: Data

- Time: 1990-2016
- The high-frequency intraday trading data for the S&P 500 Index is obtained from Tickdata.
- the VIX (VXO) is only available from CBOE.

Research Design: Variables

- **RV** is computed by first calculating squared log returns from the last tick of each five-minute interval. A subsampling scheme at one-minute intervals (Zhang, Mykland, and Ait-Sahalia, 2005) is used to reduce microstructure noise. Hansen and Lunde (2006), for example, study the impact of subsampling and note that, theoretically, it is always beneficial in reducing microstructure noise. I rescale the RVs to the monthly level so that they match the variance of a month.
- As Bekaert and Hoerova (2014) argue, the VRP’s ability to predict returns may depend on the particular model used to compute the real-world expectation component.

Research Design: Variables

- As Bekaert and Hoerova (2014) argue, the VRP's ability to predict returns may depend on the particular model used to compute the real-world expectation component.
- The constructed RV series is then used to compute the variance forecasts. Corsi (2009) proposes a Heterogeneous Autoregressive Realized Volatility (HAR-RV) model. The model assumes that the predicted value of volatility is linear in its autoregressive components – daily, weekly and monthly realized volatility.

$$RV_{\tau+k} = a_0 + a_d RV_\tau + a_w(\sum_{j=0}^{4} RV_{\tau-j}) + a_m(\sum_{j=0}^{21} RV_{\tau-j}) + \phi_{1,\tau+k}$$

Research Design: Variables

- The **VRP** is the price of variance risk and is commonly interpreted as a proxy of time-varying aggregate risk aversion , for example, Todorov (2010), Drechsler and Yaron (2011), Bekaert, Hoerova, and LoDuca (2013), and Bekaert and Hoerova (2014), among others.
- The **VRP** is essentially the expected unit cost of variance risk, and the contemporaneous variance beta is the number of swap contracts required to hedge the market portfolio against variance movement.

Research Design: Variables

- The VRP is measured by taking the difference between the square of **VIX** and the **monthly forecast of RV**. While the end-of-month values of the VIX squared is typically used for the first component, in the literature, the second component is estimated by **using a forecast model on the RV** computed by **summing up** daily observations over the entire month.

$$RV_{\tau+k} = a_0 + a_d RV_\tau + a_w(\sum_{j=0}^{4} RV_{\tau-j}) + a_m(\sum_{j=0}^{21} RV_{\tau-j}) + \phi_{1,\tau+k}$$

Research Design: Variables

- I estimate the VRP as the difference between the VIX2 and the 22-day cumulative forecast of daily realized variance. However, to deal with the mismatch, I either average the daily observations or take the end-of-month values. These two measures are parametric and denoted by VRPP and VRPPE, respectively, observations over the entire month.

$$VRP_{\overline{P},t} = \sum_{\tau \in t} \left(\frac{VIX_{\tau}^{2}}{252} - \frac{\widehat{RV}_{\tau+1,\tau+22|\tau}}{22} \right)$$

$$VRP_{PE,t} = \frac{VIX_{m(t)}^{2}}{12} - \widehat{RV}_{m(t)+1,m(t)+22|m(t)}$$

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$$VRP_{P^{E},t} = \frac{VIX_{m(t)}^{2}}{12} - \widehat{RV}_{m(t)+1,m(t)+22|m(t)}$$

- where m(t) is the last trading day of month t. Both of the variance components are rescaled to match the one-month variance.

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Research Design:
Contemporaneous Betas and Correlations Estimation

- The daily innovation of market variance is calculated by computing the unexpected changes in RV scaled so that it matches the one-month interval. Then, the monthly contemporaneous beta is estimated from the regression of market returns on variance innovations, using only observations that belong to that particular month.

$$R_{m,\tau} = \beta_{v,0,t} + \beta_{v,t}(RV_\tau - \widehat{RV}_{\tau|\tau-1}) + \epsilon_\tau$$

Research Design:
Contemporaneous Betas and Correlations Estimation

- The choice of the estimation window follows that of Ang, Hodrick, Xing, and Zhang (2006) and Chang, Christoffersen, and Jacobs (2013). They also use a single month of data to estimate the variance betas for individual stocks.
- I consider weighted least squares (WLS) in addition to ordinary least squares (OLS).

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Research Design:
Contemporaneous Betas and Correlations Estimation

- The contemporaneous correlation (ˆρt) is the correlation between the two variables in the above equation. The correlations are closely connected to the betas because they are transformations of each other.

$$\hat{\rho}_t = \hat{\beta}_{v,t} \times \frac{\hat{\sigma}_t(RV_\tau - \widehat{RV}_{\tau|\tau-1})}{\hat{\sigma}_t(R_{m,\tau})}$$

- Because each regression is based on observations from a single month, the monthly series of betas and a correlations are estimated from non-overlapping samples.

Research Design:
Out-of-sample Predictions

- **The traditional approach** to providing OOS forecasts of time T + 1 returns consists of two stages. First, we run a predictive regression using the past k months of historical data (from time T − k + 1 up to time T) as

$$R_{m,t} = \beta_0 + \beta_p VRP_{t-1} + \epsilon_t.$$

- We use the coefficient estimated at time T to forecast returns at time T+1. The one-step-ahead predicted value of the excess market returns $(\widehat{R}_{m,T+1|T})$ is given as

$$\hat{\beta}_{0,T} + \hat{\beta}_{p,T} VRP_T$$

Research Design:
Out-of-sample Predictions

- The **new approach** deviates in one critical dimension. The OOS forecast of month T + 1 returns is formed by using the contemporaneous variance beta from month T in place of the predictive beta estimated over the past k periods.
- The first OOS forecast is formed by multiplying the VRP with the negative of the contemporaneous beta. The one-step-ahead predicted value of market excess returns is then

$$\widehat{R}_{m,T+1|T} = -\hat{\beta}_{v,T} VRP_T.$$

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Research Design:
Explaining the Orthogonal Premium

- Constant Only(Including intercept only)
- Select several predictors that are well-known to predict market returns. These include: dividend yield (D/Y) (Campbell and Shiller, 1988; Fama and French, 1988a), the term (TERM) (Campbell, 1987) and the default premia (DEF) (Keim and Stambaugh, 1986), the short rate (Campbell, 1987), short interest (Rapach, Ringgenberg, and Zhou, 2016), cay (Lettau and Ludvigsen, 2001), and new orders-to-shipment (NO/S) (Jones and Tuzel, 2013). All of these are known to perform well in predicting market returns over a long historical sample.

Research Design:
Out-of-sample Predictions hybrid approach

- Hence, it is possible that the orthogonal premium is explained by other well-known predictors of market returns, such as the dividend yields, that is related to a more persistent component of the risk premium.

$$R_{m,t+1} = -\hat{\beta}_{v,t} VRP_t + \delta_0 + \delta_1 \sqrt{1 - \hat{\rho}_t^2} X_t + \eta_{t+1}$$

Research Design:
Out-of-sample Predictions

- To find estimates of ˆδ0 and ˆδ1 on a rolling basis. Here, Xt can be any predictor of market returns, including the VRP. Under this approach, the OOS forecast at time T is then,

$$\hat{R}_{m,T+1|T} = -\hat{\beta}_{v,T} VRP_T + \hat{\delta}_0 + \hat{\delta}_1 \sqrt{1 - \hat{\rho}_T^2} X_T$$

Research Design: Indicator $OOS\text{-}R^2$

$$\text{OOS-}R^2 = \quad 1 - \frac{\sum_t (\widehat{R}_{m,t+1|t} - R_{m,t+1})^2}{\sum_t (\overline{R}_{m,t} - R_{m,t+1})^2},$$

- where $\bar{R}_{mt}$ is the historical average of the market returns up to time t. Finally, we compute a test statistic, for example, a Wald statistic, to test the significance of the predictor. Diebold and Mariano (1995) provide a formal test for such OOS prediction errors.

Research Design: Indicator *Wald test*

$$W = T\left(T^{-1}\sum_{t=1}^{T}\Delta L_{t+1}\right)\hat{\Omega}^{-1}\left(T^{-1}\sum_{t=1}^{T}\Delta L_{t+1}\right)$$

$$\Delta L_{t+1} = (\overline{R}_{m,t} - R_{m,t+1})^2 - (\widehat{R}_{m,t+1|t} - R_{m,t+1})^2$$

$$\hat{\Omega} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T}(\Delta L_{t+1} - \overline{\Delta L})^2$$

Research Design: Indicator *COF*

- To better understand when the new approach especially performs especially better over the traditional approach, I develop a measure that computes the cumulative improvements in the loss function over the benchmark. I define the Cumulative Out-Performance of the Forecast (COF) as:

$$\mathrm{COF}_T = \sum_{t=1}^{T} \Delta L_t$$

Research Design:
Time-varying Out-of-sample Predictability

- I also study the connection between contemporaneous correlations and predictive R2s. I do so by dividing the full sample into different non-overlapping subsamples. Each of the 288 months in the full sample period of 1993-2016 is classified into one of three groups according to the monthly series of the contemporaneous correlations between market returns and variance innovations

The correlation is more positive than the second tercile

High	**Medium**	**Low**

The correlation is more negative than the first tercile

Research Design:
Evaluating Economic Significance - A Trading Strategy

- I use the one-step-ahead OOS forecasts to calculate optimal weight on the stock market as

$$w_T = \frac{\hat{R}_{m,T+1|T}}{\gamma \hat{\sigma}_T^2}$$

- where $\gamma = 3$ is assumed for the risk aversion coefficient and the monthly square of VIX is used as a proxy for ˆσ2. The remaining proportion $1 - w_T$ is invested in the risk-free asset.

Research Design:
Evaluating Economic Significance - A Trading Strategy

- The previous tables on the predictive performance show that predictions can be made more accurately when the absolute correlation between returns and variance is high. A concern is that the weights might rely too much on the VRP-based forecasts during periods when returns and variance innovations are unrelated.

$$w_T = \frac{\hat{R}_{m,T+1|T}}{\gamma \hat{\sigma}_T^2} \sqrt{\hat{\rho}_T^2} + \frac{\overline{R}_{m,T}}{\gamma \hat{\sigma}_T^2} \sqrt{1 - \hat{\rho}_T^2}$$

Research Design:
Evaluating Economic Significance - A Trading Strategy

- Therefore, I also consider an alternative strategy, in which a fraction of the allocation of stocks depends on the model-based predicted returns and the rest on the historical average of past returns. The weight invested in the risky asset becomes:

$$w_T = \frac{\hat{R}_{m,T+1|T}}{\gamma \hat{\sigma}_T^2} \sqrt{\hat{\rho}_T^2} + \frac{\overline{R}_{m,T}}{\gamma \hat{\sigma}_T^2} \sqrt{1 - \hat{\rho}_T^2}$$

Research Design:
Evaluating Economic Significance - A Trading Strategy

- The certainty equivalent (CE) of the return is computed as

$$CE = \overline{R_p} - \frac{\gamma}{2}\widehat{\text{Var}}(R_p)$$

where $\gamma = 3$ is assumed for the risk aversion coefficient

- Sharpe Ratios： $SR = \frac{R_p - R_f}{\sigma_p}$

Research Design: Robustness

- *Alternative Measures of the Variance Risk Premium*

- *Alternative Specifications for the Traditional Approach*

A. In a recent study, Johnson (2017) examines the OOS performance of a number of common return predictors and shows that using WLS as the first-stage regression improves OOS performance.

B. Selecting the optimal first-stage estimation interval

Empirical Result: VRP

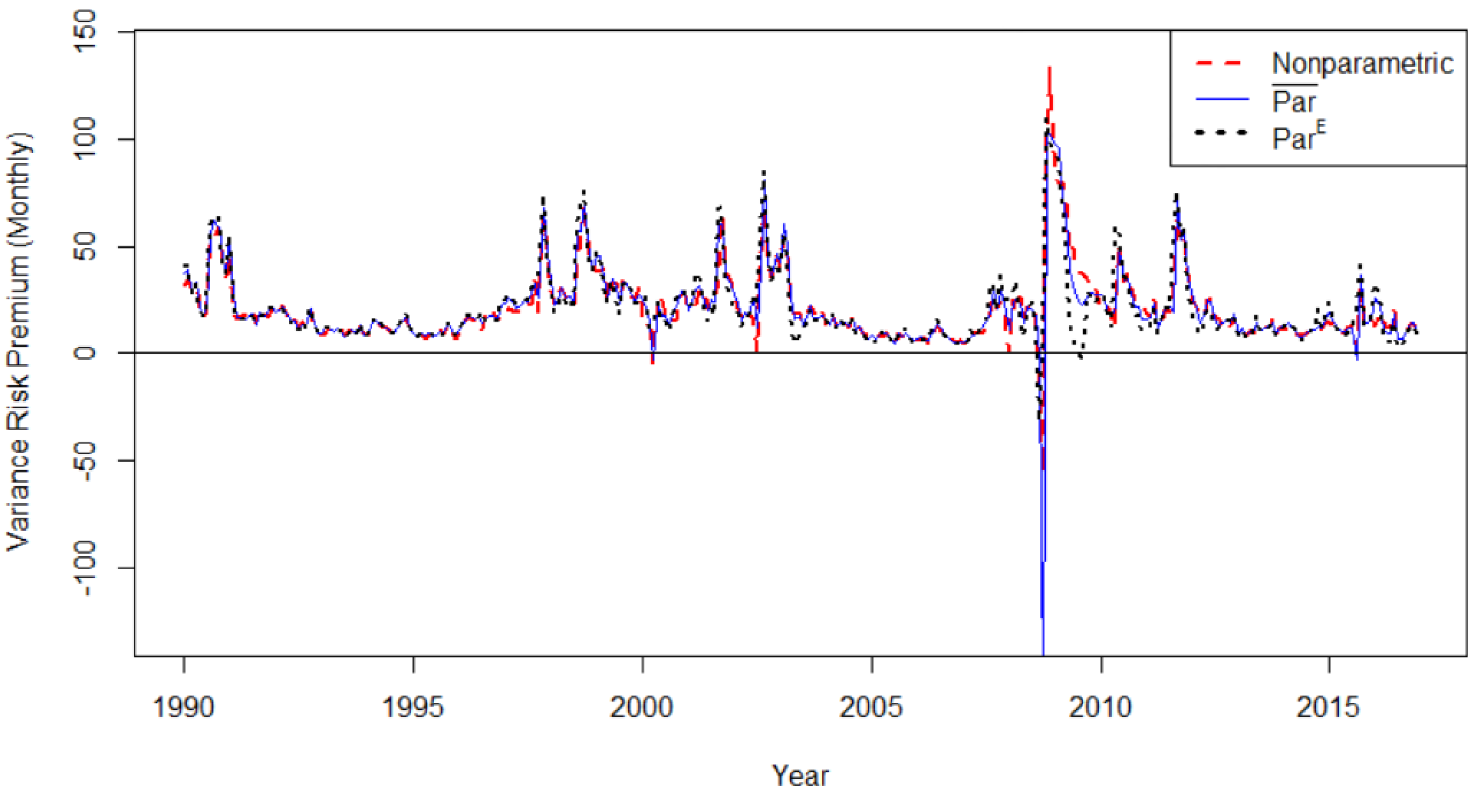
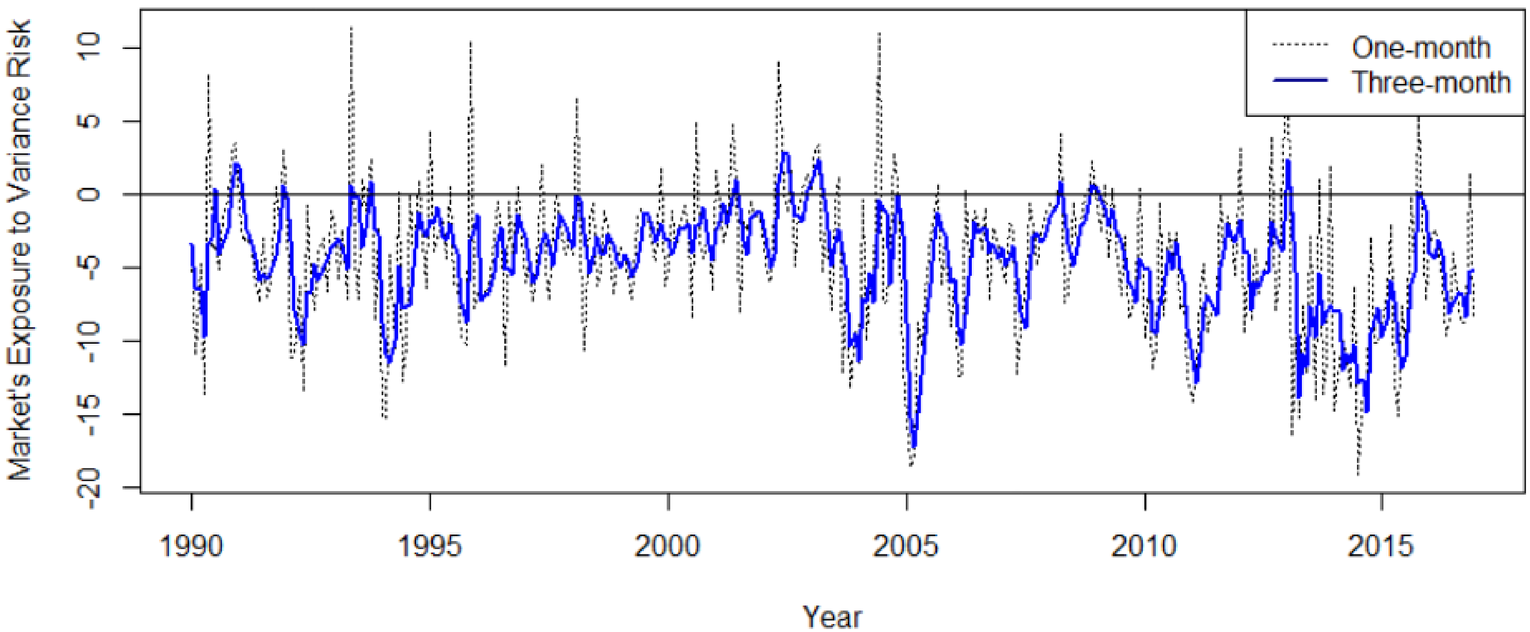


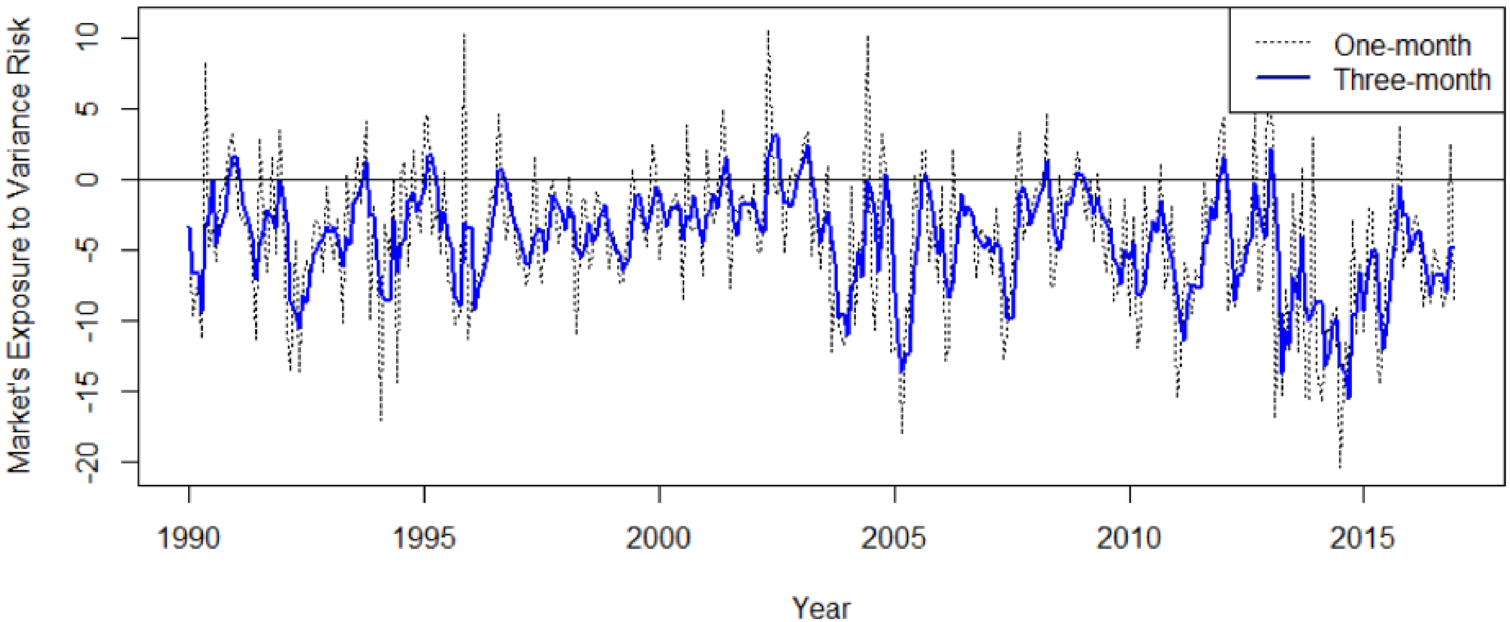
Fig. 1. Time Series of the Variance Risk Premium

Empirical Result: Beta



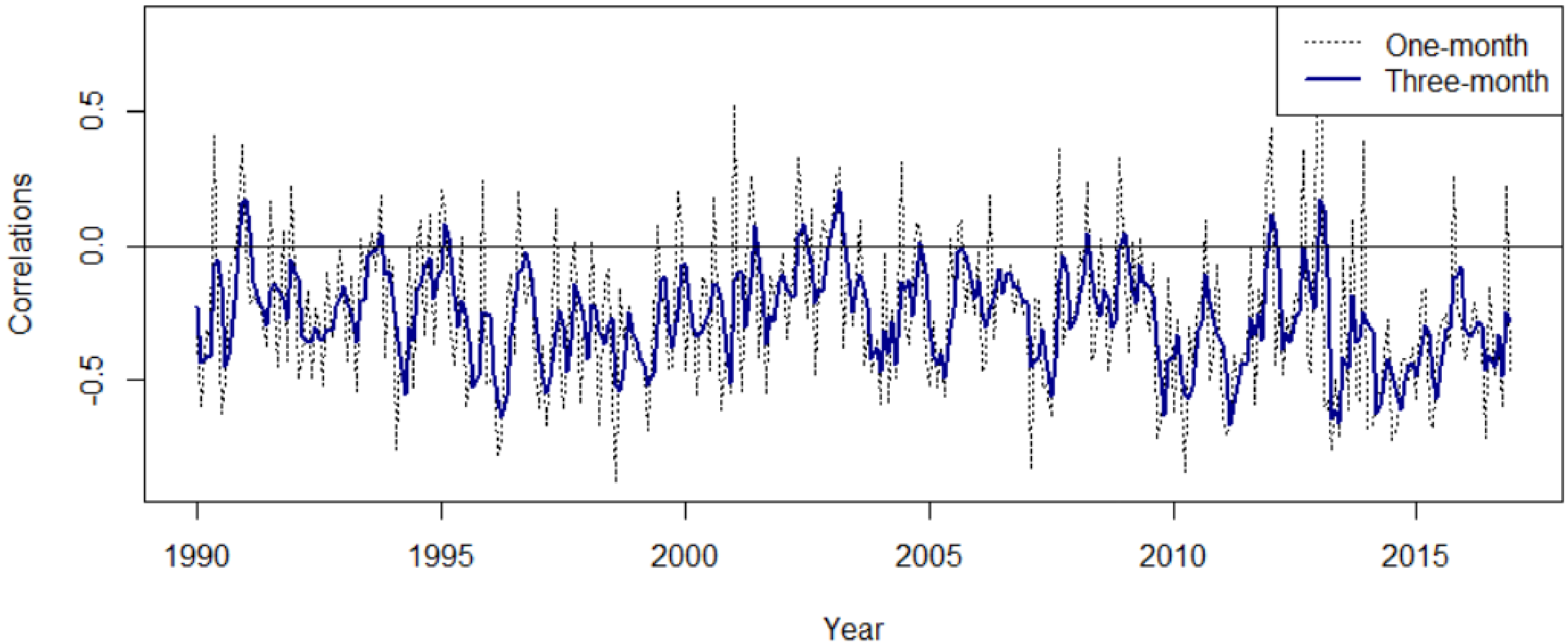
(a) Weighted Least Squares

Empirical Result: Beta



(b) Ordinary Least Squares

Empirical Result: Correlation



Empirical Result: Summary Statistics

Panel A. Summary Statistics

			$\hat{\rho} \leq$ median		NBER Recession		
	Mean	StDev	Mean	StDev	Mean	StDev	Autocorr.
RV (Monthly)	16.43	29.51	16.67	34.68	47.00	72.52	0.643
Implied Variance (Monthly)	19.71	7.49	19.89	7.34	29.02	10.30	0.840
$\mathrm{VRP}_{\overline{P}}$	21.74	17.86	21.43	16.38	39.76	31.01	0.764
VRP_{PE}	21.30	18.77	21.86	19.98	31.81	29.04	0.601
$\hat{\beta}_v$	-4.19	5.10	-7.72	3.92	-1.49	3.05	0.200
$\hat{\beta}_{v,WLS}$	-4.56	5.08	-7.72	3.81	-1.60	3.01	0.203
$\hat{\rho}$	-0.259	0.281	-0.486	0.132	-0.130	0.226	0.124
Number of Month	324		162		37		

Empirical Result: Summary Statistics

Panel B. The Leverage Effect (Correlations)

	$\hat{\beta}_{v,t+1}$	$\hat{\rho}_{t+1}$
Contemporaneous Retur ($R_{M,t+1}$)	0.120	0.280
Lagged Annual Return ($\sum_{k=0}^{11} R_{M,t-k}$)	-0.261	-0.287
RV_t	0.196	0.142
VIX_t	0.297	0.165
VIX Trend_t	-0.121	-0.259
SKEW_t	-0.300	-0.222
Tail Risk_t	-0.128	-0.131
$\mathrm{VRP}_{N,t}$	0.199	0.152
$\mathrm{VRP}_{\overline{P},t}$	0.274	0.214
$\mathrm{VRP}_{P^E,t}$	0.220	0.128

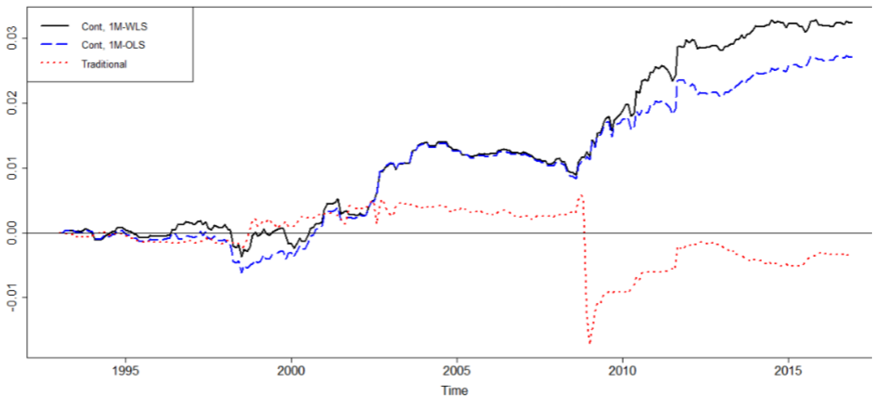
Empirical Result: Out-of-Sample

		VRP Measures					
		VRP_N		$\text{VRP}_{\overline{P}}$		VRP_{PE}	
		Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained	Constrained
A. The Traditional Approach							
	OOS-R^2	0.010	−0.007	0.002	−0.008	0.052	0.032
	Wald	0.064	0.033	0.004	0.060	1.501	0.875
	p-value	(0.800)	(0.857)	(0.952)	(0.807)	(0.221)	(0.350)
B. The Contemporaneous Beta Approach							
B-1. No Intercept							
1-month WLS	OOS-R^2	0.065	0.061	0.079	0.076	0.084	0.079
	Wald	5.027	6.836	7.956	10.838	5.996	6.984
	p-value	(0.025)	(0.009)	(0.005)	(0.001)	(0.014)	(0.008)
1-month OLS	OOS-R^2	0.054	0.051	0.068	0.066	0.085	0.069
	Wald	3.686	5.325	8.056	8.964	4.544	5.665
	p-value	(0.055)	(0.021)	(0.005)	(0.003)	(0.033)	(0.017)
3-month WLS	OOS-R^2	0.064	0.066	0.049	0.052	0.064	0.064
	Wald	11.460	12.232	3.554	3.818	4.130	4.122
	p-value	(0.001)	(0.000)	(0.059)	(0.051)	(0.042)	(0.042)
3-month OLS	OOS-R^2	0.053	0.060	0.041	0.047	0.054	0.059
	Wald	10.096	11.462	2.735	3.552	3.339	3.862
	p-value	(0.001)	(0.001)	(0.098)	(0.059)	(0.068)	(0.049)
B-2. Including Intercept							
1-month WLS	OOS-R^2	0.059	0.053	0.074	0.068	0.080	0.073
	Wald	4.078	5.317	3.668	8.991	5.096	6.018
	p-value	(0.043)	(0.021)	(0.055)	(0.003)	(0.024)	(0.014)
1-month OLS	OOS-R^2	0.047	0.045	0.062	0.061	0.065	0.064
	Wald	4.920	4.084	6.485	7.543	5.096	4.899
	p-value	(0.027)	(0.043)	(0.011)	(0.006)	(0.024)	(0.027)

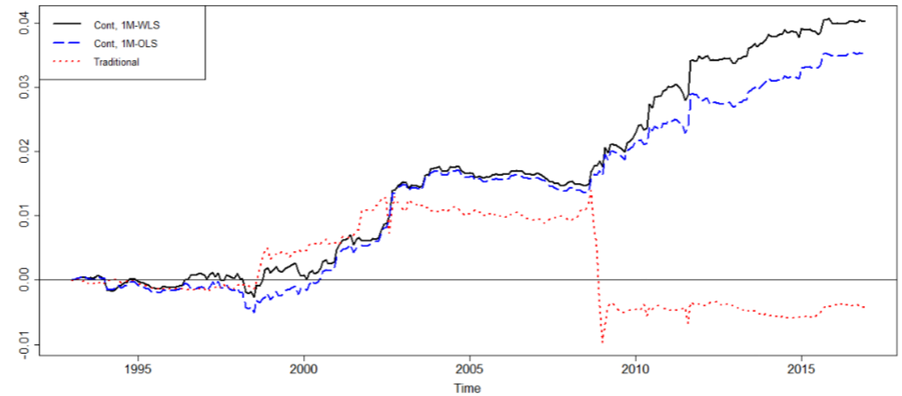
Empirical Result: The Price of Orthogonal Risk

Additional Variable (X_t)		1993-2016			Subsamples			
		VRP_N	$\text{VRP}_{\overline{P}}$	VRP_{PE}	High	Medium	Low	H-L
Intercept Only	OOS-R^2	0.047	0.062	0.065	0.070	0.041	0.021	0.049
	p-value	(0.027)	(0.011)	(0.024)				
D/Y	OOS-R^2	0.016	0.042	0.033	0.017	0.019	0.013	0.004
	p-value	(0.464)	(0.167)	(0.235)				
TERM	OOS-R^2	0.021	0.053	0.028	0.058	0.019	-0.021	0.079
	p-value	(0.260)	(0.061)	(0.234)				
DEF	OOS-R^2	−0.015	0.018	0.002	0.000	0.045	-0.079	0.079
	p-value	(0.639)	(0.614)	(0.962)				
Short Rate	OOS-R^2	−0.010	0.008	-0.310	0.007	0.031	-0.063	0.069
	p-value	(0.731)	(0.839)	(1.000)				
Short Interest	OOS-R^2	0.020	0.052	0.023	0.048	0.028	-0.020	0.068
	p-value	(0.361)	(0.096)	(0.397)				
cay	OOS-R^2	0.025	0.054	0.038	0.089	-0.067	0.023	0.066
	p-value	(0.359)	(0.125)	(0.240)				
NO/S	OOS-R^2	0.015	0.048	0.025	0.036	0.027	-0.019	0.055
	p-value	(0.434)	(0.106)	(0.300)				
LJV	OOS-R^2	−0.104	−0.189	-0.103	-0.013	-0.108	-0.212	0.199
	p-value	(0.092)	(0.276)	(0.205)				
VRP	OOS-R^2	−0.001	−0.070	-0.074	0.069	0.023	-0.100	0.169
	p-value	(0.995)	(0.312)	(0.150)				

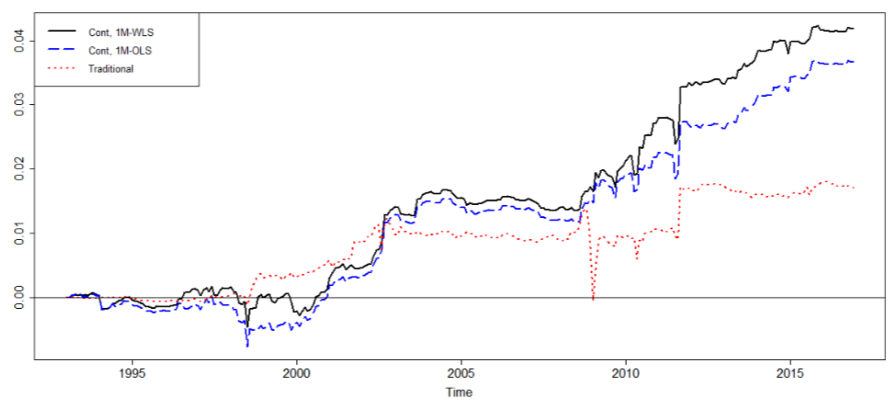
Empirical Result: COF constrained



(a) Variance Risk Premium (Non-parametric)

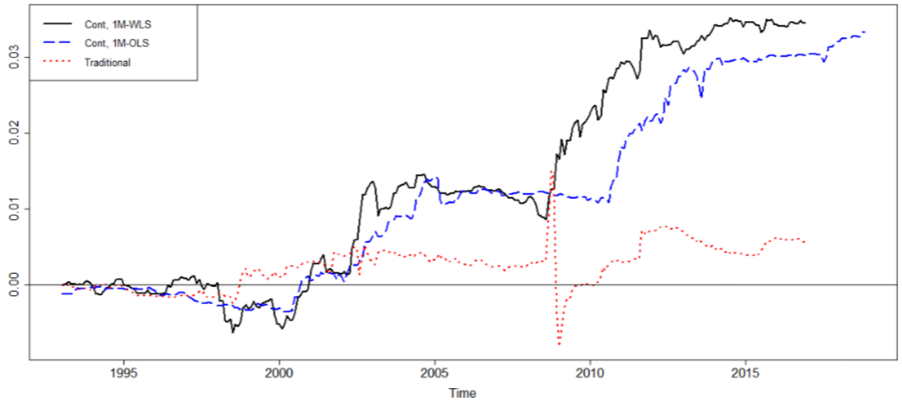


(b) Variance Risk Premium (Parametric, Averaged)

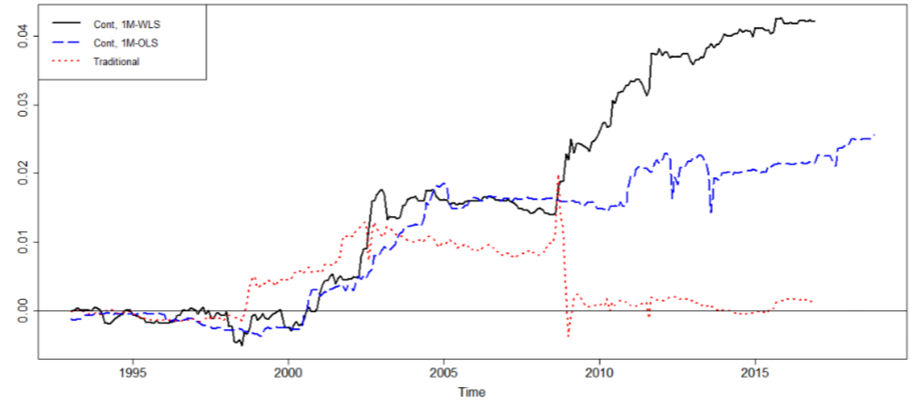


(c) Variance Risk Premium (Parametric, End of Month)

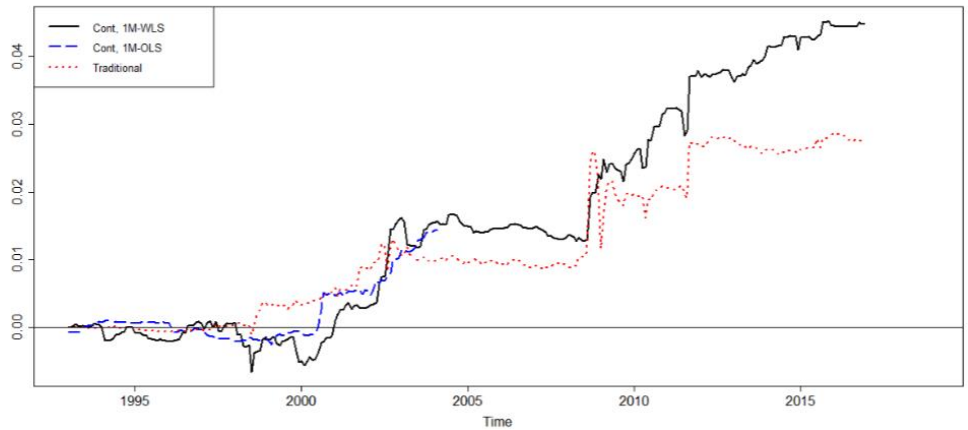
Empirical Result: COF unconstrained



(a) Variance Risk Premium (Non-parametric)



(b) Variance Risk Premium (Parametric, Averaged)



(c) Variance Risk Premium (Parametric, End of Month)

Empirical Result: Conditional Out-of-sample Predictions

		OOS-R^2					
		VRP_N		$VRP_{\overline{P}}$		VRP_{PE}	
		1-month	3-month	1-month	3-month	1-month	3-month
A. The Traditional Approach							
	High	0.068	0.162	0.096	0.065	0.164	0.207
	Medium	0.049	0.069	−0.035	0.057	−0.050	0.060
	Low	−0.124	−0.135	−0.097	−0.079	0.000	−0.056
	High-Low	0.193	0.298	0.193	0.144	0.164	0.263
B. The Contemporaneous Beta Approach							
B-1. No Intercept							
WLS	High	0.082	0.131	0.122	0.065	0.161	0.128
	Medium	0.056	0.054	0.053	0.048	0.031	0.053
	Low	0.050	0.047	0.045	0.059	0.036	0.048
	High-Low	0.032	0.085	0.077	0.006	0.125	0.080
OLS	High	0.079	0.126	0.120	0.064	0.156	0.123
	Medium	0.042	0.042	0.038	0.037	0.009	0.041
	Low	0.029	0.039	0.024	0.050	0.020	0.040
	High-Low	0.050	0.088	0.096	0.014	0.136	0.083
B-2. Including Intercept							
WLS	High	0.074	0.147	0.114	0.084	0.139	0.149
	Medium	0.057	0.033	0.055	0.024	0.050	0.031
	Low	0.040	0.035	0.035	0.050	0.024	0.038
	High-Low	0.034	0.112	0.079	0.034	0.115	0.111
OLS	High	0.070	0.140	0.111	0.080	0.131	0.141
	Medium	0.041	0.021	0.037	0.014	0.032	0.019
	Low	0.021	0.025	0.016	0.039	0.004	0.028
	High-Low	0.049	0.114	0.095	0.040	0.128	0.112

Empirical Result: Trading Strategies

	Unconditional Weighting				Conditional Weighting			
	SR	CE	Δ SR	Δ CE	SR	CE	Δ SR	Δ CE
Fixed Weight	0.527	0.046						
Average (Benchmark)	0.632	0.040						
The Traditional Approach								
VRP_N	0.524	0.033	-0.108	-0.007	0.673	0.048	+0.042	+0.009
$VRP_{\overline{P}}$	0.667	0.044	+0.035	+0.004	0.739	0.054	+0.107	+0.014
VRP_{PE}	0.672	0.053	+0.040	+0.013	0.741	0.059	+0.109	+0.019
The Contemporaneous Beta Approach								
No Intercept								
VRP_N	0.760	0.078	+0.129	+0.039	0.729	0.071	+0.097	+0.031
$VRP_{\overline{P}}$	0.922	0.098	+0.290	+0.058	0.836	0.084	+0.204	+0.044
VRP_{PE}	0.782	0.090	+0.151	+0.050	0.749	0.078	+0.117	+0.039
Including Intercept								
VRP_N	0.753	0.081	+0.121	+0.041	0.715	0.070	+0.083	+0.030
$VRP_{\overline{P}}$	0.902	0.098	+0.270	+0.059	0.817	0.081	+0.185	+0.042
VRP_{PE}	0.812	0.097	+0.180	+0.057	0.750	0.079	+0.119	+0.039

Robustness

Panel A. OOS Performance of the Traditional Approach (New Measures)

	VRP_{BTZ}	VRP_{BH}	$VRP_{VXO,N}$		$VRP_{VXO,\overline{P}}$		VRP_{VXO,P^E}	
	1993-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016
OOS-R^2	0.037	0.024	−0.015	−0.009	−0.010	−0.007	0.011	0.016
Wald	2.176	0.367	0.233	0.082	0.233	0.082	0.186	0.339
p-value	(0.140)	(0.545)	(0.629)	(0.775)	(0.629)	(0.775)	(0.666)	(0.561)

Panel B. OOS Performance of the Contemporaneous Beta Approach

	Statistics	VRP_{BTZ}	VRP_{BH}	$VRP_{VXO,N}$		$VRP_{VXO,\overline{P}}$		VRP_{VXO,P^E}	
		1993-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016
1-month WLS	OOS-R^2	0.050	0.073	0.068	0.070	0.083	0.086	0.084	0.087
	Wald	2.245	3.877	5.993	5.723	9.145	8.781	6.906	6.666
	p-value	(0.134)	(0.049)	(0.014)	(0.017)	(0.002)	(0.003)	(0.009)	(0.010)
1-month OLS	OOS-R^2	0.041	0.054	0.055	0.057	0.071	0.073	0.072	0.073
	Wald	2.907	2.863	4.098	3.960	6.885	6.602	4.982	4.619
	p-value	(0.088)	(0.091)	(0.043)	(0.047)	(0.009)	(0.010)	(0.026)	(0.032)
3-month WLS	OOS-R^2	0.060	0.072	0.059	0.064	0.046	0.049	0.053	0.054
	Wald	4.130	4.951	10.298	10.890	3.404	3.568	3.063	2.979
	p-value	(0.042)	(0.026)	(0.001)	(0.001)	(0.065)	(0.059)	(0.080)	(0.084)
3-month OLS	OOS-R^2	0.052	0.057	0.049	0.053	0.041	0.044	0.047	0.049
	Wald	4.574	3.997	7.637	8.120	2.898	3.034	2.649	2.516
	p-value	(0.032)	(0.046)	(0.006)	(0.004)	(0.089)	(0.082)	(0.104)	(0.113)

Robustness

Panel A. OOS Performance of the Traditional Approach (New Measures)

	VRP_{BTZ}	VRP_{BH}	$\text{VRP}_{\text{VXO},N}$		$\text{VRP}_{\text{VXO},\overline{P}}$		$\text{VRP}_{\text{VXO},P^E}$	
	1993-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016
OOS-R^2	0.037	0.024	−0.015	−0.009	−0.010	−0.007	0.011	0.016
Wald	2.176	0.367	0.233	0.082	0.233	0.082	0.186	0.339
p-value	(0.140)	(0.545)	(0.629)	(0.775)	(0.629)	(0.775)	(0.666)	(0.561)

Panel B. OOS Performance of the Contemporaneous Beta Approach

	Statistics	VRP_{BTZ}	VRP_{BH}	$\text{VRP}_{\text{VXO},N}$		$\text{VRP}_{\text{VXO},\overline{P}}$		$\text{VRP}_{\text{VXO},P^E}$	
		1993-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016	1991-2016	1993-2016
1-month WLS	OOS-R^2	0.050	0.073	0.068	0.070	0.083	0.086	0.084	0.087
	Wald	2.245	3.877	5.993	5.723	9.145	8.781	6.906	6.666
	p-value	(0.134)	(0.049)	(0.014)	(0.017)	(0.002)	(0.003)	(0.009)	(0.010)
1-month OLS	OOS-R^2	0.041	0.054	0.055	0.057	0.071	0.073	0.072	0.073
	Wald	2.907	2.863	4.098	3.960	6.885	6.602	4.982	4.619
	p-value	(0.088)	(0.091)	(0.043)	(0.047)	(0.009)	(0.010)	(0.026)	(0.032)
3-month WLS	OOS-R^2	0.060	0.072	0.059	0.064	0.046	0.049	0.053	0.054
	Wald	4.130	4.951	10.298	10.890	3.404	3.568	3.063	2.979
	p-value	(0.042)	(0.026)	(0.001)	(0.001)	(0.065)	(0.059)	(0.080)	(0.084)
3-month OLS	OOS-R^2	0.052	0.057	0.049	0.053	0.041	0.044	0.047	0.049
	Wald	4.574	3.997	7.637	8.120	2.898	3.034	2.649	2.516
	p-value	(0.032)	(0.046)	(0.006)	(0.004)	(0.089)	(0.082)	(0.104)	(0.113)

Robustness

C. Conditional OOS Performance (Traditional Approach, 1993-2016)

	OOS-R^2				
	VRP_{BTZ}	VRP_{BH}	$\text{VRP}_{\text{VXO},N}$	$\text{VRP}_{\text{VXO},\overline{P}}$	$\text{VRP}_{\text{VXO},P^E}$
C-1. *1-Month Correlations*					
High	0.060	0.129	0.054	0.062	0.083
Medium	0.043	−0.057	0.045	0.011	−0.001
Low	0.002	−0.002	−0.124	−0.101	−0.049
High-Low	0.058	0.131	0.178	0.163	0.131
C-2. *3-Month Correlations*					
High	0.135	0.121	0.075	0.036	0.078
Medium	0.060	0.034	0.034	0.039	0.027
Low	−0.041	−0.040	−0.121	−0.089	−0.047
High-Low	0.176	0.161	0.196	0.125	0.125

Conclusion

- First, this article shows that the slope that determines the **contemporaneous relationship** between market and variance risk resembles the relationship between the risk premium of the market and market variance. As a result, when the beta of the contemporaneous regression of market returns on changes in its variance is used as the predictive slope for the VRP, one-month market returns can be predicted in a statistically and economically significant manner, even out of sample.

Conclusion

- Second, the predictive power strongly depends on the contemporaneous correlation between returns and variance innovations. When correlations are highly negative, predictions can be made more accurately. This result holds both in sample and out of sample. Since the predicted strength of the leverage effect can be estimated exante, we can anticipate this predictive power. The combination of the contemporaneous beta and the VRP outperforms the average returns consistently over time, regardless of the strength of the asymmetry in the market.