因子动量与动量因子

Factor Momentum and the Momentum Factor

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文献介绍

Factor Momentum and the Momentum Factor

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- **First Prize** to Valentin Haddad and Tyler Muir for Do Intermediaries Matter for Aggregate Asset Prices? in the December 2021 issue.
- **Distinguished Paper** to Sina Ehsani and Juhani Linnainmaa for Factor Momentum and the Momentum Factor in the June issue.
- **Distinguished Paper** to Andrés Schneider for Risk-Sharing and the Term Structure of Interest Rates in the August issue.

文献介绍

文献内容

- Section I measures autocorrelations in the returns of well-known factors.
- Section II shows that factors in the KNS model are autocorrelated when sentiment is sufficiently persistent and that factor momentum concentrates in high-eigenvalue factors.
- Section III shows that factor momentum explains other forms of momentum.
- Section IV shows that factor momentum is not incidental to individual stock momentum.

Section1
Autocorrelation in Off-the-Shelf Factors

- This table reports estimates from regressions in which the dependent variable is a factor's monthly return and the independent variable takes the value of one if the factor's average return over the prior year is positive and zero otherwise.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.06	0.72	0.45	4.22
U.S. Factors				
Size	−0.10	−0.62	0.58	2.51
Value	0.04	0.20	0.41	1.78
Profitability	0.04	0.22	0.34	1.67
Investment	0.12	0.97	0.24	1.55
Momentum	0.72	2.70	−0.09	−0.29
Accruals	0.15	1.18	0.10	0.65
Betting against beta	−0.22	−0.63	1.32	3.53
Cash flow to price	0.13	0.78	0.24	1.16
Earnings to price	0.10	0.62	0.30	1.46
Liquidity	0.16	0.74	0.36	1.29
Long-term reversals	−0.25	−1.66	0.76	3.85
Net share issues	0.17	1.32	0.09	0.49
Quality minus junk	0.09	0.65	0.43	2.51
Residual variance	−0.46	−1.64	1.06	2.74
Short-term reversals	0.49	1.43	0.01	0.04

Section1
Autocorrelation in Off-the-Shelf Factors

- This table reports estimates from regressions in which the dependent variable is a factor’s monthly return and the independent variable takes the value of one if the factor’s average return over the prior year is positive and zero otherwise.

	Global Factors			
Size	−0.06	−0.39	0.28	1.33
Value	0.04	0.15	0.47	1.77
Profitability	0.14	1.03	0.26	1.62
Investment	−0.06	−0.41	0.38	1.94
Momentum	0.67	1.77	0.02	0.04
Betting against beta	0.19	0.58	0.84	2.30
Quality minus junk	0.39	1.76	0.12	0.49

Section2
Factor Momentum and the Covariance Structure of Returns

KNS Model (Kozak, Nagel, and Santosh (2018))

- **Fully Rational Arbitrageurs**
- **Sentiment Investors** : distorted beliefs about asset returns' true distributions. Sentiment investors' demand has an additional sentiment-driven demand component.
- KNS study the extent to which, and under what conditions, sentiment distorts asset prices.*(Sentiment investor demand results in substantial mispricing only if arbitrageurs are exposed to factor risk when taking the other side of these trades.)*
- **KNS conclusion** is that the absence of near-arbitrage opportunities together with the substantial commonality in asset returns ensures that the stochastic discount factor (SDF) can be represented as a function of a few dominant factors.

Section2
Factor Momentum and the Covariance Structure of Returns

KNS Model

- We now derive the condition under which asset returns and the factors in this model are autocorrelated.
- KNS(2018,equation(C5))

$$R_{t+1} = D_{t+1} + a_1(\xi_{t+1} - \xi_t) - R_f(a_0 + a_1\xi_t)$$

- ξ_t is sentimentinvestor demand

This demand follows an AR(1) process, $\xi_{t+1} = \mu + \phi\xi_t + \nu_{t+1}$ with $\mathrm{var}(\nu_{t+1}) = \omega^2$

- Two properties:

$\sigma^2 \equiv \mathrm{var}(\xi_t) = \frac{\omega^2}{1-\phi^2}$ $\quad$ $\mathrm{cov}(\xi_t, \xi_{t+h}) = \phi^{|h|}\sigma^2$

Section2
Factor Momentum and the Covariance Structure of Returns

KNS Model

- The return autocovariance matrix is

$$\begin{aligned}\operatorname{cov}(R_t, R_{t+1}) &= a_1 a_1' \operatorname{cov}(\xi_t - R_f\,\xi_{t-1}, \xi_{t+1} - R_f\,\xi_t)\\ &= a_1 a_1' \sigma^2\Big[(1+R_f^2)\phi - R_f - R_f\,\phi^2\Big],\end{aligned}$$

$$a_1 a_1' = \frac{\gamma^2\theta^2\Gamma\delta\delta'\Gamma}{\left[R_f + \frac{1}{1+2b_2\omega^2}\left(\frac{\gamma\theta\delta' a_1}{2b_2} - \phi\right) - \frac{\gamma\theta\delta' a_1}{2b_2}\right]^2} = \Gamma\delta\delta'\Gamma c_0.$$

$$\Gamma = Q\Lambda Q$$

- Following KNS, we consider factor q_k,whichis the k^{th} PC.

Section2
Factor Momentum and the Covariance Structure of Returns

KNS Model

- The autocovariance of this factor is

$$
\begin{aligned}
\operatorname{cov}(PC_t^k, PC_{t+1}^k) &= \operatorname{cov}(q_k' R_t, q_k' R_{t+1}) = q_k' \operatorname{cov}(R_t, R_{t+1}) q_k \\
&= q_k' a_1 a_1' q_k \sigma^2 \Big[(1 + R_f^2)\phi - R_f - R_f \phi^2 \Big] \\
&= q_k' \Gamma \delta \delta' \Gamma q_k c_0 \sigma^2 \Big[(1 + R_f^2)\phi - R_f - R_f \phi^2 \Big].
\end{aligned}
$$

- KNS(2018,equation(16)) $\delta = Q\beta$

$$
q_k' \Gamma \delta \delta' \Gamma q_k = q_k' Q \Lambda \beta \beta' \Lambda Q' q_k = \iota_k' \Lambda \beta \beta' \Lambda \iota_k = \lambda_k^2 \beta_k^2
$$

$$
\boxed{\operatorname{cov}(PC_t^k, PC_{t+1}^k) = \lambda_k^2 \beta_k^2 c_0 \sigma^2 \Big[(1 + R_f^2)\phi - R_f - R_f \phi^2 \Big]}
$$

Section2
Factor Momentum and the Covariance Structure of Returns

KNS Model

- The autocovariance of this factor is

$$\mathrm{cov}(PC_t^k, PC_{t+1}^k) = \lambda_k^2 \beta_k^2 c_0 \sigma^2 \Big[(1 + R_f^2)\phi - R_f - R_f \phi^2 \Big]$$

- When sentiment is sufficiently persistent, factors are positively correlate.

$$\phi \in (\frac{1}{R_f}, 1]$$

Section2
Factor Momentum and the Covariance Structure of Returns

High-Variance PCs and Factor Momentum

- We use data on 54 factors from Kozak, Nagel, and Santosh (2020) to measure factor momentum's concentration in high-eigenvalue PCs.
- We exclude the seven predictors that relate to momentum or that combine momentum with other characteristics.
- We exclude all-but-microcaps from the analysis.
- Weight:

$$w_{i,t} = \frac{rc_{i,t} - \overline{rc}_{i,t}}{\sum_{i=1}^{n_t} |rc_{i,t} - \overline{rc}_{i,t}|}$$

- The month t return on a factor based on characteristic j is then

$$f_t = \sum_{i=1}^{n_{t-1}} w_{i,t-1} r_{i,t}$$

Section2
Factor Momentum and the Covariance Structure of Returns

High-Variance PCs and Factor Momentum

- (i) compute eigenvectors from the correlation matrix of daily factor returns from July 1963 up to the end of month t
- (ii) compute monthly returns for PC factors up to month t + 1 using these eigenvectors
- (iii) demean and lever up or down all PC factors so that their average returns up to month t are zero and their timeseries variances match that of the average original factor up to month t
- (iv) take long positions in the PC factors with positive average returns from month t − 11 to t and short positions in factors with negative average returns
- (v) compute the return on the resulting strategy in month t + 1.

Section2
Factor Momentum and the Covariance Structure of Returns

High-Variance PCs and Factor Momentum

Table III

Factor Momentum in High- and Low-Eigenvalue Factors

Panel A: Factor Momentum in Subsets of PC Factors Ordered by Eigenvalues

Set of PCs	Full Sample $\bar{r}$	Full Sample $t(\bar{r})$	First Half $\bar{r}$	First Half $t(\bar{r})$	Second Half $\bar{r}$	Second Half $t(\bar{r})$
1–10	0.19	7.07	0.27	8.49	0.11	2.60
11–20	0.13	5.23	0.20	6.13	0.05	1.50
21–30	0.10	5.02	0.18	7.93	0.02	0.63
31–40	0.10	4.05	0.16	5.07	0.04	1.08
41–47	0.07	2.51	0.09	2.71	0.06	1.17

Section2
Factor Momentum and the Covariance Structure of Returns

High-Variance PCs and Factor Momentum

Panel B: Explaining Factor Momentum in Low-Eigenvalue PC Factors

Explanatory Variable	Set of PCs 11–20	21–30	31–40	41–47
$\alpha_{\text{first half}}$	0.12	0.12	0.06	–0.01
	(3.50)	(4.27)	(1.86)	(–0.31)
$\alpha_{\text{second half}}$	0.02	0.00	0.00	0.03
	(0.56)	(–0.16)	(–0.10)	(0.78)
$\text{FMOM}_{\text{PC1–10}}$	0.34	0.28	0.34	0.43
	(9.78)	(9.50)	(9.50)	(10.64)
FF5	Y	Y	Y	Y
N	558	558	558	558
Adj. R^2	20.8%	21.7%	20.3%	22.4%

Section2
Factor Momentum and the Covariance Structure of Returns
High-Variance PCs and Factor Momentum

Panel C: Explaining Factor Momentum in High-Eigenvalue PC Factors

Explanatory Variable	Regression (1)	(2)	(3)	(4)	(5)
α first half	0.17	0.16	0.20	0.22	0.10
	(4.63)	(4.36)	(5.38)	(6.19)	(2.92)
α second half	0.08	0.09	0.09	0.08	0.06
	(2.30)	(2.59)	(2.57)	(2.16)	(1.94)
FMOM $_{\text{PC11–20}}$	0.43				0.26
	(9.78)				(6.26)
FMOM $_{\text{PC21–30}}$		0.51			0.29
		(9.50)			(5.83)
FMOM $_{\text{PC31–40}}$			0.41		0.20
			(9.50)		(4.65)
FMOM $_{\text{PC41–47}}$				0.39	0.21
				(10.64)	(5.66)
FF5	Y	Y	Y	Y	Y
N	558	558	558	558	558
Adj. R^2	24.6%	24.0%	24.0%	26.6%	40.5%

Section3
Factor Momentum and Individual Stock Momentum
Decomposition of Individual Stock Momentum

- Consider a factor model in which asset excess returns obey an F-factor structure

$$R_{i,t} = \sum_{f=1}^{F} \beta_i^f r_t^f + \varepsilon_{i,t}$$

- The expected payoff to the position in stock i is:

$$\begin{aligned}
\mathbf{E}[\pi_{i,t}^{\text{mom}}] &= \mathbf{E}\big[(R_{i,-t} - \bar{R}_{-t})(R_{i,t} - \bar{R}_t)\big] \\
&= \sum_{f=1}^{F} \left[\text{cov}(r_{-t}^f, r_t^f)(\beta_i^f - \bar{\beta}^f)^2\right] \\
&\quad + \sum_{f=1}^{F} \sum_{g \neq f}^{F} \left[\text{cov}(r_{-t}^f, r_t^g)(\beta_i^g - \bar{\beta}^g)(\beta_i^f - \bar{\beta}^f)\right] \\
&\quad + \text{cov}(\varepsilon_{i,-t}, \varepsilon_{i,t}) + (\eta_i - \bar{\eta})^2,
\end{aligned}$$

Section3
Factor Momentum and Individual Stock Momentum

Decomposition of Individual Stock Momentum

$$\mathrm{E}[\pi_t^{\mathrm{mom}}] = \boxed{\underbrace{\sum_{f=1}^{F} \left[\operatorname{cov}(r_{-t}^f, r_t^f)\, \sigma_{\beta^f}^2\right]}_{\text{factor autocovariances}}} + \underbrace{\sum_{f=1}^{F} \sum_{g \neq f}^{F} \left[\operatorname{cov}(r_{-t}^f, r_t^g)\operatorname{cov}(\beta^f, \beta^g)\right]}_{\text{factor cross-serial covariances}}$$

$$+ \underbrace{\frac{1}{N} \sum_{i=1}^{N} \left[\operatorname{cov}(\varepsilon_{i,-t}, \varepsilon_{i,t})\right]}_{\substack{\text{autocovariances} \\ \text{in residuals}}} + \underbrace{\sigma_\eta^2}_{\substack{\text{variation in} \\ \text{mean returns}}},$$

Section3
Factor Momentum and Individual Stock Momentum

Stock Momentum and Factor Momentum

Table IV
Pricing Momentum-Sorted Portfolios with Momentum and Factor Momentum

Panel A: Pricing Decile Portfolios Sorted on Past Returns

	Asset Pricing Model						
	FF5	FF5 + UMD		FF5 + FMOM $_{ind.}$		FF5 + FMOM $_{PC1-10}$	
Decile	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{b}_{umd}$	$\hat{\alpha}$	$\hat{b}_{fmom}$	$\hat{\alpha}$	$\hat{b}_{fmom}$
Winners	1.33	0.27	1.51	0.20	3.88	0.02	6.03
– Losers	(4.91)	(2.43)	(56.81)	(0.99)	(23.13)	(0.09)	(15.84)
Avg. $\|\hat{\alpha}\|$	0.26	0.12		0.11		0.10	
GRS F-value	4.24	3.10		2.33		1.30	
GRS p-value	0.00%	0.04%		1.06%		20.29%	

Section3
Factor Momentum and Individual Stock Momentum

Stock Momentum and Factor Momentum

Panel B: Pricing UMD with Momentum in Subsets of PC Factors

Subset of PCs	Alpha $\hat{\alpha}$	Alpha $t(\hat{\alpha})$	Factor Momentum $\hat{b}_{\text{fmom}}$	Factor Momentum $t(\hat{b}_{\text{fmom}})$	FF5	R^2
None	0.62	3.36			Y	10.7%
1–10	−0.09	−0.59	3.90	17.50	Y	42.5%
11–20	0.35	1.95	2.05	6.89	Y	17.6%
21–30	0.28	1.60	3.14	9.07	Y	22.1%
31–40	0.34	2.01	3.03	10.91	Y	26.4%
41–47	0.40	2.33	2.56	10.54	Y	25.5%

Section3
Factor Momentum and Individual Stock Momentum
Alternative Momentum Factors: Spanning Tests

Table V
Alternative Definitions of Momentum: Spanning Tests

Panel B: Regressions of Individual Stock Momentum Strategies on Factor Momentum

Individual Stock Momentum, UMD*	Factor Momentum: Individual Factors $\hat{\alpha}$	Individual Factors $\text{FMOM}_{\text{ind.}}$	PC Factors 1–10 $\hat{\alpha}$	PC Factors 1–10 $\text{FMOM}_{\text{PC1–10}}$	FF5
Standard momentum	0.00	2.43	−0.09	3.90	Y
	(−0.04)	(24.72)	(−0.60)	(17.52)	
Industry-adjusted momentum	0.14	1.23	0.10	1.90	Y
	(1.67)	(17.63)	(0.99)	(12.68)	
Industry momentum	0.02	2.32	−0.16	4.10	Y
	(0.12)	(18.83)	(−0.85)	(15.51)	
Intermediate momentum	0.15	1.41	0.17	2.20	Y
	(1.51)	(17.72)	(1.40)	(12.64)	
Sharpe ratio momentum	0.02	2.12	−0.05	3.63	Y
	(0.19)	(25.45)	(−0.39)	(19.74)	

Section3
Factor Momentum and Individual Stock Momentum
Alternative Momentum Factors: Spanning Tests

Table V

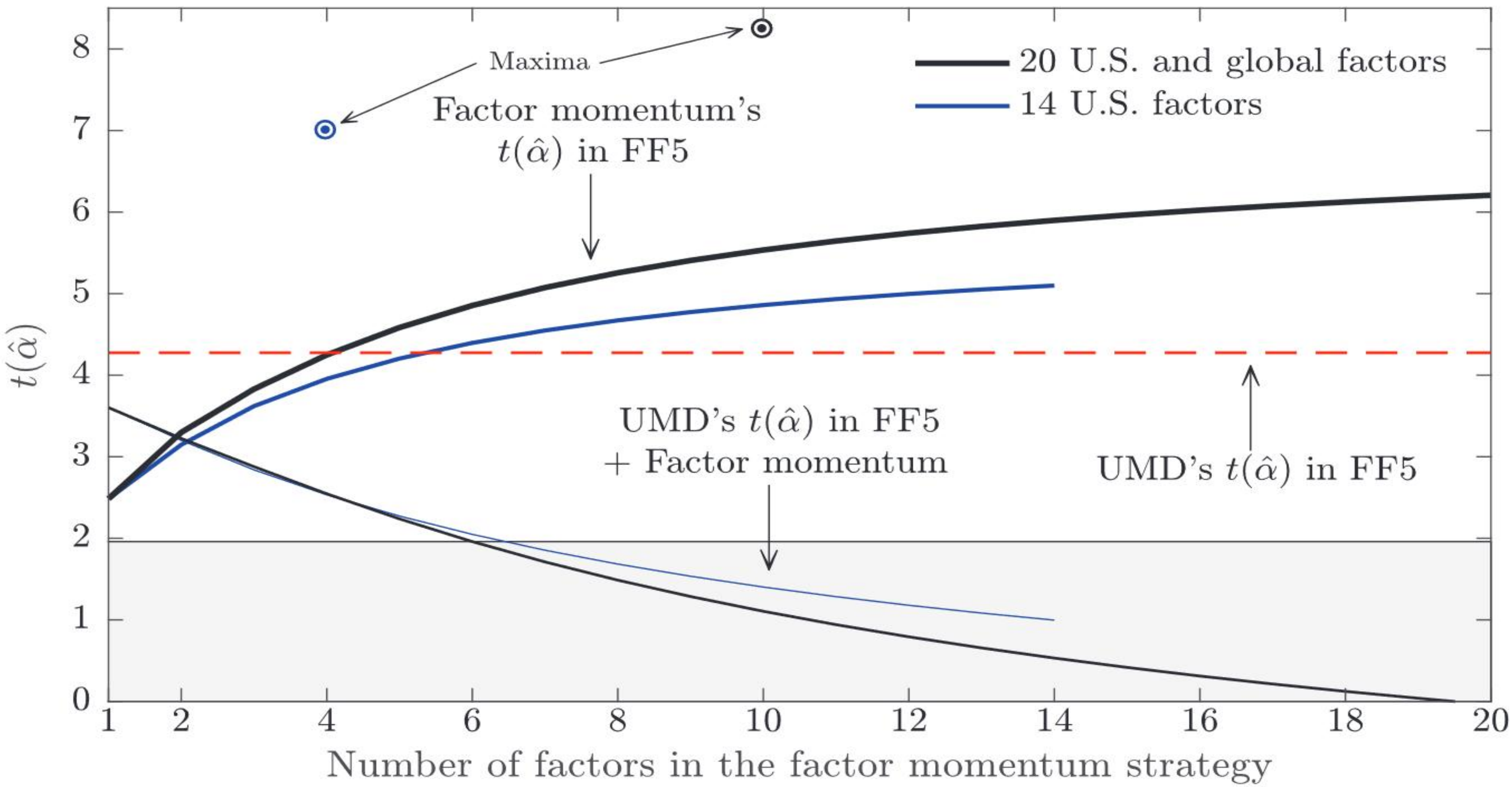
Alternative Definitions of Momentum: Spanning Tests

Panel C: Regressions of Factor Momentum on Individual Stock Momentum Strategies

Individual Stock Momentum, UMD*	Dependent Variable: Momentum in Individual Factors $\hat{\alpha}$	Momentum in Individual Factors UMD*	Momentum in PC Factors 1–10 $\hat{\alpha}$	Momentum in PC Factors 1–10 UMD*	FF5
Standard momentum	0.15	0.20	0.13	0.09	Y
	(4.44)	(24.72)	(5.53)	(17.52)	
Industry-adjusted momentum	0.16	0.26	0.13	0.12	Y
	(4.07)	(17.63)	(5.17)	(12.68)	
Industry momentum	0.19	0.15	0.14	0.07	Y
	(4.88)	(18.83)	(5.88)	(15.51)	
Intermediate momentum	0.16	0.23	0.13	0.10	Y
	(4.15)	(17.72)	(4.95)	(12.64)	
Sharpe ratio momentum	0.14	0.23	0.11	0.11	Y
	(4.20)	(25.45)	(5.17)	(19.74)	
All of above	0.14	-	0.12	-	Y
	(4.30)		(5.32)		

Section3
Factor Momentum and Individual Stock Momentum

Alternative Momentum Factors: Spanning Tests



Section3
Factor Momentum and Individual Stock Momentum

Do Firm-Specific Returns Display Momentum?

- To illustrate the issue arising from omitted factors, suppose that two systematic factors drive excess stock returns,

$$R_{i,t} = \beta_{i,1} F_{1,t} + \beta_{i,2} F_{2,t} + \varepsilon_{i,t}.$$

- In the **“Symmetric factors”** specification all 10 factors have the same variance and all factors’risk premiums are equally persistent.
- In the **“Uncorrelated market factor”** specification, the first factor explains five times as much of the cross section of returns as the other nine factors and its risk premium is uncorrelated.

Section3
Factor Momentum and Individual Stock Momentum
Do Firm-Specific Returns Display Momentum?

	Symmetric Factors		Uncorrelated Market Factor	
Number of Known Factors	Residual Momentum	Factor Momentum	Residual Momentum	Factor Momentum
1	$\boxed{5.65}$	1.55	$\boxed{5.62}$	−0.02
2	5.28	2.23	5.21	0.84
3	4.86	2.72	4.82	1.46
4	4.45	3.14	4.38	2.00
5	4.01	3.49	3.94	2.46
6	3.52	3.85	3.42	2.88
7	2.97	4.16	2.91	3.23
8	2.33	4.44	2.26	3.56
9	1.51	4.69	1.42	3.88
10	0.00	4.95	−0.01	4.16
Individual stock momentum	$\boxed{6.01}$		$\boxed{4.68}$	

Section3
Factor Momentum and Individual Stock Momentum
Do Firm-Specific Returns Display Momentum?

Sorting Variable	Average Return	Control for Factor Momentum: Individual Factors $\hat{\alpha}$	Individual Factors $\hat{b}_{\text{fmom}}$	PC Factors 1–10 $\hat{\alpha}$	PC Factors 1–10 $\hat{b}_{\text{fmom}}$
Raw returns	0.45	−0.19	1.96	−0.29	3.69
	(2.88)	(−1.45)	(19.16)	(−2.04)	(17.18)
CAPM residuals	0.58	0.08	1.53	−0.05	3.08
	(4.29)	(0.67)	(16.68)	(−0.38)	(16.58)
FF3 residuals	0.44	0.15	0.90	0.00	2.09
	(3.83)	(1.35)	(10.27)	(0.04)	(11.92)
FF5 residuals	0.37	0.17	0.63	0.00	1.72
	(3.39)	(1.52)	(7.32)	(−0.03)	(10.13)

Section4
Momentum vis-à-vis Other Factors

Unconditional and Conditional Correlations with the Momentum Factor

- The puzzling feature of individual stock momentum is its low correlations with other factors.
- The adjusted R2 from regressing UMD on the Fama-French five-factor model is just 9%.
- In particular, we report three correlations:

  A.the unconditional correlation,

  B.the correlation conditional on the factor’s return over the prior year being positive,

  C.and the correlation conditional on this return being negative.

Section4
Momentum vis-à-vis Other Factors
Unconditional and Conditional Correlations with the Momentum Factor

	Unconditional Correlation	Conditional Correlations		H_0: $\hat{\rho}^+ = \hat{\rho}^-$
Factor	$\hat{\rho}$	$\hat{\rho}^+$	$\hat{\rho}^-$	z-Value
Pooled	0.04	0.45	–0.51	18.37
U.S. Factors				
Size	–0.04	0.16	–0.39	7.20
Value	–0.20	0.17	–0.58	10.45
Profitability	0.11	0.46	–0.41	11.22
Investment	–0.03	0.19	–0.37	7.13
Accruals	0.13	0.30	–0.15	5.46
Betting against beta	0.18	0.41	–0.22	6.70
Cash flow to price	–0.13	0.23	–0.59	11.38
Earnings to price	–0.17	0.20	–0.61	11.50
Liquidity	–0.03	0.03	–0.14	2.15
Long-term reversals	–0.09	0.10	–0.43	7.02
Net share issues	0.11	0.36	–0.42	10.44
Quality minus junk	0.28	0.46	–0.41	11.00
Residual variance	0.21	0.67	–0.56	18.44
Short-term reversals	–0.30	–0.39	–0.19	–2.28

Section4
Momentum vis-à-vis Other Factors
Momentum in Momentum-Neutral Factors

- We construct momentum-neutral factors by taking the Kozak, Nagel, and Santosh (2020) factors and twisting the factor weights as little as possible to render them orthogonal with respect to past returns.
- The objective is to find new weights x_i such that

$$\min_{x_i} \sum_i (w_i - x_i)^2 \quad \text{s.t.} \sum_{i=1}^{N} x_i = 0 \;\text{ and }\; \sum_{i=1}^{N} x_i r_{i,t-12,t-2} = 0.$$

Section4
Momentum vis-à-vis Other Factors
Momentum in Momentum-Neutral Factors

Table IX
Factor Momentum in Momentum-Neutral Factors

	Dependent Variable			
	Momentum in Original Factors		Momentum in Momentum-Neutral Factors	
Independent Variable	(1)	(2)	(3)	(4)
Alpha	0.18 (6.51)	$\boxed{0.03}$ $\boxed{(1.45)}$	0.15 (7.53)	$\boxed{0.06}$ $\boxed{(3.91)}$
Momentum in original factors				0.52 (24.83)
Momentum in momentum-neutral factors		1.01 (24.83)		
FF5 factors	Y	Y	Y	Y
N	558	558	558	558
R^2	2.4%	53.9%	5.9%	55.5%

Section4
Momentum vis-à-vis Other Factors
Momentum in Momentum-Neutral Factors

The finding that the factor momentum in the momentum-neutral factors subsumes that in the original factors rejects the possibility that factor momentum is merely incidental momentum.

In fact, the results indicate that incidental momentum explains none of the factor momentum profits.