Case 3: $n = 3q + 2$

$n^2 = (3q+2)^2$ by substitution

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

Let $k = 3q^2 + 4q + 1$

Then k is an integer because it's the sum and product of integers

Therefore $n = 3k + 1$ by substitution

Hence there is an integer k such that $n^2 = 3k + 1$