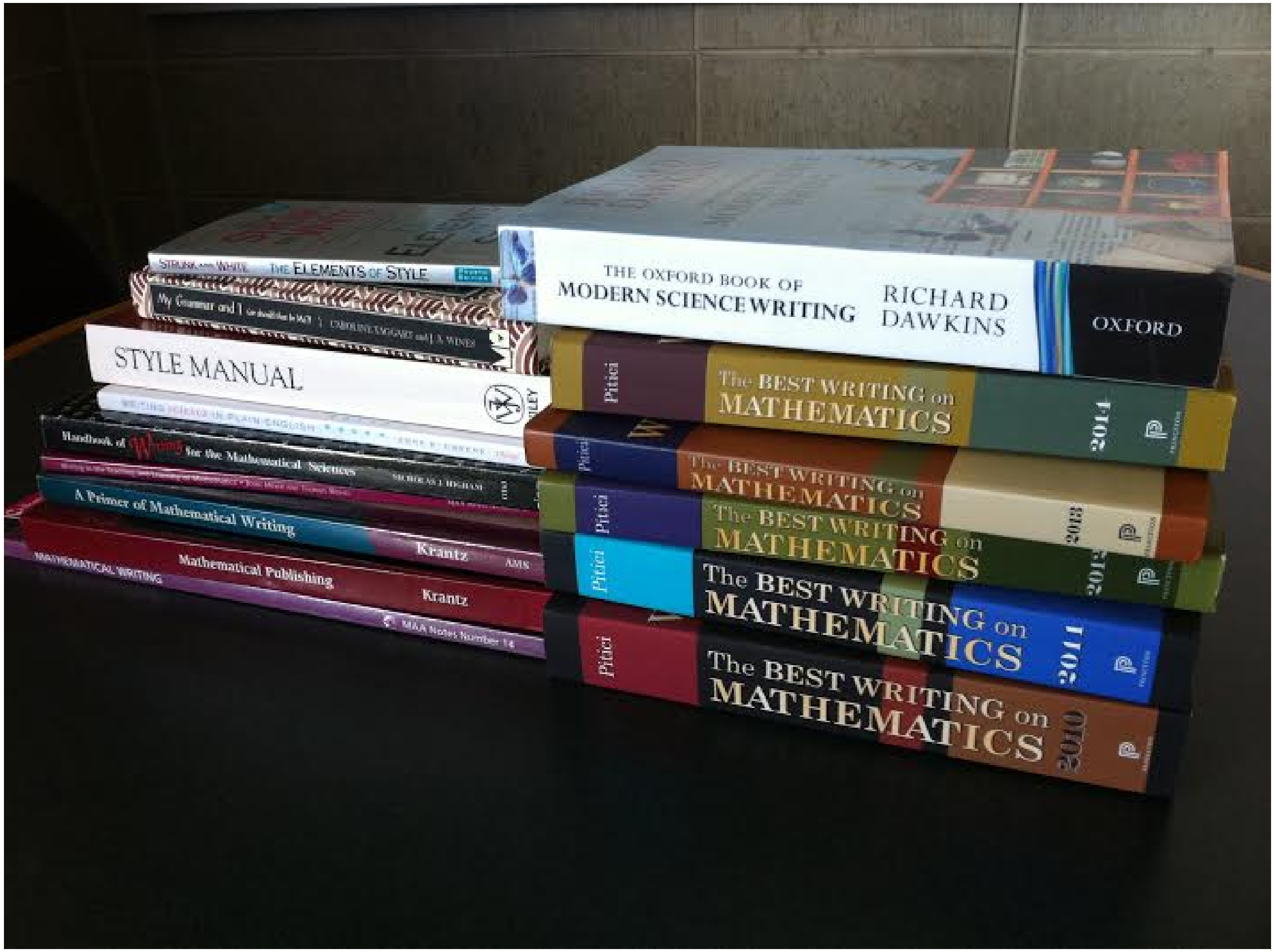
How to Write a Research Report

What is Good Writing?

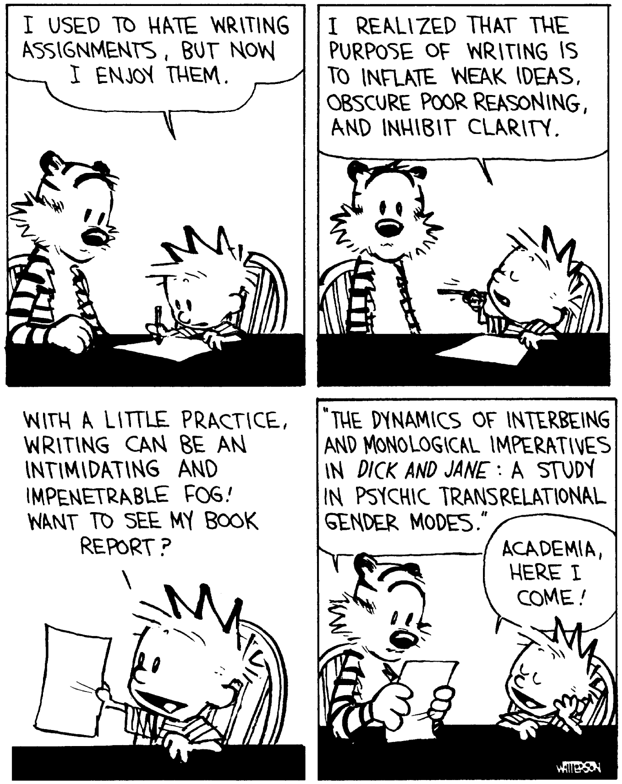
There seem to be as many opinions about writing as there are writers.



What is Good Writing?

There are many, many opinions about what constitutes good writing and how to produce it. Here are some generally agreed upon guidelines which apply to any kind of writing, mathematical or otherwise:

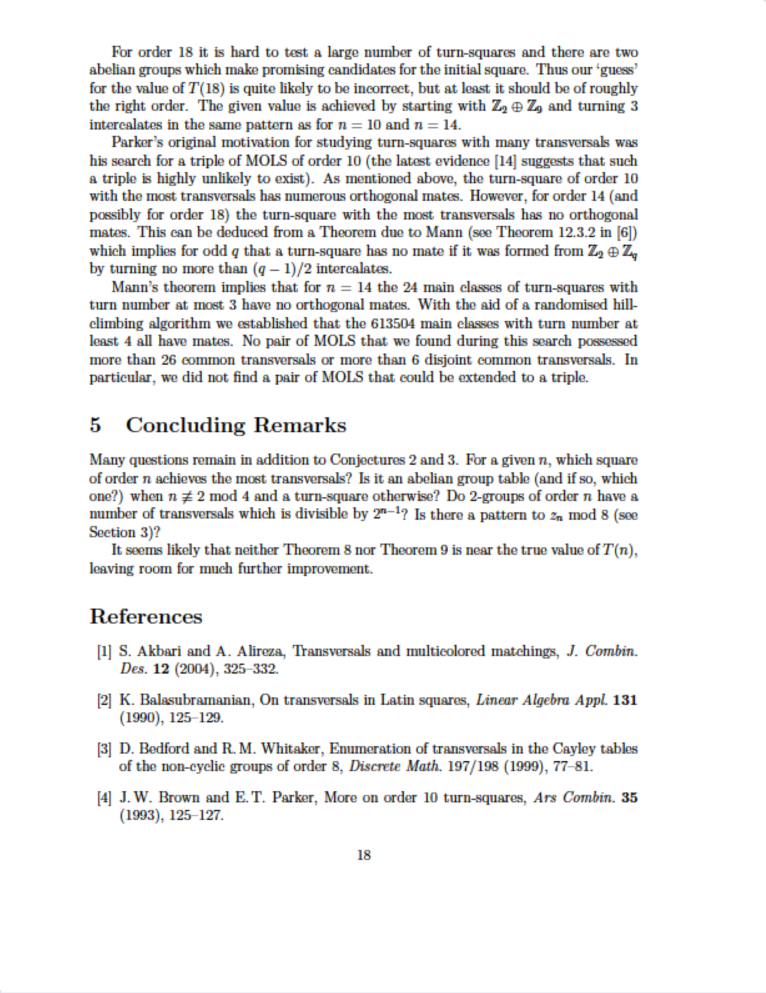
- Tell a story. Do so clearly and concisely.
- Identify your audience. Be nice to them.
- Have a plan.
- Use good English.
- Make sure it flows (read it aloud).
- Write many drafts.



The Report Format

Mathematics papers have clearly defined components. You may not need all of these.

- Title
- Author(s)
- Postal Address (University)
- Email Address (University)
- Date
- Abstract
- Introduction
- Body
- Conclusion
- Acknowledgements
- Bibliography/References



For order 18 it is hard to test a large number of turn-squares and there are two abelian groups which make promising candidates for the initial square. Thus our 'guess' for the value of $T(18)$ is quite likely to be incorrect, but at least it should be of roughly the right order. The given value is achieved by starting with $\mathbb{Z}_2 \oplus \mathbb{Z}_9$ and turning 3 intercalates in the same pattern as for $n = 10$ and $n = 14$.

Parker's original motivation for studying turn-squares with many transversals was his search for a triple of MOLS of order 10 (the latest evidence [14] suggests that such a triple is highly unlikely to exist). As mentioned above, the turn-square of order 10 with the most transversals has numerous orthogonal mates. However, for order 14 (and possibly for order 18) the turn-square with the most transversals has no orthogonal mates. This can be deduced from a Theorem due to Mann (see Theorem 12.3.2 in [6]) which implies for odd q that a turn-square has no mate if it was formed from $\mathbb{Z}_2 \oplus \mathbb{Z}_q$ by turning no more than $(q-1)/2$ intercalates.

Mann's theorem implies that for $n = 14$ the 24 main classes of turn-squares with turn number at most 3 have no orthogonal mates. With the aid of a randomised hill-climbing algorithm we established that the 613504 main classes with turn number at least 4 all have mates. No pair of MOLS that we found during this search possessed more than 26 common transversals or more than 6 disjoint common transversals. In particular, we did not find a pair of MOLS that could be extended to a triple.

5 Concluding Remarks

Many questions remain in addition to Conjectures 2 and 3. For a given n, which square of order n achieves the most transversals? Is it an abelian group table (and if so, which one?) when $n \not\equiv 2 \bmod 4$ and a turn-square otherwise? Do 2-groups of order n have a number of transversals which is divisible by 2^{n-1}? Is there a pattern to z_n mod 8 (see Section 3)?

It seems likely that neither Theorem 8 nor Theorem 9 is near the true value of $T(n)$, leaving room for much further improvement.

References

[1] S. Akbari and A. Alireza, Transversals and multicolored matchings, *J. Combin. Des.* **12** (2004), 325–332.

[2] K. Balasubramanian, On transversals in Latin squares, *Linear Algebra Appl.* **131** (1990), 125–129.

[3] D. Bedford and R. M. Whitaker, Enumeration of transversals in the Cayley tables of the non-cyclic groups of order 8, *Discrete Math.* 197/198 (1999), 77–81.

[4] J. W. Brown and E. T. Parker, More on order 10 turn-squares, *Ars Combin.* **35** (1993), 125–127.

The Report Format

Let's breakdown some of the more difficult components of the report.

- **Abstract**

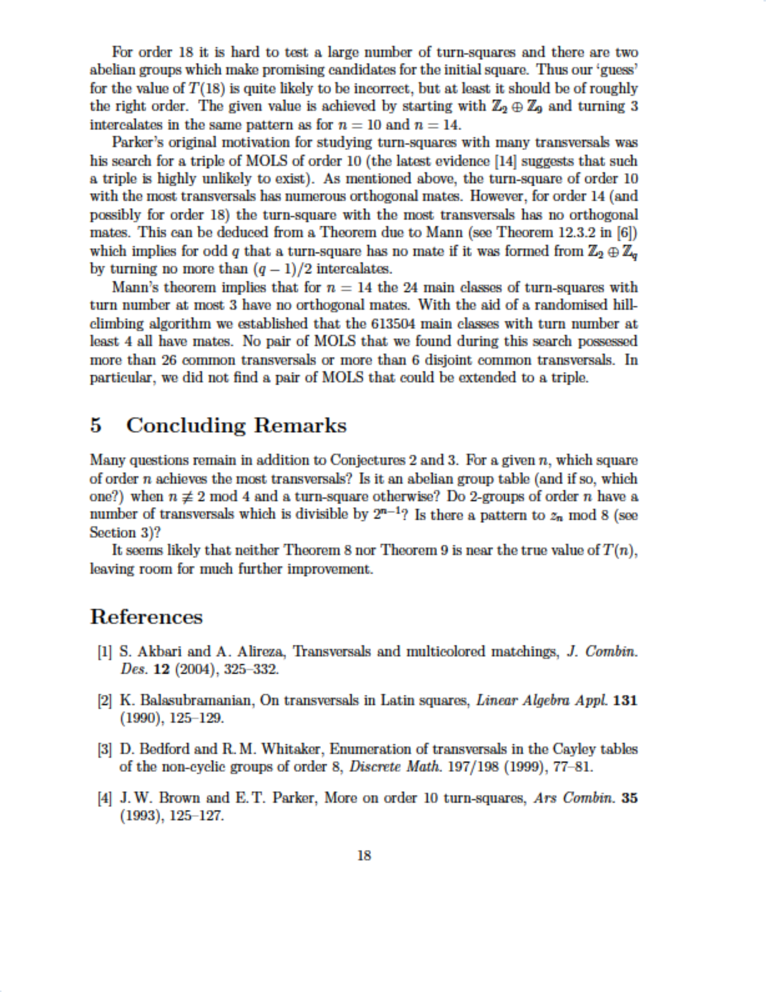
  Two to 10 lines summarising the paper. Self-contained. Uses a minimum of notation and jargon. Simple short declarative sentences.

- **Introduction**

  Defines the problem and explains what the report attempts to do. Outlines the plan of attack. Summarises previous results.

- **Conclusion**

  Sums up without too much repetition. Offers another viewpoint. Discusses/defends limitations. Suggests further research problems. Outlines open problems. Mentions work in progress.



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Mathematical Writing

The first step is the learn how to write like a mathematician.

Literature:

Mr. Utterson the lawyer was a man of a rugged countenance, that was never lighted by a smile; cold, scanty and embarrassed in discourse; backward in sentiment; lean, long, dusty, dreary, and yet somehow lovable.

Literature written like mathematics:

Mr. Utterson was a reserved but likeable lawyer.

Mathematics written like literature:

The symbol x is one amongst that infinite collection of numbers which can be written without a fractional component, and whose cardinality is constrained to always be in excess of naught.

Mathematics:

Let x be a positive integer.

Mathematical Writing

Writing mathematics is like writing English prose.

Bad:

Pythagoras's Theorem:

$$a^2 + b^2 = c^2$$

Better:

Pythagoras's Theorem states that

$$a^2 + b^2 = c^2.$$

Best:

Pythagoras's Theorem states that

$$a^2 + b^2 = c^2,$$

where c is the length of the hypotenuse in a right-angled triangle, and a and b the lengths of the triangle's other two sides.

Mathematical Writing

Here are some guidelines (there are many more):

- Write in short sentences.
- Do not use contractions.
- Use “we” rather than “I” (and never “you”).
- Use adjectives sparingly.
- State only facts (no opinions) and reference where appropriate.
- Define your terms, but define only what you need.
- Use examples to illustrate general ideas.
- Be consistent with your notation.
- Use words rather than symbols if this simplifies exposition.
- Display an equation when it needs to be numbered, it’s hard to read or important.
- Mathematical expressions are part of a sentence and need to be punctuated.
- Avoid starting sentences with a mathematical expression or symbol.

Remember, even though you are writing a technical paper about mathematics, you are still telling a story. Endeavour to make that story interesting.

Mathematical Writing

Correct but tedious:

$$\forall x \exists y, x \geq 0 \Rightarrow y^2 = x.$$

Correct and clear:

Every nonnegative real number has a square root.

Correct but repetitive:

A *forest F* is a graph containing no cycles. A *tree T* is a connected graph containing no cycles. Each component of a forest *F* is a tree.

Correct and concise:

A *tree* T is a connected graph containing no cycles. A forest is a graph whose components are trees.

Advice On Writing

> "There is nothing to writing. All you do is sit down at a typewriter and bleed."
> — Ernest Hemingway

- Persevere. Writing is hard work.
- Writing improves with experience (of both writing and reading).
- Listen to feedback and criticism. Especially that of your supervisor.
- Don't wait for inspiration to strike. Write little, write often.
- Get perspective. If you read your writing after a week, does it still make sense?
- Check your work. Is it accurate? Are your results correct?
- Use a spell checker!
- Expect to receive conflicting advice... Sometimes from the same person.

> "There's no such thing as writer's block. That was invented by people in California who couldn't write." — Terry Pratchett

Further Reading

Higham, Nicholas J., *Handbook of Writing for the Mathematical Sciences*, Philadelphia, Siam, 1998.

Krantz, Steven G., *A Primer of Mathematical Writing,* American Mathematical Society, 1997.

Strunk, William and White, E. B., *The Elements of Style* (4th ed.), Allyn & Bacon, 1999.