Define the linear transformation T as follows,

$$T(a,b,c,d,e,f,g,h,i) = [i,a,b+i,c,d,e,f,g,h] \tag{1}$$

We can see that T is a transition function that maps the current day's state to the next day's state. a is the number of fish with internal timer 8, b is the number of fish with internal timer 7 and so on...
The matrix representation of T is,

$$A_T = \begin{bmatrix} 0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \end{bmatrix} \tag{2}$$

Let $\vec{v_1}$ be the state on day 1. We wish to compute $\vec{v_{256}}$, which can be expressed as

$$\vec{v_{256}} = (A_T)^{256}\vec{v_1} \tag{3}$$

Computing $(A_T)^{256}$ is incredibly expensive. Let us diagonalize A_T by computing it's eigenvectors

$$A_T\vec{v} = \lambda v \tag{4}$$
$$\det(A_T - \lambda I) = 0 \tag{5}$$

$$\det \begin{bmatrix} -\lambda&0&0&0&0&0&0&0&1 \\ 1&-\lambda&0&0&0&0&0&0&0 \\ 0&1&-\lambda&0&0&0&0&0&1 \\ 0&0&1&-\lambda&0&0&0&0&0 \\ 0&0&0&1&-\lambda&0&0&0&0 \\ 0&0&0&0&1&-\lambda&0&0&0 \\ 0&0&0&0&0&1&-\lambda&0&0 \\ 0&0&0&0&0&0&1&-\lambda&0 \\ 0&0&0&0&0&0&0&1&-\lambda \end{bmatrix} = -\lambda^9 + \lambda^2 + 1 = 0 \tag{6}$$

You get the point, I'm not solving a nonic-polynomial...