1 Part 1

We would like to minimize the total fuel usage of all the crab submarines. The fuel usage $F_i(y)$ for the submarine i is calculated as,

$$F_i(y) = |y - x_i| \tag{1}$$

where x_i is the original position of submarine i.
The total fuel usage is then,

$$F(y) = \sum_i F_i(y) = \sum_i |y - x_i| \tag{2}$$

We find the value y such that $F(y)$ is at a minimum, this can be computed with

$$F'(y) = \sum_i sign(|y - x_i|) = 0 \tag{3}$$

We see that $F'(y)$ is as close as possible to 0 when approximately half the x_i's are greater than y and half the x_i's are less than y. Thus $y = median(x_i)$ where $1 \leq i \leq N$

2 Part 2

We simply redefine $F_i(y)$ to be

$$F_i(y) = \frac{|y - x_i|(|y - x_i| + 1)}{2} \tag{4}$$

The total fuel usage is then,

$$\begin{align}
F(y) &= \sum_i F_i(y) \tag{5}\\
&= \sum_i \frac{|y - x_i|(|y - x_i| + 1)}{2} \tag{6}\\
&= \frac{1}{2}\sum_i (y - x_i)^2 + |y - x_i| \tag{7}
\end{align}$$

We find the value y such that $F(y)$ is at a minimum, this can be computed with

$$\begin{align}
F'(y) &= \frac{1}{2}\sum_i 2(y - x_i) + sign(y - x_i) \tag{8}\\
&= \sum_i (y - x_i) + \frac{sign(y - x_i)}{2} \tag{9}\\
&= Ny - \sum_i x_i + \frac{1}{2}\sum_i sign(y - x_i) \tag{10}
\end{align}$$