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A refined weighted method for forecasting based on type 2 fuzzy time series

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ABSTRACT
In this paper, we proposed a method for type 2 fuzzy time series forecasting which is an extension of type 1 fuzzy time series model to enhance the accuracy in forecasts. The proposed method uses frequency distribution approach to define the appropriate length of intervals. High and low observations are used to define type 2 fuzzy time series and different fuzzy logical relationship groups (FLRGs) have been obtained for both high and low observations. Further, weight function are defined with the help of FLRGs to compute forecasted outputs by a simple arithmetic mean rather than complicated union and intersection operator of type 2 fuzzy sets. The proposed method has been applied for forecasting university enrollments and crop (wheat) production. It is shown that the proposed method has higher accuracy in terms of mean absolute percent error and root-mean-square error (RMSE) as compared to the other fuzzy time series methods.

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Frequency-based partitioning; triangular fuzzy sets; weight function; type 2 fuzzy time series; fuzzy logical relationship groups (FLRGs) for type 2

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1. Introduction

Fuzzy time series is an application of fuzzy set theory in the field of time series analysis. The fuzzy set was first introduced by Zadeh [1] and since then it has been used in many application areas. Fuzzy time series were first introduced by Song and Chissom [2,3] to forecast enrollments of the University of Alabama. In recent years, many researchers have proposed different forecasting method based on fuzzy time series to deal with forecasting problems with imprecision. According to the literature, decomposition of the universe of discourse plays an important role in fuzzy time series forecasting. Many authors like Huarng [4] and Chen [5,6] suggested the importance of interval length on forecasting and proposed new technique for finding the length of intervals based on the mean and distribution method.

To improve the accuracy in the forecasted value several researchers like Cheng et al. [7] and Lee et al. [8] proposed a trend-weighted fuzzy time series model. Further, many researchers have developed several forecasting methods using various approaches. Chen and Hwang [9], Singh [10], Yolcu et al. [11] and Chou [12] presented different methods of forecasting technique based on fuzzy time series. Singh [13,14] presented a computational method of forecasting based on high-order fuzzy time series and time variant method to improve the forecasting in fuzzy time series. Ismail and Efendi [15] developed a modified weighted model for enrollment forecasting. Own [16] presented an application of enhanced knowledge model to fuzzy time series. Further Uslu et al. [17] gave a fuzzy time series approach based on weights determined by number of recurrence of fuzzy relations. Pathak and Singh [18] and Gangwar and Kumar [19] introduced a forecasting method based on bandwidth interval and probabilistic and intuitionistic fuzzy sets-based method, respectively, to enhanced the accuracy in forecasted outputs. But all the above models are based on type 1 fuzzy time series. A type 2 fuzzy time series model is an extension of type 1 fuzzy time series model. In type 1 the degree of membership grade is regarded as a crisp value but in type 2 the degree of membership is regarded to be a fuzzy set.

The application of type 2 fuzzy logic in decision-making problems is given in [20,21]. However, Huarng and Yu [22] proposed a type 2 fuzzy time series model for stock index forecasting by defining extra variable as high and low and these extra variables play a very vital role in type 2 fuzzy time series model. Later Lertworaprachaya et al. [23] proposed a novel improved model for fuzzy time series forecasting using high-order and type 2 fuzzy time series. Bajestani and Zare [24] used a model as type 2 fuzzy set and fourth order fuzzy time series for TAIEX forecasting. Also Koca et al. [25] introduced a type-2 fuzzy sliding mode control of a four-bar mechanism and Akpolat and Altinors [26] presented a model of type-2 fuzzy for speed control of an electric drive.

In this paper, a new method for forecasting has been developed for type 2 fuzzy time series. It provides simple computational algorithms to obtain forecasted outputs. In the process, we define a triangular fuzzy sets and obtain membership grades for each observations in type 2 fuzzy historical data rather than randomly chosen a membership grades for each data. Huarng and Yu [22] considered three variables to obtain forecasted values as closing, high and low in their type 2 fuzzy model and defined

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fuzzy logical relationship groups (FLRGs) for type 1 fuzzy models. In this study we choose only two variables as high and low, then obtained FLRGs for type 2 fuzzy models as different high and low observations. Proposed method minimizes the computation to compute a forecasted outputs by apply weighted function rather than Huarng and Yu model which apply complicated union and intersection operator for type 2 fuzzy sets to compute forecasted outputs. The proposed algorithm has been implemented on the enrollments of university of Alabama and wheat crop production to examine its suitability in forecasting over the other available models. The proposed model shows its superiority as compared to other existing models in terms of RMSE and mean absolute percent error (MAPE).

The objective of this study is to propose a weighted type 2 fuzzy time series model to improve the forecasting results by modifying the Huarng and Yu [22] model. The paper is organized as follows: Sections 2 and 3 review the basic concept of fuzzy time series and type 2 fuzzy time series. In Section 4, the weighted functions are introduced. Section 5 proposes an algorithm for weighted type 2 fuzzy time series forecasting. In Sections 6 and 7 verification and comparison have been implemented to forecast the university enrollments data and crop (wheat) production with comparison of RMSE and MAPE to other methods in the literature, and finally Section 8 presents the conclusion.

2. Fuzzy time series and related definitions

In this section, we briefly review some basic concepts of fuzzy time series as introduced by Song and Chissom [2,3] and its forecasting framework.

2.1. Fuzzy set

Let U be the universe of discourse where $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set $\tilde{A}_i$ of U can be defined as

$$\tilde{A}_i = \frac{\mu_{\tilde{A}_i}(u_1)}{u_1} + \frac{\mu_{\tilde{A}_i}(u_2)}{u_2} + \cdots + \frac{\mu_{\tilde{A}_i}(u_n)}{u_n}, \tag{1}$$

where $\mu_{\tilde{A}_i}$ is the membership function of fuzzy set $\tilde{A}_i$ and $\mu_{\tilde{A}_i}:U \to [0, 1]$.

2.2. Fuzzy time series

Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots)$ are defined. If $F(t)$ is a collection of $f_i(t)$ $(i = 1, 2, \ldots)$, then $F(t)$ is called a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$.

2.3. Fuzzy time series relationships

Suppose $F(t)$ is said to be caused by $F(t-1)$ only and is denoted by $F(t-1) \to F(t)$, then the relationship can be expressed as $F(t) = F(t-1) * R(t-1, t)$, which is the fuzzy relationship between $F(t)$ and $F(t-1)$ where $*$ represent an operator. Suppose $F(t-1) = \tilde{A}_i$ and $F(t) = \tilde{A}_j$, a fuzzy logical relationship (FLR) can be defined as $\tilde{A}_i \to \tilde{A}_j$ where, $\tilde{A}_i$ is called the current state and $\tilde{A}_j$ is called the next state of FLR, respectively. Further, these FLRs can be grouped to establish different fuzzy relationships.

3. Type 2 fuzzy sets

3.1. Definition

A type 2 fuzzy set can be defined as an extension of type 1 fuzzy set. Let U be the universe of discourse. Let $\tilde{A}(U)$ be the set of fuzzy set in U. Then, a type 2 fuzzy set $\tilde{A}$ in X is fuzzy set whose membership grades are themselves fuzzy and

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \,|\, \mu_{\tilde{A}}(x) \in \tilde{A}(U): x \in X\} \tag{2}$$

where $\mu_{\tilde{A}_i}:X \to \tilde{A}(U)$. In type 1 fuzzy set, the degree of membership is characterized by a crisp value, where as in type 2 fuzzy set the degree of membership is regarded as a fuzzy.

In Figure 1, type 1 fuzzy set, we have crisp degree of membership value of 1.0 for $x = 2$ and a crisp degree of membership value of 0.5 for $x = 1$. Based on these type 2 fuzzy set, there can be fuzzy set for any degree of membership. For example, there is a triangular fuzzy set (0.4, 0.5, 0.6) as a degree of membership for $x = 1$ in Figure 2. In other words, for the same x, there can be multiple degree of membership.

4. Weighted fuzzy time series model

In this section, we considered Own [16] model for weighted fuzzy time series and its algorithm. Own presented the weighted fuzzy time series models for type 1 fuzzy time series forecasting and we extended this weighted fuzzy time series model for type 2 fuzzy time series forecasting by defining high and low observations from type 1 historical time series data. This model is based on the weighted measure of historical information and

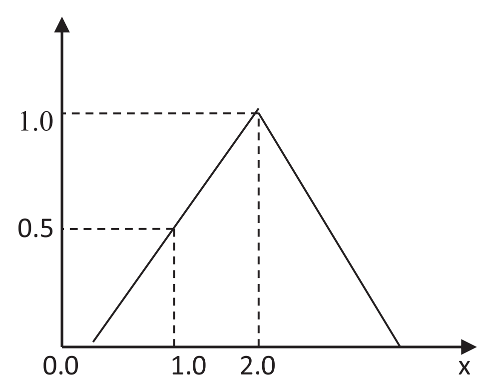

Figure 1. Type 1 fuzzy set.

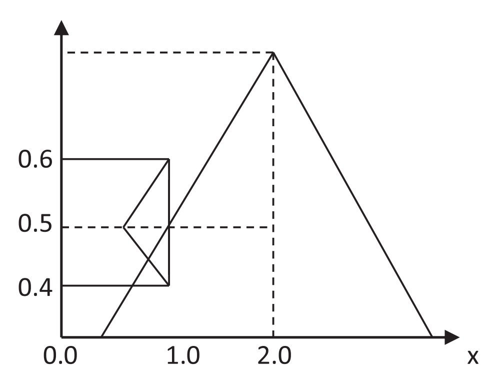

Figure 2. Type 2 fuzzy set.

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the frequencies of the fuzzy set to adjust their ratios. Here, this study considers the support of weighted measure and knowledge which is introduced by Own in following definitions.

4.1. Measure the frequency of fuzzy sets

Measure the frequency of fuzzy sets as shown in fuzzy relationships. For simplicity, the frequency for fuzzy set $\tilde{A}_i$ is denoted as f_i.

Example: Suppose that the fuzzy relationships calculated from any data-set are obtained as follows:

$$\tilde{A}_r \rightarrow \tilde{A}_{r_1}$$

$$\tilde{A}_s \rightarrow \tilde{A}_{r_3}$$

$$\tilde{A}_r \rightarrow \tilde{A}_{r_1}$$

$$\tilde{A}_r \rightarrow \tilde{A}_{r_1}$$

$$\tilde{A}_r \rightarrow \tilde{A}_{r_2}$$

Hence, fuzzy set $\tilde{A}_{r_1}$ shown in relationship $\tilde{A}_r \rightarrow \tilde{A}_{r_1}$ occurs three times denoted as $f_{r_1} = 3$. Fuzzy set $\tilde{A}_{r_2}$ shown in relationship $\tilde{A}_r \rightarrow \tilde{A}_{r_2}$ occurs once, denoted as $f_{r_2} = 1$, Similarly, we find the frequency for $f_{r_3} = 1$.

4.2. Definition

If the grouped fuzzy relationship of $\tilde{A}_i$ is $\tilde{A}_i \rightarrow \tilde{A}_{i_1}, \tilde{A}_{i_2}, \dots, \tilde{A}_{i_k}, \dots, \tilde{A}_{i_l}$, where $i_1 < i_2 < \cdots < i_k < \cdots < i_l$. Accordingly, all the fuzzy sets are partitioned into two parts, high and low parts. The high part includes the fuzzy sets in high ranking. On the contrary, the low part includes the fuzzy sets in low ranking. If $i_{k-1} < i < i_k$, then fuzzy sets in the low part are $\{\tilde{A}_{i_1}, \tilde{A}_{i_2}, \dots, \tilde{A}_{i_{k-1}}\}$ and fuzzy sets in the high part are $\{\tilde{A}_{i_k}, \tilde{A}_{i_{k+1}}, \dots, \tilde{A}_{i_l}\}$. Note that if the low/ high part is empty set, then the low/ high part need to include their current state of the relationships, that is $\tilde{A}_i$. If $i = i_k$, then fuzzy sets in the low part are $\{\tilde{A}_{i_1}, \tilde{A}_{i_2}, \dots, \tilde{A}_{i_k}\}$ and fuzzy sets in the high part are $\{\tilde{A}_{i_k}, \tilde{A}_{i_{k+1}}, \dots, \tilde{A}_{i_l}\}$.

Hence, the selection strategy of fuzzy sets is: If the trend of time series leads to an increase, the fuzzy sets in the high part are all selected; else if the trend of the time series leads to a decrease, the fuzzy set in the low part are all selected; else if the trend of the time series leads to no change, the existing state of affairs would be preferred, and the origin fuzzy set $\tilde{A}_i$ is selected.

4.3. Establish the weighted function

Suppose that the grouped fuzzy relationships of $\tilde{A}_i$ are $\tilde{A}_i \rightarrow \tilde{A}_{i_1}, \tilde{A}_{i_2}, \dots, \tilde{A}_{i_j}, \dots, \tilde{A}_{i_k}, \dots, \tilde{A}_{i_m}$ where, $j < k < l < m$.

The weighting of the fuzzy set $\tilde{A}_{i_j}$ in the low part is computed as the probability of frequency defined by

$$l_{i_j} = \frac{f_{i_j}}{f_{i_1} + f_{i_2} + \cdots + f_{i_Q}} \tag{3}$$

where $Q = k - 1$, if $i_{k-1} < i < i_k$
and $Q = k$, if $i = i_k$.

f_{i_j} is denoted as the frequency of the fuzzy set $\tilde{A}_{i_j}$.

Similarly, the weighting of the fuzzy set $\tilde{A}_{i_l}$ in the high part is computed as the probability of frequency defined by

$$q_{i_j} = \frac{f_{i_l}}{f_{i_k} + \cdots + f_{i_m}} \tag{4}$$

Hence for each grouped relationships, the weighted function is established as follows

$$w_i|_\alpha = \frac{\alpha(\alpha-1)}{2} \sum_{j=i_1,\dots,i_Q} l_j m_j + \frac{2(1+\alpha)(1-\alpha)}{2} m_i + \frac{\alpha(\alpha+1)}{2} \sum_{j=i_k,\dots,i_m} q_j m_j, \tag{5}$$

where $Q = k - 1$, if $i_{k-1} < i < i_k$
and $Q = k$, if $i = i_k$.

Note that $w_i|_\alpha$ is derived corresponding to the original fuzzy set $\tilde{A}_i$ of each grouped fuzzy relationships, $i = 1, 2, \dots, n$. Here parameter α is the difference between time t - 1 and t. It leads to an increase, a decrease and no change, denoted as $\alpha = 1, -1$ and 0, respectively. Accordingly, the output weighted function is derived as $w_i|_{\alpha=-1,0,1}$ and m_j is the mid-point of the corresponding interval v_j.

Example: Consider the same example as discuss in Section 4.1, in which five FLRs are included. The grouped fuzzy relationships are

$$\tilde{A}_r \rightarrow \tilde{A}_{r_1}, \tilde{A}_{r_2}$$

$$\tilde{A}_s \rightarrow \tilde{A}_{r_3}$$

Here, the probability of frequency is $f_{r_1} = 3$, $f_{r_2} = 1$, $f_{r_3} = 1$.

Hence, the weighted function is defined by Equation (5) corresponding to fuzzy set $\tilde{A}_r$ and $\tilde{A}_s$ are

$$w_r|_\alpha = \frac{\alpha(\alpha-1)}{2} m_r + \frac{2(1+\alpha)(1-\alpha)}{2} m_r + \frac{\alpha(\alpha+1)}{2} \left(\frac{3}{4} m_{r_1} + \frac{1}{4} m_{r_2}\right),$$

$$w_s|_\alpha = \frac{\alpha(\alpha-1)}{2} m_{r_3} + \frac{2(1+\alpha)(1-\alpha)}{2} m_s + \frac{\alpha(\alpha+1)}{2} m_s.$$

where the ranking of fuzzy sets are $r < r_1 < r_2$ and $r_3 < s$. And the parameter α is 1, 0 or −1.

5. The proposed algorithm of refined weighted type 2 fuzzy time series method

In this section, we present a new method of forecasting using type 2 fuzzy time series. Here, we define variables as high and low to pick type 2 observations, rather than all three variables as high, low and closing observations because the closing value is the approximate mean value of high and low observations in type 2 fuzzy sets. Further, we apply a weighted function for high and low observations of each FLRGs. We proposed a methodology of type 2 fuzzy time series forecasting with a weighted computational method as refinement of Huarng and Yu [22] type 2 fuzzy time series

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model. The proposed algorithm is presented stepwise as follows:

Step 1. Define the universe of discourse U.

$U = [D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are small number assigned for easy partitioning of U. As the length of interval is determined, U can be partitioned in to several equal length of intervals $u_i (i = 1, 2, \ldots, n)$.

Step2. Apply the frequency statistical distribution method of historical time series data for each interval $u_i (i = 1, 2, \ldots, n)$. The interval has the maximum number of historical distribution data, then it is re-divided into four equal lengths of subintervals. Now again we find the interval having the second maximum number of historical distribution data, then it is re-divided into three equal lengths of subintervals. Find the interval having the third maximum number of historical distribution data and it re-divide into two equal lengths of subintervals. Finally, find the interval with the fourth maximum number of historical distribution data, and then the length of this interval remains unchanged. If there is an interval having no historical distribution data then discard that interval. The re-divided intervals are in the form of $v_1, v_2, \ldots, v_n$.

Step 3. Let n triangular fuzzy sets $(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$ are constructed corresponding to length of re-divided intervals $v_1, v_2, \ldots, v_n$ and define membership grades of each historical time series data corresponding to n triangular fuzzy sets.

Step 4. *Fuzzify the historical time series data*

For fuzzification of historical time series data, we choose maximum degree of membership grades of x_l (for fixed l) to $\tilde{A}_i$. i.e. we fuzzify the data to $\tilde{A}_i (i = 1, 2, \ldots, n)$ for maximum membership grades in triangular fuzzy sets $(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$ of x_l (for fixed l).

Step 5. *Choose type 2 observations*

Choose variable high and low as type 2 observations. Here high is the highest enrollment/ production and low is the lowest enrollment/ production of that year from the actual data of time series as type 1 observations. To convert a historical time series data as type 2 we choose high and low observations for each x_l (fixed l) from type 1 historical data, as data in type 1 are actual observations. Then, we fuzzify the data of high and low observations as type 2 accordingly defined in step 4.

Step 6. After fuzzification of a type 2 historical time series data, we established different FLR for both high and low observations as:

If $\tilde{A}_i$ is the fuzzified value of year n and $\tilde{A}_k$ is the fuzzified value of year $n + 1$, then fuzzy logical relationship FLR is defined as $\tilde{A}_j \rightarrow \tilde{A}_k$, where, $\tilde{A}_j$ is the current state and $\tilde{A}_k$ is the next state of time series historical data. After FLR, we established the different FLRGs from both high and low FLR observations for type 2 historical time series data.

Step 7. *Compute the weighted function*

Apply the weighted function as defined in Section 4, on each FLRGs for both high and low observations. The weighted function are defined for high and low observations corresponding to current state of FLRGs. Suppose that the current states of grouped fuzzy relationship for high/ low observations are $\tilde{A}_i, \tilde{A}_j$ and $\tilde{A}_k$. Then, we established a weighted function for high/ low observations are $w_i|_\alpha, w_j|_\alpha$ and $w_k|_\alpha$ corresponding to each fuzzy set $\tilde{A}_i, \tilde{A}_j$ and $\tilde{A}_k$, where α is a parameter and its value is −1, 0 and 1.

Step 8. *Calculate the forecasted outputs*

To calculate the forecasted value at time t in high observations, we assume that fuzzified value at time $t - 1$ is $\tilde{A}_j$. Then after we choose the weight function $w_j|_\alpha$ corresponding to fuzzified value $\tilde{A}_j$, where the value of α is difference between time $t - 1$ and t. It leads to an increase, a decrease and no change, denoted as parameter α is 1, −1 and 0, respectively. Therefore, defuzzified output at time t is derived as weight $w_j|_{\alpha=-1,0,1}$.

Similarly, we calculate the defuzzified output for low observations at time t, which is derived as weight $w_k|_{\alpha=-1,0,1}$, if fuzzified value in low observation at time $t - 1$ is $\tilde{A}_k$.

Finally, the forecasted output at time t is obtained to take the mean of higher and lower observations at time t:

$$\text{Forecasted output (time } t) = \frac{w_j|_{\alpha=-1,0,1} + w_k|_{\alpha=-1,0,1}}{2}. \quad (6)$$

Thus, forecasted output of each year for high and low observations can be computed in a similar way.

Step 9. In time series, MAPE and root-mean-square error (RMSE), the common tool to measuring the forecasted accuracy, are defined as follows.

$$\text{MAPE} = \frac{1}{n} \sum \frac{|\text{forecast value} - \text{actual value}|}{\text{actual value}} \times 100, \quad (7)$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum (\text{forecast value} - \text{actual value})^2}. \quad (8)$$

6. Numerical illustration 1

In this section we considered the problem of enrollment of the University of Alabama for forecasting illustration of algorithm presented in Section 5. The stepwise procedures are as follows.

Step 1. Define the universe of discourse U by taking two proper positive value $D_1 = 55$ and $D_2 = 663$ as

$$U = [13000, 20000]$$

Further U can be partitioned into seven equal lengths of intervals $u_1, u_2, \ldots, u_7$.

where, $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$, $u_7 = [19000, 20000]$.

Step 2. According to frequency statistical distribution defined in above section, the intervals u_i, $i = 1, 2, \ldots, 7$ of equal length re-divided into linguistic subintervals $v_1, v_2, \ldots, v_{13}$ of different lengths as follows.

Table 1 shows that interval u_3 has maximum number of historical enrollment data. Then, from step 2 of proposed algorithm the interval u_3 is divided in to four subintervals of equal length. Now, second maximum number of historical enrollment data is in interval u_4, and u_4 is divided in to three subinterval of equal

Table 1. Frequency distribution data.

Intervals	Number of historical distribution data
u_1	3
u_2	1
u_3	9
u_4	4
u_5	0
u_6	3
u_7	2

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length. Similarly, interval u_1 and u_6 are divided into two equal lengths of subintervals. Interval u_2 remains unchanged. And finally, interval u_5 has no historical enrollment data, so we discard this interval.

So intervals $u_1, u_2, \ldots, u_7$ can be re-divided into different length of subintervals $v_1, v_2, \ldots, v_{13}$: $v_1 = [13000, 13500]$, $v_2 = [13500, 14000]$, $v_3 = [14000, 15000]$, $v_4 = [15000, 15250]$, $v_5 = [15250, 15500]$, $v_6 = [15500, 15750]$, $v_7 = [15750, 16000]$, $v_8 = [16000, 16333]$, $v_9 = [16333, 16667]$, $v_{10} = [16667, 17000]$, $v_{11} = [18000, 18500]$, $v_{12} = [18500, 19000]$, $v_{13} = [19000, 20{,}000]$.

Step 3. The triangular fuzzy sets $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_{13}$ are defined with corresponding linguistic interval $v_1, v_2, \ldots, v_{13}$ as follows.

$$\begin{aligned}
&\tilde{A}_1 = (13000, 13000, 13500), \quad \tilde{A}_2 = (13000, 13500, 14000),\\
&\tilde{A}_3 = (14000, 15000, 16000), \quad \tilde{A}_4 = (14750, 15000, 15250),\\
&\tilde{A}_5 = (15000, 15250, 15500), \quad \tilde{A}_6 = (15250, 15500, 15750),\\
&\tilde{A}_7 = (15500, 15750, 16000), \quad \tilde{A}_8 = (15667, 16000, 16333),\\
&\tilde{A}_9 = (16000, 16333, 16667), \quad \tilde{A}_{10} = (16333, 16667, 17000),\\
&\tilde{A}_{11} = (17500, 18000, 18500), \quad \tilde{A}_{12} = (18000, 18500, 19000),\\
&\tilde{A}_{13} = (19000, 20{,}000, 20{,}000).
\end{aligned}$$

The membership grades of each enrollment are defined as

$$\tilde{A}_1 = \frac{0.89}{13055},$$

$$\tilde{A}_2 = \frac{0.11}{13055} + \frac{0.87}{13563} + \frac{0.26}{13867},$$

$$\vdots$$

$$\tilde{A}_{11} = \frac{0.70}{18150},$$

$$\tilde{A}_{12} = \frac{0.3}{18150} + \frac{0.06}{18970} + \frac{0.24}{18876},$$

$$\tilde{A}_{13} = \frac{0.32}{19328} + \frac{0.33}{19337}.$$

Step 4. The actual and fuzzified enrollments are placed in Table 2.

Step 5. *Choose type 2 observations.*

Considering Table 2 as basis, the approximate high and low variables in fuzzy type 2 observations are given in Table 3.

Step 6. FLRGs for high and low observations are placed in Table 4.

Step 7. Assign a weighted function to corresponding FLRGs defined in Table 4 for high and low observations.

Weights for high observations

$$\begin{aligned}
w_3|_\alpha = {} & \frac{\alpha(\alpha-1)}{2} m_3 + \frac{2(1+\alpha)(1-\alpha)}{2} m_3\\
& + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{4} m_3 + \frac{1}{4} m_8 + \frac{1}{4} m_{10}\right),
\end{aligned}$$

$$w_7|_\alpha = \frac{\alpha(\alpha-1)}{2} m_3 + \frac{2(1+\alpha)(1-\alpha)}{2} m_7 + \frac{\alpha(\alpha+1)}{2} m_{10},$$

$$\begin{aligned}
w_8|_\alpha = {} & \frac{\alpha(\alpha-1)}{2}\left(\frac{2}{3} m_7 + \frac{1}{3} m_8\right) + \frac{2(1+\alpha)(1-\alpha)}{2} m_8\\
& + \frac{\alpha(\alpha+1)}{2}\left(\frac{1}{2} m_8 + \frac{1}{2} m_{11}\right),
\end{aligned}$$

$$w_{10}|_\alpha = \frac{\alpha(\alpha-1)}{2} m_8 + \frac{2(1+\alpha)(1-\alpha)}{2} m_{10} + \frac{\alpha(\alpha+1)}{2} m_{11},$$

Table 2. Year wise actual and fuzzified enrollments.

Years	Actual enroll-ments	Fuzzified enrollments	Years	Actual enroll-ments	Fuzzified enroll-ments
1971	13055	$\tilde{A}_1$	1982	15433	$\tilde{A}_6$
1972	13563	$\tilde{A}_2$	1983	15497	$\tilde{A}_6$
1973	13867	$\tilde{A}_2$	1984	15145	$\tilde{A}_3$
1974	14696	$\tilde{A}_3$	1985	15163	$\tilde{A}_3$
1975	15460	$\tilde{A}_6$	1986	15984	$\tilde{A}_8$
1976	15311	$\tilde{A}_5$	1987	16859	$\tilde{A}_{10}$
1977	15603	$\tilde{A}_6$	1988	18150	$\tilde{A}_{11}$
1978	15861	$\tilde{A}_7$	1989	18970	$\tilde{A}_{12}$
1979	16807	$\tilde{A}_{10}$	1990	19328	$\tilde{A}_{13}$
1980	16919	$\tilde{A}_{10}$	1991	19337	$\tilde{A}_{13}$
1981	16388	$\tilde{A}_9$	1992	18876	$\tilde{A}_{12}$

Table 3. Type 2 observations.

Years	High observations	Low observations
1971	14048 $(\tilde{A}_3)$	13015 $(\tilde{A}_1)$
1972	15028 $(\tilde{A}_3)$	13063 $(\tilde{A}_1)$
1973	14900 $(\tilde{A}_3)$	13448 $(\tilde{A}_2)$
1974	15989 $(\tilde{A}_8)$	13882 $(\tilde{A}_2)$
1975	16032 $(\tilde{A}_8)$	14062 $(\tilde{A}_3)$
1976	15784 $(\tilde{A}_7)$	14331 $(\tilde{A}_3)$
1977	16772 $(\tilde{A}_{10})$	14789 $(\tilde{A}_3)$
1978	16038 $(\tilde{A}_8)$	15861 $(\tilde{A}_8)$
1979	17834 $(\tilde{A}_{11})$	16002 $(\tilde{A}_8)$
1980	17837 $(\tilde{A}_{11})$	16042 $(\tilde{A}_8)$
1981	17589 $(\tilde{A}_{11})$	15405 $(\tilde{A}_6)$
1982	16518 $(\tilde{A}_{10})$	14470 $(\tilde{A}_3)$
1983	15997 $(\tilde{A}_8)$	13349 $(\tilde{A}_2)$
1984	15745 $(\tilde{A}_7)$	13589 $(\tilde{A}_2)$
1985	15163 $(\tilde{A}_3)$	14159 $(\tilde{A}_3)$
1986	16900 $(\tilde{A}_{10})$	15032 $(\tilde{A}_3)$
1987	17517 $(\tilde{A}_{11})$	15891 $(\tilde{A}_8)$
1988	18962 $(\tilde{A}_{12})$	18150 $(\tilde{A}_{11})$
1989	19668 $(\tilde{A}_{13})$	18052 $(\tilde{A}_{11})$
1990	19697 $(\tilde{A}_{13})$	18559 $(\tilde{A}_{12})$
1991	19892 $(\tilde{A}_{13})$	19337 $(\tilde{A}_{13})$
1992	19172 $(\tilde{A}_{13})$	18163 $(\tilde{A}_{11})$

Table 4. Fuzzy logical relationship groups.

High observations	Low observations
$\tilde{A}_3 \to \tilde{A}_3, \tilde{A}_8, \tilde{A}_{10}$	$\tilde{A}_1 \to \tilde{A}_1, \tilde{A}_2$
$\tilde{A}_7 \to \tilde{A}_3, \tilde{A}_{10}$	$\tilde{A}_2 \to \tilde{A}_2, \tilde{A}_3$
$\tilde{A}_8 \to \tilde{A}_7, \tilde{A}_8, \tilde{A}_{11}$	$\tilde{A}_3 \to \tilde{A}_2, \tilde{A}_3, \tilde{A}_8$
$\tilde{A}_{10} \to \tilde{A}_8, \tilde{A}_{11},$	$\tilde{A}_6 \to \tilde{A}_3$
$\tilde{A}_{11} \to \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}$	$\tilde{A}_8 \to \tilde{A}_6, \tilde{A}_8, \tilde{A}_{11}$
$\tilde{A}_{12} \to \tilde{A}_{13}$	$\tilde{A}_{11} \to \tilde{A}_{11}, \tilde{A}_{12}$
$\tilde{A}_{12} \to \tilde{A}_{13}$	$\tilde{A}_{12} \to \tilde{A}_{13}$
	$\tilde{A}_{13} \to \tilde{A}_{11}$

$$w_{11}|_\alpha = \frac{\alpha(\alpha-1)}{2}\left(\frac{1}{3}m_{10}+\frac{2}{3}m_{11}\right)+\frac{2(1+\alpha)(1-\alpha)}{2}m_{11} + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{3}m_{11}+\frac{1}{3}m_{12}\right),$$

$$w_{12}|_\alpha = \frac{\alpha(\alpha-1)}{2}m_{12}+\frac{2(1+\alpha)(1-\alpha)}{2}m_{12}+\frac{\alpha(\alpha+1)}{2}m_{13},$$

$$w_{13}|_\alpha = \frac{\alpha(\alpha-1)}{2}m_{13}+\frac{2(1+\alpha)(1-\alpha)}{2}m_{13}+\frac{\alpha(\alpha+1)}{2}m_{13}.$$

Weights for low observations

$$w_1|_\alpha = \frac{\alpha(\alpha-1)}{2}m_1+\frac{2(1+\alpha)(1-\alpha)}{2}m_1 + \frac{\alpha(\alpha+1)}{2}\left(\frac{1}{2}m_1+\frac{1}{2}m_2\right),$$

$$w_2|_\alpha = \frac{\alpha(\alpha-1)}{2}m_2+\frac{2(1+\alpha)(1-\alpha)}{2}m_2 + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{4}m_2+\frac{2}{4}m_3\right),$$

$$w_3|_\alpha = \frac{\alpha(\alpha-1)}{2}\left(\frac{1}{4}m_2+\frac{3}{4}m_3\right)+\frac{2(1+\alpha)(1-\alpha)}{2}m_3 + \frac{\alpha(\alpha+1)}{2}\left(\frac{3}{5}m_3+\frac{2}{5}m_8\right),$$

$$w_6|_\alpha = \frac{\alpha(\alpha-1)}{2}m_6+\frac{2(1+\alpha)(1-\alpha)}{2}m_6+\frac{\alpha(\alpha+1)}{2}m_3,$$

$$w_8|_\alpha = \frac{\alpha(\alpha-1)}{2}\left(\frac{1}{3}m_6+\frac{2}{3}m_8\right)+\frac{2(1+\alpha)(1-\alpha)}{2}m_8 + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{3}m_8+\frac{1}{3}m_{11}\right),$$

$$w_{11}|_\alpha = \frac{\alpha(\alpha-1)}{2}m_{11}+\frac{2(1+\alpha)(1-\alpha)}{2}m_{11} + \frac{\alpha(\alpha+1)}{2}\left(\frac{1}{2}m_{11}+\frac{1}{2}m_{12}\right),$$

$$w_{12}|_\alpha = \frac{\alpha(\alpha-1)}{2}m_{12}+\frac{2(1+\alpha)(1-\alpha)}{2}m_{12}+\frac{\alpha(\alpha+1)}{2}m_{13},$$

$$w_{13}|_\alpha = \frac{\alpha(\alpha-1)}{2}m_{13}+\frac{2(1+\alpha)(1-\alpha)}{2}m_{13}+\frac{\alpha(\alpha+1)}{2}m_{11}.$$

Here parameter α is difference between time $t-1$ and t. It leads to an increase, a decrease and no change, accordingly. Its value is 1, −1 and 0, respectively.

Step 8. Forecasted enrollments of each year by the proposed method are placed in Table 6. We list some data for the year 1980 to 1982 for explanatory purposes in Table 5.

An example to compute a forecasted value of year 1981: A fuzzified enrollment of year 1980 in high observation is $\tilde{A}_{11}$. Then we choose a weight function $w_{11}|_\alpha$ corresponding to fuzzy set $\tilde{A}_{11}$ to obtain a forecasted value for high observation.

$$w_{11}|_\alpha = \frac{\alpha(\alpha-1)}{2}\left(\frac{1}{3}m_{10}+\frac{2}{3}m_{11}\right)+\frac{2(1+\alpha)(1-\alpha)}{2}m_{11} + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{3}m_{11}+\frac{1}{3}m_{12}\right),$$

where, the data for high observations of this year will decrease so we take $\alpha = -1$.

$$w_{11}|_{\alpha=-1} = \frac{1}{3}\times 16833.5+\frac{2}{3}\times 18250 = 5611.16+12166.6 = 17777.7.$$

And a fuzzified enrollment of year 1980 in low observation is $\tilde{A}_8$. Then we choose a weight function $w_8|_\alpha$ corresponding to fuzzy set $\tilde{A}_8$ to obtain a forecasted value for low observation.

Table 5. Forecasted enrollments by proposed method.

Years	High forecasted value	Low forecasted value	Forecasted value
…	…	…	…
1980	18416.6	16860.9	17638.7
1981	17777.7	15985.9	16881.8
1982	17777.7	15625	16701.3
…	…	…	…

Table 6. Comparison of forecasted results for enrollments with other existing methods.

Year	Actual enroll-ments	Chen method [27]	Yolcu et al. [11]	Chou's method [12]	Pathak and Singh [18]	Gangwar and Kumar [19]	Proposed method
1971	13055						
1972	13563	14000	14031.35	14025	13250	14586	14463.5
1973	13867	14000	14795.36	14568	13750	14586	14000
1974	14696	15500	14795.36	14568	13750	15363	14776
1975	15460	16000	14795.36	15654	14500	15363	15666.6
1976	15311	16000	16406.57	15654	15375	15442	15569.3
1977	15603	16000	16406.57	15654	15375	15442	16000
1978	15861	16000	16406.57	15654	15625	15442	15666.5
1979	16807	16833	16406.57	16197	15875	15442	17034.5
1980	16919	16833	17315.29	17283	16833	17064	17638.7
1981	16388	16833	17315.29	17283	16833	17064	16881.8
1982	15433	16833	17315.29	16197	16500	15438	16701.3
1983	15497	16000	16406.57	15654	15500	15442	15239.5
1984	15145	16000	16406.57	15654	15500	15442	15048.5
1985	15163	16000	16406.57	15654	15125	15363	14312.5
1986	15984	16000	16406.57	15654	15125	15363	15333.2
1987	16859	16000	16406.57	15654	16833	15438	16708.3
1988	18150	16833	17315.29	16197	16667	17064	17117.9
1989	18970	19000	19132.79	17283	18125	19356	18875
1990	19328	19000	19132.79	18369	18750	19356	18458.3
1991	19337	19000	19132.79	19454	19500	19356	19500
1992	18876	19000	19132.79	19454	19500	19356	19500

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$$w_8|_\alpha = \frac{\alpha(\alpha-1)}{2}\left(\frac{1}{3}m_6 + \frac{2}{3}m_8\right) + \frac{2(1+\alpha)(1-\alpha)}{2}m_8 + \frac{\alpha(\alpha+1)}{2}\left(\frac{2}{3}m_8 + \frac{1}{3}m_{11}\right),$$

where, the data for low observation will also decrease so we take $\alpha = -1$.

$$w_8|_{\alpha=-1} = \frac{1}{3} \times 15625 + \frac{2}{3} \times 16166.5 = 15985.9.$$

All m_i are mid-point of an interval v_i $(i = 1, 2, \dots, 13)$. Finally, the total forecasted value of year 1981 is obtained to take the weighted mean of high and low observations, i.e.

$$\text{Forecast value year (1981)} = \frac{w_{11}|_{\alpha=-1} + w_8|_{\alpha=-1}}{2} = \frac{17777.7 + 15985.9}{2} = 16881.8.$$

Now forecasted enrollments by the proposed method is now computed as average of high forecasted value and low forecasted value and are listed in Table 5.

Step 9. The comparison of RMSE and MAPE are placed in Table 7. These two metrics are used to evaluate the forecasting performance and also show the superiority of the proposed method.

7. Numerical illustration 2

In view of examining its suitability as presented in Section 5, the proposed method is also being implemented for forecasting the historical time series data of wheat production in term of productivity in kg per hectare. For the sake of simplicity, we considered the same example of wheat crop production as undertaken by Singh [10]. The proposed method has been implemented and computations are presented below.

Step 1. Define universe of discourse $U = [1400, 3500]$, by taking two proper positive values $D_1 = 31$ and $D_2 = 66$.

Further, universe of discourse can be partitioned into seven equal intervals of linguistic values.

$$U_1 = [1400, 1700],\quad u_2 = [1700, 2000],\quad u_3 = [2000, 2300],$$
$$u_4 = [2300, 2600],\quad u_5 = [2600, 2900],\quad u_6 = [2900, 3200],$$
$$u_7 = [3200, 3500].$$

Step 2. See Table 8. According to frequency distribution data, intervals $u_1, u_2, \dots, u_7$ can be re-divided into different lengths of subintervals $v_1, v_2, \dots, v_{16}$:

$$v_1 = [1400, 1700],\quad v_2 = [2000, 2100],$$
$$v_3 = [2100, 2200],\quad v_4 = [2200, 2300],$$
$$v_5 = [2300, 2400],\quad v_6 = [2400, 2500],$$
$$v_7 = [2500, 2600],\quad v8 = [2600, 2675],$$
$$v_9 = [2675, 2750],\quad v_{10} = [2750, 2825],$$
$$v_{11} = [2825, 2900],\quad v_{12} = [2900, 3000],$$
$$v_{13} = [3000, 3100],\quad v_{14} = [3100, 3200],$$
$$v_{15} = [3200, 3350],\quad v_{16} = [3350, 3500].$$

Table 7. A comparison table in terms of RMSE and MAPE.

Models	Chen method [27]	Yolcu et al. [11]	Chou's method [12]	Pathak and Singh [18]	Gang-war and Kumar [19]	Pro-posed method
RMSE	630.9	805.1	785	646.67	642.68	545.7
MAPE (%)	3.08	4.28	3.60	2.98	2.97	2.83

Table 8. Frequency distribution data.

Intervals	Number of historical distribution data
u_1	1
u_2	0
u_3	3
u_4	3
u_5	9
u_6	3
u_7	2

Table 9. Actual production and fuzzified production.

Years	Actual production (kg/ha)	Fuzzified production	Years	Actual production (kg/ha)	Fuzzified production
1981	2730	$\tilde{A}_{10}$	1992	1431	$\tilde{A}_1$
1982	2957	$\tilde{A}_{13}$	1993	2248	$\tilde{A}_4$
1983	2382	$\tilde{A}_6$	1994	2857	$\tilde{A}_{12}$
1984	2572	$\tilde{A}_8$	1995	2318	$\tilde{A}_5$
1985	2642	$\tilde{A}_9$	1996	2617	$\tilde{A}_8$
1986	2700	$\tilde{A}_9$	1997	2254	$\tilde{A}_5$
1987	2872	$\tilde{A}_{12}$	1998	2910	$\tilde{A}_{12}$
1988	3407	$\tilde{A}_{16}$	1999	3434	$\tilde{A}_{16}$
1989	2238	$\tilde{A}_4$	2000	2795	$\tilde{A}_{11}$
1990	2895	$\tilde{A}_{12}$	2001	3000	$\tilde{A}_{13}$
1991	3276	$\tilde{A}_{16}$			

Step 3. The triangular fuzzy sets $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{16}$ are defined with corresponding linguistic interval $v_1, v_2, \dots, v_{16}$ as follows.

$$\tilde{A}_1 = (1400, 1550, 1700),\quad \tilde{A}_2 = (1900, 2000, 2100),$$
$$\tilde{A}_3 = (2000, 2100, 2200), \tilde{A}_4 = (2100, 2200, 2300),$$
$$\tilde{A}_5 = (2200, 2300, 2400),\quad \tilde{A}_6 = (2300, 2400, 2500),$$
$$\tilde{A}_7 = (2400, 2500, 2600),\quad \tilde{A}_8 = (2525, 2600, 2675),$$
$$\tilde{A}_9 = (2600, 2675, 2750),\quad \tilde{A}_{10} = (2675, 2750, 2825),$$
$$\tilde{A}_{11} = (2750, 2825, 2900),\quad \tilde{A}_{12} = (2800, 2900, 3000),$$
$$\tilde{A}_{13} = (2900, 3000, 3100),\quad \tilde{A}_{14} = (3000, 3100, 3200),$$
$$\tilde{A}_{15} = (3050, 3200, 3350),\quad \tilde{A}_{16} = (3200, 3350, 3500).$$

Step 4. The historical time series data along with triangular fuzzy sets in order to have a maximum membership grade to each actual wheat production data in kg/ha, is placed in Table 9.

Step 5. *Choose type 2 observations*: In this step, we choose high as the highest production of the year and low as the lowest production of the year in type 2 observations. We list some data for high and low observations (production in kg/ha) in Table 10 with their corresponding fuzzy sets.

Step 6. With these fuzzified wheat production data, the FLRGs for type 2 have been constructed for both high and low observations, as given in Table 11.

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Table 10. High and low observations in type 2 fuzzy.

Years	High observations	Low observations	Years	High observations	Low observations
1981	2968 $(\tilde{A}_{13})$	2552 $(\tilde{A}_7)$	1992	1689 $(\tilde{A}_1)$	1428 $(\tilde{A}_1)$
1982	3291 $(\tilde{A}_{16})$	2659 $(\tilde{A}_9)$	1993	2355 $(\tilde{A}_6)$	2020 $(\tilde{A}_2)$
1983	2561 $(\tilde{A}_8)$	2225 $(\tilde{A}_4)$	1994	3233 $(\tilde{A}_{15})$	2612 $(\tilde{A}_8)$
1984	2572 $(\tilde{A}_8)$	2309 $(\tilde{A}_5)$	1995	2583 $(\tilde{A}_8)$	2115 $(\tilde{A}_3)$
1985	2949 $(\tilde{A}_{12})$	2539 $(\tilde{A}_7)$	1996	2858 $(\tilde{A}_{12})$	2516 $(\tilde{A}_7)$
1986	2979 $(\tilde{A}_{13})$	2404 $(\tilde{A}_6)$	1997	2587 $(\tilde{A}_8)$	1983 $(\tilde{A}_2)$
1987	3219 $(\tilde{A}_{15})$	2553 $(\tilde{A}_7)$	1998	3148 $(\tilde{A}_{15})$	2910 $(\tilde{A}_{12})$
1988	3458 $(\tilde{A}_{16})$	3335 $(\tilde{A}_{16})$	1999	3468 $(\tilde{A}_{16})$	3215 $(\tilde{A}_{15})$
1989	2589 $(\tilde{A}_8)$	2207 $(\tilde{A}_4)$	2000	2868 $(\tilde{A}_{12})$	2518 $(\tilde{A}_7)$
1990	2895 $(\tilde{A}_{12})$	2575 $(\tilde{A}_8)$	2001	3298 $(\tilde{A}_{16})$	2852 $(\tilde{A}_{11})$
1991	3410 $(\tilde{A}_{16})$	3010 $(\tilde{A}_{13})$			

Table 11. Fuzzy logical relationship groups of wheat production.

High observations	Low observations
$\tilde{A}_{13} \to \tilde{A}_{15}, \tilde{A}_{16}$	$\tilde{A}_7 \to \tilde{A}_2, \tilde{A}_6, \tilde{A}_9, \tilde{A}_{11}, \tilde{A}_{16}$
$\tilde{A}_{16} \to \tilde{A}_1, \tilde{A}_8, \tilde{A}_{12}$	$\tilde{A}_2 \to \tilde{A}_8, \tilde{A}_{12}$
$\tilde{A}_8 \to \tilde{A}_8, \tilde{A}_{12}, \tilde{A}_{15}$	$\tilde{A}_1 \to \tilde{A}_2$
$\tilde{A}_{12} \to \tilde{A}_8, \tilde{A}_{13}, \tilde{A}_{16}$	$\tilde{A}_3 \to \tilde{A}_7$
$\tilde{A}_{15} \to \tilde{A}_8, \tilde{A}_{16}$	$\tilde{A}_4 \to \tilde{A}_5, \tilde{A}_8$
$\tilde{A}_1 \to \tilde{A}_6$	$\tilde{A}_5 \to \tilde{A}_7$
$\tilde{A}_6 \to \tilde{A}_{15}$	$\tilde{A}_6 \to \tilde{A}_7$
	$\tilde{A}_8 \to \tilde{A}_3, \tilde{A}_{13}$
	$\tilde{A}_9 \to \tilde{A}_4$
	$\tilde{A}_{12} \to \tilde{A}_{15}$
	$\tilde{A}_{13} \to \tilde{A}_1$
	$\tilde{A}_{15} \to \tilde{A}_7$
	$\tilde{A}_{16} \to \tilde{A}_4$

Table 12. Actual wheat production vs. forecasted production.

Years	Actual production (kg/ha)	Forecasted by Chen model [27]	Forecasted by proposed method
1981	2730		
1982	2957	2750	3174.9
1983	2382	2900	2346.8
1984	2572	2600	2723.4
1985	2642	2600	2731.5
1986	2700	2750	2774.9
1977	2872	2750	2950
1988	3407	2750	3212.4
1989	2238	2150	2346.8
1990	2895	2900	2723.4
1991	3276	2750	3174.9
1992	1431	2150	1996.8
1993	2248	2150	2250
1994	2857	2900	3034.3
1995	2318	2750	2393.7
1996	2617	2600	2751.5
1997	2254	2750	2443.7
1998	2910	2900	2873.3
1999	3434	2900	3350
2000	2795	2150	2496.8
2001	3000	2750	3149.8

Table 13. Comparison of RMSE and MAPE of wheat production by proposed method and Chen model.

Models	Chen model [27]	Proposed method
RMSE	369.7	189.1
MAPE (%)	11.08	6.26

Step 7. Assign a weight function by Equation (5) to a current state of each FLRGs defined in step 6 for each high and low observations.

Step 8. Forecasted wheat production by proposed method of each year and comparison with Chen model are given in Table 12.

Step 9. See Table 13.

8. Conclusion

In this paper, we have proposed a computational method of forecasting based on type 2 fuzzy time series which is an extension of type 1 fuzzy time series model. Here in the proposed method we obtain FLRGs which is based on type 2 fuzzy by converting a type 1 historical time series data into type 2 fuzzy and we obtain FLRGs for both high and low observations. The modified weighted function for both high and low observations are assigned using FLRGs of type 2 fuzzy model. We define triangular fuzzy sets to obtain membership grades of each type 2 fuzzy data (named as high and low). Further, the forecasted output is obtained by assigning a simple weight function rather than complicated union and intersection operator of type 2 fuzzy sets. The method has been implemented on the historical time series data of enrollments in university of Alabama and on crop (wheat) production to forecast. The comparison Tables 7 and 13 show that the proposed method can achieve a higher forecasting accuracy over the existing methods.

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Disclosure statement

No potentials conflict of interest was reported by the authors.

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