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High-Order Type-2 Fuzzy Time Series

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***Abstract*—This paper proposes a novel improved model for fuzzy time series forecasting using high-order and Type-2 fuzzy time series. Some Type-1 fuzzy time series model based on first order and high-order fuzzy time series have already been developed, however the accuracy of the forecasting values sill needs to be improved. In this paper, we present a high-order fuzzy time series model using Type-2 fuzzy sets. The employment of Type-2 fuzzy sets enables the proposed method to achieve a better result compared with existing fuzzy time series models. Our experimental results demonstrate the improved forecasting accuracy and reliability in comparison with other existing fuzzy time series models.**

***Index Terms*—type-2 fuzzy sets; high-order fuzzy time series; type-2 fuzzy time series.**

I. Introduction

One of the big challenges in time series forecasting is the poor accuracy of the prediction in comparison with real data, especially for fuzzy time series forecast. There have been different fuzzy time series methods investigated to improve the forecasting accuracy. Song and Chissom (1993) proposed a first-order fuzzy time series model which deals with vague values under uncertain circumstance [6], [7], [9]. For example, fuzzy time series can be applied to linguistic data [4], [7], [9]. Singh (2005) presented a simple method of forecasting based on fuzzy time series using the stepwise procedure and historical time series data [5]. Singh's stepwise procedure was applied in various fuzzy time series forecasting [1] - [9]. Huarng (2005) put forward Type-2 fuzzy time series model to refine the fuzzy relationships obtained from a Type-1 model [2]. The Type-2 models of Huarng's model consist of two critical steps: defining the simple operations for utilising extra observations and computing the forecasting [2]. Huarng's method suffers by not producing good forecasts. Huarng applied the union and intersection operators, which are used in Type-2 model, to the Fuzzy Logical Relationship Groups (FLRGs) for all the observations by using the middle point of an interval value for each sample for forecasting [2]. The forecasting results by Huarng's method is far from actual data and Root Mean Square Error (RMSE) is higher.

In this paper, we propose a novel model for fuzzy time series forecasting using high order and Type-2 fuzzy time series. It is called HO-T2FTS. The HO-T2FTS model builds on the basis of Singh's method and Huarng's method [2], [5]. The model is split into two parts: high-order Type-1 fuzzy time series forecasting and Type-2 time series forecasting. The high-order Type-1 fuzzy time series model is employed to define the universe of discourse, partition the universe into equal length of intervals, construct the fuzzy sets, fuzzify the historical data and establish the fuzzy logical relationships by the rules for forecasting in [5]. Then the rule in the high-order Type-1 fuzzy time series is used in Type-2 fuzzy time series forecasting. The Type-2 time series forecasting is consisted of several steps, and we applied the operators of fuzzy logical group relationships (FLRGs) for defuzzification [2]. The complex min-max composition operators are applied to all forecast and then the forecasting performance is evaluated by Root Mean Square Error (RMSE). The advantage of the proposed model is that the model leads to a higher forecasting performance and a lower RMSE compared with previous models.

Section II reviews the definitions and properties of fuzzy time series forecasting. Section III proposes a novel HO-T2FTS model and algorithm. Section V demonstrates some example forecasting results and RMSEs for forecasting performance. Section V concludes the study.

II. Preliminaries

A. Fuzzy Time Series

There are various fuzzy time series concepts proposed by many authors in [1]-[9]. Let U be the universe of discourse on which U=$\{u_1$,u_1,...,$u_n\}$, where u_i are elements of U. A_i is a fuzzy set of U, which is defined by A_i=$f_{A_i}(u_1)/u_1$+$f_{A_i}(u_2)/u_2$+...+$f_{A_i}(u_n)/u_n$, where $f_{A_i}(u_i)$ is the membership function of the fuzzy set A_i, such that $f_{A_i}(u_i)$: $U \to$ [0,1] (see [1], [2], [3], [5], [8]).

Definition 1: [1], [2], [3], [5], [8] Let $Y(t)$; $(t = \ldots, 1, 2, \ldots)$, be a subset of real number R, be the universe of discourse. Assume that $f_i(t)$; $(i = 1, 2, 3, \ldots)$ is defined in the universe of discourse $Y(t)$ and $F(t)$ is a collection of $f_i(t)$; $(i = 1, 2, 3, \ldots)$, then $F(t)$ is called a fuzzy time series of $Y(t)$. At that, $F(t)$ can be understood as a linguistic variables, where $f_i(t)$; $(i = 1, 2, 3, \ldots)$, possible linguistic values of $F(t)$.

Definition 2: [5], [8] Suppose $F(t)$ is caused by $F(t-1)$ only or by $F(t-1)$ or $F(t-2)$ or ... or $F(t-m)$. It is denoted by $F(t-1) \to F(t)$ or $F(t-m) \to F(t)$, respectively. A fuzzy relationship between $F(t)$ and $F(t-1)$ or $F(t)$ and $F(t-1)$, $F(t)$ and $F(t-2)$,...,$F(t)$ and $F(t-m)$ can be expressed as the following equation:

$$F(t) = (F(t-1) \cup F(t-2) \cup \ldots \cup F(t-m)) \circ R(t, t-m). \quad (1)$$

where "$\cup$" is the union operator, "$\circ$" is the max-min composition operator.

Definition 3: [3], [7], [8] Suppose $F(t)$ is caused by $F(t-1)$ and $F(t-2)$ and ... and $F(t-m)$ simultaneously. A fuzzy relationship between $F(t)$ and $F(t-1)$,$F(t)$ and $F(t-2)$,... , $F(t)$ and $F(t-m)$ is defined by

$$F(t) = (F(t-1) \cap F(t-2) \cap ... \cap F(t-m)) \circ R(t, t-m). \quad (2)$$

$$F(t) = (F(t-1) \times F(t-2) \times ... \times F(t-m)) \circ R(t, t-m). \quad (3)$$

where "$\cap$" is the intersection operator, "$\times$" is the Cartesian product and m>0 is a parameter of the m^{th}-order model of such as number of years [9]. Then relation $R(t, t-m)$ of $F(t)$ in equation (1) is called the first-order model of $F(t)$. The relation $R(t, t-m)$ of $F(t)$ in equation (2) or (3) is called the m^{th}-order model of $F(t)$. For different times t_1 and t_2, if $R(t_1, t_1-1)$=$R(t_2, t_2-1)$ or $R(t_1, t_1-m)$=$R(t_2, t_2-m)$ then $F(t)$ is called a time-invariant fuzzy time series. Otherwise it is called a time-variant fuzzy time series [8].

B. Fuzzification and High-Order Fuzzy Time Series

In the real world, some historical data may be linguistic values in addition to real numbers. Fuzzy time series are able to deal well with linguistic values [7]. If historical data are real numbers rather than linguistic values, the data should be fuzzified first. The fuzzification process of data is as follows:

Define the universe of discourse U and then partitioning U into several even length intervals from historical data. Let D_{min} and D_{max} be the minimum and maximum of historical data. By the universe $U = [D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers [3], [5], [7]. Then partitioning U into n equal intervals as D_1,D_2,...,D_n, which the number of intervals must correspond with the number of linguistic variables as A_1,A_2,...,A_m [1], [5]. Then the fuzzified time series for time was treated as A_i [3]. The membership grades to fuzzy sets of linguistic values are defined as:
A_1 = $f_{A_1}(u_1)/(u_1)$+$f_{A_1}(u_2)/(u_2)$+...+$f_{A_1}(u_n)/(u_n)$.
A_2 = $f_{A_2}(u_1)/(u_1)$+$f_{A_2}(u_2)/(u_2)$+...+$f_{A_2}(u_n)/(u_n)$.
... = ... + ... + ... + ...
A_m = $f_{A_m}(u_1)/(u_1)$+$f_{A_m}(u_2)/(u_2)$+...+$f_{A_m}(u_n)/(u_n)$.

The next step establishes fuzzy logical relations and fuzzy logical relationship groups as following:

Definition 4: [4] Let $F(t-1) = A_i$ and $F(t) = A_j$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by $A_i \rightarrow A_j$, which A_i is called the left-hand side (LHS) and A_j is called the right-hand side (RHS). $A_i \rightarrow A_j$ is called Fuzzy Logical Relationship (FLRs) and is called first-order fuzzy time series model. Here A_i is current state and A_j is next state.

Further, FLRs are grouped into FLRGs as following:

Definition 5: [5] If $\hat{F}(t)$ is caused by more fuzzy sets, $\hat{F}(t-n)$, $\hat{F}(t-n+1)$, ...,$\hat{F}(t-1)$, the fuzzy relationship is represented by $\hat{A}_{i1}$, $\hat{A}_{i2}$,..., $\hat{A}_{in} \rightarrow \hat{A}_j$. Here, $\hat{F}(t-n)$=$\hat{A}_{i1}$, $\hat{F}(t-n+1)$ = $\hat{A}_{i2}$,..., $\hat{F}(t-1)$ = $\hat{A}_{in}$. This relationship is called n^{th}-order fuzzy time series model.

C. Type-2 Fuzzy Time Series

Type-1 fuzzy time series models use only one variable for forecasting [1]. A triangular function is applied to construct the triangular rule in [1] and a triangular function is adopted to generate a triangular fuzzy set in [7]. However, Type-2 fuzzy sets are applied to predict the forecasting values for multiple variables in [5]. Type-2 fuzzy logic systems (FLSs), can be utilized to provide the information about uncertainty in the training data and to express more information about the output [1]. We applied the Type-2 fuzzy time series concept as follows.

Huarng (2005) defined a Type-2 fuzzy time series model an extension of a Type-1 model. This Type-2 model utilises the fuzzy relationships established by a Type-1 model based on Type-1observations. Operators are used to include or screen out fuzzy relationships obtained from Type-1 and Type-2 observations. Type-2 forecasts are then calculated from these fuzzy relationships [1].

The operators in definition 6 as for example, union operator ($\vee$) involves including and intersection ($\wedge$) operator entails screening out fuzzy relation. The operations of $\vee$ and $\wedge$ are commutative and associative as the theorem in [1]. The two operators are proposed in Type-2 fuzzy time series model and are extended for multiple FLRGs, $\vee_m$ and $\wedge_m$.

Definition 6: [1] When the results of $\vee_m$ and $\wedge_m$ are empty sets. (a) If $\vee_m(LHS_c, LHS_d, LHS_e, ...)$=$\Phi$, then let $\vee_m(LHS_c, LHS_d, LHS_e, ...) = LHS_x$, where LHS_x is obtained from the FLRG established by Type-1 observations. (b) If $\wedge_m(LHS_c, LHS_d, LHS_e, ...)$=$\Phi$, then let $\wedge_m(LHS_c, LHS_d, LHS_e, ...) = LHS_x$, where LHS_x is obtained from the FLRG established by Type-1 observations. Then, defuzzification of forecasting a Type-2 model in [1] is applied to the rule for forecasting in [5] and discussed in next section.

D. Defuzzification of Forecasting a Type-2 Model

Using results of $\vee_m$ and $\wedge_m$ to get the defuzzified forecasts, let A_{q1},A_{q2},...,A_{qj} be the forecasting and m_{q1},m_{q2},...,m_{qj} be the midpoints of interval u_{q1},u_{q2},...,u_{qj}, where $u_{qz}, z = 1, ..., j$. Huarng employs the average of each midpoint of each interval to defuzzify for forecasting [1]. The forecasting calculation of Type-2 model is defined by

$$D(t) = \frac{\sum_{z=1}^{j} defuzzification_z(t)}{j} = \frac{\sum_{z=1}^{j} m_{qz}}{j}. \quad (4)$$

where $D(t)$ is a defuzzified forecast from a Type-2 observation and j is a total of Type-2 observation at time t, where m_{qz} is the midpoint of u_{qz}, is a defuzzified forecast from a Type-2 observation and there are a total of j Type-2 observation at time t. Evaluating the forecasting performance is defined by

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(actual_i - forecasting_i)^2}{n}}. \quad (5)$$

The accuracy in forecasted values is compared by the RMSE (Root Mean Square Error). The smallest RMSE value has a high accuracy.

III. A Novel Computation of High-Order and Type-2 Fuzzy Time Series

As two different models [1], [6], high-order fuzzy time series forecasting is independent from Type-2 fuzzy time series forecasting in the existing models. Each of them has its own merits and they cannot replace each other. Here, we combine the two methods together to fully make use their advantages. This novel HO-T2FTS method is expected to improve fuzzy time series prediction to obtain better performance. Considering fuzzy time series and fuzzification and high-order fuzzy time series in section II, we have the following definition for the novel model of high-order and Type-2 fuzzy time series.

Definition 7: Let $\tilde{F}(t)$ be caused by more fuzzy subsets, $\tilde{F_1}(t)$, $\tilde{F_2}(t)$, ..., $\tilde{F_i}(t)$. The fuzzy logical relationships for $\tilde{F_i}(t)$ is caused by $\tilde{F_i}(t-1)$, $\tilde{F_i}(t-2)$,..., $\tilde{F_i}(t-n)$; where $i = 1, 2, ..., p$. Here, a fuzzy subset at time t is denoted by $\tilde{F_i}(t-n)$, $\tilde{F_i}(t-n+1)$,...,$\tilde{F_i}(t-1)$, the fuzzy logical relationship is represented by

$$\tilde{A}_{i1}, \tilde{A}_{i2}, ..., \tilde{A}_{in} \rightarrow \tilde{A}_{ij}. \quad (6)$$

where $i = 1, 2, ..., p$; $j = 1, 2, ..., q$. Here, $\tilde{F_i}(t-n)$=$\tilde{A}_{i1}$, $\tilde{F_i}(t-n+1)$=$\tilde{A}_{i2}$, ..., $\tilde{F_i}(t-1)$ = $\tilde{A}_{in}$. $\tilde{A}_{in}$ is a linguistic variable or fuzzy relationship. This relationship is called n^{th}-order Type-2 fuzzy time series model.

Definition 8: Let $H_i(t)$ be the different values of actual data E_t of time t and E_{t-1} of time $t-1$ at time t for fuzzy subset i is defined by

$$H_i(t) = ||(E_t - E_{t-1})| - |(E_{t-1} - E_{t-2})| - ... - |(E_{t-r+l} - E_{t-r})||. \quad (7)$$

where $i = 1, 2, .., p$; $t = 1, 2, ..., n$; $r = 1, 2, ..., l$. r is a number of term of moving average of data and l is a maximum number of term of moving average.

Definition 9: Let $G_{min/max}(t)$; $(t = 1, 2, ..., n)$ be the minimum and maximum values of $H_i(t)$ for subattribute i at time t is defined by

$$G_{min/max}(t) = min/max(H_1(t), H_2(t), ..., H_p(t)). \quad (8)$$

The fuzzy logical relation for time t to $t+1$ can be obtained by selection of membership functions such as triangular function. The $G_{min}(t)$ and $G_{max}(t)$ in Equation (8) are calculated for V_{min_t}, V_{max_t}, W_{min_t}, W_{max_t}, X_{min_t}, X_{max_t}, Y_{min_t} and Y_{max_t} is defined by

$$V_{min_t/max_t} = E_t + \frac{G_{min/max}(t)}{2}. \quad (9)$$

$$W_{min_t/max_t} = E_t - \frac{G_{min/max}(t)}{2}. \quad (10)$$

$$X_{min_t/max_t} = E_t + G_{min/max}(t). \quad (11)$$

$$Y_{min_t/max_t} = E_t - G_{min_/max}(t). \quad (12)$$

where V_{min_t}, V_{max_t}, W_{min_t} and W_{max_t} are the triangular membership function when the membership grade of them is 0.5 and X_{min_t}, X_{max_t}, Y_{min_t} and Y_{max_t} are also the triangular membership function when the membership grade of them is 1. We use V_{min_t}, V_{max_t}, W_{min_t}, W_{max_t}, X_{min_t}, X_{max_t}, Y_{min_t} and Y_{max_t} to compare with the conditions for each interval $U[A_j]$ in each fuzzy subsets $\tilde{A}_{in}$, for example, $V_{min_t} \geq L[A_j]$, $V_{min_t} \leq U[A_j]$ and $V_{max_t} \geq L[A_j]$, $V_{max_t} \leq U[A_j]$, where, $L[A_j]$ is the lower bound of interval and $U[A_j]$ is the upper bound of interval. If $V_{min_t} \geq L[A_j]$ and $V_{min_t} \leq U[A_j]$ is true then V_{min_t} is selected and the membership in A_j is Supremum and V_{min} is Supremum value which V_{min} is 1, otherwise V_{min} is 0. Similarly, the remaining of parameters is computed by the same way. Following this if $W_{min_t} \geq L[A_j]$ and $W_{min_t} \leq U[A_j]$ is true then W_{min_t} is selected and W_{min_t} is 1, otherwise W_{min_t} is 0; if $X_{min_t} \geq L[A_j]$ and $X_{min_t} \leq U[A_j]$ is true then X_{min_t} is selected and X_{min_t} is 1, otherwise X_{min_t} is 0 and if $Y_{min_t} \geq L[A_j]$ and $Y_{min_t} \leq U[A_j]$ is true then Y_{min_t} is selected and Y_{min_t} is 1, otherwise Y_{min_t} is 0.

Definition 10: Let $P_{min/max}(t)$ be a parameter in equation (9), (10), (11) and (12) and $C_{min/max}$ is a supremum value of the membership in A_j of $P_{min/max}(t)$ at time t is defined by

$$P_{min/max}(t) = \begin{cases} P_{min/max}(t); & L(A_j) \leq P_{min/max}(t) \leq U(A_j) \\ 0; & \text{Otherwise.} \end{cases} \quad (13)$$

$$C_{min/max}(t) = \begin{cases} 1; & L(A_j) \leq P_{min/max}(t) \leq U(A_j) \\ 0; & \text{Otherwise.} \end{cases} \quad (14)$$

where $P_{min/max}(t)$ is a value of V_{min_t}, V_{max_t}, W_{min_t}, W_{max_t}, X_{min_t}, X_{max_t}, Y_{min_t} and Y_{max_t}, respectively. In equation (14), $C_{min/max}(t)$ is a value of 1 or 0 for V_{min_t}, V_{max_t}, W_{min_t}, W_{max_t}, X_{min_t}, X_{max_t}, Y_{min_t} and Y_{max_t}, respectively. For example,$P_{min/max}(t)$ = V_{min_t}, if $V_{min_t} \leq U[A_j]$ then $P_{min}(t)$ = $V_{min}(t)$ and $C_{min}(t)$ = 1 else $P_{min}(t)$ = 0 and $C_{min}(t)$ = 0.

Definition 11: Let $T_{min}(t)$ be summation of V_{min_t}, W_{min_t}, X_{min_t} and Y_{min_t} and T_{max_t} be summation of V_{max_t}, W_{max_t}, X_{max_t} and Y_{max_t}, respectively. $T_{min}(t)$ and T_{max_t} are defined by

$$T_{min_t/max_t} = \sum_{h=1}^{n} P_{min_h/max_h}(t). \quad (15)$$

In Equation (15), if h=1,2,3 and 4 then the example of $P_{min_{h=1}}(t)$=V_{min_t} is denoted by h=1; $P_{min_{h=2}}(t)$=W_{min_t} is denoted by h=2; $P_{min_{h=3}}(t)$=X_{min_t} is denoted by h=3 and $P_{min_{h=4}}(t)$=Y_{min_t} is denoted by h=4, respectively. Similarly, $P_{max_{h=1}}(t)$=V_{max_t} is computed by the same way as $P_{min_{h=1}}(t)$=V_{min_t}. Other $P_{min_h}(t)$=W_{min_t}, $P_{min_h}(t)$=X_{min_t} and $P_{min_h}(t)$=Y_{min_t} are similar. h=2,3,4 for P_{max} are defined in a similarly way.

Definition 12: Let cT_{min} be summation of V_{min_t}, W_{min_t}, X_{min_t} and Y_{min_t} and cT_{max} be the summation of V_{max_t}, W_{max_t}, X_{max_t} and Y_{max_t}, respectively. cT_{min} and cT_{max} are defined by

$$cT_{min_t/max_t} = \sum_{i=1}^{n} C_{min_i/max_i}(t) + 1. \quad (16)$$

Definition 13: Let $\hat{F}_{min}$ and $\hat{F}_{max}$ be forecasting values at time t for minimum and maximum values of $G_i(t)$, respectively. The defuzzification of $\hat{F}_{min}$ and $\hat{F}_{max}$ are defined by

$$\hat{F}_{min/max}(t) = \begin{cases} M(A_j); & T_{min_t/max_t}\text{=0.} \\ \frac{T_{min_t/max_t}+M(A_j)}{cT_{min_t/max_t}}; & Otherwise. \end{cases} \quad (17)$$

where $M(A_j)$ is the mid value of the interval u_j having Supremum value in A_j (i.e.1).

Definition 14: Let forecasting(t) be final forecasting values at time t. The computation of high-order and Type-2 fuzzy time series forecasting values is defined by

$$Forecasting_i(t) = \frac{\hat{F}_{min}(t) + \hat{F}_{max}(t)}{2}; i = 1, ..., p. \quad (18)$$

The procedure of high-order and Type-2 fuzzy times series forecasting as listed.

Step 1: Define the universe of discourse U and partition U into equal length of intervals.

Step 2: Applying Type-1 fuzzy sets to construct the fuzzy sets A_i in accordance with the intervals in step 1.

Step 3: Fuzzify the historical time series data and obtain the $FLRs$.

Step 4: Rule for forecasting is derived from [7].

Step 5: Pick Type-2 observations by using rules in step 4 to calculate G_t values and then all G_t values are compared by splitting maximum comparison and minimum comparison and defuzzified the produced forecast.

Step 6: Calculate forecasts for high-order and Type-2 model and evaluate the performance.

TABLE I
DATA FOR FORECASTING DAILY INDEX PRICE (ACTUAL DATA)(NT$MILLION)

Date	Time (t)	Closing	High	Low
2007/11/2	1	5626.08	5796.08	5212.73
2007/11/3	2	5796.08	5941.85	5252.83
2007/11/4	3	5677.30	5805.17	5284.41
2007/11/6	4	5657.48	5772.51	5380.09
2007/11/7	5	5877.77	5877.77	5720.89
2007/11/8	6	6067.94	6164.62	5889.01
2007/11/9	7	6089.55	6089.55	5926.64
Maximum		6089.55	6164.42	5926.64
Minimum		5626.08	5772.51	5212.73

TABLE II
FUZZY SETS (NT$MILLION)

Date	Closing	FS	High	FS	Low	FS
2007/11/2	5626.08	A_5	5796.08	A_6	5212.73	A_1
2007/11/3	5796.08	A_6	5941.85	A_8	5252.83	A_1
2007/11/4	5677.30	A_5	5805.17	A_7	5284.41	A_1
2007/11/6	5657.48	A_5	5772.51	A_6	5380.09	A_2
2007/11/7	5877.77	A_7	5877.77	A_7	5720.89	A_6
2007/11/8	6067.94	A_9	6164.62	A_{10}	5889.01	A_7
2007/11/9	6089.55	A_9	6089.55	A_6	5926.64	A_8

Note: FS stands for fuzzy sets.

TABLE IV
HUARNG'S METHOD FORECASTING

Date	FLRGs	Forecasting after		Final forecasting		
		$\wedge_m$	$\vee_m$	$\wedge_m$	$\vee_m$	Type-2
11/7	C:$A_7 \to A_6, A_8, A_9, A_{10}$	A_8	A_5, A_6,	5950.0	5900.0	5925.0
	H:$A_7 \to A_6, A_8, A_9, A_{10}$		A_7, A_8,	5950.0	5900.0	5925.0
	L:$A_6 \to A_5, A_7, A_8$		A_9, A_{10}	5950.0	5900.0	5925.0
11/8	C:$A_9 \to A_9$	A_9	A_6, A_8,	6050.0	5975.0	6012.5
	H:$A_{10} \to A_9$		A_9, A_{10}	6050.0	5975.0	6012.5
	L:$A_7 \to A_6, A_8, A_9, A_{10}$			6050.0	5975.0	6012.5
11/9	C:$A_9 \to A_9$	A_9	A_6, A_7,	6050.0	5950.0	6000.0
	H:$A_8 \to A_7$		A_8, A_9,	5950.0	5950.0	5950.0
	L:$A_7 \to A_6, A_8, A_9, A_{10}$		A_{10}	5850.0	5950.0	5900.0

Note: C, H and L stand for closing, high and low, respectively.

IV. APPLICATIONS

A. Initial Test Data

The data in Table I shows some daily TAIEX data for forecasting from [1]. TAIEX stands for **TAI**wan Stock **EX**change. It is a apart of Taiwan Stock Exchange Corporation (TWSE), which is a private corporation. TWSE works with their competent authority, Financial Supervisory Commission (FSC), to deregulate and liberalize Taiwan's stock market, and gears itself up more in line with major international market [11]
.

Table II illustrates fuzzy sets for fuzzy subsets: closing, high and low. FLRs are done and FLRGs are grouped by FLRs in Table III. Using the FLRGs from Huarng's method defuzzified the forecast of $\wedge_m$ and $\vee_m$ [4]. Final results are obtained by Type-2 forecasting. We use the proposed method to forecast the high-order Type-2 fuzzy time series by using equation (18). Thus, the Closing, High and Low columns forecasting are shown in Table IV.

The Type-2 forecasting between Huarng's method and the proposed method are compared (see Table IV and V). The forecasting values from the proposed method have better results and fit to the original data in Table1 than Huarng's method. We evaluate the forecasting performance for both methods and RMSEs are shown as listed:

RMSE of Huarng's method-Closing: 67.73, High: 108.22 and Low: 179.64.

RMSE of the proposed method-Closing: 29.99, High: 28.97 and Low: 32.91.

B. Second Experiment

In the experiment, we obtain data from daily trading value/volume of individual security of TAIEX for forecasting [11]. 747 instances of a data set recorded during January 2007 - December 2009 is employed to forecast and evaluate the performance of the forecasting methods.

C. Analysis

In the experiment, we tested the performance of HO-T2FTS method and Huarng's method. Figure 1 and Table VI illustrate the comparison between the two methods. The results from HO-T2FTS method are obviously better than the results from Haurng's method.

TABLE III
FUZZY LOGICAL RELATIONSHIPS AND FUZZY LOGICAL RELATIONSHIPS GROUPS

	FLRs	FLRGs (Closing+High+Low)	
Closing	$A_5 \to A_6$, $A_6 \to A_5$, $A_5 \to A_5$, $A_5 \to A_7$, $A_7 \to A_9$, $A_9 \to A_9$	$A_1 \to A_1, A_2$	$A_7 \to A_6, A_8, A_9, A_{10}$
High	$A_6 \to A_8$, $A_8 \to A_7$, $A_7 \to A_6$, $A_6 \to A_7$, $A_7 \to A_{10}$, $A_{10} \to A_9$	$A_2 \to A_6$	$A_8 \to A_7$
Low	$A_1 \to A_1$, $A_1 \to A_1$, $A_1 \to A_2$, $A_2 \to A_6$, $A_6 \to A_7$, $A_7 \to A_8$	$A_5 \to A_5, A_6, A_7$	$A_9 \to A_9$
		$A_6 \to A_5, A_7, A_8$	$A_{10} \to A_9$

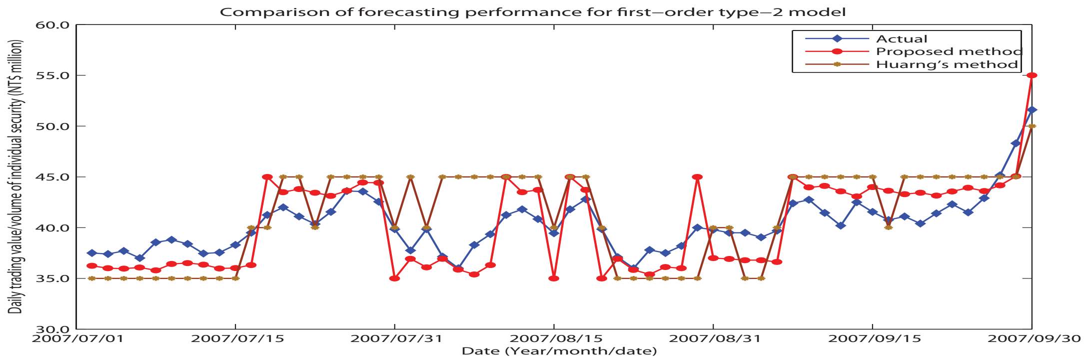

Fig. 1. Actual, Huarng's method and proposed method are compared for forecasting performance

TABLE V
THE PROPOSED (HO-T2FTS) METHOD FORECASTING

Date	G_t		Min	Max	Final forecasting		
					$\wedge_m$	$\vee_m$	Type-2
11/7	Closing:	19.82	19.82	-	5850.0	5850.0	5850.0
	Hight:	32.66	-	-	6050.0	6050.0	6060.0
	Low:	95.68	-	95.68	6032.7	6050.5	6041.3
11/8	Closing:	220.29	-	-	5827.0	5845.4	5836.2
	Hight:	105.26	105.26	-	6150.0	6150.0	6150.0
	Low:	340.80	-	340.80	6054.7	6050.0	6052.3
11/9	Closing:	190.17	-	-	5750.0	5750.0	5750.0
	Hight:	286.85	-	286.85	5850.0	5833.3	5841.6
	Low:	168.12	168.12	-	5962.0	5950.0	5956.0

We use Type-2 model to forecast first order forecasting between three subattributes: Closing, High and Low. In Table VI, the forecasting errors of Huarng's method are between 0.64 and 252.81, and most small errors are greater than 1. In opposite, the proposed method in the same table shows a much smaller forecasting errors: 0.032 - 22.562, and most small errors are smaller than 1. For example, forecasting values on 03/01/2007 to 05/01/2007 in Table VI are 25.00 for all closing, high and low subattributes when actual data for Closing, High and Low on the 03/01/2007 are 29.45, 29.85 and 29.45, respectively. The Closing, High and Low values on 04/01/2007 are 29.35, 29.45 and 29.15, respectively, and Closing, High and Low values on 05/01/2007 are 29.05, 29.40 and 28.95, respectively. In the same table, forecasting values of proposed method for Closing, High and Low subattributes on 03/01/2007 are 28.80, 28.84 and 28.48, respectively. The Closing, High and Low forecasting values on 04/01/2007 are 26.78, 26.94 and 27.78, respectively and Closing, High and Low forecasting values on 05/01/2007 are 26.74, 28.78 and 26.66, respectively. However, Table VII shows the different forecasting values in the same period and subattributes. Obviously, the overall error forecasting values (RMSE) of the proposed method is better than Huarng's method due to the smaller error values.

V. CONCLUSION

In this paper, the proposed HO-T2FTS model combines Singh's high order Type-1 fuzzy time series model with the Type-2 fuzzy time series model proposed by Huarng. Huarng's method applied Type-2 fuzzy logic in the first order. However, the accuracy is limited. Singh's method employed high-order fuzzy logic in Type-1 fuzzy sets. Thus, HO-T2FTS possesses the advantages of both methods and is able to give better performance for data with multiple sub attributes. The experiments in the paper demonstrate that the proposed method outperforms the existing fuzzy times series models.

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TABLE VI

FORETASTED DAILY TRADING VALUE/VOLUME OF INDIVIDUAL SECURITY (1^{th}-ORDER TYPE-2 FUZZY TIME SERIES) BY HUARNG'S METHOD AND THE PROPOSED METHOD (NT$MILLION)

Date	Huarng's method									proposed method								
	Closing	F	Error	High	F	Error	Low	F	Error	Closing	F	Error	High	F	Error	Low	F	Error
2007/01/02	29.75			29.80			29.35			29.75			29.80			29.35		
2007/01/03	29.45	25.00	19.80	29.85	25.00	23.52	29.45	25.00	19.80	29.45	28.80	0.42	29.85	28.84	0.94	29.45	28.48	0.94
2007/01/04	29.35	25.00	18.92	29.45	25.00	19.80	29.15	25.00	17.22	29.35	26.78	6.60	29.45	26.94	5.62	29.15	26.78	5.62
2007/01/05	29.05	25.00	16.40	29.40	25.00	19.36	28.95	25.00	15.60	29.05	26.74	5.36	29.40	28.78	5.24	28.95	26.66	5.24
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2009/12/31	34.00	35.00	1.00	34.25	35.00	0.56	33.95	35.00	1.10	34.00	35.00	0.15	34.25	34.28	0.15	33.95	34.08	0.35
RMSE			3.15			3.13			3.31			2.06			1.94			2.05

Note: F stands for forecasting.

TABLE VII

HIGH-ORDER BASED ON TYPE-2 MODEL FORECASTING (NT$MILLION)

Date	Order (Closing)						Order (High)						Order (Low)					
	Actual	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	Actual	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	Actual	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2007/01/02	29.75						29.80						29.35					
2007/01/03	29.45	28.80					29.85	28.84					29.45	28.48				
2007/01/04	29.35	26.78	28.56				29.45	26.94	28.88				29.15	26.78	28.56			
2007/01/05	29.05	26.74	28.48	28.48			29.40	26.78	28.56	28.56			28.95	26.66	26.66	34.08		
2007/01/08	29.00	26.62	28.24	28.24	28.24		29.10	26.76	28.52	28.52	28.52		28.50	26.58	28.16	28.16	28.16	
2007/01/09	30.25	35.00	35.00	35.00	35.00	35.00	30.25	35.00	35.00	35.00	35.00	35.00	29.10	26.40	27.80	27.80	27.80	27.80
2007/01/10	29.75	25.00	26.20	26.25	25.00	25.00	30.25	25.00	31.38	31.20	31.20	32.20	29.65	26.64	28.80	28.80	28.80	28.80
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2007/12/31	34.00	34.60	34.20	34.20	34.20	34.20	34.25	34.64	34.28	34.28	34.28	34.28	33.95	34.54	34.08	34.08	34.08	34.08
RMSE		2.06	1.50	1.49	1.47	1.53		1.94	1.31	1.31	1.31	1.38		2.05	1.44	1.46	1.42	1.48

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