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# An Interval Type-2 Fuzzy Logic System for Stock Index Forecasting Based on Fuzzy Time Series and a Fuzzy Logical Relationship Map

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**ABSTRACT** This paper proposes an interval type-2 fuzzy logic system (IT2FLS) for stock index forecasting based on a fuzzy time series and a fuzzy logical relationship map (FLRM). First, variations within the data are found with the maximum and minimum variations used for the interval settings of the universe of discourse. The time series variations are fuzzified into fuzzy sets in order to form fuzzy logical relationships, which are then used to construct the FLRM. Second, the input interval type-2 fuzzy sets (IT2FSs) and the output intervals of the IT2FLS are defined based on the maximum and minimum variations found. Third, the data variation between time $t-1$ and time t, and the input IT2FSs are used as input for the IT2FLS, and the output of the IT2FLS is the forecasting variation, which is found between time t and time $t+1$. An output interval is formed using the IT2FLS rule-base based on the FLRM. Finally, the forecast value at time $t+1$ is defined as the data point at time t plus the forecast variation. In this paper, the proposed method is applied to data from the Taiwan Stock Exchange Capitalization Weighted Stock Index, the Dow Jones Industrial Average, and the National Association of Securities Dealers Automated Quotation. Existing methods are then compared with the proposed method using Wilcoxon non-parametric statistical testing, as opposed to simply comparing the average root-mean-square error. Based on the statistical analysis results, the proposed method is found to typically outperform the other methods.

**INDEX TERMS** Fuzzy logical relationship, fuzzy time series, interval type-2 fuzzy logic system, stock index, TAIEX.

## I. INTRODUCTION

Individual investors, stock fund managers, and financial analysts must rely upon their own professional knowledge or stock analysis tools to predict the daily movement of stock indices in various stock markets. Increased profits can be made if their accuracy of stock index prediction is high. To make this easier, it is necessary to develop an accurate method for prediction of stock indices which can change rapidly and dramatically, causing the forecasting problem of stock volatility. Several technical indicators have been adopted to predict stock trends. For example, index fluctuations have been used to predict stock indices, because stock analysts assume that recurring patterns in the stock market will appear again in the future [1]. Another example is observation of the US stock market, which plays an important role worldwide, and its trends and movements influence stock markets around the world [2]. Two of the most well-known indicators in the US stock market used for forecasting of stock trends are the Dow Jones Industrial Average (DJIA) and the National Association of Securities Dealers Automated Quotation (NASDAQ) [3]–[13]. Trading volume, the proportion of short sellers, and various options that are contracted on a stock can also be adopted in order to forecast stock trends [1].

In recent years, many time series forecasting models have been proposed and applied to help make economic predictions, such as the autoregressive conditional heteroscedasticity model, the autoregressive moving average model, and the autoregressive integrated moving average model [1]. However, traditional time series forecasting methods are based on observations of historical data, and these data must be stationary in order to obtain higher accuracy with the forecasting method. If the data are not stationary, then they may contain uncertainty [14]. In order to solve the problem of uncertainty, the data should be presented in linguistic terms, which are represented by fuzzy sets. However, traditional time series forecasting methods are unable to solve linguistic term problems, thus, fuzzy time series (FTS) methods are used, as presented by Song and Chissom [15].

Since introduction of FTS forecasting models, this method has been used in numerous studies to solve stock index forecasting problems [1], [3]–[14], [16]–[29]. For example, Chen and Chang [3] presented a multi-variable fuzzy forecasting method that uses fuzzy clustering and fuzzy rule interpolation to forecast the temperature and the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). Chen *et al.* [4], [5] and Chen and Jian [6] used two factors, namely second order fuzzy trend logical relationship groups and particle swarm optimization (PSO) techniques, for fuzzy forecasting. This method was used to forecast the TAIEX and the NTD/USD exchange rates. Peng *et al.* [13] proposed a neuro-fuzzy modeling scheme with a hybrid learning algorithm for refining fuzzy rules to predict fluctuations of the TAIEX. Wong *et al.* [14] presented an adaptive time variant model, which automatically adapts the analysis window size of a fuzzy time series to forecast enrollments at the University of Alabama as well as movement of the TAIEX. A complex neuro-fuzzy autoregressive integrated moving average (ARIMA) forecasting method was presented by Li and Chiang [20]. The method integrates a neuro-fuzzy system with complex fuzzy sets with ARIMA models in order to make forecasts of the TAIEX. However, none of the aforementioned studies include a suitable weighting method for fuzzy logical relationships (FLRs). Yu presented weighted models which reflect the importance of each individual FLR in relation to forecasting [30]. Chen and Chen proposed a FTS model based on the granular computing approach with binning-based partitioning and entropy-based discretization methods in order to forecast the TAIEX [28]. The granular computing approach is able to determine reasonable interval lengths suited to the TAIEX datasets. This model also adopts the trend-weighted approach in order to enhance forecasting accuracy. Chen and Chen proposed a FTS method, integrating the entropy discretization technique with a Fast Fourier Transform algorithm in order to forecast the TAIEX [29]. The proposed model is used to resolve issues which result from the reasonable partition of discourse and the defuzzification method. Chen and Chen sought to strengthen forecasting accuracy by applying the trend-weighted approach with the fuzzy time series [29]. The fuzzy trend relationship group is another method for dealing with FLRs. Chen and Jian [6] Chen and Chen [7] proposed a method for TAIEX forecasting based on two-factor second-order fuzzy trend logical relationship groups, as well as the probabilities of trends of fuzzy trend logical relationships. Chen and Chen [9] forecast movement of the TAIEX using FTS and fuzzy variation groups. Chen *et al.* [11] used the variation in magnitude of adjacent historical data in order to generate fuzzy variation groups for the fuzzy time series, and the method was then applied in order to forecast the TAIEX.

In all the studies mentioned above, only type-1 fuzzy sets are utilized, in which the degree of membership is represented in a range from zero to one. The membership functions of type-1 fuzzy sets are a precise point, which means that each element of the universe of discourse is assigned a precise degree of membership. This precise membership degree means, however, that type-1 fuzzy sets are not able to handle problems with the incorporation of uncertainty, such as noisy or non-stationary conditions. Hence, type-2 fuzzy sets, which allow related membership degrees to be uncertain, have been presented in order to handle problems with uncertainty [12]. For example, Huarng and Yu [23] proposed a type-2 FTS model for TAIEX forecasting. In this model, additional observations were made to refine the FLRs obtained from the type-1 model, thereby yielding better forecasting performance compared to the type-1 model. Liu *et al.* [12] presented a type-2 neuro-fuzzy model for TAIEX forecasting. Their results showed that the type-2 neuro-fuzzy modeling method yielded higher forecasting accuracy without using additional information. Bajestani and Zare [27] proposed a high-order type-2 FTS method to obtain higher TAIEX forecasting accuracy. Moreover, type-2 fuzzy systems have also been applied in different areas. For example, Jafarzadeh *et al.* [31] used an interval type-2 Takagi-Sugeno-Kang fuzzy system to model and predict solar power generation. These interval type-2 Takagi-Sugeno-Kang fuzzy systems show the best performance among the tested methods. Khosravi *et al.* [32] used an interval type-2 fuzzy logic system (IT2FLS) to deal with uncertainties and improve short-term load forecasting. Zarandi *et al.* [33] presented an interval type-2 fuzzy c-means (FCM) algorithm for the IT2FLS to construct the footprint of uncertainties. This method was applied to the forecasting of carbon monoxide concentrations, with high forecasting accuracy.

The findings from the literature review are summarized as follows: 1) A suitably weighted approach to the FLRs of fuzzy time series can properly reflect the importance of each individual FLR and strengthen forecasting accuracy [28], [29]. 2) The precise membership degree of the type-1 fuzzy sets means they are unable to handle problems relating to uncertainty, such as the fluctuations of stock indices. Type-2 fuzzy sets, on the other hand, which allow related membership degrees to be uncertain, are able to handle these types of problems [12]. This study integrates these two advantages in a hybrid method. First, based on the concept of the weighted approach, a fuzzy logical relationship map (FLRM) is proposed, and an IT2FLS is used to calculate the type-2 fuzzy sets. Thus, an IT2FLS for stock index forecasting which combines fuzzy time series with an FLRM method is proposed. This method is dependent on the variation concept, specifically, that current variations are related to past trends [9], [17].

The computational procedure for the proposed method is described briefly here. In the computation, the variation of the data between time $t-1$ and time $t$ is calculated first. Then, the variation between time $t-1$ and time $t$ is used as an input to IT2FLS. Fuzzy rules are tuned based on the FLRM, which is the distribution of the FLRs $A_i \rightarrow A_j$. According to the tuned fuzzy rules, the output of IT2FLS is the forecasting variation between time $t$ and time $t+1$. Finally, the forecasting value at time $t+1$ is the data point at time $t$ plus the forecasting variation. The proposed method is then applied to the TAIEX, DJIA, and NASDAQ data, which have also been used in investigation in numerous other studies. The forecasted results obtained in these studies are compared with the results obtained with the proposed method using a Wilcoxon non-parametric statistical test. Based on the statistical results, the proposed method is shown to generally outperform the other methods.

## II. FUZZY TIME SERIES

Fuzzy time series, first presented by Song and Chissom, are able to handle linguistic terms [15]. In this study, the advantages of FTS are utilized to solve the linguistic term problem. The FTS concept is defined below.

A fuzzy set $A$ is defined in the universe of discourse $U = \{u_1, u_2, \ldots, u_n\}$ as

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n, \tag{1}$$

where $f_A$ is the membership function of fuzzy set $A$. The range of $f_A$ in the universe of discourse $U$ is from zero to one; $f_A(u_i)$ is the degree of membership of $u_i$ in fuzzy set $A$ and can be expressed as $f_A(u_i) \in [0, 1]$, where $1 \le i \le n$ [8].

Let $Y(t)(t = \ldots, 0, 1, 2, \ldots)$, a subset of $R^1$, where $R^1$ is a set of real numbers, either naturally or artificially defined, be the universe of discourse on which the fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots)$ are defined. Then $F(t)$, which is the collection of $f_i(t)$ $(i = 1, 2, \ldots)$, is called a fuzzy time series in $Y(t)(t = 0, 1, 2, \ldots)$.

Suppose that a first-order model is considered, where $F(t)$ is only caused by $F(t-1)$, and the fuzzy logical relationship is $F(t-1) \rightarrow F(t)$. Suppose that $F(t-1) = A_i$ and $F(t) = A_j$. The relationship between these fuzzy sets can be expressed as $A_i \rightarrow A_j$, where $A_i$ is represented on the left-hand side, and $A_j$ is represented on the right-hand side.

## III. INPUT INTERVAL TYPE-2 FUZZY SETS, OUTPUT INTERVALS AND THE INTERVAL TYPE-2 FUZZY LOGIC SYSTEM

Unlike a type-1 fuzzy set (T1FS), the membership of the interval type-2 fuzzy set (IT2FS) is an uncertain interval. This uncertain interval membership allows the IT2FS to model and minimize the effect of uncertainties. An IT2FS is bounded by the upper T1FS and the lower T1FS ($\overline{X}$ and $\underline{X}$). $\overline{X}$ and $\underline{X}$ are called the upper and lower membership functions, respectively and the area between $\overline{X}$ and $\underline{X}$ is referred to as the footprint of uncertainty. An example of the IT2FS is represented by a 9-point vector $(p_1, p_2, \ldots, p_9)$ as shown in Fig. 1 (a) [34].

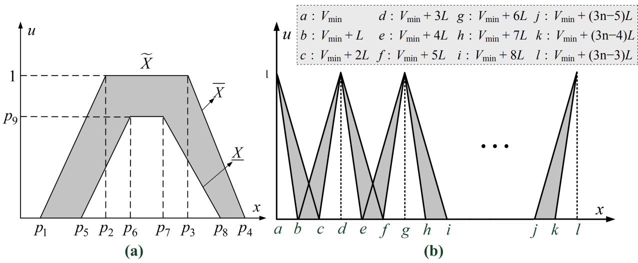


**FIGURE 1.** An example of the IT2FS. (a) An IT2FS represented by a 9-point vector. (b) The IT2FSs of the IT2FLS.

In the proposed forecasting method, the IT2FSs are established through observations of historical data. If there are $N$ historical data $D$, there will be $N-1$ variations $V$ computed by

$$V_n = D_n - D_{n-1}, \tag{2}$$

where $n = 2, 3, \ldots, N$. Then, the maximum variation $V_{\max}$ and minimum variation $V_{\min}$ are obtained from variation set $V$ as below:

$$V_{\max} = \max(V); \tag{3}$$
$$V_{\min} = \min(V). \tag{4}$$

The variations obtained from the historical data can then be mapped into corresponding fuzzy sets $(I_1, I_2, I_3, \ldots, I_{n-1}, I_n)$. The interval length $L$ of IT2FSs is defined as follows:

$$L = \frac{1}{3n-3}[(V_{\max} + D_2) - (V_{\min} - D_1)], \tag{5}$$

where $n$ is the number of fuzzy sets, and two proper positive numbers ($D_1$ and $D_2$) are used to cover the noise of the testing data [7], [8]. The methods of IT2FSs are defined by a 9-point vector as follows:

$$\begin{aligned} I_1 = [&V_{\min}, V_{\min}, V_{\min}, \\ &V_{\min}+2L, V_{\min}, V_{\min}, \\ &V_{\min}, V_{\min}+L, 1] \end{aligned} \tag{6}$$

$$\begin{aligned} I_i = [&V_{\min}+(3i-5)L, V_{\min}+(3i-3)L, V_{\min}+(3i-3)L, \\ &V_{\min}+(3i-1)L, V_{\min}+(3i-4)L, V_{\min}+(3i-3)L, \\ &V_{\min}+(3i-3)L, V_{\min}+(3i-2)L, 1] \end{aligned} \tag{7}$$

$$\begin{aligned} I_n = [&V_{\min}+(3n-5)L, V_{\min}+(3n-3)L, V_{\min}+(3n-3)L, \\ &V_{\min}+(3n-3)L, V_{\min}+(3n-4)L, V_{\min}+(3n-3)L, \\ &V_{\min}+(3n-3)L, V_{\min}+(3n-3)L, 1] \end{aligned} \tag{8}$$

where $i = 2, 3, \ldots, n-1$. A triangular shape is one of the most common for membership functions in fuzzy models [35], [36], and so is adopted in this study. In order to create a triangular shape, some points of the IT2FS overlap and $p_9$ of each IT2FS equals one. Thus, $p_1, p_2, p_3, p_5, p_6$, and $p_7$ of $I_1$ are overlapping; $p_2$ and $p_3$ of $I_2$ to $I_{n-1}$ are overlapping; and $p_6$ and $p_7$ of $I_2$ to $I_{n-1}$ are overlapping. $p_2, p_3, p_4, p_6, p_7$, and $p_8$ of $I_n$ are also overlapping. Fig. 1 (b) shows an example of IT2FSs.

The output intervals can also be divided into $n$ intervals ($O_1$, $O_2$, $O_3, \ldots, O_{n-1}$, $O_n$). The interval length $L$ of the output intervals is defined as follows:

$$L = \frac{1}{n+1}[(V_{\max} + D_2) - (V_{\min} - D_1)], \tag{9}$$

where $n$ is the number of intervals. The output intervals are defined by a 2-point vector as follows:

$$O_i = [(V_{\min} - D_1) + (i-1)L, (V_{\min} - D_1) + (i+1)L], \tag{10}$$

where $i = 1, 2, \ldots, n$. Similar to type-1 fuzzy logic systems, type-2 fuzzy logic systems include a fuzzifier, fuzzy inference engine, rule-base, type reducer and defuzzifier, as shown in Fig. 2.

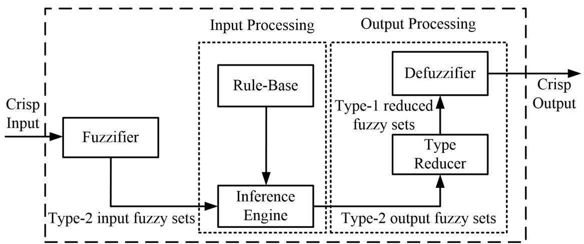


**FIGURE 2. The structure of a type-2 fuzzy logic system.**

$N$ rules of IT2FLS are assumed to have the following form:

$$Rule^n: \text{ If } x_1 \text{ is } X_1^n \text{ and } \cdots \text{ and } x_I \text{ is } X_I^n, \text{ Then } y \text{ is } Y^n,$$

where $n = 1, 2, \ldots, N$, $X_i^n (i = 1, \ldots, I)$ are input IT2FSs, and $Y^n = [\underline{y}^n, \bar{y}^n]$ is an output interval, which can be seen as the centroid of a consequent IT2FS. The computation in the IT2FLS is shown in the following steps:

Step 1) The input vector is assumed to be $x = (x_1, x_2, \ldots, x_I)$.

Step 2) Compute the corresponding membership level of $x_i$ in $X_i^n$, $\left[\mu \underline{X}_i^n(x_i), \mu \overline{X}_i^n(x_i)\right]$, where $i = 1, 2, \ldots, I$ and $n = 1, 2, \ldots, N$.

Step 3) Compute the firing interval of the $n^{\text{th}}$ rule with minimum $*$, $F^n(x)$:

$$F^n(x) = [\mu \underline{X}_1^n(x_1) * \cdots * \mu \underline{X}_I^n(x_I), \mu \overline{X}_1^n(x_1) * \cdots * \mu \overline{X}_I^n(x_I)]. \tag{11}$$

The firing interval can be defined as

$$F^n(x) \equiv [\underline{f}^n, \bar{f}^n], \tag{12}$$

where $n = 1, \ldots, N$.

Step 4) The center-of-sets type-reducer is used to combine the firing interval $F^n(x)$ and the corresponding rule consequents of IT2FLS:

$$Y(x) = \bigcup_{\substack{f^n \in F^n(x) \\ y^n \in Y^n}} \frac{\sum_{n=1}^{N} f^n y^n}{\sum_{n=1}^{N} f^n} = [yl, yr]; \tag{13}$$

$fl(k)$ can be defined as

$$fl(k) = \frac{\sum_{n=1}^{k} \bar{f}^n \underline{y}^n + \sum_{n=k+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{k} \bar{f}^n + \sum_{n=k+1}^{N} \underline{f}^n}; \tag{14}$$

$yl$ can be re-expressed as

$$yl = \min_{k \in [1, N-1]} fl(k) \equiv fl(L) = \frac{\sum_{n=1}^{L} \bar{f}^n \underline{y}^n + \sum_{n=L+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{L} \bar{f}^n + \sum_{n=L+1}^{N} \underline{f}^n}; \tag{15}$$

$fr(k)$ can be defined as

$$fr(k) = \frac{\sum_{n=1}^{k} \underline{f}^n \bar{y}^n + \sum_{n=k+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{k} \underline{f}^n + \sum_{n=k+1}^{N} \bar{f}^n}; \tag{16}$$

and $yr$ can be re-expressed as

$$yr = \max_{k \in [1, N-1]} fr(k) \equiv fr(R) = \frac{\sum_{n=1}^{R} \underline{f}^n \bar{y}^n + \sum_{n=R+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \bar{f}^n}, \tag{17}$$

where $\{\underline{y}^n\}$ and $\{\bar{y}^n\}$ are sorted in ascending order and the switch points $L$ and $R$ are determined by

$$\underline{y}^L \leq yl \leq \underline{y}^{L+1}, \tag{18}$$

$$\bar{y}^R \leq yr \leq \bar{y}^{R+1}; \tag{19}$$

$yl$ and $yr$ can be computed using a more efficient algorithm, which is an enhanced iterative algorithm with the stop condition (EIASC) [34].

Step 5) Compute the defuzzified output as

$$y = \frac{yl + yr}{2}. \tag{20}$$

## IV. AN INTERVAL TYPE-2 FUZZY LOGIC SYSTEM FOR FORECASTING BASED ON FUZZY TIME SERIES AND A FUZZY LOGICAL RELATIONSHIP MAP

The proposed model involves five major steps: 1) defining and partitioning the universe of discourse. The reason this proposed fuzzy logic system decides to use the technique for partitioning of the universe of discourse is because this technology is well developed in lots of lectures even for recent years [37]–[40]; 2) defining the fuzzy sets and fuzzifying the time series; 3) defining the input interval type-2 fuzzy sets (IT2FSs) and output intervals of the interval type-2 fuzzy logic system (IT2FLS); 4) establishing fuzzy logical relationships (FLRs) and the fuzzy logical relationship map (FLRM); and 5) forecasting results. Each step is described below:

Step 1) Define the universe of discourse $U$ and the intervals. The universe of discourse is composed of the variations between each data set. If there are $N$ historical data $D$, the variation $V$ is computed by

$$V_{n-1} = D_n - D_{n-1}, \tag{21}$$

where $n = 2, 3, \ldots, N$. For $U = [V_{\min} - D_1, V_{\max} + D_2]$; $V_{\max}$ and $V_{\min}$ are the maximum and the minimum value of the variations in the training data; $D_1$ and $D_2$ are two proper positive numbers used so that the universe of discourse $U$ covering the noise of the testing data and can be divided into $n$ intervals of equal length [6]; and for the intervals $u_i = [(V_{\min} - D_1) + (i-1)L, (V_{\max} + D_2) + iL]$, where $i = 1, 2, \ldots, n$; $L = [(V_{\max} + D_2) - (V_{\min} - D_1)]/n$; and $n$ is the number of intervals.

Step 2) Define the fuzzy sets and fuzzify the data. Each fuzzy set $A_i$ is assigned a linguistic term and can be defined by the interval $u_i$. For example:

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n, \tag{22}$$

where $i = 1, 2, \ldots, n$, and $n$ is the number of intervals.

Step 3) Define the input IT2FSs and the output intervals of the IT2FLS. Each input $I_i$ of the IT2FLS is assigned a linguistic term, and each $O_i$ is an output interval of the IT2FLS, where $i = 1, 2, \ldots, n$, and $n$ is the number of intervals. The definitions of the input IT2FSs and the output intervals have been mentioned in section 3.

Step 4) Establish the FLR of time $t$ and time $t+1$, and the FLRM. In the training phase, the FLR is supposed to be $A_i \to A_j$ ($A_i$ is on the left-hand side of time $t$ and $A_j$ is on the right-hand side of time $t+1$), where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$, and $n$ is the number of intervals. In the testing phase, the FLR is supposed to be $A_i \to *$($*$ is the unknown value). The FLRM $F$ is initialized to create an $n \times n$ matrix of zeros, and $n$ is the number of intervals. The rows and columns of the FLRM show the right-hand and left-hand linguistic terms, respectively. The FLRM is established by

$$F(j, i) = C, \tag{23}$$

where $C$ is the count of each FLR. For example, there are some FLRs as follows: $A_1 \to A_2$, $A_2 \to A_3$, $A_2 \to A_3$, and $A_3 \to A_1$. If the count of $A_1 \to A_2$ is 1, then $F(2, 1) = 1$. If the count of $A_2 \to A_3$ is 2, then $F(3, 2) = 2$. If the count of $A_3 \to A_1$ is 1, then $F(1, 3) = 1$. An example of establishing the FLRM is shown in Fig 3.

Step 5) Calculate the forecasted results. In the training phase, the FLRM is established based on the FLR count. In the testing phase, the forecasted values are calculated by using Algorithm 1, as shown in Fig. 4.

Initialize the FLRM to form a zero matrix.

| Right-hand side \ Left-hand side | $A_1$ | $A_2$ | $A_3$ | … | $A_n$ |
|---|---|---|---|---|---|
| $A_1$ | 0 | 0 | 0 | 0 | 0 |
| $A_2$ | 0 | 0 | 0 | 0 | 0 |
| $A_3$ | 0 | 0 | 0 | 0 | 0 |
| ⋮ | 0 | 0 | 0 | 0 | 0 |
| $A_n$ | 0 | 0 | 0 | 0 | 0 |

→

Train the FLRM by the count of the fuzzy logical relationship.

| Right-hand side \ Left-hand side | $A_1$ | $A_2$ | $A_3$ | … | $A_n$ |
|---|---|---|---|---|---|
| $A_1$ | 0 | 0 | 1 | 0 | 0 |
| $A_2$ | 1 | 0 | 0 | 0 | 0 |
| $A_3$ | 0 | 2 | 0 | 0 | 0 |
| ⋮ | 0 | 0 | 0 | 0 | 0 |
| $A_n$ | 0 | 0 | 0 | 0 | 0 |

**FIGURE 3. An example of establishing the FLRM.**

Some notations are defined as follows:

| | |
|---|---|
| *IT2FS* | a vector containing input IT2FSs [$I_1, I_2, I_3, \ldots, I_7$]; |
| *OI* | a vector containing output intervals [$O_1, O_2, O_3, \ldots, O_7$]; |
| IT2FLS | the interval type-2 fuzzy logic system; |
| $AV_t$ | actual value at time $t$; |
| $FV_{t+1}$ | forecasted value at time $t+1$; |
| $A_t$ | Fuzzified variation at time $t$; |
| *N* | numbers of intervals; |
| *FLRM* | a fuzzy relationship map; |

**Algorithm 1.** The FLR of time $t$ and time $t+1$ is supposed to be $A_i \to *$.

```
Let FLRMS = 0 and dis = positive infinite;
Compute the difference v between AV_t and AV_{t-1};
it2fs is the A of the IT2FS at time t;
FOR i = 1 to N
    FLRMS is FLRMS plus ith row of OI multiplied by the ith
    row and the A_t th column of FLRM;
END FOR
IF FLRMS equals 0 THEN
    [Aj, Ai] = find the position of maximum value in the FLRM;
    IF there is only one maximum value in the FLRM
        odi is the Aj of the OI;
    ELSE
        FOR j = 1 to length of A_i minus 1
            IF the |Ai(j) - A_t| is smaller than dis
                dis is |Ai(j) - A_t|;
                oi is the j of the Aj of the OI;
            END IF
        END FOR
    END IF
ELSE
    odi is FRLMS divided by the summation of the A_t th column
    of FLRM;
END IF
Compute the df by the IT2FLS(v, it2fs, oi);
Compute the FV_{t+1} by AV_t plus df;
```

**FIGURE 4. Pseudocode of the Algorithm 1.**

## V. RESULTS AND DISCUSSION

The forecasting results obtained with the proposed method and previous existing methods are summarized in this section. First, the proposed method was applied to forecast fluctuations in the TAIEX from 1999 to 2004, and from 1990 to 1999, then moving to predict the variations in the DJIA from 1999 to 2004 and in the NASDAQ from January 3, 2007 to December 20, 2010. These three stock indices are selected to verify the forecasting accuracy of the proposed method. A comparison is drawn with other forecasting methods, using the root-mean-square error (RMSE) to measure forecasting accuracy:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (Forecasted\_Value_i - Actual\_Value_i)^2}{n}}, \tag{24}$$

note that $n$ is the number of forecasted values. The RMSEs generated by each method are used to calculate the average RMSE and the standard deviation (SD), and are then

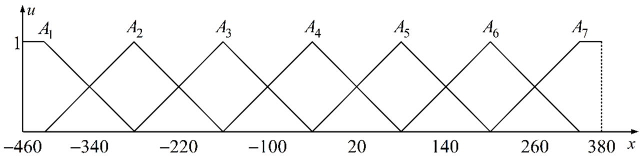


**FIGURE 5.** The intervals of the universe of discourse.

subjected to the statistical analysis. The coefficient of variation (CV) is also calculated. Previous studies only considered the average RMSE when deciding which method would yield better forecasting performance. However, this would make it difficult to verify whether our forecasting method is better than other methods, because the same average RMSE might have a different SD. Hence, it is important to employ statistical analysis to verify forecasting performance. The Wilcoxon non-parametric statistical test is chosen as the statistical analysis method for examining differences in the forecasting performance between the proposed method and other existing methods. If a statistically significant difference is found, that is if the RMSE is smaller, then the forecasting performance of the proposed method is better than that of other methods.

### A. TAIEX FORECASTING

An example of TAIEX forecasting, with data from 2004, is used to illustrate the proposed method. The training data set is from January to October 2004 and the testing data set is from November to December 2004.

Step 1) Define the universe of discourse $U$ and the intervals. The variation $V$ is computed as shown in Table 1. Consider the historical variation in training data from January 2 to October 29, 2004 [8], the maximum value of variations $V_{max}$ and the minimum value of variations $V_{min}$ are 341.49 and $-455.17$, respectively. Let $D_1 = 4.83$ and $D_2 = 38.51$, so that $U$ becomes $[V_{\min} - D_1, V_{\max} + D_2] = [-455.17 - 4.83, 341.49 + 38.51] = [-460, 380]$. $U$ is divided into 7 intervals $u_1$, $u_2$, $u_3$, $u_4$, $u_5$, $u_6$, and $u_7$ of equal length, and the interval length $L$ is [380 – (–460)] / 7 = 120. The intervals are $u_1 = [-460, -340)$, $u_2 = [-340, -220)$, $u_3 = [-220, -100)$, $u_4 = [-100, 20)$, $u_5 = [20, 140)$, $u_6 = [140, 260)$, and $u_7 = [260, 380]$. Fig. 5 shows the intervals of the universe of discourse.

Step 2) Define the fuzzy sets and fuzzify the data. The fuzzy sets are based on the 7 generated intervals $u_1$, $u_2$, $u_3$, $u_4$, $u_5$, $u_6$, and $u_7$, and the fuzzy sets $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, and $A_7$ are defined as follows:

$$A_1 = 1/u_1+0.5/u_2+0/u_3+0/u_4+0/u_5+0/u_6+0/u_7; \tag{25}$$

$$A_2 = 0.5/u_1+1/u_2+0.5/u_3+0/u_4+0/u_5+0/u_6+0/u_7; \tag{26}$$

$$A_3 = 0/u_1+0.5/u_2+1/u_3+0.5/u_4+0/u_5+0/u_6+0/u_7; \tag{27}$$

$$A_7 = 0/u_1+0/u_2+0/u_3+0/u_4+0/u_5+0.5/u_6+1/u_7; \tag{28}$$

The variation data are fuzzified into the corresponding fuzzy set as shown in Table 1.

**TABLE 1.** Historical data and fuzzified historical variation of the TAIEX from January 2 to December 29, 2004 [8].

| Date | TAIEX | Variation | Fuzzified Variation |
|---|---|---|---|
| 1/2 | 6041.56 | – | – |
| 1/5 | 6125.42 | 83.86 | $A_5$ |
| 1/6 | 6144.01 | 18.59 | $A_4$ |
| 1/7 | 6141.25 | –2.76 | $A_4$ |
| 1/8 | 6169.17 | 27.92 | $A_4$ |
| 1/9 | 6226.98 | 57.81 | $A_5$ |
| 1/12 | 6219.71 | –7.27 | $A_4$ |
| 1/13 | 6210.22 | –9.49 | $A_4$ |
| 1/14 | 6274.97 | 64.75 | $A_5$ |
| 1/15 | 6264.37 | –10.6 | $A_4$ |
| 1/16 | 6269.71 | 5.34 | $A_4$ |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 11/16 | 5910.85 | 4.16 | $A_4$ |
| 11/17 | 6028.68 | 117.83 | $A_5$ |
| 11/18 | 6049.49 | 20.81 | $A_4$ |
| 11/19 | 6026.55 | –22.94 | $A_4$ |
| 11/22 | 5838.42 | –188.13 | $A_3$ |
| 11/23 | 5851.1 | 12.68 | $A_4$ |
| 11/24 | 5911.31 | 60.21 | $A_5$ |
| 11/25 | 5855.24 | –56.07 | $A_4$ |
| 11/26 | 5778.65 | –76.59 | $A_4$ |
| 11/29 | 5785.26 | 6.61 | $A_4$ |
| 11/30 | 5844.76 | 59.5 | $A_5$ |
| ⋮ | ⋮ | ⋮ | ⋮ |

Step 3) Define the input IT2FSs and the output intervals of the IT2FLS. The maximum value of the variations $V_{max}$ and the minimum value of the variations $V_{min}$ are 341.49 and –455.17, respectively, and the two proper positive numbers are $D_1 = 4.83$ and $D_2 = 38.51$. The interval length of IT2FSs is [(341.49 + 38.51) – (–455.17 – –4.83)] / (3 × 7 – 3) = 46.67. The IT2FSs are defined by a 9-point vector, and the output intervals are defined by a 2-point vector. The IT2FSs and the output intervals are shown in Fig. 6.

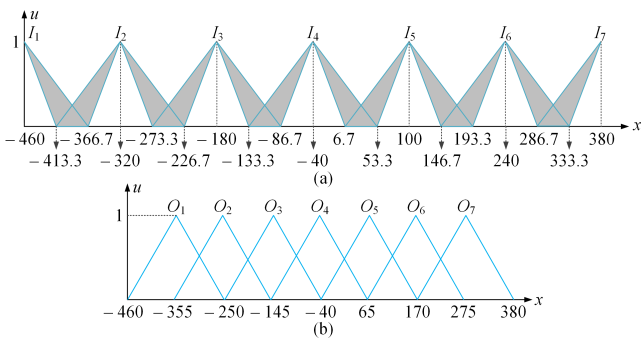


**FIGURE 6.** The IT2FSs and the output intervals used in this study. (a) IT2FSs (b) Output intervals.

Step 4) Establish the FLR of time $t$ and time $t + 1$, and the FLRM. Based on the historical data and fuzzified historical variations of the TAIEX index shown in Table 1, the FLR is supposed to be $A_i \rightarrow A_j$ in the training phase, where $A_i$ is on the left-hand side of time $t$ and $A_j$ is on the right-hand side of time $t + 1$; $i$ and $j$ are $1, 2, \ldots, n$; and $n$ is the number of intervals, respectively. The count of the FLRs for the TAIEX forecasting is listed in Table 2. For example, there are some FLRs as follows: $A_2 \rightarrow A_5$, $A_3 \rightarrow A_5$, $A_4 \rightarrow A_4$, and $A_4 \rightarrow A_5$ which the corresponding counts of the FLRs are 2, 6, 58, and 35, respectively.

**TABLE 2.** The count of the FLR.

| FLR | Count | FLR | Count |
|---|---|---|---|
| $A_1 \rightarrow A_3$ | 1 | $A_5 \rightarrow A_1$ | 1 |
| $A_2 \rightarrow A_4$ | 1 | $A_5 \rightarrow A_3$ | 3 |
| $A_2 \rightarrow A_5$ | 2 | $A_5 \rightarrow A_4$ | 40 |
| $A_3 \rightarrow A_2$ | 2 | $A_5 \rightarrow A_5$ | 18 |
| $A_3 \rightarrow A_3$ | 2 | $A_5 \rightarrow A_6$ | 3 |
| $A_3 \rightarrow A_4$ | 5 | $A_5 \rightarrow A_7$ | 1 |
| $A_3 \rightarrow A_5$ | 6 | $A_6 \rightarrow A_2$ | 1 |
| $A_4 \rightarrow A_3$ | 9 | $A_6 \rightarrow A_4$ | 4 |
| $A_4 \rightarrow A_4$ | 58 | $A_6 \rightarrow A_5$ | 3 |
| $A_4 \rightarrow A_5$ | 35 | $A_7 \rightarrow A_4$ | 1 |
| $A_4 \rightarrow A_6$ | 5 | $A_7 \rightarrow A_5$ | 1 |
| $A_4 \rightarrow A_7$ | 1 | – | – |

The FLRM $F$ is initialized to create an $n \times n$ matrix of zeros, and $n$ is the number of intervals. Based on the Eq. (23), the FLRM $F(j, i)$ is established by the count $C$ of the each FLR. The rows ($j$) and columns ($i$) of the FLRM $F(j, i)$ show the right-hand and left-hand linguistic terms, respectively. The FLRM established by the count of the FLR for the TAIEX forecasting is listed in Table 3. For example, If the count of $A_2 \rightarrow A_5$ is 2, then $F(5, 2) = 2$. If the count of $A_3 \rightarrow A_5$ is 6, then $F(5, 3) = 6$. If the count of $A_4 \rightarrow A_4$ is 58, then $F(4, 4) = 58$. If the count of $A_4 \rightarrow A_5$ is 35, then $F(5, 4) = 35$.

**TABLE 3.** The FLRM established by the count of the FLR.

| | | Left-hand side | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ |
| Right-hand side | $A_1$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| | $A_2$ | 0 | 0 | 2 | 0 | 0 | 1 | 0 |
| | $A_3$ | 1 | 0 | 2 | 9 | 3 | 0 | 0 |
| | $A_4$ | 0 | 1 | 5 | 58 | 40 | 4 | 1 |
| | $A_5$ | 0 | 2 | 6 | 35 | 18 | 3 | 1 |
| | $A_6$ | 0 | 0 | 0 | 5 | 3 | 0 | 0 |
| | $A_7$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

Step 5) Calculate the forecasted results. The forecasted values are calculated using algorithm 1. For example, assume that the forecasted value for November 22, 2004 is calculated. According to Table 1, the variation $V$ between November 18, 2004 and November 19, 2004 (November 20th and November 21st is the weekend.) is –22.94, and the fuzzified variation is $A_4$. The IT2FS is that the fuzzified variation $A_4$ corresponds to the IT2FS $I_4$. FLRMS becomes $[(-460 \times 0) + (-355 \times 0) + (-250 \times 9) + (-145 \times 58) + (-40 \times 35) + (65 \times 5) + (170 \times 1), (-250 \times 0) + (-145 \times 0) + (-40 \times 9) + (65 \times 58) + (170 \times 35) + (275 \times 5) + (380 \times 1)] = [-11565, 11115]$. The output interval $O_i$ is $[-11565, 11115]/(9 + 58 + 35 + 5 + 1) = [-107.08, 102.92]$. Then, the forecasted variation between November 19, 2004 and November 22, 2004 is $-2.08$. Finally, the forecasted value of November 22, 2004 is the actual value of November 19, 2004 plus the forecasted variation, and $6026.55 + (-2.08) = 6024.47$. Fig. 7 shows the TAIEX forecasting for testing phase in 2004.

There are two cases discussed here for comparison of TAIEX forecasting. *Case 1* uses data from 1999 to 2004 for TAIEX forecasting; *Case 2* uses data from 1990 to 1999 for TAIEX forecasting. The TAIEX data are divided into two parts for each year; the training phase was from January to October and the testing phase was from November to December.

### 1) CASE I

During the testing phase, the proposed method is compared with many existing forecasting methods [3]–[13], [18], [20], [24]–[36], [41]. Here, Fig. 7 shows the TAIEX forecasts for the testing phase from 1999 to 2004. In Fig. 7, the blue solid line indicates the actual stock value, and the red dashed line indicates the forecasted stock values obtained with the proposed method. The $Y$-axis represents the stock value and the $X$-axis represents the number of trading days in the testing period. During the six testing periods, it is found that the forecasted values are close to the actual values for both uptrends and downtrends in TAIEX variation. Also, similar results are obtained for TAIEX forecasts from 1990 to 1998, DJIA forecasts from 1999 to 2004, and NASDAQ forecasts from January 3, 2007 to December 20, 2010.

Table 4 shows the forecasting performance of the different forecasting models [8]. The test results show significant differences between the proposed method and previous methods including the multivariate heuristic fuzzy time series model proposed by Huarng *et al.* [10], the univariate fuzzy time series model (U_FTS model) [18], [26], the univariate conventional regression model (U_R model) [26], the univariate neural network model (U_NN model) [26], the univariate neural network-based fuzzy time series model (U_NN_FTS model) [25], [26], the univariate neural network-based fuzzy time series model with substitutes (U_NN_FTS_S model) [25], [26], the bivariate conventional regression model (B_R model) [26], the bivariate neural network model (B_NN model) [26], the bivariate neural network-based fuzzy time series model (B_NN_FTS model) [24], [26], the multivariable fuzzy forecasting method proposed by Chen and Chang [3], and the forecasting method based on fuzzy time series and fuzzy variation groups [9]. The results show that the proposed method outperforms these methods. However, there are no statistically significant differences between the proposed method and those methods using fuzzy time series and the automatically generated weights of multiple factors such as in the method proposed by Cheng *et al.* [8] and Chen *et al.* [11], the fuzzy forecasting method based on two-factor second-order fuzzy-trend logical relationship groups and the PSO techniques proposed by Chen *et al.* [5], the type-2 neuro-fuzzy modeling method (T2NFS) [12], the complex neuro-fuzzy system and autoregressive integrated moving average models (CNFS-ARIMA) [20], the direct and

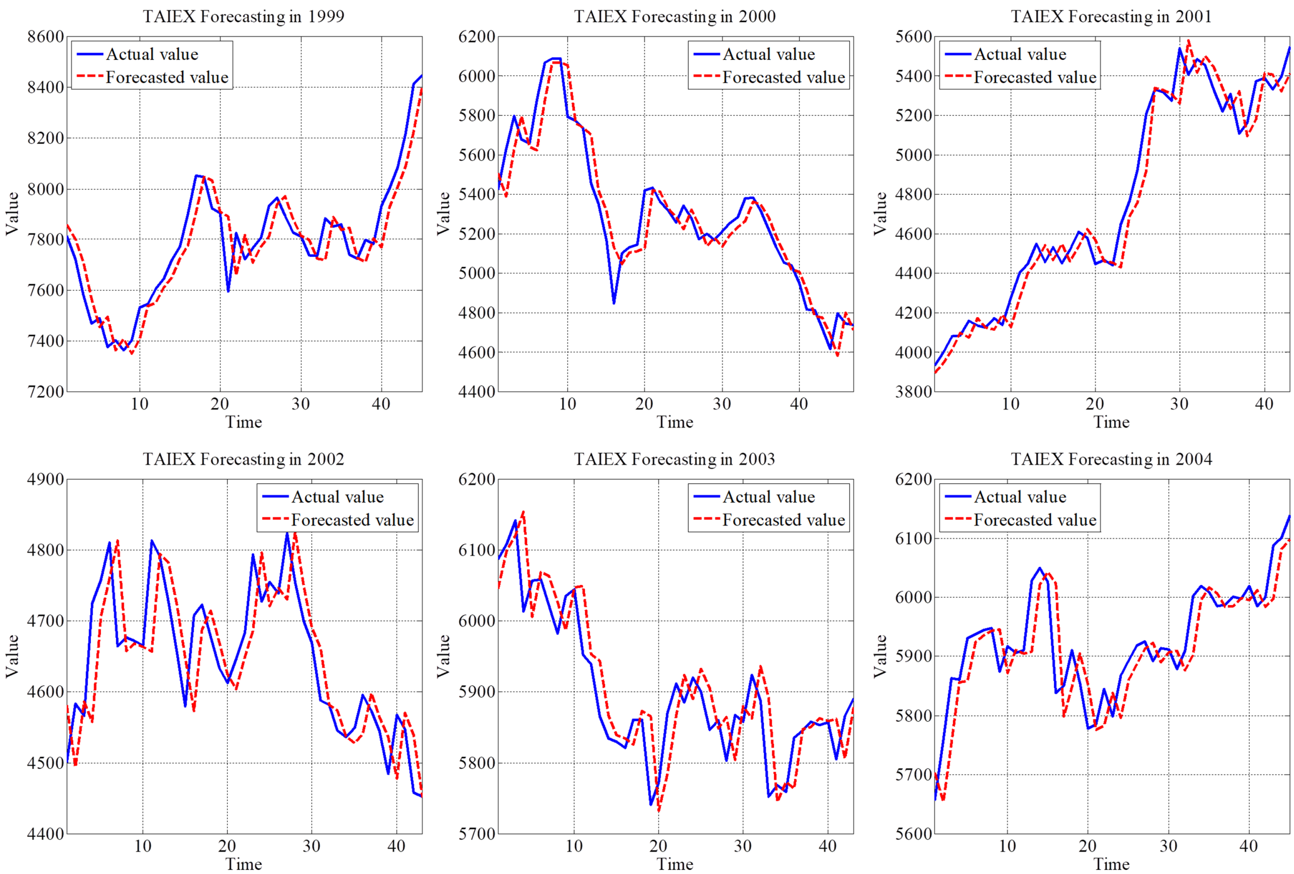


**FIGURE 7. TAIEX forecasting from 1999 to 2004.**

iterative local modeling based on the neuro-fuzzy forecasting model (LMNF-D/I) [13], the forecasting method based on two-factor second-order fuzzy-trend logical relationship groups and the probabilities of trends of FLRs proposed by Chen and Chen [7], and the new fuzzy time series model combined with ant colony optimization and auto-regression proposed by Cai *et al.* [41]. It should be noted that only one factor is considered in the proposed model for predicting the future index. Although there are no statistically significant differences between the proposed method and the afore-mentioned methods, it is reasonable to believe that with additional factors, the proposed method might even outperform the existing methods.

The three findings particularly worthy of discussion are detailed below. First, no statistically significant difference is found between the B_NN model [26] and the proposed method. The larger average RMSE and the SD of the B_NN method should lead to a possible difference between the proposed method and the B_NN method, but this contradicts the statistical results. Thus, the CV is calculated to compare the performance of the two methods. The fact that the CV of the proposed model is smaller than that of the B_NN model [26], shows it to have better forecasting accuracy. Second, there are no statistically significant differences between Chen and Chang's method [3] (with the indices of DJIA, NASDAQ, and M1b) and the proposed method. The RMSE obtained with Chen and Chang's method [3] for 2002 is smaller than that obtained with the proposed method, but for 2001 and 2003 they are very close in value to the RMSEs obtained with the proposed method. Finally, a statistically significant difference is found between LMNF-D/I [13] (with the indices of DJIA and NASDAQ) and the proposed method with the RMSEs obtained with the proposed method being smaller than those obtained with Chen and Chen's method [8], [13] for 1999, 2000, 2001, 2002, and 2003. In addition, although Chen and Chen's RMSEs [13] are smaller than those obtained with the proposed method for 2004, they are very close in value to those obtained with the proposed method.

### 2) CASE 2

Table 5 shows the forecasting performance of the different methods [8]. The statistical tests show that there are significant differences between conventional models [30], weighted models [30], the forecasting method based on FTS and fuzzy variation groups proposed by Chen and Chen [9], the forecasting method using FTS and automatically generated weights of multiple factors proposed by Cheng *et al.* [8] and Chen *et al.* [11] and the proposed method. The proposed method yields higher forecasting performance than any of these methods. However, no significant differences are found

**TABLE 4.** The forecasting accuracy with different methods based on the average RMSE, the SD, and the CV using the RMSE of the TAIEX data from 1999 to 2004 [8].

| Methods | | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | Avg. RMSEs ± SDs | CVs |
|---|---|---|---|---|---|---|---|---|---|
| Huarng *et al.*'s method [10] | Using A | – | 165.8 | 138.25 | 93.73 | 72.59 | 73.49 | 108.84 ± 41.54* | 0.38 |
| | Using B | – | 158.7 | 136.49 | 95.15 | 65.51 | 73.57 | 105.88 ± 40.36* | 0.38 |
| | Using C | – | 169.19 | 133.26 | 97.1 | 75.23 | 82.01 | 111.36 ± 39.35* | 0.35 |
| | Using A and B | – | 157.64 | 131.98 | 93.48 | 65.51 | 73.49 | 104.42 ± 39.31* | 0.38 |
| | Using B and C | – | 155.51 | 128.44 | 97.15 | 70.76 | 73.48 | 105.07 ± 36.49* | 0.35 |
| | Using A, B, and C | – | 154.42 | 124.02 | 95.73 | 70.76 | 72.35 | 103.46 ± 35.76* | 0.35 |
| U_FTS model [18][26] | – | 120 | 176 | 148 | 101 | 74 | 84 | 117.17 ± 39.10* | 0.33 |
| U_R model [26] | – | 164 | 420 | 1070 | 116 | 329 | 146 | 374.17 ± 360.89* | 0.96 |
| U_NN model [26] | – | 107 | 309 | 259 | 78 | 57 | 60 | 145.00 ± 110.27* | 0.76 |
| U_NN_FTS model [25][26] | – | 109 | 255 | 130 | 84 | 56 | 116 | 125.00 ± 68.85* | 0.55 |
| U_NN_FTS_S model [25][26] | – | 109 | 152 | 130 | 84 | 56 | 116 | 107.83 ± 33.96* | 0.31 |
| B_R model [26] | – | 103 | 154 | 120 | 77 | 54 | 85 | 98.83 ± 35.18* | 0.36 |
| B_NN model [26] | – | 112 | 274 | 131 | 69 | 52 | 61 | 116.50 ± 83.11 | 0.71 |
| B_NN_FTS model [26] | – | 108 | 259 | 133 | 85 | 58 | 67 | 118.33 ± 74.16* | 0.63 |
| B_NN_FTS_S model [24][26] | – | 112 | 131 | 130 | 80 | 58 | 67 | 96.33 ± 32.18* | 0.33 |
| Chen and Chang's method [3] | Using A | 101.97 | 148.85 | 113.7 | 79.81 | 64.08 | 82.32 | 98.46 ± 30.25* | 0.31 |
| | Using B | 123.64 | 131.10 | 115.08 | 73.06 | 66.36 | 60.48 | 94.95 ± 31.69* | 0.33 |
| | Using C | 156.92 | 142.70 | 132.76 | 96.06 | 90.27 | 100.10 | 119.80 ± 27.91* | 0.23 |
| | Using A and B | 106.34 | 130.13 | 113.33 | 72.33 | 60.29 | 68.07 | 91.75 ± 28.56* | 0.31 |
| | Using B and C | 116.22 | 134.63 | 116.59 | 76.48 | 53.51 | 69.29 | 94.45 ± 32.28* | 0.34 |
| | Using A, B, and C | 111.7 | 129.42 | 113.67 | 66.82 | 56.1 | 64.76 | 90.41 ± 31.33 | 0.35 |
| Chen and Chen's method [9] | Using A | 115.47 | 127.51 | 121.98 | 74.56 | 66.02 | 58.89 | 94.09 ± 30.86* | 0.33 |
| | Using B | 119.32 | 129.87 | 123.12 | 71.01 | 65.14 | 61.94 | 95.07 ± 32.12* | 0.34 |
| | Using C | 120.01 | 129.87 | 117.61 | 85.85 | 63.10 | 67.29 | 97.29 ± 28.95* | 0.3 |
| | Using A and B | 116.64 | 123.62 | 123.85 | 71.98 | 58.06 | 57.73 | 91.98 ± 32.71* | 0.36 |
| | Using B and C | 116.59 | 127.71 | 115.33 | 77.96 | 60.32 | 65.86 | 93.96 ± 29.27* | 0.31 |
| | Using B and C | 114.87 | 128.37 | 123.15 | 74.05 | 67.83 | 65.09 | 95.56 ± 29.57* | 0.31 |
| | Using A, B, and C | 112.47 | 131.01 | 117.86 | 77.38 | 60.65 | 65.09 | 94.08 ± 30.01* | 0.32 |
| Chen *et al.*'s method [11] | Using A | 99.87 | 122.75 | 117.18 | 68.45 | 53.96 | 52.55 | 85.79 ± 31.52 | 0.37 |
| | Using B | 102.60 | 119.98 | 114.81 | 69.07 | 53.16 | 53.57 | 85.53 ± 30.58 | 0.36 |
| | Using C | 101.22 | 123.99 | 117.75 | 70.63 | 54.92 | 55.29 | 87.30 ± 31.04 | 0.36 |
| | Using A and B | 101.33 | 121.27 | 114.48 | 67.18 | 52.72 | 52.27 | 84.88 ± 31.25 | 0.37 |
| | Using A and C | 100.59 | 124.10 | 116.28 | 68.11 | 53.5 | 53.33 | 85.99 ± 31.7 | 0.37 |
| | Using B and C | 102.25 | 122.47 | 115.02 | 68.51 | 52.82 | 53.99 | 85.84 ± 31.2 | 0.36 |
| | Using A, B, and C | 101.47 | 122.88 | 114.47 | 67.17 | 52.49 | 52.84 | 85.22 ± 31.57 | 0.37 |
| Chen *et al.*'s method [5] | Using A | 102.11 | 131.30 | 113.83 | 66.45 | 52.83 | 54.17 | 86.78 ± 33.4 | 0.38 |
| | Using B | 102.34 | 131.25 | 113.62 | 65.77 | 52.23 | 56.16 | 86.89 ± 33.2 | 0.38 |
| | Using C | 103.52 | 131.36 | 112.55 | 66.23 | 53.20 | 55.36 | 87.04 ± 33.07 | 0.38 |
| T2NFS [12] | – | 100 | 128 | 111 | 64 | 51 | 54 | 84.67 ± 32.58 | 0.38 |
| | Using A | 97.3 | 124.28 | 110.01 | 60.05 | 52.24 | 51.8 | 82.61 ± 31.89 | 0.39 |
| | Using B | 99.63 | 121.94 | 108.68 | 64.83 | 51.05 | 51.81 | 82.99 ± 30.91 | 0.37 |
| | Using A and B | 99.04 | 120.9 | **103.84** | **58.1** | 52.49 | 51.73 | 81.02 ± 30.44 | 0.38 |
| CNFS-ARIMA [20] | – | 100.01 | 122.58 | 115.82 | 64.34 | 57.69 | 55.56 | 86.00 ± 30.40 | 0.35 |
| LMNF-D/I [13] | – | 97.24 | 123.33 | 118.76 | 67.54 | 52.35 | 53.63 | 85.47 ± 31.98 | 0.37 |
| | Using A | 97.36 | 130.22 | 122.08 | 66.99 | **50.9** | 56.28 | 87.31 ± 34.21 | 0.39 |
| | Using B | **92.19** | 129.88 | 116.73 | 63.66 | 54.78 | 56.21 | 85.58 ± 32.46 | 0.38 |
| | Using A and B | 98.68 | 129.14 | 122.77 | 68.42 | 56.14 | 56.06 | 88.54 ± 32.96* | 0.37 |
| Chen and Chen's method [7] | Using A | 103.90 | 127.32 | 115.37 | 64.71 | 52.84 | 53.36 | 86.25 ± 33.19 | 0.38 |
| | Using B | 104.99 | 124.52 | 114.66 | 64.79 | 53.63 | 52.96 | 85.93 ± 32.42 | 0.38 |
| | Using C | 105.61 | 127.37 | 115.46 | 66.07 | 53.67 | 53.30 | 86.91 ± 33.08 | 0.38 |
| Cai *et al.*'s method [41] | – | 102.22 | 131.53 | 112.59 | 60.33 | 51.54 | **50.33** | 84.75 ± 35.08 | 0.41 |
| Proposed method | – | 97.61 | **119.73** | 113.26 | 67.39 | 54.95 | 56.21 | 84.86 ± 29.00 | 0.34 |

***Note**: A (DJIA); B (NASDAQ); C (M1b); "*" means statistically significant difference against the proposed method.*

between the forecasting method based on FTS, PSO techniques, and the support vector machine (SVM) method proposed by Chen and Kao [42], the forecasting method based on two-factor second-order fuzzy-trend logical relationship groups and the probabilities of trends of FLRs using DJIA and NASDAQ proposed by Chen and Chen [7], the new fuzzy time series model combined with ant colony optimization and auto-regression proposed by Cai *et al.* [41] and the proposed method. It is noted that in the proposed model, Chen and Kao's method [42], and Cai *et al.*'s method [41], only one factor is considered when predicting the future index. Chen and Kao [42] used PSO techniques, which

**TABLE 5.** The forecasting accuracy with different methods based on the average RMSE, the SD, and the CV using the RMSE of the TAIEX data from 1990 to 1999 [8].

| Methods | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | Avg. RMSEs ± SDs | CVs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Conventional models [30] | 220 | 80 | 60 | 110 | 112 | 79 | 54 | 148 | 167 | 149 | 117.90 ± 52.91* | 0.45 |
| Weighted models [31] | 227 | 61 | 67 | 105 | 135 | 70 | 54 | 133 | 151 | 142 | 114.5 ± 54.12* | 0.47 |
| Chen and Chen's method [9] | 172.89 | 72.87 | 43.44 | 103.21 | 78.63 | 66.66 | 59.75 | 139.68 | 124.44 | 115.47 | 97.70 ± 40.56* | 0.42 |
| Chen *et al.*'s method [11] | 174.62 | 43.22 | 42.66 | 104.17 | 94.6 | 54.24 | 50.5 | 138.51 | 117.87 | 101.33 | 92.17 ± 44.55* | 0.48 |
| Chen and Kao's method [42] | **156.47** | 56.5 | **36.45** | 126.45 | **62.57** | 105.52 | 51.5 | 125.33 | **104.12** | **87.63** | 91.25 ± 38.94 | 0.43 |
| Chen and Chen's method using DJIA [7] | 180.36 | 43.8 | 43.06 | 104.89 | 75.35 | 55.06 | 50.06 | 133.82 | 112.11 | 103.9 | 90.24 ± 45.16 | 0.50 |
| Chen and Chen's method using NASDAQ [5] | 174.15 | 45.04 | 42.1 | 104.94 | 76.4 | 54.96 | 50.17 | 133.45 | 113.37 | 104.99 | 89.96 ± 43.82 | 0.49 |
| Cai *et al.*'s method [41] | 187.1 | **39.58** | 39.37 | 101.8 | 76.32 | 56.05 | **49.45** | **123.98** | 118.41 | 102.34 | 89.44 ± 46.87 | 0.52 |
| Proposed method | 175.41 | 43.24 | 41.43 | **95.53** | 88.08 | **53.07** | 49.52 | 136.74 | 112.89 | 97.61 | 89.35 ± 44.27 | 0.50 |

*Note: "*" means statistically significant difference against the proposed method.*

**TABLE 6.** The forecasting accuracy with different methods based on the average RMSE, the SD, and the CV using the RMSE of the DJIA data from 1999 to 2004.

| Methods | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | Avg. RMSEs ± SDs | CVs |
|---|---|---|---|---|---|---|---|---|
| SVR (two models, each with single output) [43][44] | 109.05 | 134.78 | 101.44 | 117.95 | 82.76 | 71.49 | 102.91 ± 21.13* | 0.21 |
| ANFIS (two models, each with single output) [45] | 111.2 | 135.76 | 105.56 | 111.69 | 72.09 | 68 | 100.72 ± 25.97* | 0.26 |
| ANFIS (one model with two outputs) [45] | 120.15 | 138.71 | 128.2 | 142.05 | 90.37 | 83.69 | 117.2 ± 24.71* | 0.21 |
| RBF (two models, each with single output) [46] | 128.38 | 143.13 | 106.33 | 131.24 | 97.58 | 81.79 | 114.74 ± 23.3* | 0.2 |
| RBF (one model with two outputs) [46] | 149.96 | 158.41 | 181.79 | 136.28 | 154.14 | 148.11 | 154.78 ± 15.19* | 0.1 |
| CNFS(4)-ARIMA(4, 2, 0) [20] | 102.05 | 130.69 | 103.06 | **103.42** | 70.7 | 66.55 | 96.08 ± 23.88* | 0.25 |
| Proposed method | **83.04** | **127.74** | **96.18** | 105.45 | **58.86** | **61.71** | 88.83 ± 26.5 | 0.3 |

***Note**: "*" means statistically significant difference against the proposed method.*

enables optimal intervals in the universe of discourse, thereby ensuring higher forecasting accuracy. Cai *et al.* [41] used ant colony optimization to suitably partition the universe of discourse, and an auto-regression method, which was able to obtain better high-order information and thereby promote forecasting performance. The proposed method only uses a traditional partitioning method for partitioning the universe of discourse and the second-order historical data to forecast the future index. It is reasonable to believe that with the inclusion of additional factors, suitable intervals in the universe of discourse, and a higher order method, the proposed method could outperform previous methods.

### B. DJIA FORECASTING

For a comparison of DJIA forecasting, data from 1999 to 2004 were chosen. The data from January to October each year were used for the training phase and the data from November to December were used for the testing phase. The proposed method is again compared with existing methods, including the support vector regression (SVR) [43], [44], adaptive neuro-fuzzy inference system (ANFIS) [45], radial basis function (RBF) [44], and CNFS-ARIMA methods [20]. From Table 6, we can see that the results obtained with proposed method are statistically and significantly different from all other existing methods. The proposed method yields better forecasting performance.

### C. NASDAQ FORECASTING

The daily opening and closing indices of NASDAQ from January 3, 2007 to December 20, 2010 were chosen for

**TABLE 7.** The forecasting accuracy with different methods using the RMSE of the NASDAQ data from January 3, 2007 to December 20, 2010.

| Methods | Opening index | Closing index |
|---|---|---|
| SVR (two models, each with single output) [43][44] | 37.23 | 40.24 |
| ANFIS (two models, each with single output) [45] | 38.8 | 42.36 |
| ANFIS (one model with two outputs) [45] | 72.52 | 85.08 |
| RBF (two models, each with single output) [46] | 37.52 | 44.08 |
| RBF (one model with two outputs)[46] | 261.37 | 258.89 |
| CNFS(5)-ARIMA(4, 2, 0) [20] | 32.52 | 33.7 |
| Proposed method | **26.62** | **29** |

comparison. The data of the training phase were from January 3, 2007 to October 26, 2008, and the data of the testing phase were from October 27, 2008 to December 20, 2010. The proposed method is again compared with existing methods, including SVR [43], [44], ANFIS [45], RBF [46], and CNFS-ARIMA [20]. The results in Table 7 show that the proposed method has the smallest RMSEs for the forecast opening and closing indices of NASDAQ compared to the other methods.

Finally, in summary, consider the first case, TAIEX forecasting from 1999 to 2004. The results show there to be no statistically significant difference between the methods published after 2012 and our proposed method. However, an important point needs to be addressed, which is that the proposed method only uses one factor to forecast the TAIEX, while other methods use one or more factors to forecast the TAIEX, which presumably improves their forecasting accuracy.

The second case features TAIEX forecasting from 1990 to 1999. The results of the statistical analysis show no statistically significant difference between the methods published after 2013 and the proposed method. It should be noted that in the proposed method only a traditional method is used to partition the universe of discourse and second-order historical data used for TAIEX forecasting. Nevertheless, our method yields a similar forecasting accuracy comparable to other methods that use an advanced partitioning method and a high-order method to forecast the TAIEX.

Moreover, the statistical results from DJIA forecasting show that the proposed method is significantly better than other methods. Furthermore, predictions of the opening and closing indices of the NASDAQ achieved by the proposed method also show the smallest RMSE values. In all three cases, the proposed method generally outperforms the other methods. Based on the analytical results, with the proposed model, a simpler method can be used to achieve a similar level of forecasting accuracy compared to other methods that adopt more complex or advanced methods. Therefore, the proposed model is capable of being applied to financial modelling.

## VI. CONCLUSIONS AND FUTURE WORK

In this study, an IT2FLS for stock index forecasting based on fuzzy time series and an FLRM is presented. The input IT2FSs and the output intervals are generated using the minimum and the maximum variations found in the training data. The FLRM is developed based on the FLRs of the training data, and is used to form the output interval of the IT2FLS. Previous studies have shown that a smaller average RMSE indicates higher forecasting accuracy, but it is problematic to declare higher forecasting accuracy based solely on the smaller average RMSE, due to the lack of standardization of measurement. Unlike previous studies, this study uses a statistical method to examine the forecasting performance of both the proposed method and other methods. In the first case of TAIEX forecasting, the proposed method is found to offer better forecasting performance than the other methods presented before 2011, and shows no difference after 2012. Moreover, no statistically significant difference is found between the methods developed after 2012 and the proposed method. One possible reason for this is that the proposed method only uses one factor to forecast the TAIEX, while other methods use one or more factors, which may improve the forecasting performance. In the second case of TAIEX forecasting, the proposed method is found to offer better forecasting performance than the other methods published before 2012, and no difference in forecasting performance for those developed after 2013. Moreover, no statistically significant difference is found between the methods presented after 2013 and the proposed method. There are two possible reasons for this; the proposed method only uses a traditional technique for partitioning of the universe of discourse and second-order historical data to forecast the TAIEX, while other methods use a more advanced partitioning method, and a higher-order method, which may improve the forecasting accuracy. Moreover, in terms of DJIA forecasting, the proposed method offers better forecasting performance than other methods. In NASDAQ forecasting, the proposed method has the smallest RMSE, for both opening and closing index forecasting. These applied forecasting areas show that the proposed method generally outperforms the other methods, and has the potential to obtain higher forecasting accuracy than other methods by using more factors, a better partitioning method, and a higher-order method.

Though the proposed method only uses one factor, a traditional technique for partitioning of the universe of discourse, and second-order historical data to forecast the stock index and from the forecasting results of this study using three databases of TAIEX, DJIA, and NASDAQ, the proposed method is found to offer better forecasting performance or no statistically significant difference than the other methods. There are also two limitations of this work that need to be addressed: 1) IT2FSs represents uncertainty and complexity and they are represented by many parameters. It is a problem to select proper parameters for type-2 fuzzy sets in order to obtain better performance [47]. 2) The universe of discourse in the fuzzy time series is partitioned into segments of equal length in this study. It is thus hard to reliably reflect the data distribution, and it is difficult to select the proper interval lengths, to improve forecasting accuracy [28]. In the future, a method using multiple factors (e.g., DJIA, NASDAQ, and M1b) could be applied to stock index forecasting. It has been shown in previous studies that adopting more factors can improve forecasting performance [3]–[13]. The limitations mentioned above could be resolved by implementing one of the following methods: 1) the piecewise-linear approach presented by Ibarra *et al.* [47] can be used for type-2 fuzzy membership function design; 2) Chen and Chen [28] presented a granular computing method with binning-based partitioning and entropy-based discretization methods to solve interval problems. 3) Lagunes *et al.* [48] reported that it could improve the type-2 fuzzy logic model by parameter optimization for the membership functions. 4) Considering more factors, more advanced partitioning methods [49]–[52] and a higher-order method. All these methods would be the possible ways to further strengthen and improve the forecasting results in the nearly future.

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