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Forecasting TAIEX using improved type 2 fuzzy time series

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ABSTRACT

This paper presents a new method to forecast TAIEX based on a high-order type 2 fuzzy time series. Extra observations are used to improve forecasting performance. Extra observations are modeled as type 2 fuzzy sets and fourth-order fuzzy time series. Our proposed model outperforms previous studies.

1. Introduction

Fuzzy time series models have been applied to handle nonlinear problems. There have been different fuzzy methods proposed to solve fuzzy time series problems. Tanaka (1987), Tanaka and Ishibuchi (1992), Tanaka, Uejima, and Asai (1982) proposed linear programming to solve problems in fuzzy regression. Watada applied fuzzy regression to solve the problems of fuzzy time series (Watada, 1992). Tseng et al. extended fuzzy regression for autoregressive integrated moving average analyses (Tseng & Tzeng, 2002; Tseng, Tzeng, Yu, & Yuan, 2001). On the other hand, Song and Chissom have proposed novel definitions for fuzzy time series (Song & Chissom, 1993b). Chen improved Song and Chissom's model (Chen, 1996). Following these definitions, fuzzy time series models have been proposed for various applications, such as enrollment (Chen, 1996; Huarng, 2001a; Hwang, Chen, & Lee, 1998; Nguyen & Wu, 2000; Song & Chissom, 1993a, 1994; Sullivan & Woodall, 1994), stock indexes (Hsu, Tse, & Wu, 2003; Huarng, 2001a, 2001b; Huarng & Yu, 2003; Li-Wei, Li-Hui, Shyi-Ming, & Yung-Ho, 2006; Ming-Tao, 2008; Yu, 2005a, 2005b), temperature (Chen & Hwang, 2000), reactors (Versaci & Morabito, 2003).

A fuzzy time series essentially consists of steps such as fuzzification, the establishment of fuzzy relationships, and defuzzification.

Fuzzy time series can model forecasting problem that the historical information are linguistic value. In addition, type 2 fuzzy time series can use more observation in forecasting.

Fuzzy time series can be used to deal with forecasting problems in which historical data are linguistic value.

This study aims to propose a new method to forecast Taiwan stock index based on optimized high-order type 2 fuzzy time series.

The rest of this paper is organized as follows. Section 2 reviews the relevant studies covered in this study, including Type 1 models and Type 2 fuzzy sets. Section 3 describes Huarng and Yu and the proposed algorithm for type 2 fuzzy time series models. Section 4 compares the empirical results from using the different model.

2. Review

Definition 1. Let U be the universe of discourse, where $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set A of U is defined by:

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n, \tag{1}$$

where f_A is the membership function of fuzzy set A, f_A: $U \to [0,1]$. u_i is the element of fuzzy set A, and $f_A(u_i)$ is the degree of belongingness of u_i to A, $f_A(u_i) \in [0,1]$, $1 \leqslant i \leqslant n$.

Definition 2. Fuzzy time series: $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, is a subset of R. Let $Y(t)$ be the universe of discourse defined by fuzzy set $f_i(t)$. If $F(t)$ consists of $f_i(t) (i = 1, 2, \ldots)$, $F(t)$ is defined as a fuzzy time series on $Y(t)$.

From Definition 1, we can see that $F(t)$ can be regarded as a linguistic variable (Chen, 1996) and $f_i(t)$ can be viewed as possible linguistic values of $F(t)$, where $f_i(t)$ are represented by fuzzy sets.

Chen (1996), has used the following two examples to explain the concepts of fuzzy time series: observe the mood of a person with normal mental conditions during a period of time, where the mood of a person can be expressed according to his own feeling using fuzzy sets "good", "very good", "very very good", "really good", "bad", "not bad", "not too bad",..., etc. This mood can be modeled with fuzzy time series.

Definition 3. Let $F(t-1) = A_i$ and $F(t) = A_j$ The relationship between $F(t-1) = A_i$ and $F(t)$(referred to as a fuzzy logical relationship, FLR) can be denoted by $A_i \to A_j$, where A_i is called the LHS (left-hand side) and the A_j, RHS (right-hand side) of the FLR.

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Definition 4. Suppose there are the following FLR:

$$\begin{aligned} &A_i \to A_{j1},\\ &A_i \to A_{j1},\\ &\vdots\\ &A_i \to A_{jl}. \end{aligned} \tag{2}$$

Following Chen's model (Chen, 1996), these FLRs can be grouped into an FLRG as:

$$A_i \to A_{j1}, A_{j2}, \ldots, A_{jl}. \tag{3}$$

Definition 5. Type 2 fuzzy sets: we explain it with an example from Kunhuang and Hui-Kuang (2005).

Suppose there is a fuzzy set for "approximate 2," as in Fig. 1. We have a crisp degree of membership value of 1 for $x = 2$ and a crisp degree of membership value of 0.5 for $x = 1$. Based on these type 2 fuzzy sets, there can be a fuzzy set for any degree of membership. For example, there is a triangular fuzzy set (0.4,0.5,06) as a degree of membership for $x = 1$ in Fig. 2. In other words, for the same x, there can be multiple degrees of membership. To apply this concept, we can adopt more observations for forecasting in each time slot.

Definition 6. A type 2 fuzzy time series model is defined as an extension of a Type 1 model. This type 2 model utilizes the fuzzy relationships established by a Type 1 model based on Type 1 observations. Operators are used to include or screen out fuzzy relationships obtained from Types 1 and 2 observations. Type 2 forecasts are then calculated from these fuzzy relationships.

Following Definition 6, we define two operators. The first involves including and the other screening out fuzzy relationships. Hence, we propose union and intersection operators in type 2 model accordingly.

Definition 7. Union ($\vee$) and intersection ($\wedge$) operators are defined to compute the relationships between two FLRGs:

$$\begin{aligned} &\vee(LHS_d, LHS_e) = RHS_d \cup RHS_e,\\ &\wedge(LHS_d, LHS_e) = RHS_d \cap RHS_e, \end{aligned} \tag{4}$$

where $\cup$ is the union and $\cap$ the intersection operator for set theory. LHS_d and RHS_d are the LHS and RHS of an FLRG respectively.

Definition 8. The union and intersection operators for multiple FLRGs, $\vee_m$ and $\wedge_m$ are defined as:

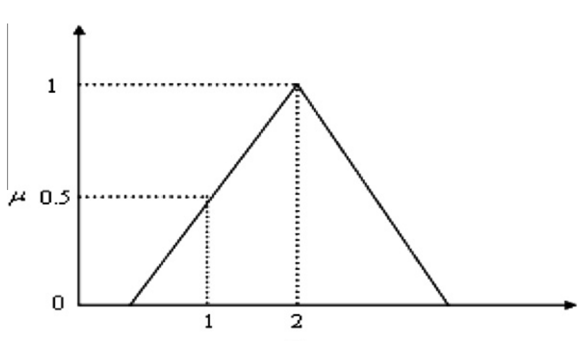

Fig. 1. Type 1 fuzzy set.

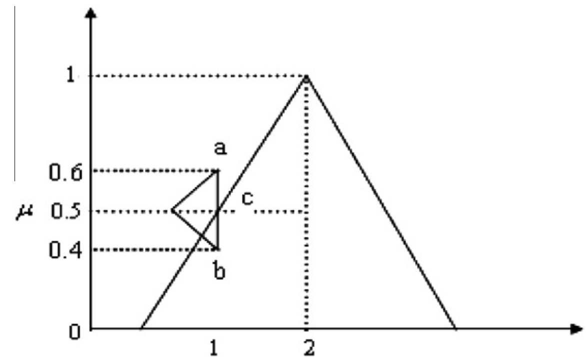

Fig. 2. Type 2 fuzzy set.

$$\begin{aligned} &\vee_m(LHS_c, LHS_d, LHS_e, \ldots) = \vee \ldots (\vee(\vee(LHS_c, LHS_d), LHS_e), \ldots),\\ &\wedge_m(LHS_c, LHS_d, LHS_e, \ldots) = \wedge \ldots (\wedge(\wedge(LHS_c, LHS_d), LHS_e), \ldots), \end{aligned} \tag{5}$$

$$\begin{aligned} &\vee_m(LHS_c, LHS_d, LHS_e, \ldots) = (RHS_c \cup RHS_d \cup RHS_e \cup \ldots),\\ &\wedge_m(LHS_c, LHS_d, LHS_e, \ldots) = (RHS_c \cap RHS_d \cap RHS_e \cap \ldots). \end{aligned} \tag{6}$$

Definition 9

(a) If $\vee_m(LHS_c, LHS_d, LHS_e, \ldots) = \phi$ then let $\vee_m(LHS_c, LHS_d, LHS_e, \ldots) = LHS_x$ where LHS_x is obtained from the FLRG established by Type 1 observations.
(b) If $\wedge_m(LHS_c, LHS_d, LHS_e, \ldots) = \phi$ then let $\wedge_m(LHS_c, LHS_d, LHS_e, \ldots) = LHS_x$ where LHS_x is obtained from the FLRG established by Type 1 observations.

Definition 10. Assume that $F(t)$ is a fuzzy time-series and $F(t)$ is caused by $F(t-1)$, $F(t-2), \ldots$, and $F(t-n)$, then the fuzzy logical relationship can be represented as follows: $F(t-1)$, $F(t-2), \ldots$, $F(t-n) \to F(t)$. This expression is called the nth-order fuzzy time-series forecasting model, where $n \geqslant 2$ (Tanaka & Ishibuchi, 1992; Watada, 1992)

3. Huarng and Yu model and proposed model for forecasting type 2 fuzzy time series

An algorithm for type 2 model is proposed by Huarng and Yu, This algorithm for type 2 models is listed below (Versaci & Morabito, 2003).

1. Choose a Type 1 fuzzy time series model.
2. Pick a variable and Type 1 observations.
3. Apply the Type 1 model to the Type 1 observations and obtain FLRGs.
4. Pick type 2 observations.
5. Map out-of-sample observations to FLRGs and obtain forecasts.
6. Apply operators to the FLRGs for all the observations.
7. Defuzzify the forecasts.
8. Calculate forecasts for type 2 model.
9. Evaluate the performance.

We apply some changes to this model:

1. Using triangular fuzzy set with indeterminate legs and optimizing these triangular fuzzy sets.
2. Using indeterminate coefficient in calculating type 2 forecasting.
3. Using center of gravity defuzzifier.
4. Using 4-order type 2 fuzzy time series.

Huarng and Yu model is described at first because our model is optimizing Huarng and Yu model. Application of Huarng and Yu model and proposed model for forecasting taiwan stock index.

3.1. Huarng and Yu model

Each step of type 2 fuzzy time series model is further illustrated below:

Step 1: Choose a Type 1 fuzzy time series model:
Among Type 1 models, Chen's model (Chen, 1996) provides simple calculations as well as better forecasting results. Hence, in this study we choose Chen's model with proposed changes as the Type 1 model.

Step 2. Pick a variable and Type 1 observations:
The TAIEX (Taiwan Stock Exchange apitalization Weighted Stock Index) is chosen as the forecasting target. The Closing price has been chosen as the forecasting target in many applications (Bajestani & Zare, 2009; Chao-Chih & Shun-Jyh, 2000; Chen & Hwang, 2000; Ching-Hsue, Tai-Liang, Hia Jong, & Chen-Han, 2009; Hsu et al., 2003; Kunhuang & Hui-Kuang, 2005; Versaci & Morabito, 2003). Hence, this study also chooses closing as Type 1 observation.

Step 3: Apply the Type 1 model to the Type 1 observation and obtain FLRGs Following Chen's model, the forecasting is conducted as follows:

Step 3–1: Define the universe of discourse and the intervals for the observations:
For example, we determine the following values for the TAIEX in the year 2000:
$D_{\min}$ = 4614.03 and $D_{\max}$ = 10202.02. Hence, the universe of discourse, U = [4600,10,300]. Then, U is partitioned into 57 intervals with equal lengths of 100:

$$u_1 = [4600, 4700],\ u_2 = [4700, 4800],\ \ldots,\ u_{57} = [10, 200, 10, 300].$$

Step 3–2: Define fuzzy sets for observations
Each A_i is defined by the intervals $u_1, u_2, \ldots, u_{57}$.

$$\begin{aligned} A_1 &= 1/u_1 + 0.5/u_2 + \cdots + 0/u_{57} \\ &\vdots \\ A_{16} &= 0/u_1 + 0/u_2 + \cdots + 1/u_{57}. \end{aligned} \tag{7}$$

Step 3–3: Fuzzify the observations
Different TAIEX values can be fuzzified into corresponding fuzzy sets. For example, the TAIEX for 2000/10/2 was 6024.07, which is fuzzified to:A_{15} and the TAIEX for 2000/10/3 was 6143.44, which is fuzzified to A_{16}. Some fuzzified TAIEX for the year 2000 are listed in Table 1.

Table 1
Fuzzy sets.

Date	TAIEX	Fuzzy sets
⋮	⋮	⋮
2000/10/2	6024.07	A_{15}
2000/10/3	6143.44	A_{16}
2000/10/4	5997.92	A_{14}
2000/10/5	6029.65	A_{15}
⋮	⋮	⋮

Step 3–4: Establish fuzzy relationships
We can establish FLRs by putting two consecutive fuzzy sets together. Following the above examples, the TAIEX for 2000/10/2 is A_{15} and for 2000/10/3 it is A_{16}. Hence, we can establish an FLR as $A_{15} \rightarrow A_{16}$. Some FLRs are listed in Table 2.
Following Table 2, we can establish the FLRGs. Some of the FLRGs are listed in Table 3.

Step 4: Pick type 2 observations
In this study, we pick high (the highest price of the day) and low (the lowest price of the day) as type 2 observations. We list some data for the year 2000 for explanatory purposes in Table 4. The first column is the date, the second is closing and its corresponding fuzzy set, and the third and fourth are high and low and their corresponding fuzzy sets, respectively.

Step 5: Map out-of-sample observations to FLRGs and obtain forecasts.
We can then map the out-of-sample observations, including Types 1 and 2 observations to FLRGs and obtain forecasts. Suppose $F(i) = A_i$ and the FLRG for $A_i \rightarrow A_{i1}, A_{i2}, \ldots, A_{iz}$. The forecast for $F(t)$ is $A_{i1}, A_{i2}, \ldots, A_{iz}$. For example if $F(t-1)$ is A_{11}, the forecast for $F(t-1)$ is A_{12}, A_{14}, A_{10}.
We can obtain the forecasts for both Types 1 and 2 observations. In Table 5, the first column consists of the date within the year 2000. The second column comprises the forecasts. The second column is divided further into three rows.
For each date, Type 1 forecasts (for closing) and type 2 forecasts (for high and low).

Step 6: Apply operators to the FLRGs for all the observations.
We then apply both the $\wedge_m$ and $\vee_m$ to these forecasts for each date. In Table 5, $\wedge_m$ and $\vee_m$ are applied to all the forecasts, including those for Types 1 and 2 observations for example, For 11/7:

$$\wedge_m(A_{13}, A_{13}, A_{12}) = \{A_{11}, A_{13}\} \cap (\{A_{11}, A_{13}\} \cap \{A_9\}) = \phi.$$

Because the result is an empty set ϕ and the LHS of Type 1 observation is A_{13}. Hence, $\wedge_m$ (A_{13}, A_{13}, A_{12}) = A_{13} And $\vee_m$ (A_{13}, A_{13}, A_{12}) = $\{A_{11}, A_{13}\} \cup (\{A_{11}, A_{13}\} \cup \{A_9\}) = \{A_{11}, A_{13}, A_9\}$Supposing the forecast is $A_{q1}, A_{q2}, \ldots, A_{qj}$ the defuzzified forecast is:

$$\text{defuzzification}_k(t) = \frac{\sum_{z=1}^{j} m_{qz}}{j},$$

m_{qz} is the midpoint of u_{qz}.
For example, in the case of $\wedge_m$, the forecast for 11/7 is A_{13} and the defuzzified forecast of A_{13} is equal to m_{13}. Thus:

$$\text{deffuzzification}_{\text{intersection}}(11/7) = 5850.$$

Similarly, in the case of $\vee_m$ the forecast for 11/7 is A_9, A_{11}, A_{13}. The defuzzified forecast is equal to:

$$\text{defuzzification}_{\text{union}}(11/7) = \frac{(5450 + 5650 + 5850)}{3} = 5650.$$

Step 8: Calculate forecasts for type 2 model The forecasts from type 2 model are calculated as follows:

Table 2
Fuzzy logic relationship.

$A_{15} \rightarrow A_{16}$,	$A_{16} \rightarrow A_{14}$,	$A_{14} \rightarrow A_{15}$
$A_{15} \rightarrow A_{18}$,	$A_{18} \rightarrow A_{18}$,	$A_{18} \rightarrow A_{17}, \ldots$

Table 3
Fuzzy logic relationship group.

group	1: $A_5 \to A_9$
group	2: $A_9 \to A_5, A_{10}$
group	3: $A_{10} \to A_{11}$
group	4: $A_{11} \to A_{12}, A_{14}, A_{10}$
⋮	

Table 4
Data for forecasting.

Date	Closing	High	Low
⋮	⋮	⋮	⋮
11/7	5877.77(A_{13})	5877.77	5720.89(A_{12})
11/8	6067.94(A_{15})	6164.62(A_{16})	5889.01(A_{13})
11/9	6089.55(A_{15})	6089.55(A_{15})	5926.64(A_{14})
⋮	⋮	⋮	⋮

$$\begin{aligned}\text{defuzzification}(11/7) &= \frac{\text{defuzzification}_{\text{intersection}} + \text{defuzzification}_{\text{union}}}{2}\\ &= \frac{5850 + 5650}{2} = 5750.\end{aligned}$$

Step 9: Evaluate the performance
The RMSE is used to evaluate the forecasting performance:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\text{actual value}(t) - \text{forecasted value}(t))^2}{n}}$$

3.2. Proposed model

In Huarng and Yu model and other old method, fuzzy sets and membership functions are determined from the beginning but we use triangular fuzzy sets with indeterminate legs in Fig. 3. This means that a1, a2, b1, b2,... are not fixed and they will be determined after the solution of an optimization problem (Bajestani & Zare, 2009).

Our optimization problem is:

$$\text{Min } RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\text{actual value}(t) - \text{forecasted value}(t))^2}{n}}$$
$$\text{s.t} \quad \text{membership function parameter.}$$

This is the RMSE of type 2 model.

It means that membership functions are selected in a way that the RMSE of type 2 model gets its minimum value.

Membership function information is the condition that must be observed for membership functions. For example:

$$\begin{aligned}& a1 = 4600, \quad b1 \geqslant 4700, \quad b1 \leqslant 10{,}300\\ & a2 \geqslant 4600, \quad a2 \leqslant 4700, \quad b2 \geqslant 4800, \quad b1 \leqslant 10{,}300\\ & \vdots\\ & a111 \geqslant 4600, \quad a111 \leqslant 10{,}100, \quad b112 \geqslant 10{,}200, \quad b1 \leqslant 10{,}300\\ & a113 \geqslant 4600, \quad a113 \leqslant 10{,}200, \quad b114 = 10300.\end{aligned}$$

Table 5
Forecast after $\wedge_m$ and $\vee_m$.

Date			Forecast after $\wedge_m$	Forecast after $\wedge_m$
11/7	Closing	$A_{13} \to A_{11}, A_{13}$	A_{13}	A_9, A_{11}, A_{13}
	High	$A_{13} \to A_{11}, A_{13}$		
	Low	$A_{12} \to A_9$		
11/8	Closing	$A_{15} \to A_{13}, A_{14}, A_{16}, A_{18}$	A_{15}	$A_{11}, A_{13}, A_{14}, A_{15}, A_{16}, A_{18}$
	High	$A_{16} \to A_{15}, A_{14}$		
	Low	$A_{13} \to A_{11}, A_{13}$		
11/9	Closing	$A_{15} \to A_{13}, A_{14}, A_{16}, A_{18}$	A_{13}	$A_{13}, A_{14}, A_{15}, A_{16}, A_{18}$
	High	$A_{16} \to A_{13}, A_{14}, A_{16}, A_{18}$		
	Low	$A_{14} \to A_{15}, A_{13}$		

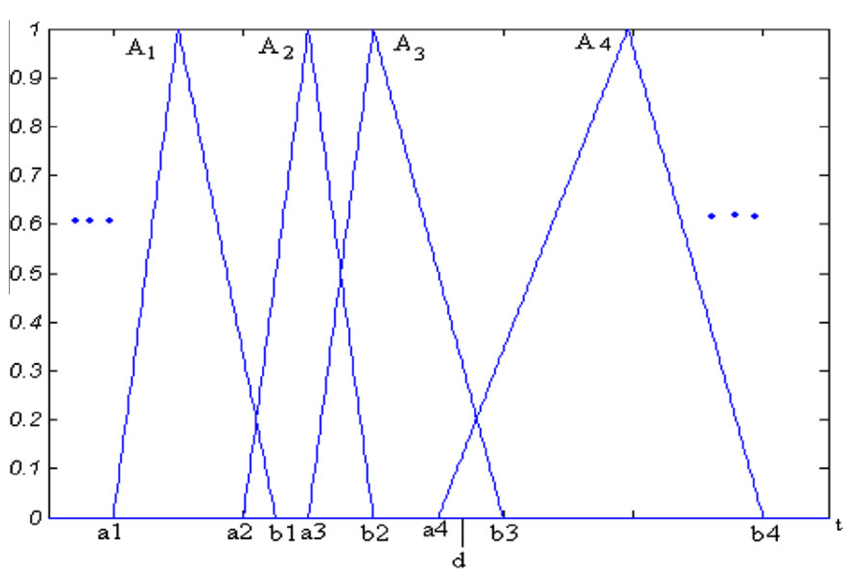

Fig. 3. Fuzzy sets.

Table 6
RMSE comparison.

Model	Type 1 (Chen's model)	Type 2 (Huarng and Yu model)	Proposed model
RMSE	176.32	138.96	23.0631

Thus, in the optimization problem, in steps 3–2 initial fuzzy sets are chosen and the next steps are continued. Then, after some episode minimum RMSE and the related membership functions are found:

$$a1 = 4600, \quad b1 = 4700 \quad a2 = 4600, \quad b2 = 4800$$
$$a3 = 4600, \quad b3 = 4900 \quad a4 = 4900, \quad b4 = 5022.4$$
$$a5 = 4892.1, \quad b5 = 5100 \quad a6 = 4885.9, \quad b7 = 5701.7$$
$$\vdots$$

And we have optimized forecasting with these membership functions.

For defuzzification, center average defuzzifier is replaced by center of gravity defuzzifier.

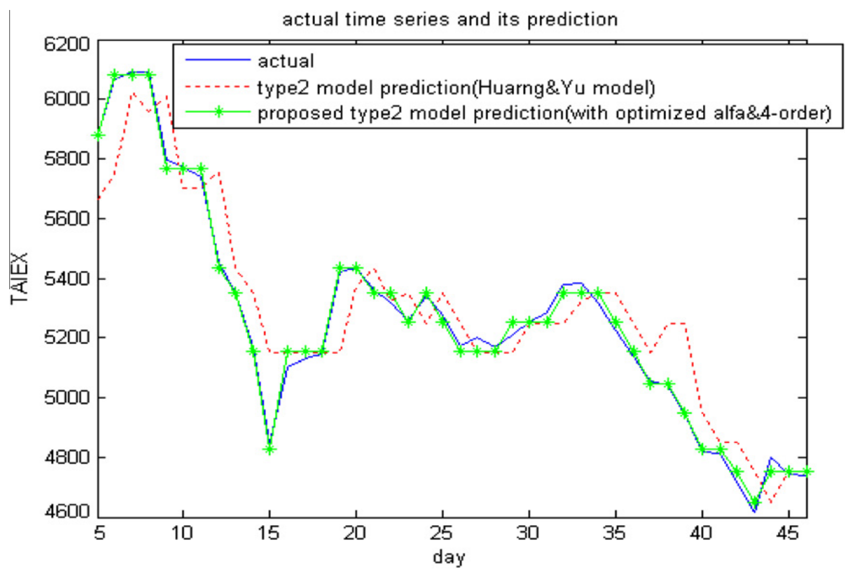

Fig. 4. TAIEX time series and its prediction with Huarng and Yu model and proposed model.

Table 7
Forecasts of the TAIEX in the year 2000.

	Date	Actual	Type 1 (Chen's model)	Type 2 (Huarng and Yu model)	Proposed Type 2 model
	⋮	⋮	⋮	⋮	⋮
5	11/7	5877.77	5750	5662.50	5877.8
6	11/8	6067.94	5750	5750.00	6082.1
7	11/9	6089.55	6075	6025.00	6082.1
8	11/10	6088.74	6075	5960.00	6082.1
9	11/13	5793.52	6075	6010.00	5767.7
10	11/14	5772.51	5450	5700.00	5767.7
11	11/15	5737.02	5450	5700.00	5767.7
12	11/16	5454.13	5450	5758.34	5436
13	11/17	5351.36	5300	5433.34	5351.4
14	11/18	5161.35	5350	5350.00	5153.2
15	11/20	4845.21	5150	5150.00	4824.6
16	11/21	5103	4850	5150.00	5153.2
⋮	⋮	⋮	⋮	⋮	⋮
33	12/13	5384.36	5350	5325.00	5351.4
34	12/14	5320.16	5350	5350.00	5251.5
35	12/15	5224.74	5350	5350.00	5251.5
36	12/16	5134.1	5250	5250.00	5153.2
37	12/18	5055.2	5150	5150.00	5047.6
38	12/19	5040.25	5450	5250.00	5047.6
39	12/20	4947.89	5450	5250.00	4947.6
40	12/21	4817.22	4950	4950.00	4824.6
41	12/22	4811.22	4850	4850.00	4824.6
42	12/26	4721.36	4850	4850.00	4750.4
43	12/27	4614.63	4750	4750.00	4650
44	12/28	4797.14	4650	4650.00	4750.4
45	12/29	4743.94	4750	4750.00	4750.4
46	12/30	4739.09	4750	4750.00	4750.4

$$\text{defuzzification}_k(t_0) = \frac{\int t\mu_A(t)dt}{\int \mu_A(t)dt}.$$

Another change which is implemented in step 8 using α as a coefficient in calculating type 2 forecasting:

$$\text{defuzzification}(t) = \alpha(\text{defuzzification}_{\text{intersection}}(t)) + (1-\alpha)(\text{defuzzification}_{\text{union}}(t)).$$

We can find α by solving the optimization problem too. It means that we have another parameter in our optimization problem. Optimum α is 0.78.

Thus we use it for calculating type 2 forecasting:

$$\text{defuzzification}(t) = 0.78^{*}(\text{defuzzification}_{\text{intersection}}(t)) + 0.22^{*}(\text{defuzzification}_{\text{union}}(t))$$

In this method we use fourth-order fuzzy time series. It means that if $F(t)$ is a fuzzy time-series and $F(t)$ is caused by $F(t-1)$, $F(t-2)$, $F(t-3)$, $F(t-4)$, then the fuzzy logical relationship can be represented as follows (Bajestani & Zare, 2009; Li-Wei et al., 2006):

$$F(t-1), \quad F(t-2), \quad F(t-3), \quad F(t-4) \rightarrow F(t).$$

Table 6 shows RMSE of this methods and in Fig. 4 and Table 7, the results of two methods are showed.

4. Conclusions

This study presents a new type 2 fuzzy time series model. From the empirical analysis, we can see that the proposed method can make very good forecast of the Taiwan stock index. The proposed method is more efficient than the previous methods, a new type 2 fuzzy time series model.

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