1. What is Probability and Event with its type? Write rules of probability. Also, explain Mutually Exclusive \and Non-Exclusive Events in Detail with help of Example.

Probability is the branch of mathematics concerning events and numerical descriptions of how likely they are to occur. The probability of an event is a number between 0 and 1; the larger the probability, the more likely an event is to occur. The higher the probability of an event, the more likely it is that the event will occur. A simple example is the tossing of a fair (unbiased) coin. Since the coin is fair, the two outcomes ('heads' and 'tails') are both equally probable; the probability of 'heads' equals the probability of 'tails'; and since no other outcomes are possible, the probability of either 'heads' or 'tails' is 1/2 (which could also be written as 0.5 or 50%).

Rules of probability

1) Possible values for probabilities range from 0 to 1

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0 = impossible event
1 = certain event
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2) The sum of all the probabilities for all possible outcomes is equal to 1.

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Note the connection to the complement rule.
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3) Addition Rule - the probability that one or both events occur

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mutually exclusive events: P(A or B) = P(A) + P(B) <br>
not mutually exclusive events: P(A or B) = P(A) + P(B) - P(A and B)
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4) Multiplication Rule - the probability that both events occur together

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independent events: P(A and B) = P(A) * P(B) <br>
P(A and B) = P(A) * P(B|A)
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5) Conditional Probability - the probability of an event happening given that another event has already happened

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 P(A|B) = P(A and B) / P(B)<br>
*Note the line | means "given" while the slash / means divide
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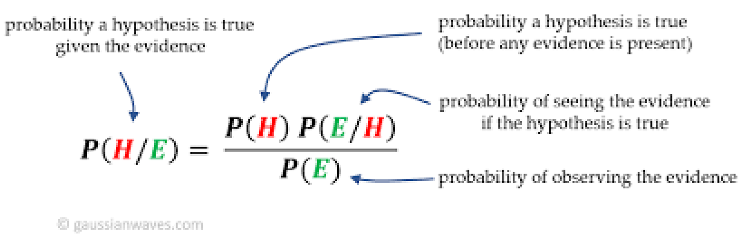
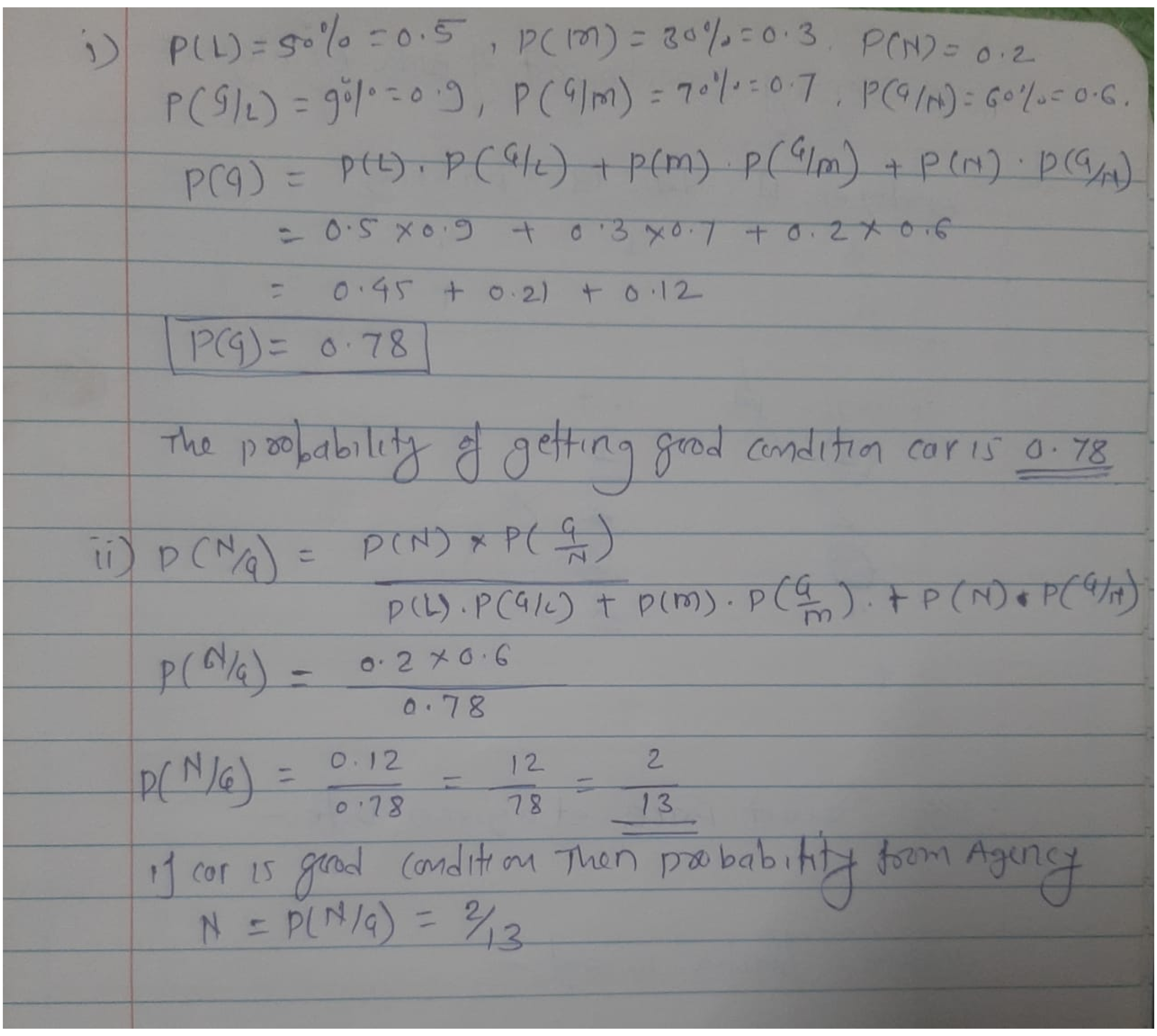
- Mutually exclusive events are events that can not happen at the same time. Examples include: right and left hand turns, even and odd numbers on a die, winning and losing a game, or running and walking.
- Non-mutually exclusive events are events that can happen at the same time. Examples include: driving and listening to the radio, even numbers and prime numbers on a die, losing a game and scoring, or running and sweating.
  Non-mutually exclusive events can make calculating probability more complex

2. State Bayes Theorem and write it's applications. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?

Bayes theorem is a formula that describes how to update the probabilities of hypotheses when given evidence. It follows simply from the axioms of conditional probability, but can be used to powerfully reason about a wide range of problems involving belief updates.
Given a hypothesis H and evidence E, Bayes theorem states that the relationship between the probability of the hypothesis before getting the evidence P(H) and the probability of the hypothesis after getting the evidence P(H|E) is



- P(H|E): Posterior Probability
- P(H): Prior Probability
- P(E): Marginal Probability
- P(E|H): Conditional Probability
- P(E|H)*P(H): Joint Probability



i) $P(L) = 50\% = 0.5$, $P(M) = 30\% = 0.3$, $P(N) = 0.2$

$P(G/L) = 90\% = 0.9$, $P(G/M) = 70\% = 0.7$, $P(G/N) = 60\% = 0.6$.

$$P(G) = P(L)\cdot P(G/L) + P(M)\cdot P(G/M) + P(N)\cdot P(G/N)$$
$$= 0.5 \times 0.9 + 0.3 \times 0.7 + 0.2 \times 0.6$$
$$= 0.45 + 0.21 + 0.12$$
$$\boxed{P(G) = 0.78}$$

The probability of getting good condition car is 0.78

ii) $$P(N/G) = \frac{P(N) \times P(\frac{G}{N})}{P(L)\cdot P(G/L) + P(M)\cdot P(\frac{G}{m})\cdot + P(N)\cdot P(G/N)}$$

$$P(N/G) = \frac{0.2 \times 0.6}{0.78}$$

$$P(N/G) = \frac{0.12}{0.78} = \frac{12}{78} = \frac{2}{13}$$

if car is good condition Then probability from Agency N $= P(N/G) = 2/13$

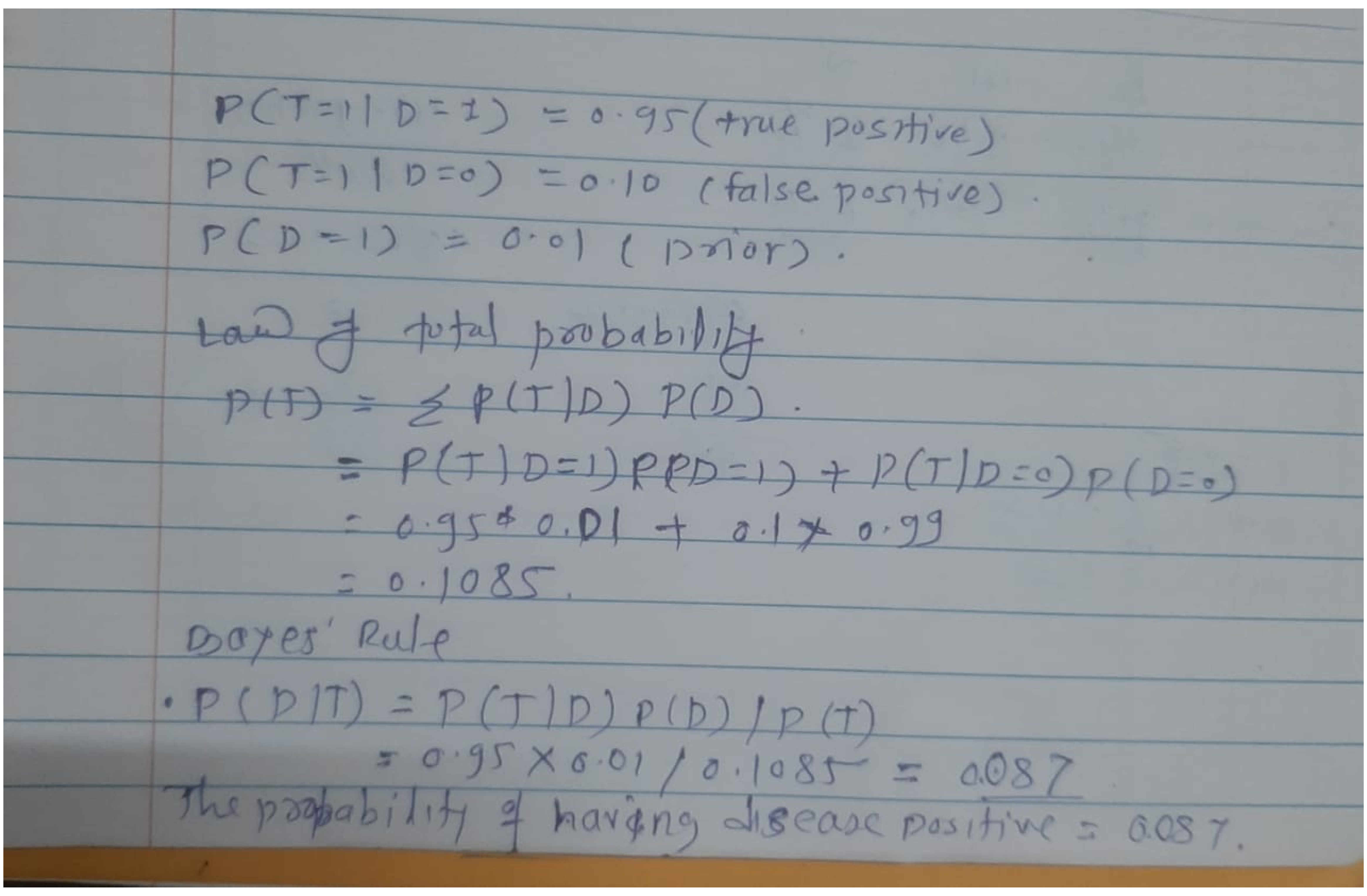
3. Suppose you have tested positive for a disease; what is the probability that you actually have the disease if:

• P(T=1|D=1) = .95 (true positive)

• P(T=1|D=0) = .10 (false positive)

• P(D=1) = .01 (prior)

Where T= Test and D=Disease.



$P(T=1 | D=1) = 0.95$ (true positive)
$P(T=1 | D=0) = 0.10$ (false positive).
$P(D=1) = 0.01$ (prior).

Law of total probability

$$P(T) = \sum P(T|D)\,P(D).$$
$$= P(T|D=1)P(D=1) + P(T|D=0)P(D=0)$$
$$= 0.95 * 0.01 + 0.1 \times 0.99$$
$$= 0.1085.$$

Bayes' Rule

• $P(D|T) = P(T|D)\,P(D) / P(T)$
$$= 0.95 \times 0.01 / 0.1085 = 0.087$$

The probability of having disease positive = 0.087.

4. Give a Brief on Probability Distribution Functions and it's Type.

A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be bounded between the minimum and maximum possible values, but precisely where the possible value is likely to be plotted on the probability distribution depends on a number of factors. These factors include the distribution's mean (average), standard deviation, skewness, and kurtosis.

Binomial

The binomial distribution, for example, evaluates the probability of an event occurring several times over a given number of trials and given the event's probability in each trial. It may be generated, for example, by keeping track of how many free throws a basketball player makes in a game, where 1 = a basket and 0 = a miss.
Another typical example would be to use a fair coin and figure out the probability of that coin coming up heads in 10 straight flips. A binomial distribution is discrete, as opposed to continuous, since only 1 or 0 is a valid response.

Normal

The most commonly used distribution is the normal distribution, which is used frequently in finance, investing, science, and engineering. The normal distribution is fully characterized by its mean and standard deviation, meaning the distribution is not skewed and does exhibit kurtosis.
This makes the distribution symmetric and it is depicted as a bell-shaped curve when plotted. A normal distribution is defined by a mean (average) of zero and a standard deviation of 1.0, with a skew of zero and kurtosis = 3.
In a normal distribution, approximately 68% of the data collected will fall within +/- one standard deviation of the mean; approximately 95% within +/- two standard deviations; and 99.7% within three standard deviations. Unlike the binomial distribution, the normal distribution is continuous, meaning that all possible values are represented (as opposed to just 0 and 1 with nothing in between).