Length of the string questions

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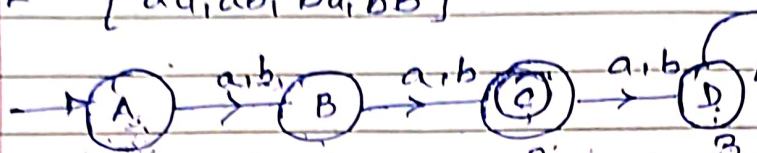
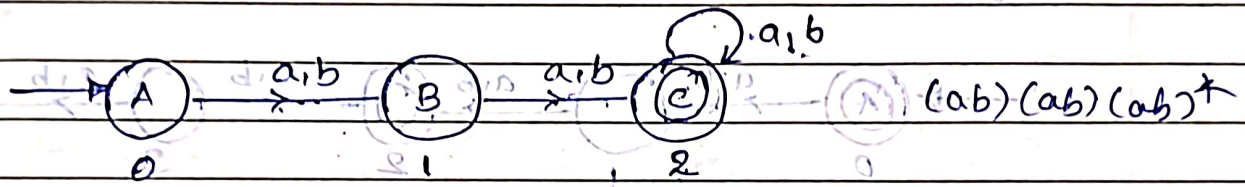
* Consider a DFA, that accepts set of all strings over {a,b} of length 2.

$L = \{aa, ab, ba, bb\}$



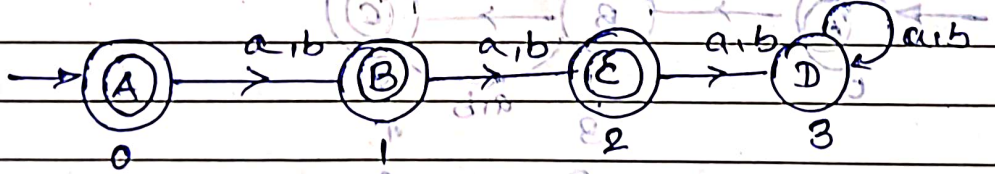
* Construct a DFA, that accepts all string over {a,b}, where $|w| \geq 2$.

$L = \{aa, ab, ba, bb, aaa, bbb, \ldots\ldots\}$



* DFA, that accept {a,b} $|w| \leq 2$

$L = \{\epsilon, a, b, aa, ab, ba, bb\}$



- L = many DFA's but it will contain only one Minimal DFA.

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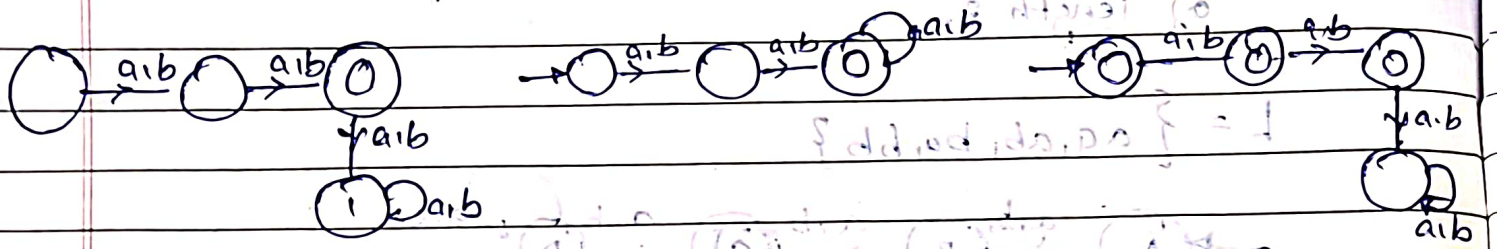
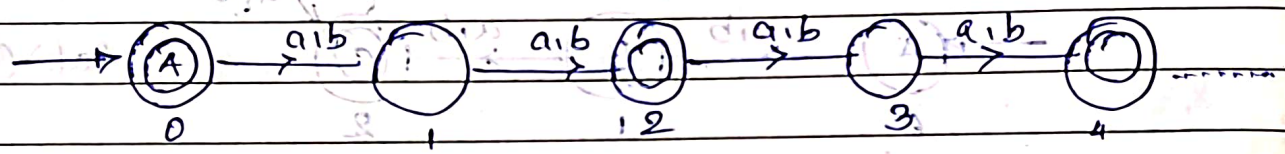
$|w| = 2$ $|w| \geq 2$ $|w| \leq 2$



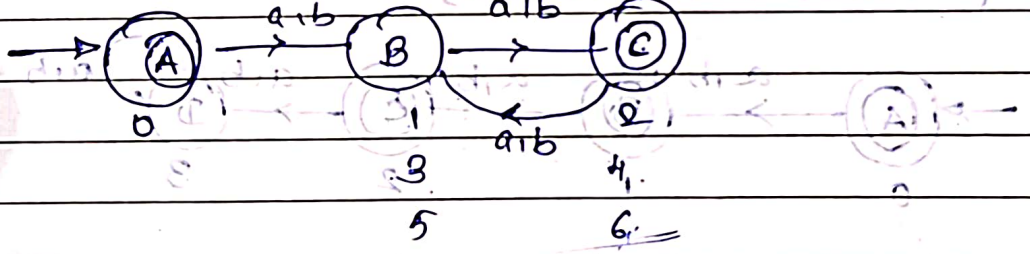
$\|w\| = n$	$\|w\| \geq n$	$\|w\| \leq n$
No. of states required is $(n+2)$	$(n+1)$ states	$(n+2)$

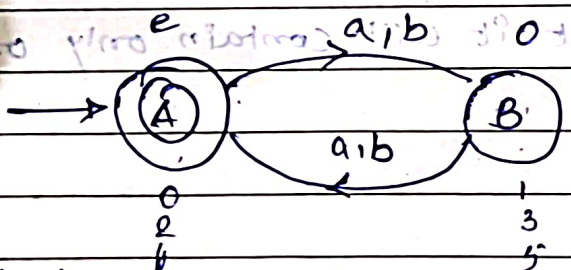
* Construct a minimal DFA, $w \in \{a,b\}$, $|w| \bmod 2 = 0$

$L = \{\epsilon, aa, ab, ba, bb, aaaa, abba, aaba, \ldots\ldots\}$

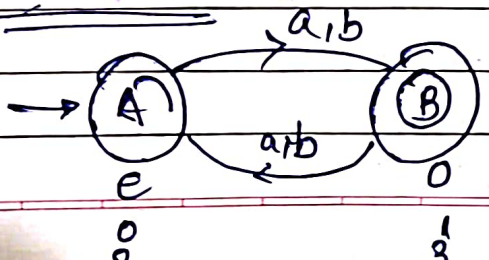


distinguish between even length and odd length





* $|w| \bmod 2 = 1$



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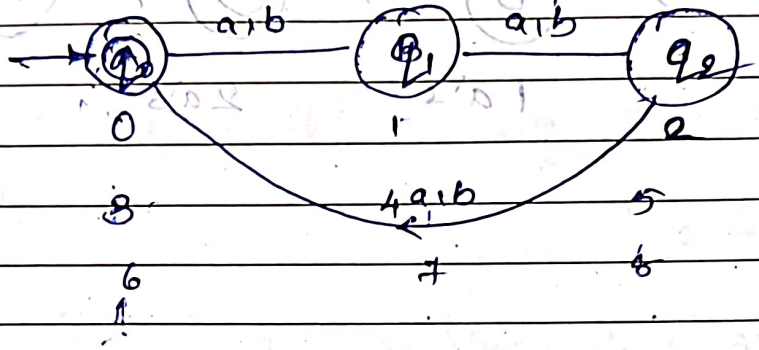
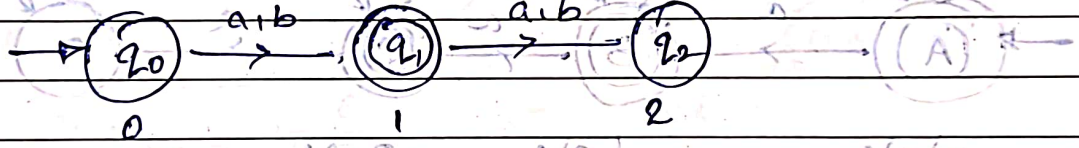
* $w \in \{a,b\}^*$ $|w| \bmod 3 = 1$

$L = \{ \epsilon, aaa, aba, abb, baa, bab, \ldots\ldots \}$

babab



* $|w| \cong 1 \bmod 3$ or $|w| \bmod 3 = 1$



Note: if $|w| \bmod n = 0$ then no. of states is equal to "n"

Restrictions on 'a'

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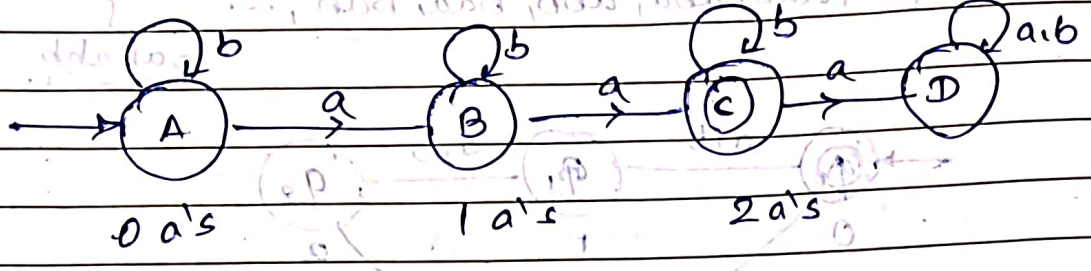
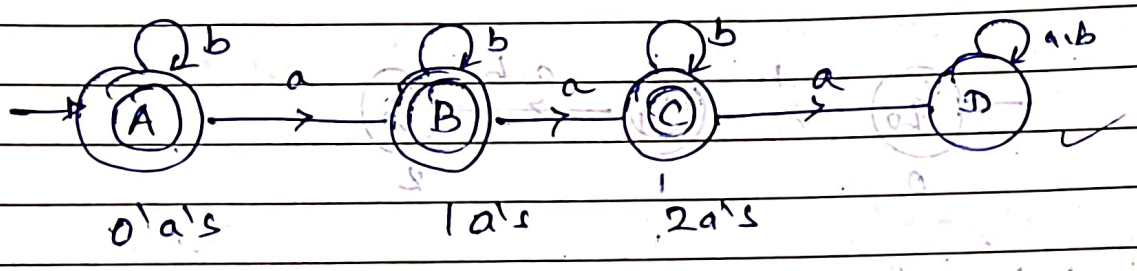
* Construct a minimal DFA $w \in \{a,b\}^*$. $n_a(w) = 2$

$L = \{aa, baa, aba, abba, bbaa, \ldots\}$



* $n_a(w) \leq 2$

$L = \{\epsilon, a, aa, b, ab, abb, aabb, \ldots\}$



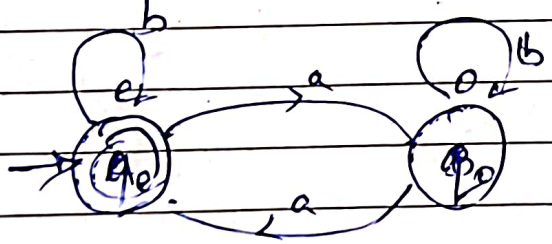
* $w \in \{a,b\}^*$ where no. of a is even.

or

$n_a(w) \bmod 2 = 0$

or

$n_a(w) \stackrel{?}{=} 0 \bmod 2$.



* $n_a(w) \bmod 2 = 1$

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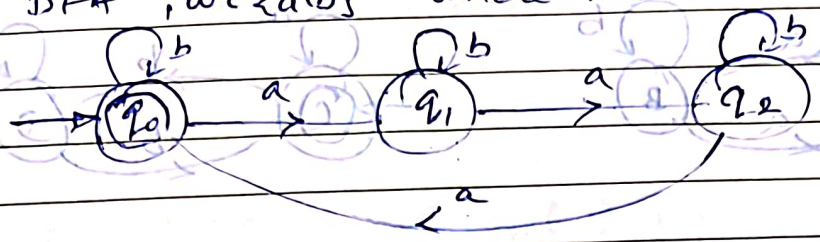
* DFA, $w \in \{a,b\}^*$ where $n_a(w) \bmod 3 = 0$

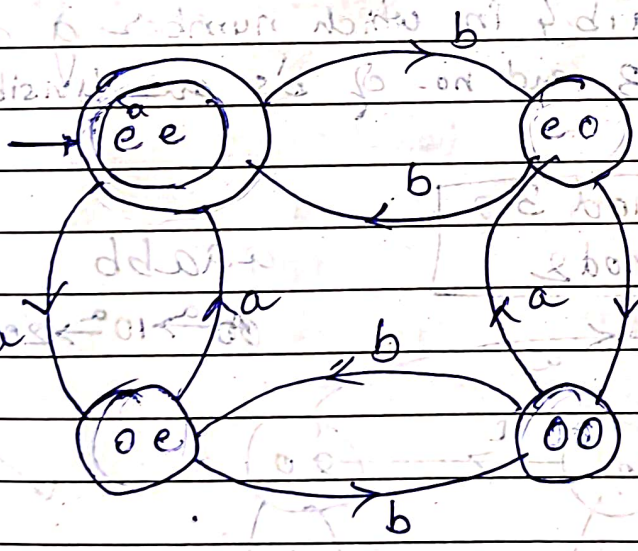


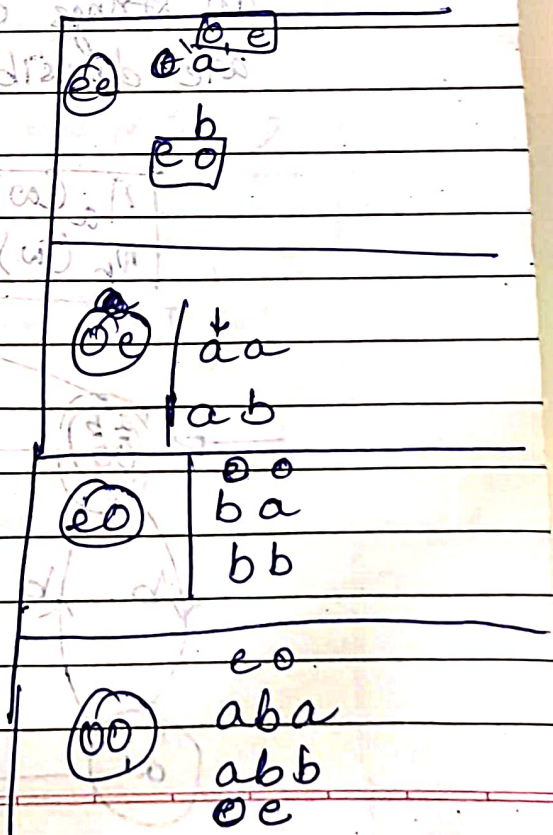
* DFA, $w \in \{a,b\}^*$, $n_a(w) \overset{2}{\equiv} 0 \bmod 2$ && $n_b(w) \overset{2}{\equiv} 0 \bmod 2$

1st Method

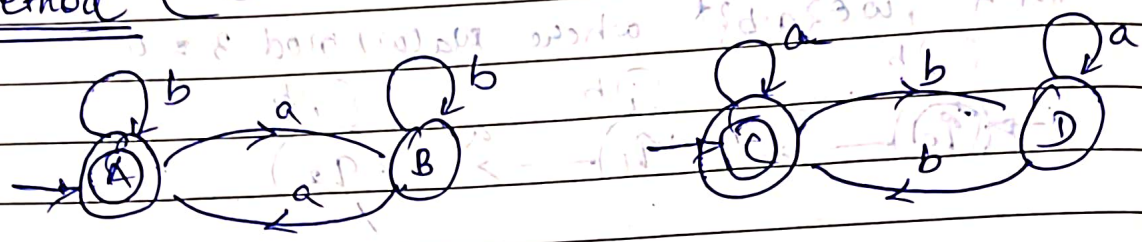
$L = \{ \epsilon, aa, bb, aabb, abab, baba, bbbb \ldots \}$

$n_a(w)$	$n_b(w)$	
e	e	ϵ, aa, bb, ...
e	o	aab
o	e	aaabb
o	o	ab

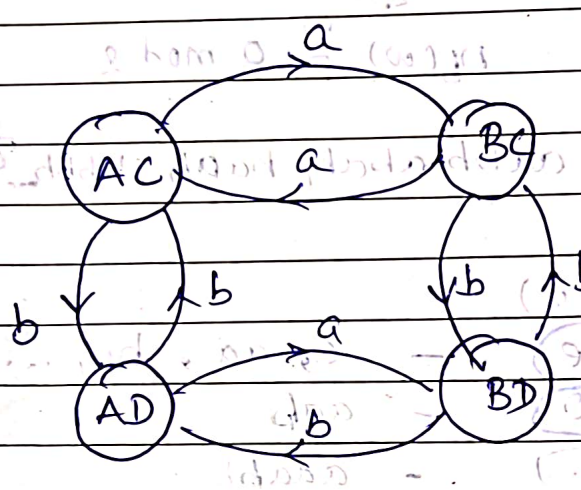




2nd Method (CROSS PRODUCT METHOD)



$\{A, B\} \times \{C, D\} = \{AC, AD, BC, BD\}$



A —a→ B
C C

A —b→ A
C D

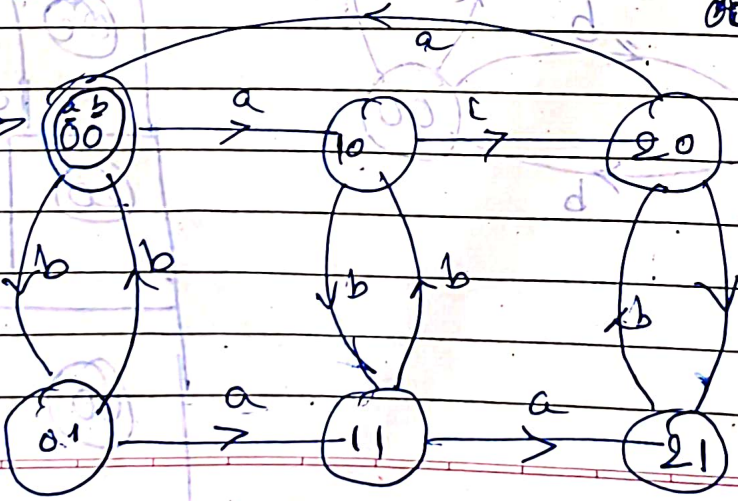
* Construct a minimal DFA which accepts set of all strings over {a,b} in which number of a's are divisible by 3 and no. of b's are divisible by 2.

or

$n_a(w) \equiv 0 \bmod 3$ &
$n_b(w) \equiv 0 \bmod 2$

aaabb

00 —a→ 10 —a→ 20 —a→ 00
b
01
b
00



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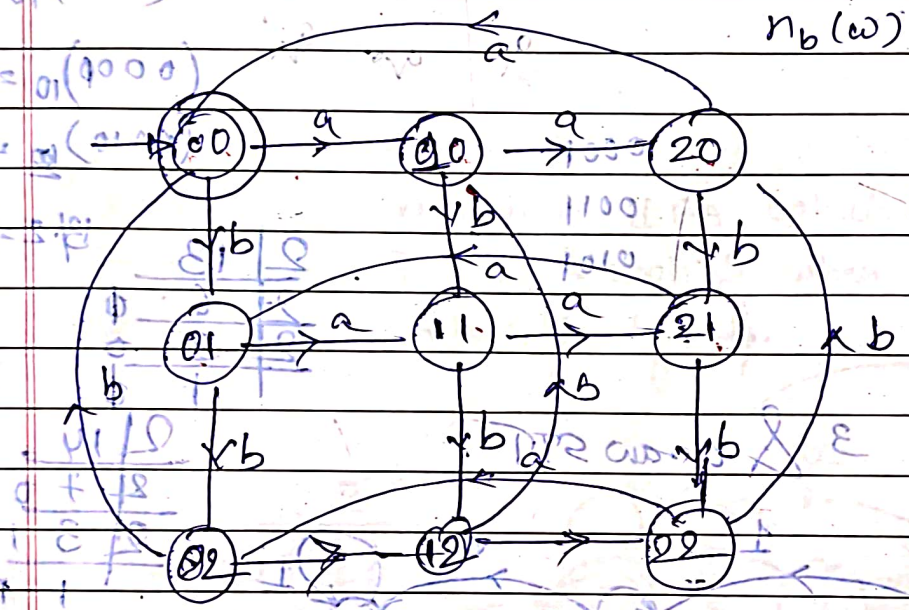
* $n_a(w) \bmod 3 \geq n_b(w) \bmod 3$
✓ $= \{10, 20, 21, 11\}$

* $n_a(w) \bmod 3 = n_b(w) \bmod 2$
✓ $= \{00, 11\}$

$n_a(w) \bmod 3 = 0$
$n_b(w) \bmod 3 = 0$

* Construct a DFA $w \in \{a,b\}^*$ $n_a(w) \cong 0 \bmod 3$
$n_b(w) \cong 0 \bmod 3$

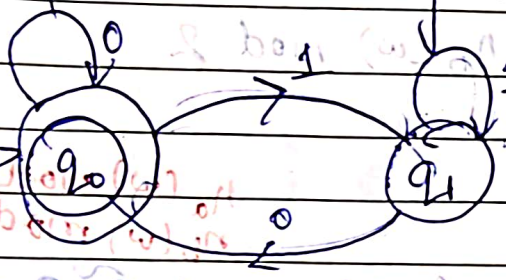


FS = {00}

$n_a(w) \bmod 3 = 1$ && $n_b(w) \bmod 3 = 2$
$= \{12\}$

$n_a(w) \bmod 3 \geq n_b(w) \bmod 3$
$= \{10, 20, 21\}$

* Construct a minimal DFA which accepts set of all strings over {0,1} ~~in which no. of a's are divisible by 3 and no. of b's are divisible by 3.~~ when interpreted as binary numbers divisible by 2.



	0	1
→ q_0	q_0	q_1
q_1	q_0	q_1

$(1110)_{10} = 14$

$(0000)_{10} = 0$

$(0001)_{10} = 1$

$(0010)_{10} = 2$

$14 \cdot 2 = 7$

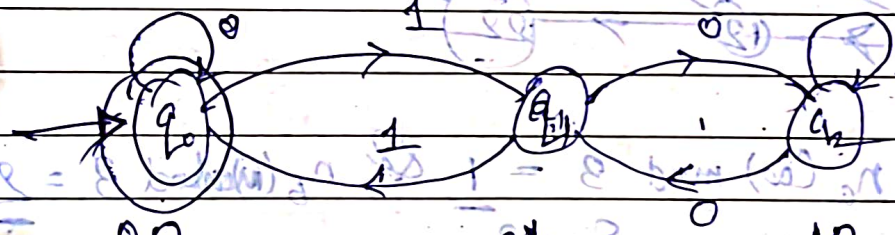
2	13
2	6 — 1
2	3 — 0
	1 — 1

1101

2	14
2	7 — 0
2	3 — 1
	1 — 1

0.000
0010
0100
0011
0101

* divisible by 3 & draw STT



00
10
100
111
1001
01
100
1001
10 - 2
1010

STT

	0	1
→ q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

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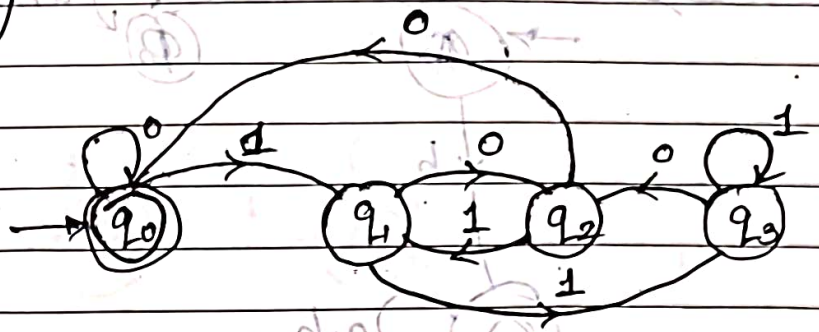
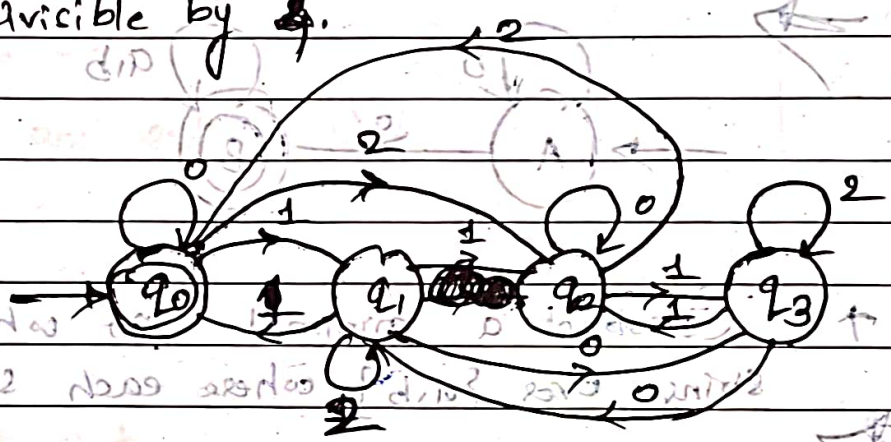
* Construct a minimal DFA which accepts set of all strings over {0,1} which when interpreted as binary no. divisible by 4. Also, draw a state Transition Table.

	0	1
→ q_0*	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3



* Construct a minimal DFA which accepts set of all strings over {0,1,2} which when interpreted as Ternary no. divisible by 4.

	0	1	2
→ q_0*	q_0	q_1	q_2
q_1	q_3	q_0	q_1
q_2	q_2	q_3	q_0
q_3	q_1	q_2	q_3



$L = \{0, 11, 22, 121, 220, \ldots\}$

$(11)_3 = 1\times3 + 1\times3 = 3+1 = 4$

$(22)_3 = 2\times3 + 2 = 6+2 = 8$

$(120)_3 = 1\times3^2 + 2\times3^1 + 1\times3^0 = 9+6+1 = 16$

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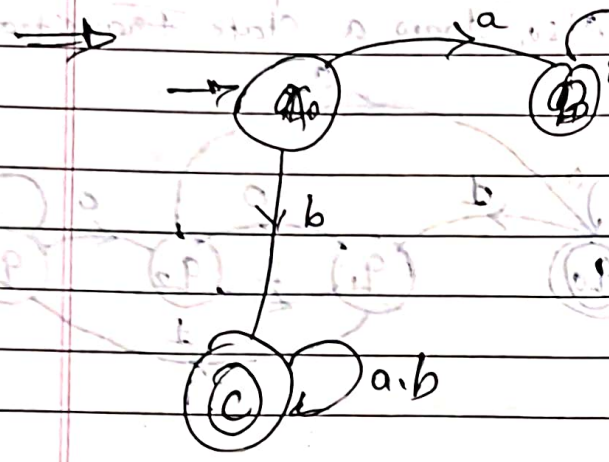
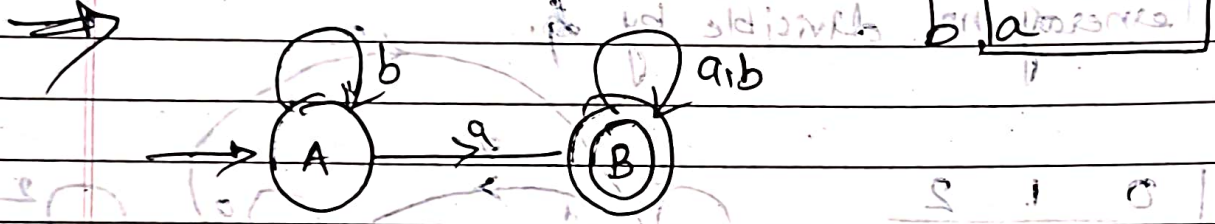
* Construct a minimal DFA for set of all strings starting with a over $\Sigma = \{a, b\}$.



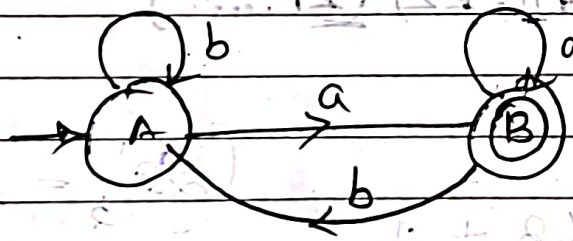
* Construct a minimal DFA which accepts set of all strings over {a,b} where each string contains 'a'.

$L = \{a, aa, ab, ba, \ldots\}$



* Construct a minimal DFA which accepts set of all strings over {a,b} where each string ends with an 'a'.

$L = \{a, aa, ba, baa, aaa \ldots\}$



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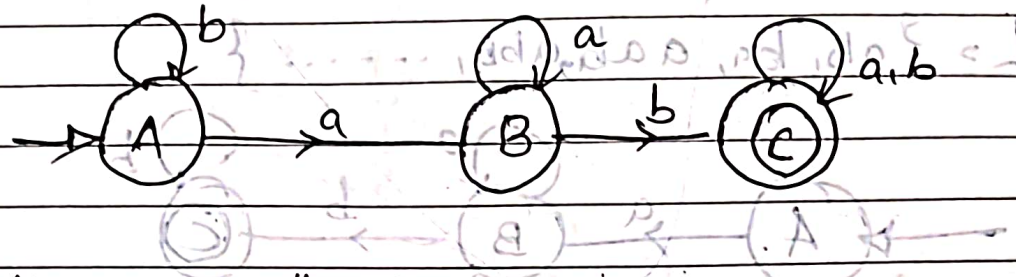
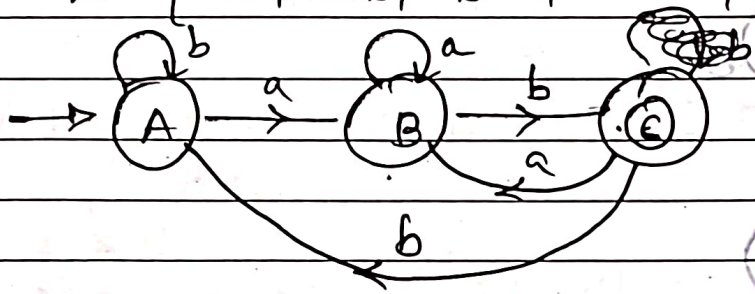
* Construct a DFA which accepts set of all strings over {a, b} which contains 'a'.

⇒ $L = \{ab, aab, aba, bab, \ldots\ldots\}$



* Ending with "ab"

⇒ $L = \{ab, aab, bab, abab, \ldots\ldots\}$

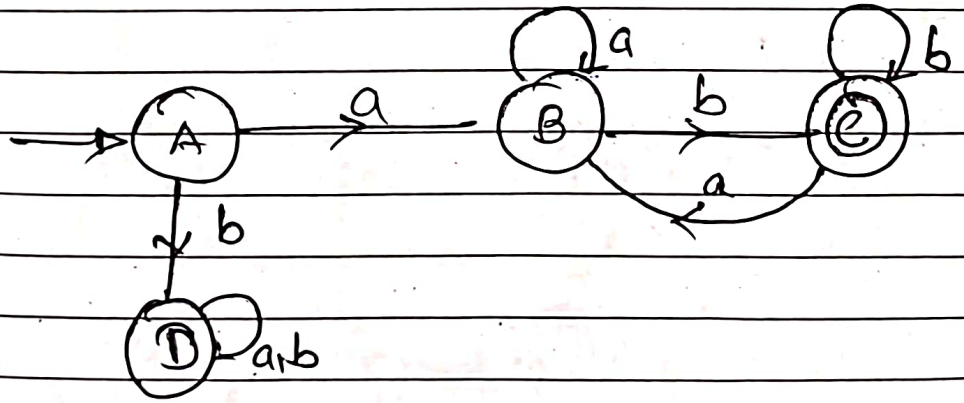


* Starts with 'a' and ends with 'b'

⇒ $L_1 = \{a, ab, aa, aaa \ldots\}$

$L_2 = \{b, ab, bb, aab, \ldots\}$

$L_1 L_2 = \{ab, aab, abb, \ldots\ldots\}$



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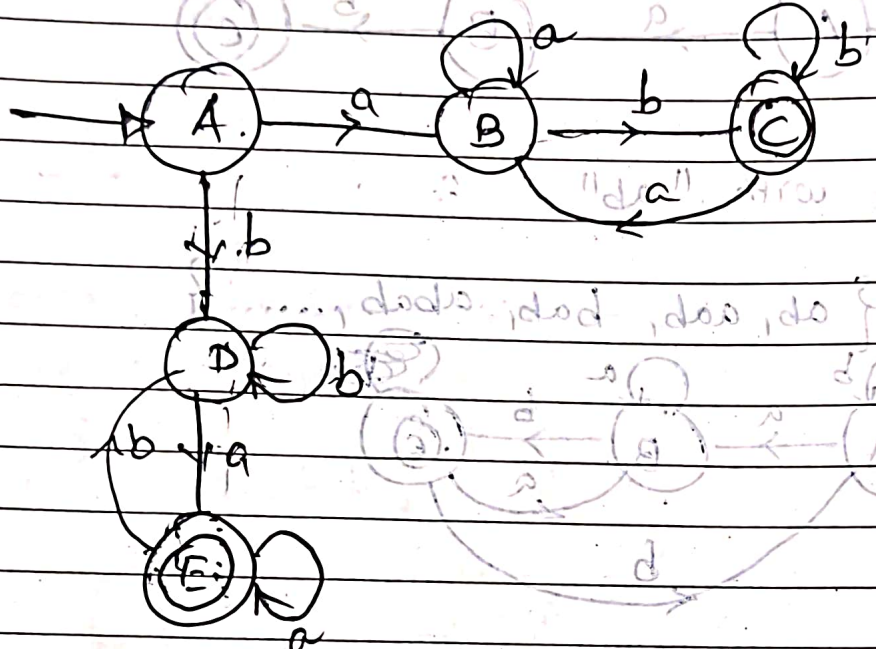
* Starts and ends with different symbols.

a — b
b — a

L = {ab, ba, aaba, abb, ... }



* Starting and ending with same symbol

NFA
Non-Deterministic Finite Automata

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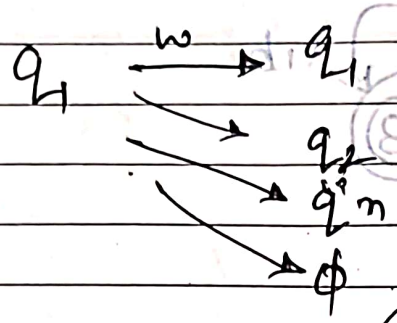
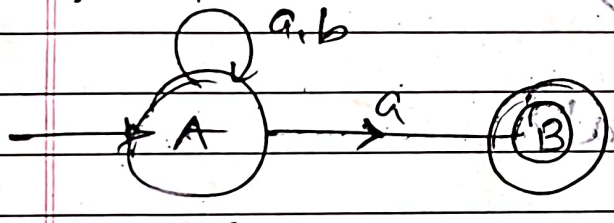
$q_1 \xrightarrow{w} q_2$



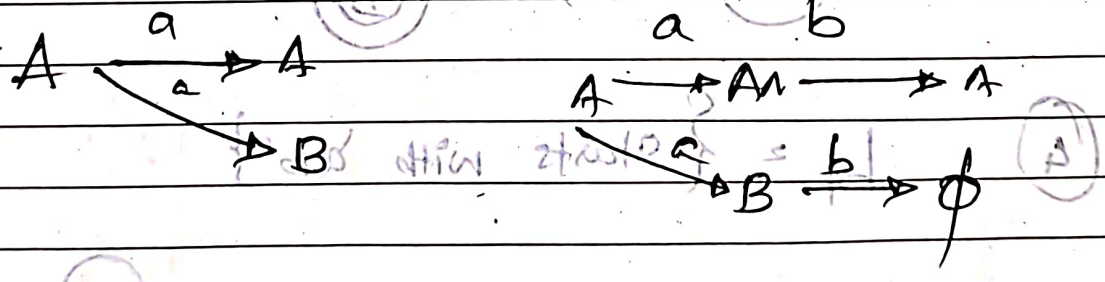
~~Computer, Mech~~ NFA $\delta : Q \times \Sigma \to 2^Q$

$(Q, \Sigma, \delta, q_0, F)$ DFA $\delta : Q \times \Sigma \to Q$

Eg $L = \{$ ends with 'a' $\}$ $\Sigma = \{a, b\}$

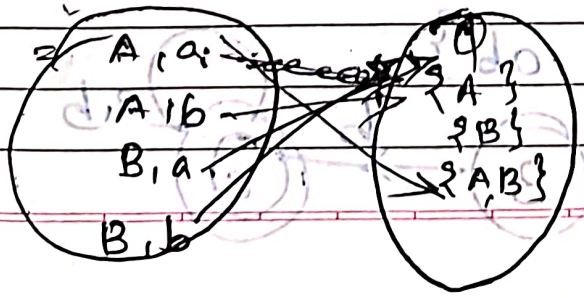


$L = \{a, aa, ba, \ldots\}$



$Q = \{A, B\}$ $\Sigma = \{a, b\}$

$Q \times \Sigma$ 2^Q



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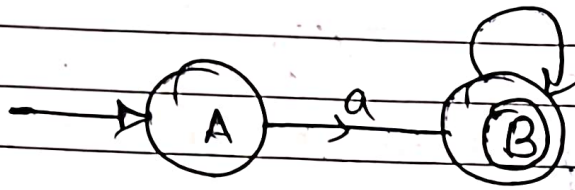
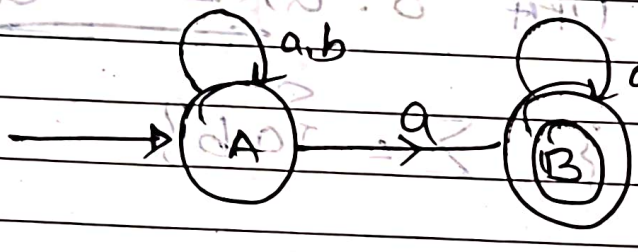
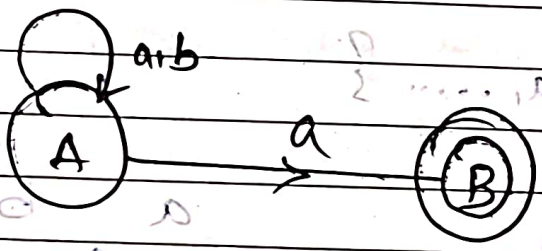
(1) $L_1 = \{$ Starts with 'a' $\}$ $\Sigma = \{a, b\}$



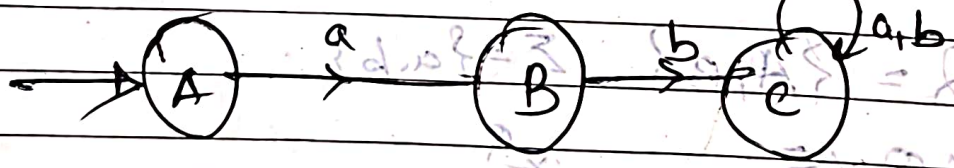
(2) $L_2 = \{$ Contains 'a' $\}$



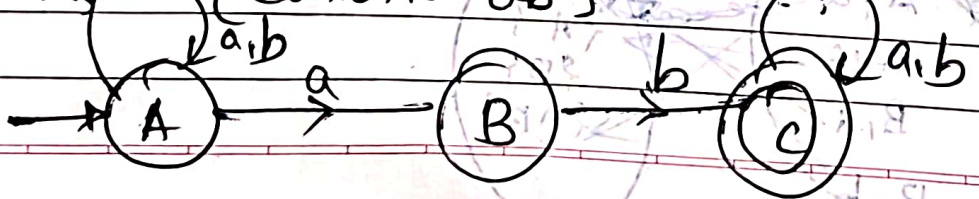
(3) $L_3 = \{$ ends with 'a' $\}$



(4) $L_4 = \{$ Starts with 'ab' $\}$



(5) $L_5 = \{$ Contains 'ab' $\}$



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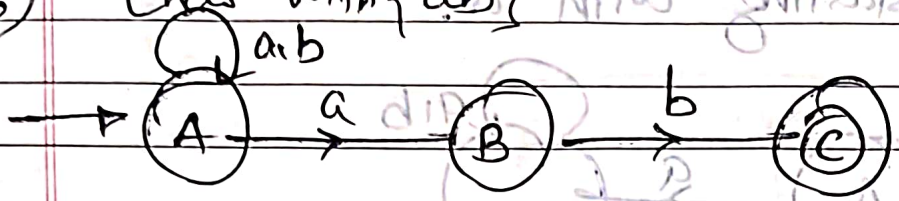
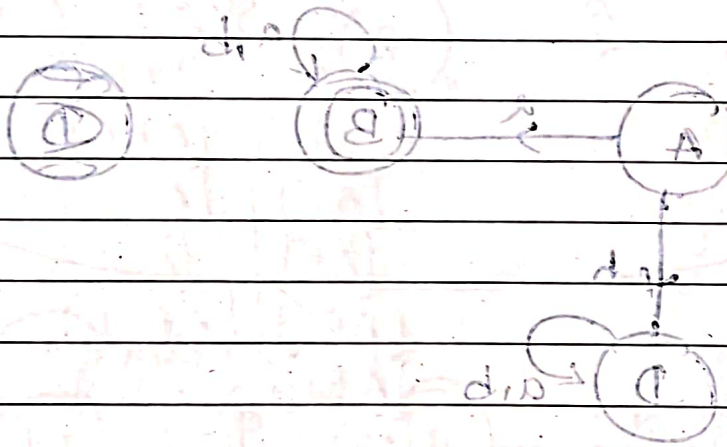
(6) Ends with {abb}



	a	b
→ A	B	A
B	B	C
*C	B	A



Conversion from NFA to DFA

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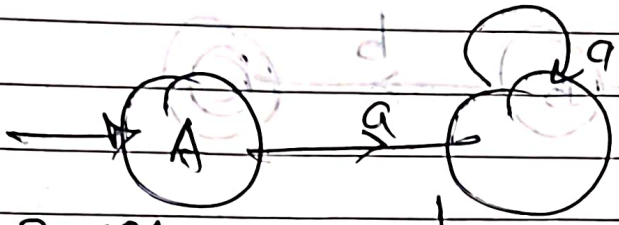
(1) All strings starting with 'a'.

NFA:



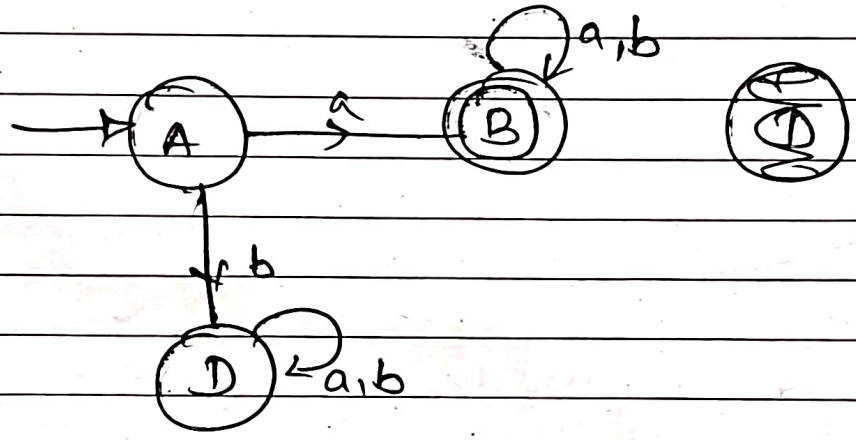
STT of NFA

	a	b
A	B	ϕ
B	B	B

STT of DFA

	a	b
A	B	D
B	B	B
D	D	D

DFA



$\delta(A, a) = \{A, B\}$

$\delta(A, b) = \{A\}$

$\delta(\{A, B\}, a) = \delta(A, a) \cup \delta(B, a)$
$= \{A, B, C\}$

$\delta(\{A, B\}, b) = \delta(A, b) \cup \delta(B, b)$
$= \{A\} \cup \{C\}$
$= \{A, C\}$

$\delta(\{A, C\}, a) = \delta(A, a) \cup \delta(C, a)$
$= \{A, B\} \cup \{A\} = \{AB\}$

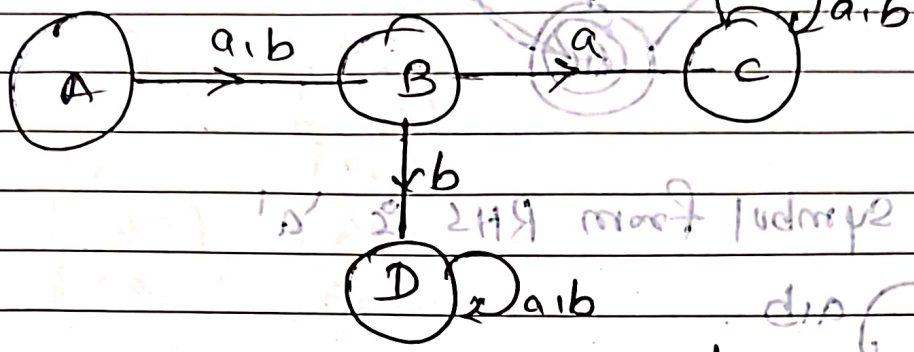
$\delta(\{A, C\}, b) = \{A\}$

$\delta(\{ABC\}, a) = \delta(A, a) \cup \delta(B, a) \cup \delta(C, a)$
$= \{A, B, C\}$

$\delta(\{A, B, C\}, b) = \delta(A, b) \cup \delta(B, b) \cup \delta(C, b)$
$= \{AC\}$

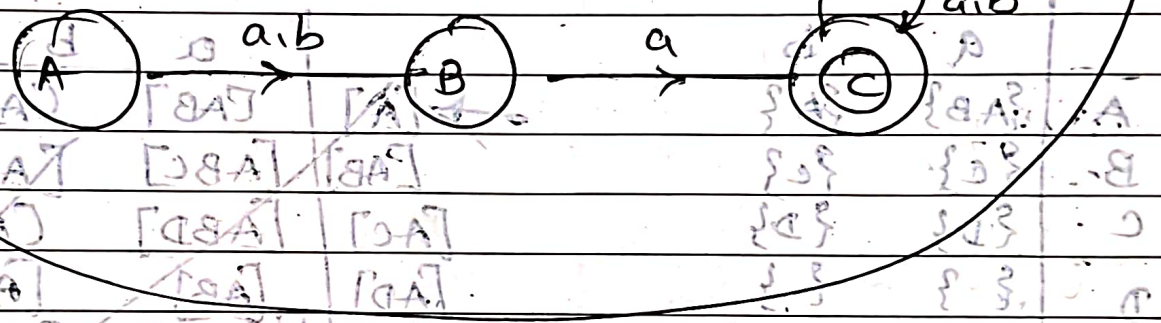
(2) "all string in which second symbol from RHS is "a"

DFA

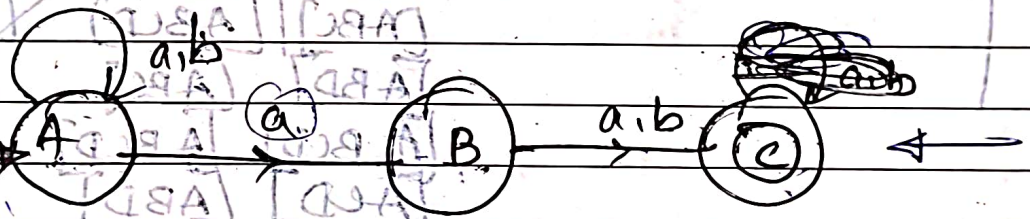


2nd symbol from LHS is a

NFA



NFA →

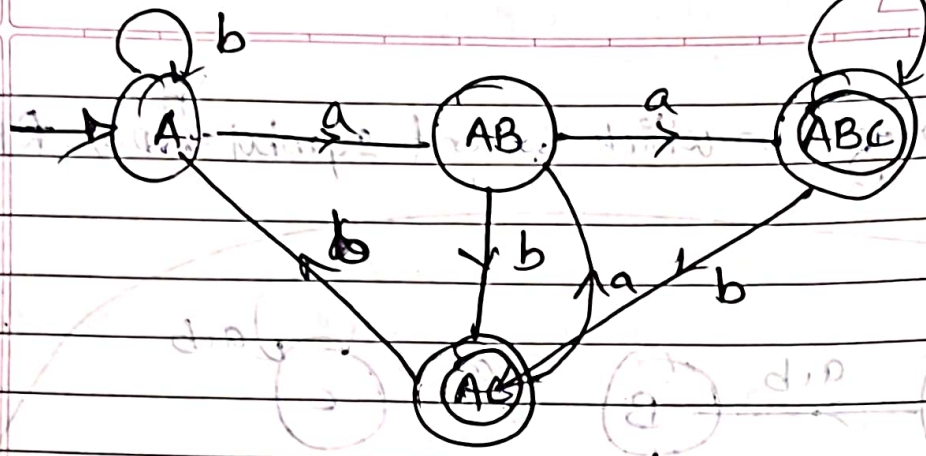


STT for NFA

	a	b
→A	{A,B}	{A}
B	{C}	{C}
*C	{-}	{-}

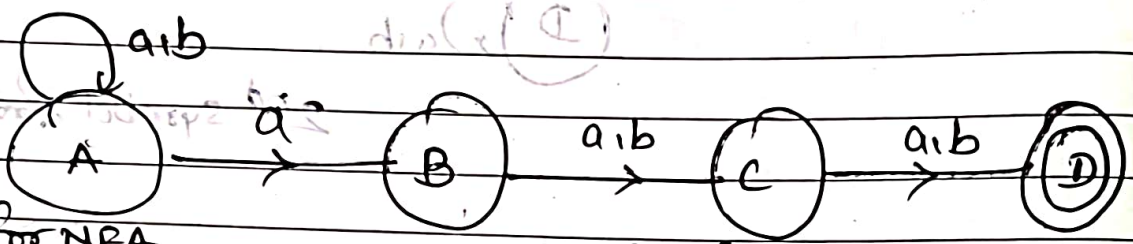
STT for DFA

	a	b
→A	[AB]	[A]
[AB]	[ABC]	[AC]
[AC]	[AB]	[A]
[ABC]	[ABC]	[AC]



(b) 3rd symbol from RHS is 'a'

NFA

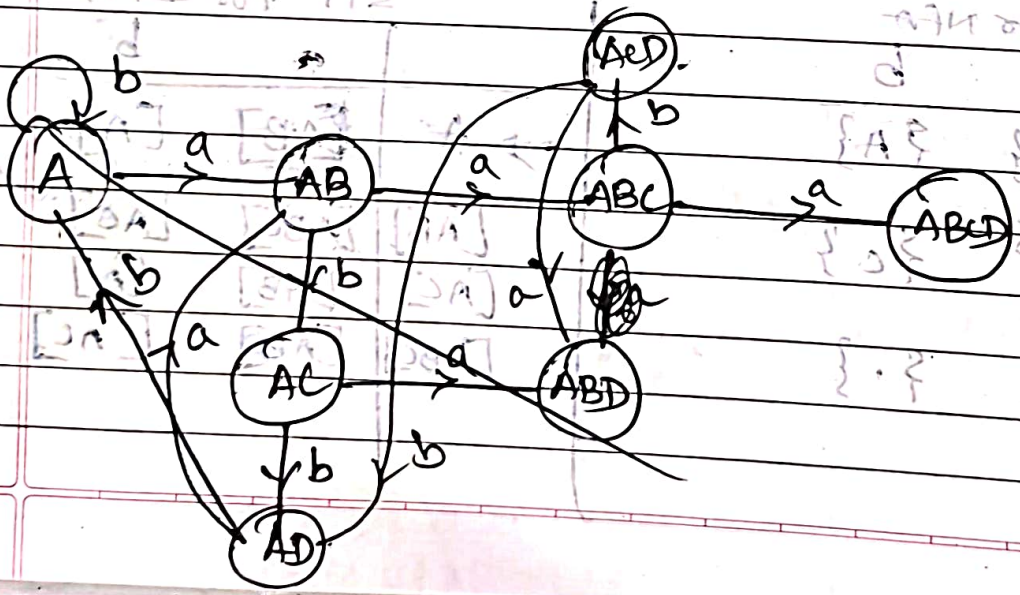


STT for NFA

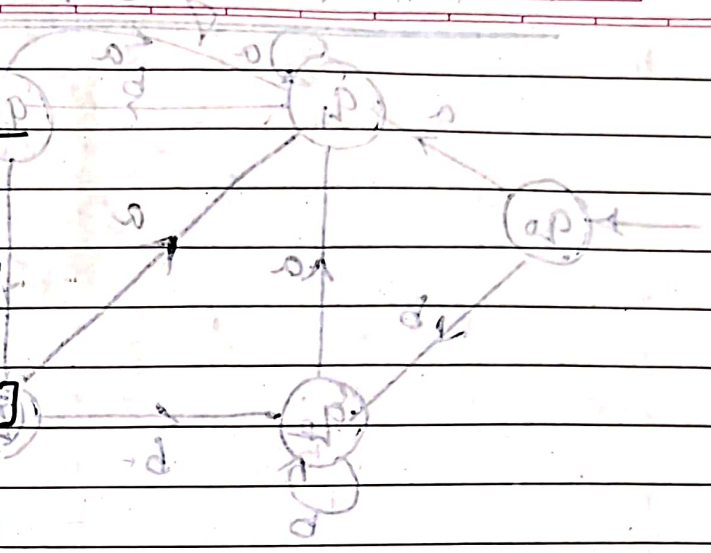
	a	b
A	{A,B}	{A}
B	{C}	{C}
C	{D}	{D}
D	{ }	{ }

STT for DFA

	a	b
→ [A]	[AB]	[A]
[AB]	[ABC]	[AC]
[AC]	[ABD]	[AD]
[AD]	[AB]	[A]
[ABC]	[ABCD]	[ACD]
[ABD]	[ABC]	[AC]
[ABCD]	[ABCD]	[ACD]
[ACD]	[ABD]	[AD]

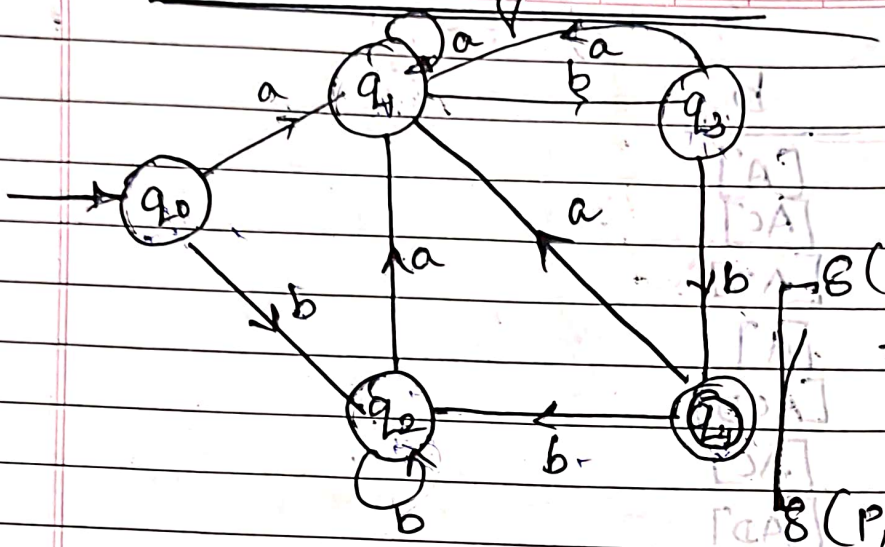


		a	b
→	A	[AB]	[A]
	[AB]	[ABC]	[AC]
	[AC]	[ABD]	[AD]
*	[AD]	[AB]	[A]
•	[ABC]	[ABCD]	[ACD]
*	[ABD]	[ABC]	[AC]
*	[ACD]	[ABD]	[AD]
*	[ABCD]	[ABD]	[ABCD]



Minimization of DFA

Initial



(P, Q)

$\delta(P, w) \in F$
$\Rightarrow \delta(q, w)$

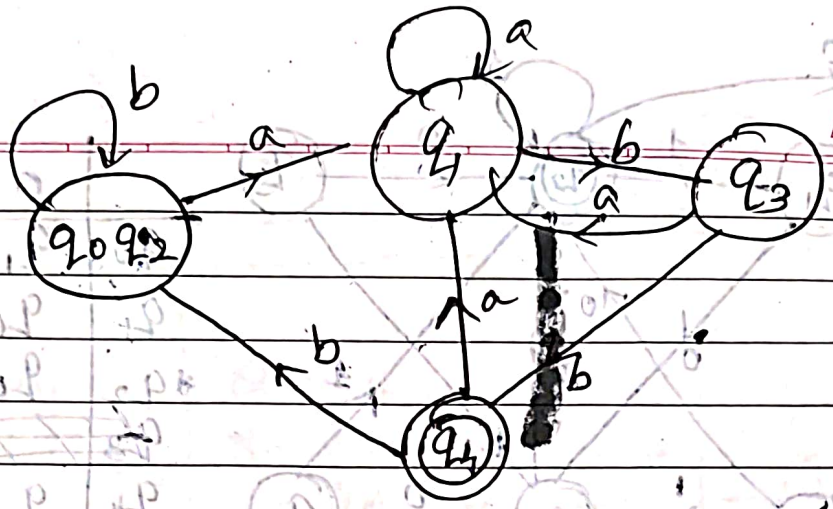
$\delta(P, w) \notin F$
$\Rightarrow \delta(q, w) \notin F$

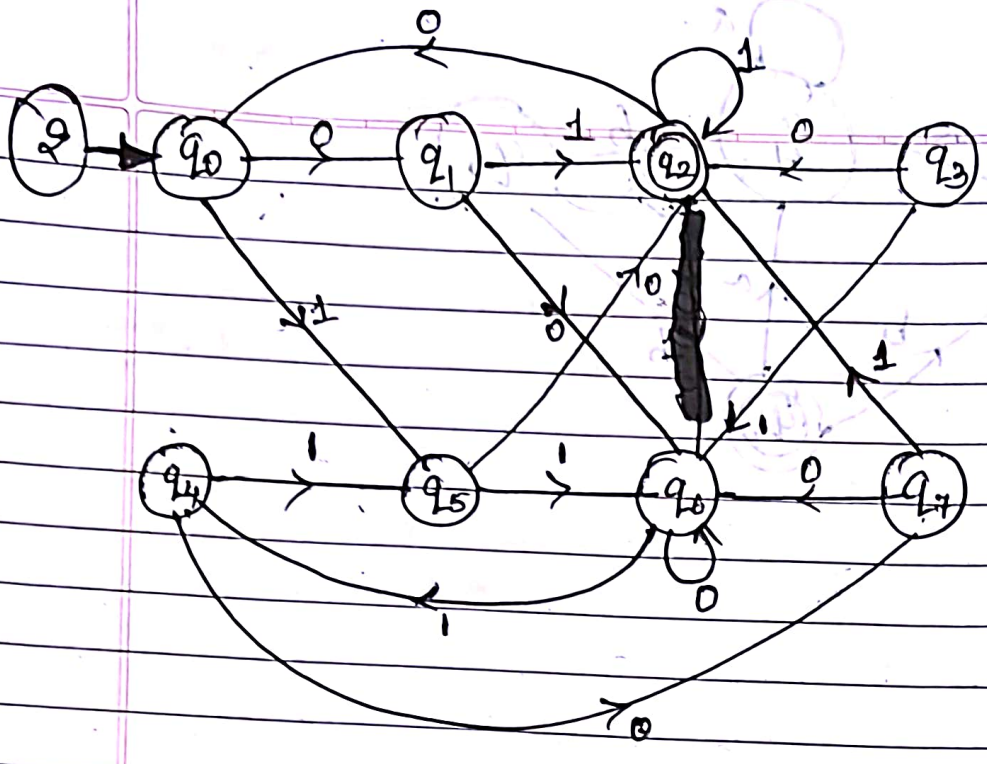
$|w| = 0$, 0 equivalent
$|w| = 1$, 1 equivalent
$|w| = 2$, 2 equivalent
⋮
$|w| = n$, n equivalent

	a	b
→ q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
* q_4	q_1	q_2

0 equivalent — $[q_0\ q_1\ q_2\ q_3]$ $[q_4]$ ←
1 equivalent — $[q_0, q_1, q_2]$ $[q_3]$ $[q_4]$
2 equivalent — $[q_0, q_2]$ $[q_1]$ $[q_3]$ $[q_4]$
3 equivalent — $[q_0, q_2]$ $[q_1]$ $[q_3]$ $[q_4]$
⋮

No changes.


a
b
a
b
a
b
a
b
q0 q2
q1
q3
q4



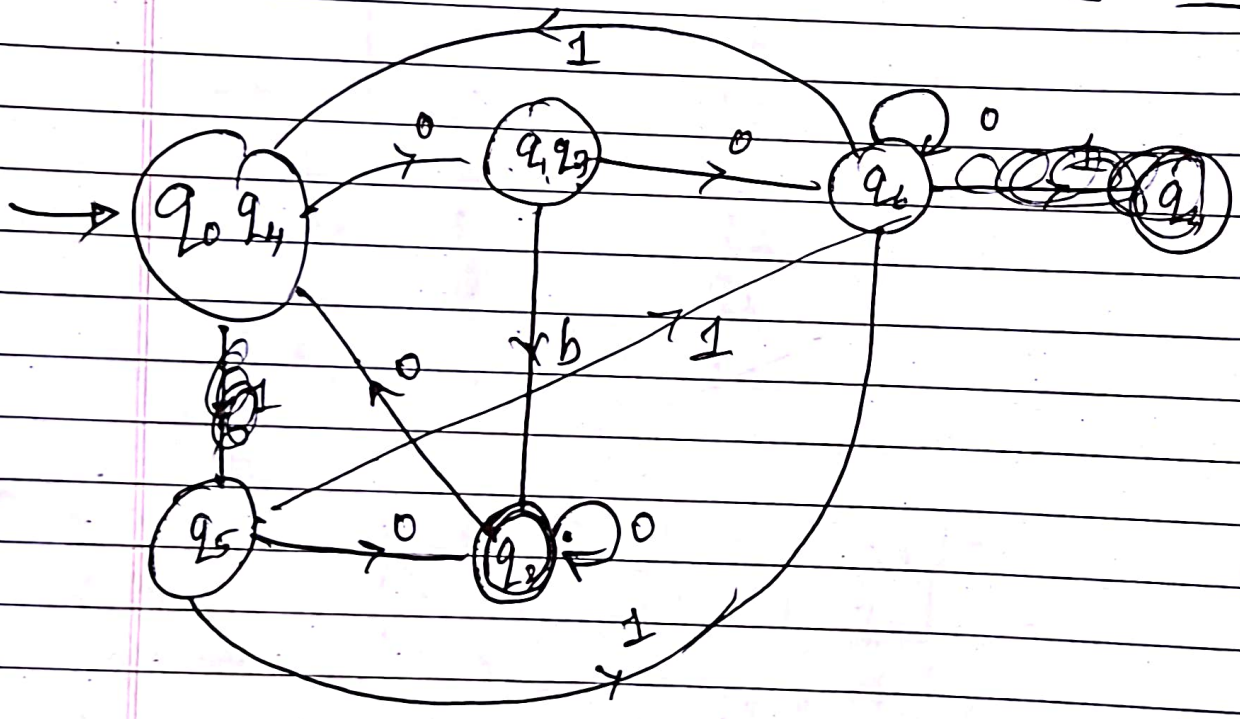
	0	1
q_0	q_1	q_5
q_1	q_6	q_2
*q_2	q_0	q_2
~~q_3~~	~~/~~	~~/~~
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

0 equivalent $[q_0 q_1 q_4 q_5 q_6 q_7]$ $[q_2]$

1 equivalent $[q_0, q_4, q_6]$ $[q_1 q_7]$ $[q_5]$ $[q_2]$

2 equivalent $[q_0 q_4]$ $[q_6]$ $[q_1 q_7]$ $[q_5]$ $[q_2]$

3 equivalent $[q_0 q_4]$ $[q_6]$ $[q_1 q_7]$ $[q_5]$ $[q_2]$



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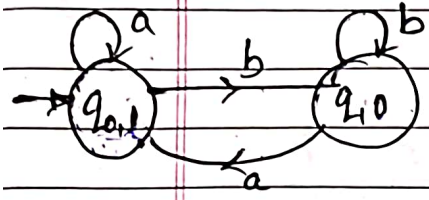
FA with O/P (1)

Moore m/c



$\lambda : Q \to \Delta$

$q_0 \to 1 \quad q_1 \to 0$

$(Q, \Sigma, \delta, q_0, \Delta, \lambda)$

Q - finite set of states

Σ - i/p alphabet

δ - Transition f^n

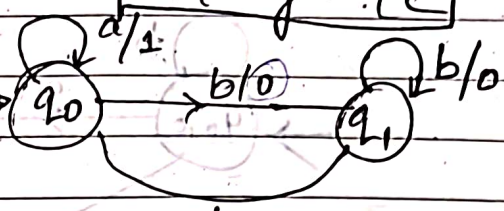
$Q \times \Sigma \to Q$

q_0 - initial state

Δ - o/p alphabet

λ - o/p function

Mealy m/c



$\lambda : Q \times \Sigma \to \Delta$

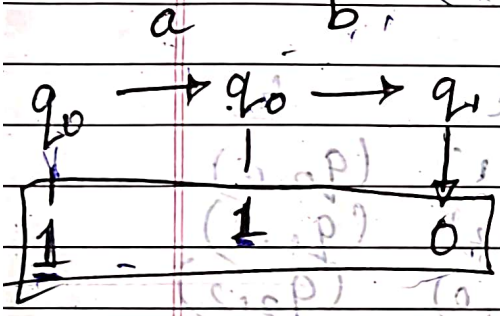
$(q_0, a) \to 1$

$(q_0, b) \to 0$

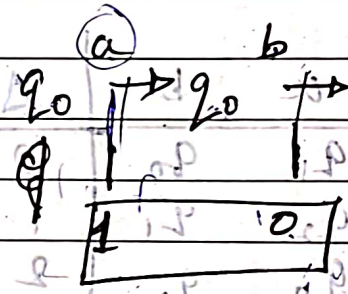
$(q_1, b) \to 0$

$(q_1, a) \to 1$

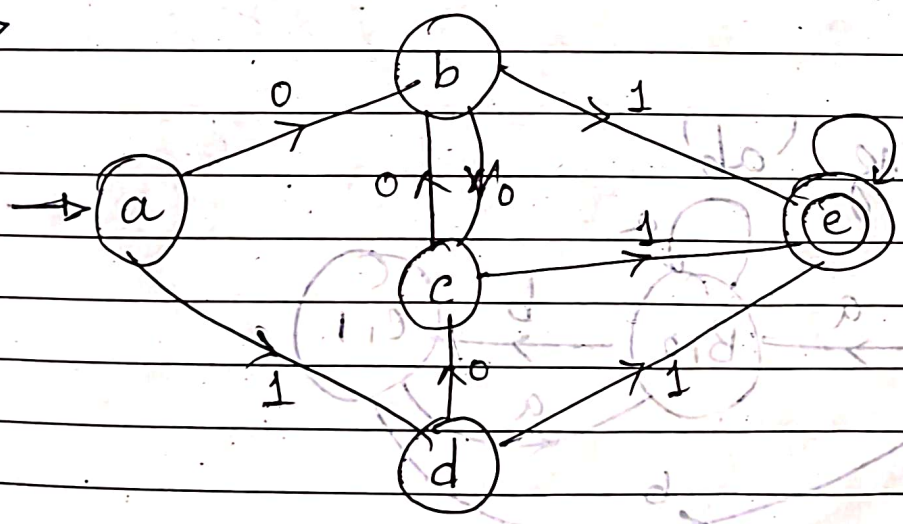
$n = 2$ (n+1)



$n = 2$ (n)



(3)



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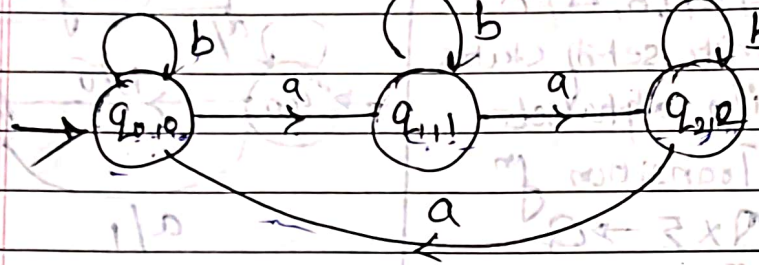
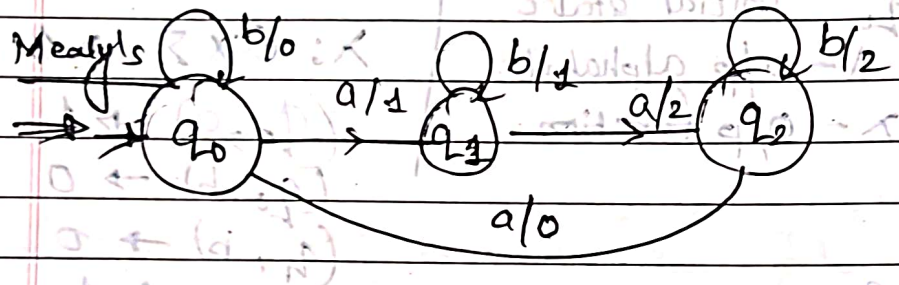
(1) Conversion of Moore's machine to Mealy's machine

1) Count no. of 1's. 3



Into Mealy's



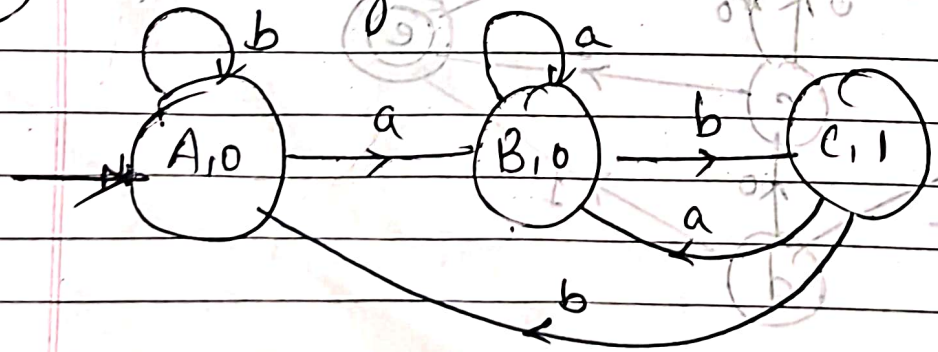
Moore's STT

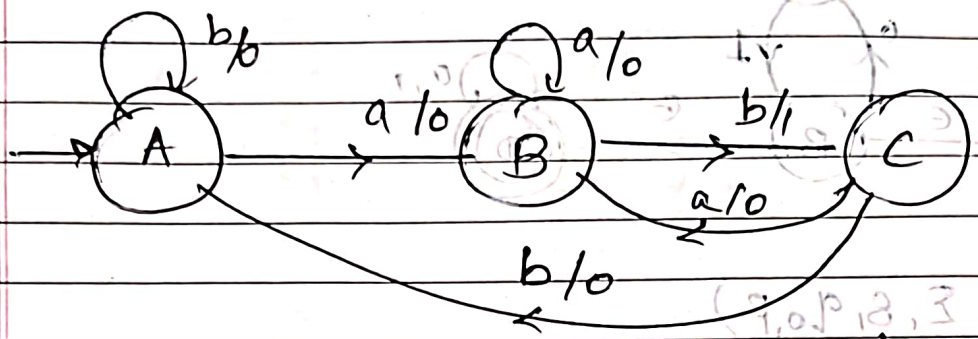
	a	b	Δ
q_0	q_1	q_0	0
q_1	q_2	q_1	1
q_2	q_0	q_2	2

Mealy's STT

	a	b
q_0	$(q_1, 1)$	$(q_0, 0)$
q_1	$(q_2, 2)$	$(q_1, 1)$
q_2	$(q_0, 0)$	$(q_2, 2)$

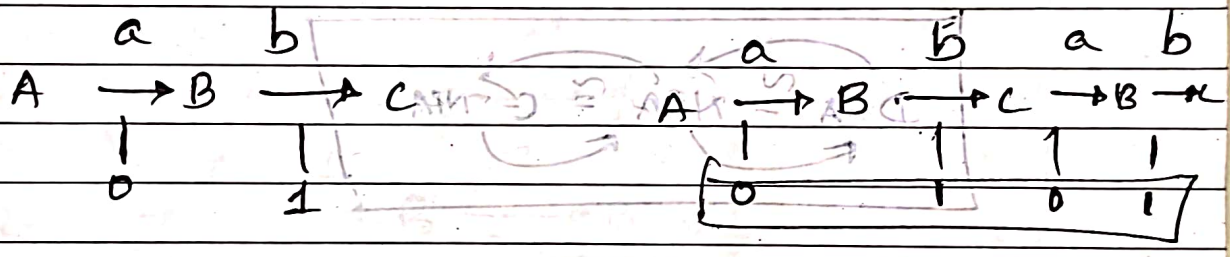
(2) Containing 'ab'



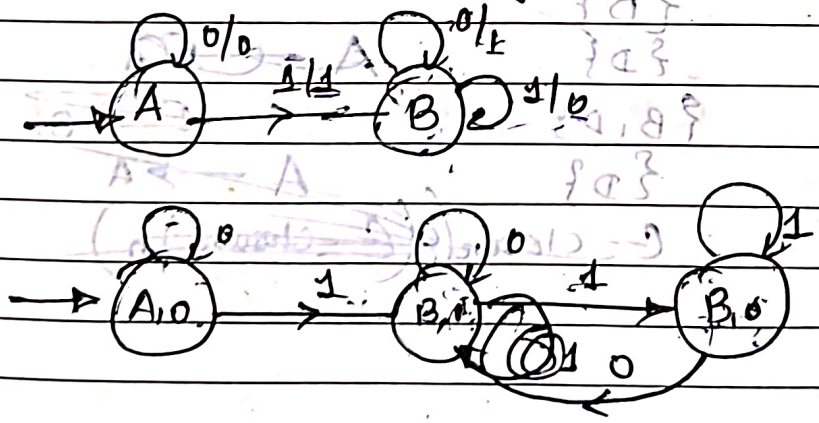


	a	b	A
A	B	A	0
B	B	C	0
C	B	A	1

	a	b
A	(B,0)	(A,0)
B	(B,0)	(C,1)
C	(B,0)	(A,0)



(3) Conversion of Mealy's M/c to Moore's Machine



A	0
B	0
A,0	0
B,1	1
B,0	1

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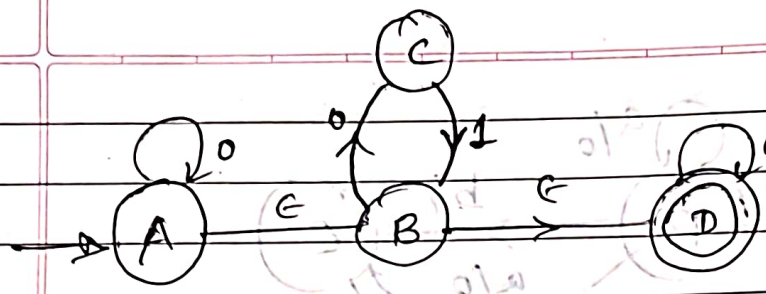
∈-NFA



$(Q, \Sigma, \delta, q_0, F)$

$$\delta : \{Q \times \Sigma\} \cup \{\epsilon\} \rightarrow 2^Q$$

* ϵ-closure $(A) = \{A, B, D\}$

$A \xrightarrow{\epsilon} A$

$A \xrightarrow{\epsilon} B \xrightarrow{\epsilon} D$

DFA $\cong$ NFA $\cong$ ϵ-NFA

ϵ-NFA $\rightarrow$ NFA

	0	1
A	{A,B,C,D}	{D}
B	{C,D}	{D}
C	{∅}	{B,D}
D	{D}	{D}

ϵ-closure

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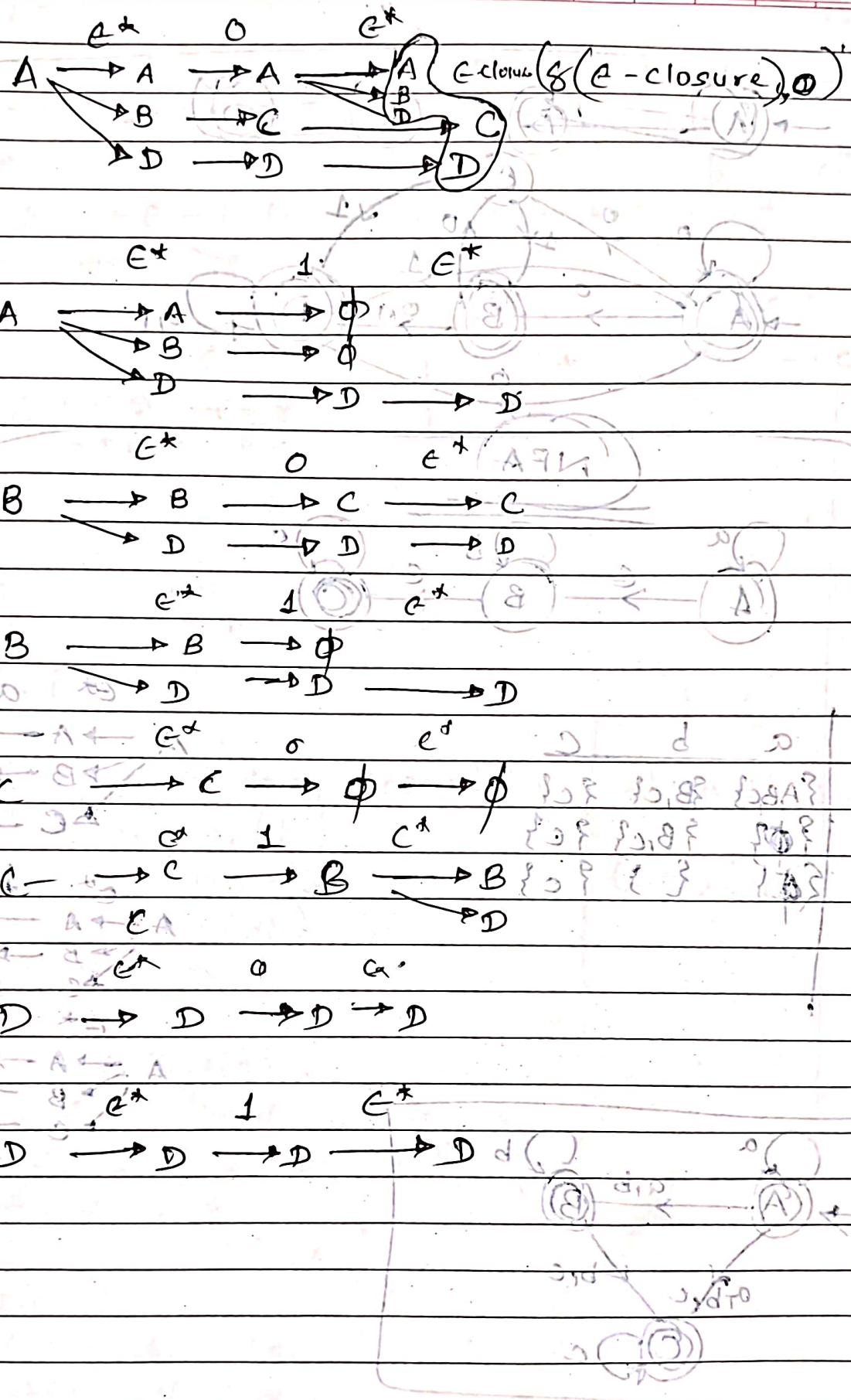
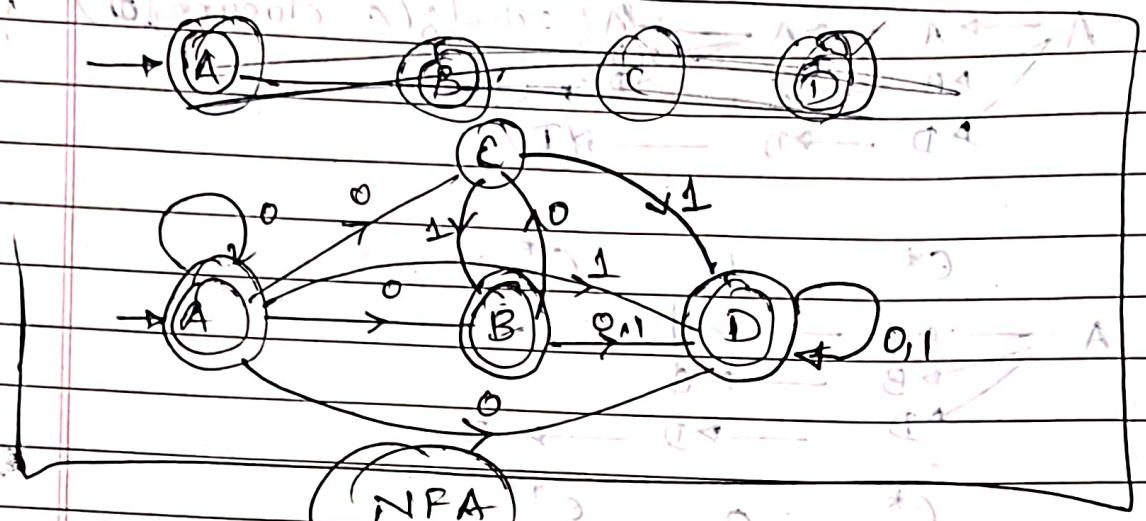
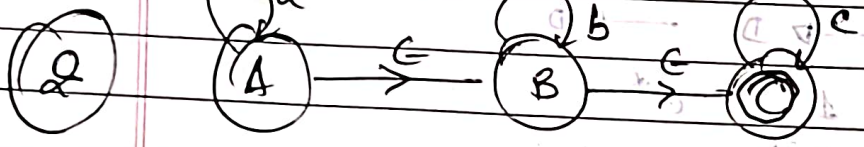
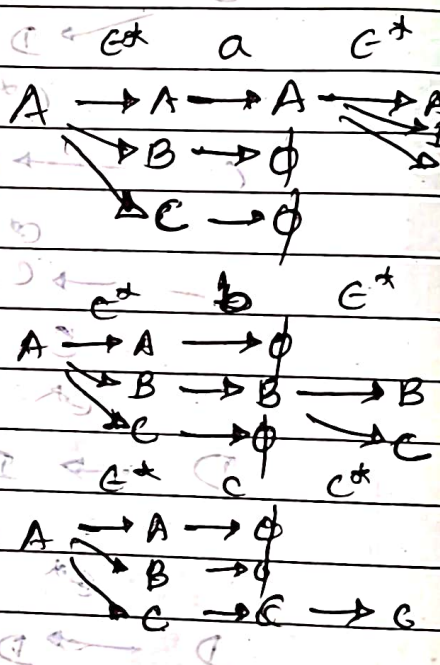
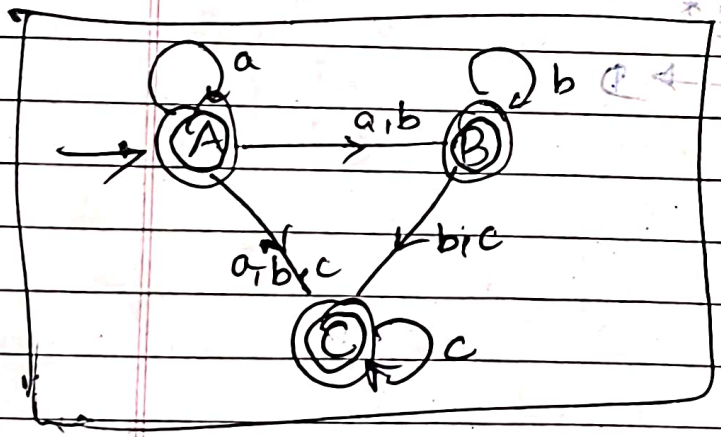


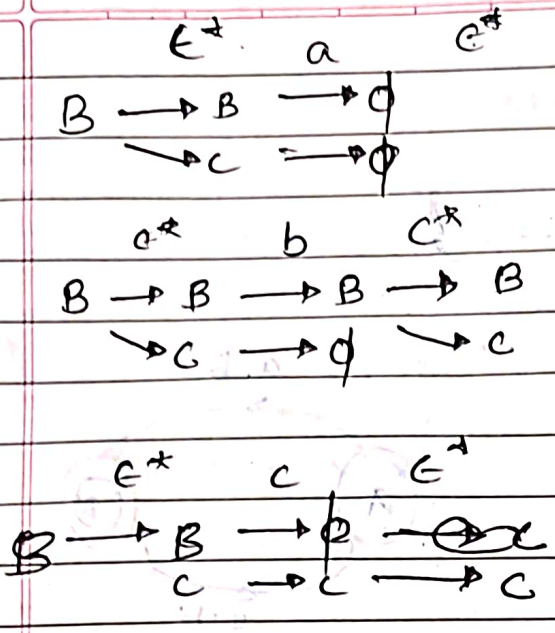


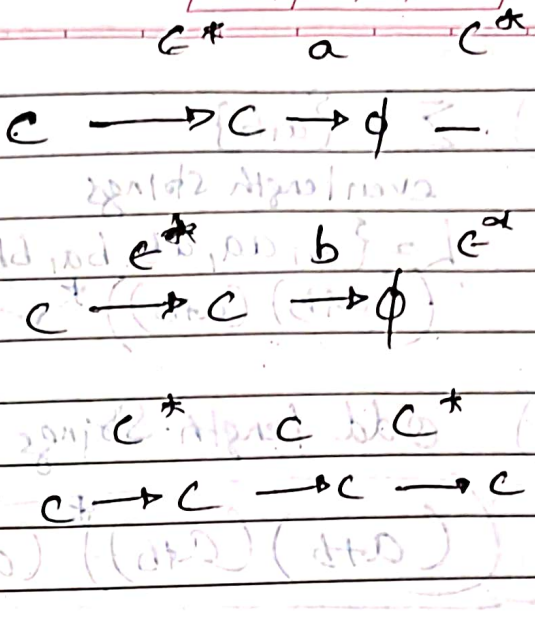


	a	b	c
A	{A,B,C}	{B,C}	{C}
B	{ϕ}	{B,C}	{C}
C	{ϕ}	{ }	{C}









Regular Expression

(1) $\Sigma = \{a, b\}$

L_1 = length of all string exactly '2'

$L = \{aa, ab, ba, bb\}$

$= aa + ab + ba + bb$

$= a(a+b) + b(a+b)$

$= (a+b)(a+b)$

length = 3

$(a+b)(a+b)(a+b)$

(2) ~~$L_2 = \{aa, ab, ba, bb\}$~~

Regular Ex. length is atleast 2

$L = \{aa, ab, ba, bb, aba \ldots\}$

$(a+b)(a+b)(a+b)^*$

(3) Atmost 2

$L = \{\epsilon, a, b, aa, ab, ba, bb\}$

~~$\epsilon + a + b + (a+b) + (a+b)$~~

$\epsilon + a + b + aa + ab + ba + bb$

$= (a+b+\epsilon)(a+b+\epsilon)$ ✓

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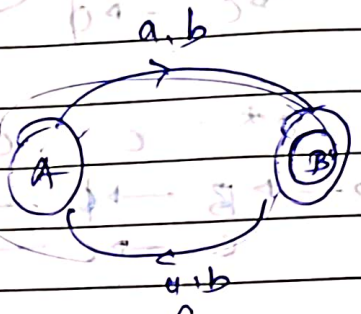
(4) $\Sigma = \{a, b\}$ →
even length strings
$L = \{\epsilon, aa, ab, ba, bb, aaaa, \ldots\}$
$((a+b)(a+b))^*$ ✓

(5) Odd length Strings
$\boxed{((a+b)(a+b))^*(a+b)}$



$(a+b)((a+b)(a+b))^*$

(6) String length divisible by 3.
$L = \{0, 3, 6, 9, 12 \ldots\}$
$\Rightarrow ((a+b)(a+b)(a+b))^*$

(7) String $L \equiv 2 \bmod 3$
$L \bmod 3 = 2$
$((a+b)(a+b)(a+b))^*(a+b)(a+b)$ ✓

(8) $\Sigma = \{a, b\}$ strings in which 'a's are exactly 2.
$\Rightarrow b^*ab^*ab^*$ ✓

(9) Strings 'a's are atleast 2
$b^*ab^*a(a+b)^*$ ✓

(10) No. of a's are atmost 2
$b^*(\epsilon+a)b^*(\epsilon+a)b^*$

(11) No. of a's are even.

~~$b^*(a\ b^*\ a\ b^*)^*$~~

$(b^*\ a\ b^*\ a\ b^*)^*$ ~~b^*~~

$(b^*\ a\ b^*a)^*\ b^*$

(12) Starts with 'a'

$a(a+b)^*$

(13) End with 'a'

$(a+b)^*\ a$

(14) Containing 'a'

$(a+b)^*\ a\ (a+b)^*$

(15) Starting & Ending with different symbols

$a\ (a+b)^*\ b + b(a+b)^*\ a$ ✓

(16) S & E with same symbols.

~~$a(a+b)^*\ a + b\ (a+b)^*\ b$~~

$L = \{\epsilon, a, b, aa, bb, aba, bab \ldots\}$

$a\ (a+b)^*\ a + b\ (a+b)^*\ b + (a+b+\epsilon)$

(17) No 2 a's come together / No 'aa' as substring

$L = \{\epsilon, b, bb, a, ab, aba, abba, abab, ba, bab, baab, baba, \ldots\}$

Building block - b, ab or ba

$(b+ab)^* + (b+ab)^*\ a$

$(b+ab)^*\ (\epsilon + a)$

(18) No 2 a's and no 2 b's should come together

$L = \{\epsilon, a, b, ab, ba, aba, bab, \ldots\ldots\}$

	Starts with	Ends with	
{a, aba, ababa....}	a	a	$(ab)^*a$ or $a(ba)^*$
{ab, abab, ababab....}	a	b	$(ab)^*$ or $a(ba)^*b$
{ba, baba, bababa....}	b	a	$(ba)^*$ or $b(ab)^*a$
{b, bab, babab, ...}	b	b	$(ba)^*b$ or $b(ab)^*$

$(ab)^*a + (ab)^* + b(ab)^*a + b(ab)^*$

$(ab)^*(a+\epsilon) + b(ab)^*(a+\epsilon)$

$(\epsilon + b)(ab)^*(a+\epsilon)$

- Identities of RE

→ $\phi + R = R + \phi = R$

→ $\phi \cdot R = R \cdot \phi = \phi$

→ $\epsilon \cdot R = R \cdot \epsilon = R$

→ $\epsilon^* = \epsilon$

→ $\phi^* = \epsilon$

→ $\epsilon + RR^* = R^*R + \epsilon = R^*$