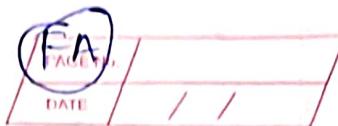


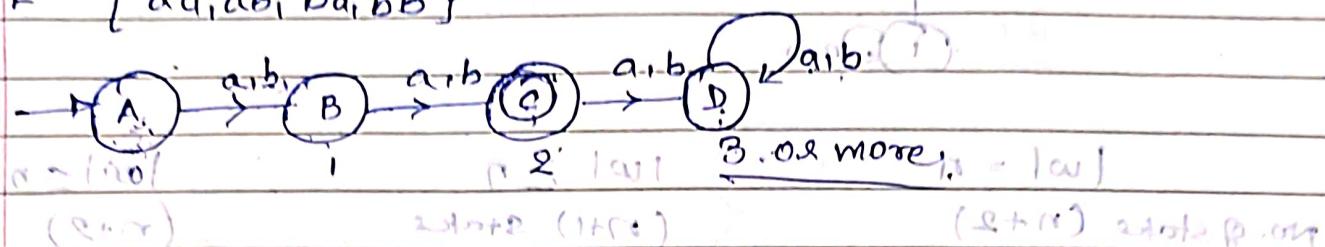
Length of the string. questions



* Construct a DFA, that accepts set of all strings over $\{a, b\}$.

Q) length ≥ 2 .

$$L = \{aa, ab, ba, bb\}$$

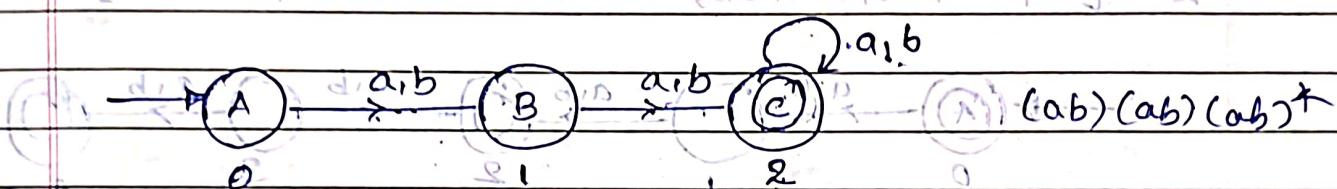


* Construct a DFA, that accepts of string over $\{a, b\}$, where $|w| \geq 2$.

Q) ~~length~~ no. of distinct consecutive pairs in word.

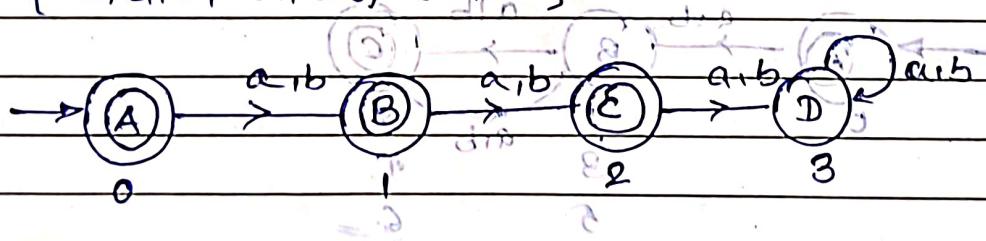
$$L = \{aa, ab, ba, bb, aaa, bbb, \dots\}$$

$\{ \dots, addaa, addab, addba, addbb, addaaa, addbbb, \dots \} = L$

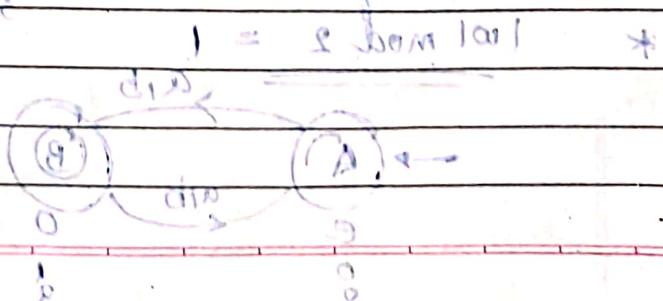
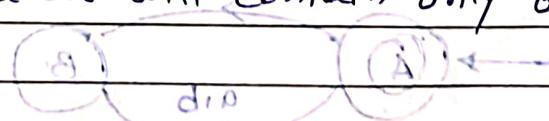


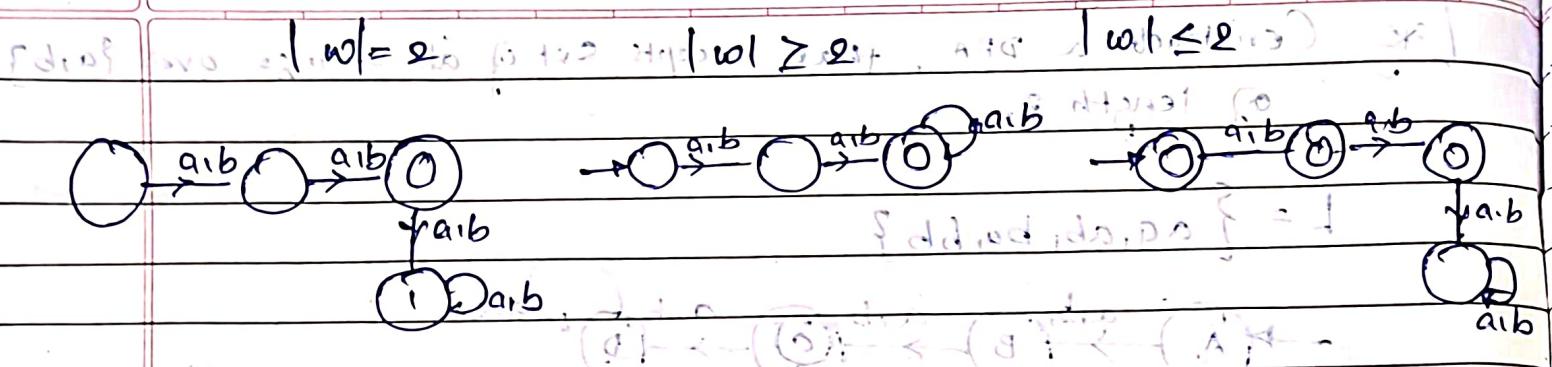
* DFA, that accept $\{a, b\}$ no. of ≤ 2 strings with it.

$$L = \{ \epsilon, a, b, aa, ab, ba, bb \}$$



- L = many DFA's but it will contain only one minimal DFA.

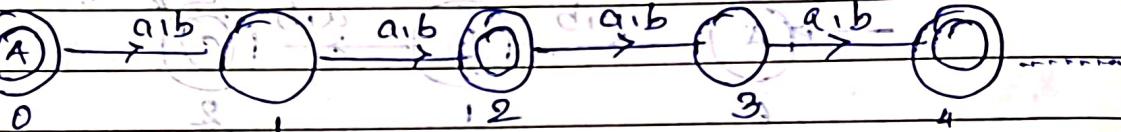




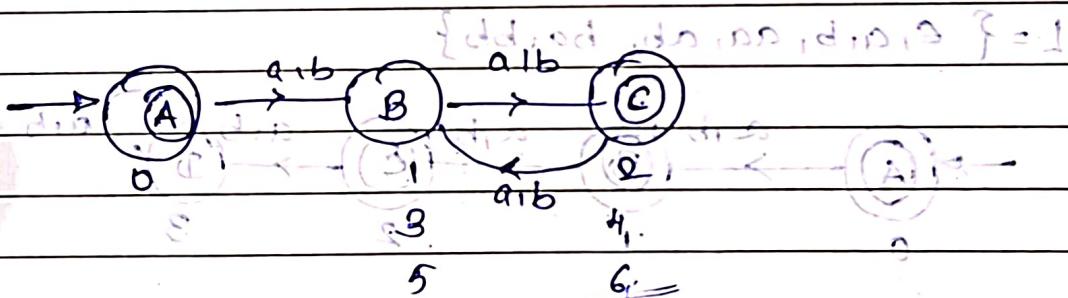
* Construct a minimal DFA, $\omega \in \{ab\}$, $|w| \bmod 2 = 0$

$$L = \{ \epsilon, aa, ab, ba, bb, aaaa, abba, aaba, \dots \}$$

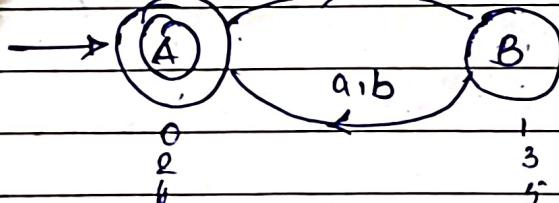
even length



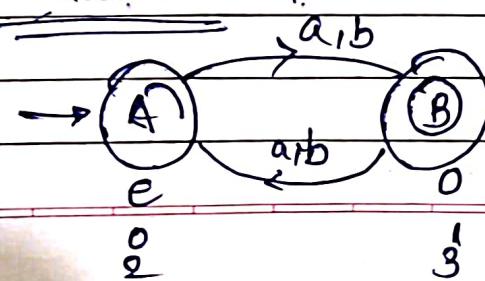
distinguish between even length and odd length



even length a, b if 0 mod 2 $\neq AFG$ $\Rightarrow 1$



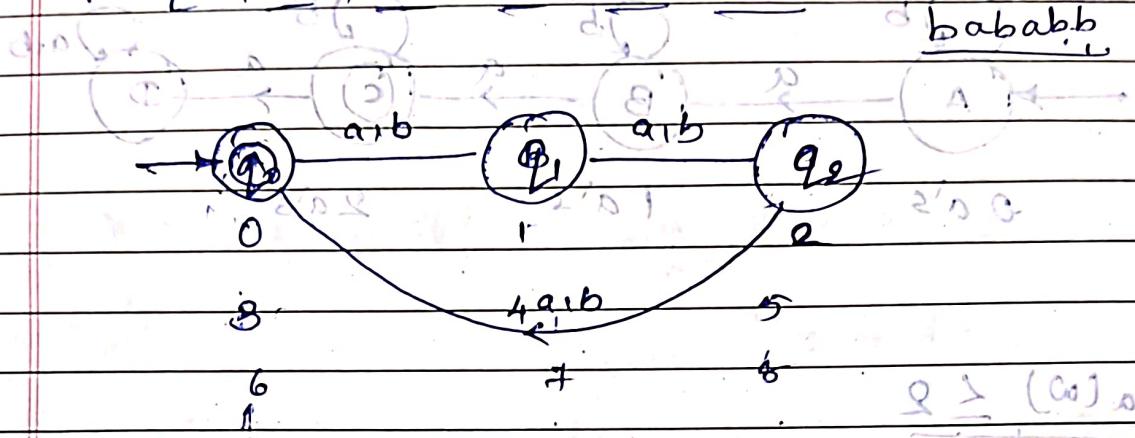
* $|w| \bmod 2 = 1$



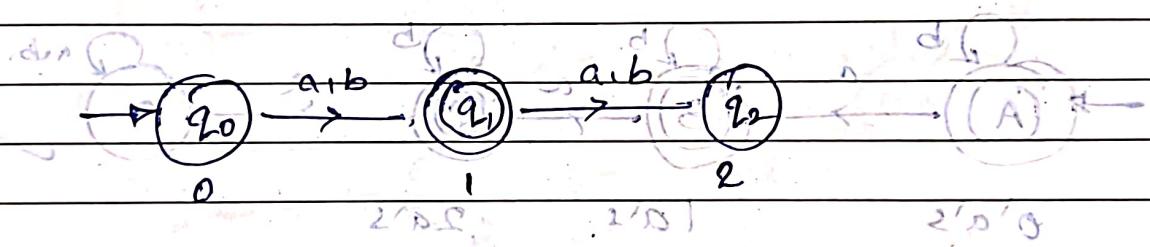
$\hat{y} = (\text{age})^{0.1} \cdot \left\{ \begin{array}{l} \text{dorming age} \quad \text{if } (\text{age}) \text{ dorming age} \\ \text{age} \quad \text{otherwise} \end{array} \right.$

$$w \in \{a,b\}^* \quad |w| \bmod 3_1 = 1$$

$$L = \{ \epsilon, \underline{aaa}, \underline{aba}, \underline{abb}, \underline{baa}, \underline{bab}, \dots \}$$



$$100 \cong 1 \pmod{3} \quad \text{and} \quad 100 \not\equiv 1 \pmod{8}$$



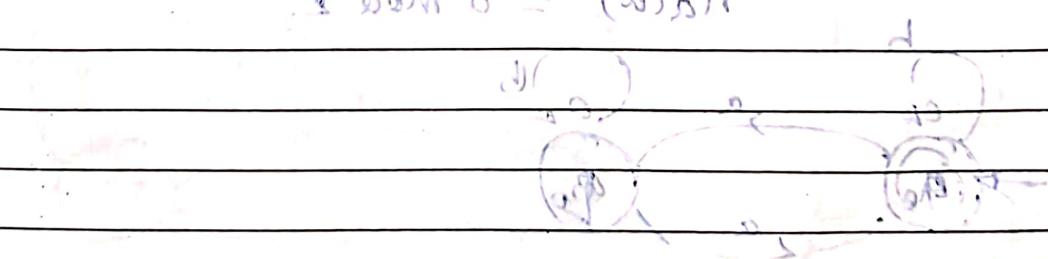
$|w| \bmod n = 0$, then no. of states is equal to \underline{n}

ANS 29 B 6-00 starting 45° incline

$$\theta = \pm \arctan(\omega_0)$$

10. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

主寫的口語 (w) 言



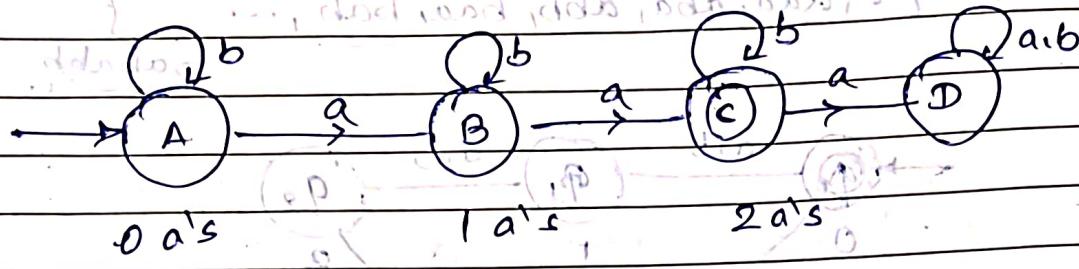
$$f = \sum_{k=0}^{\infty} b_k \sin(k\omega) e^{ikx}$$

Restrictions on 'a'

PAGE No.	11
DATE	

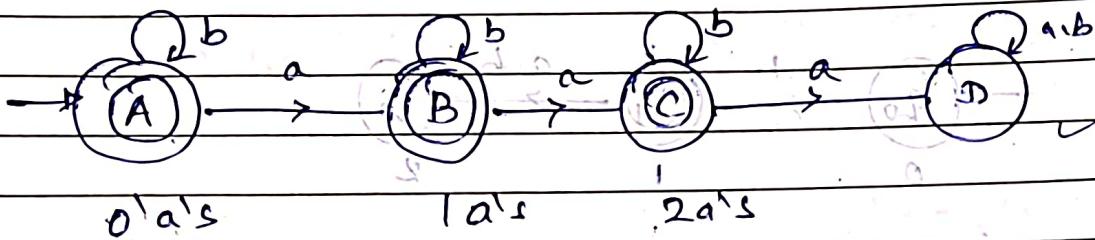
* Construct a minimal DFA $w \in \{a, b\}^*$, $na(\omega) \leq 2$

$$L = \{aa, baa, aba, abba, bbba, \dots\}$$



* $na(\omega) \leq 2$

$$L = \{ \epsilon, a, aa, b, ab, abb, aabb, \dots \}$$



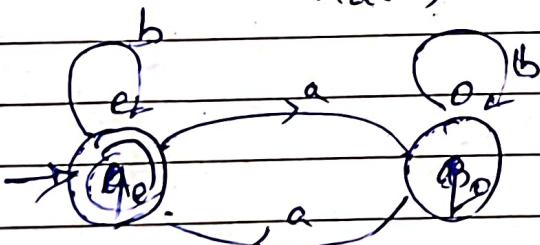
* $w \in \{a, b\}^*$ where no. of a is even.

or

$$na(\omega) \bmod 2 = 0$$

or

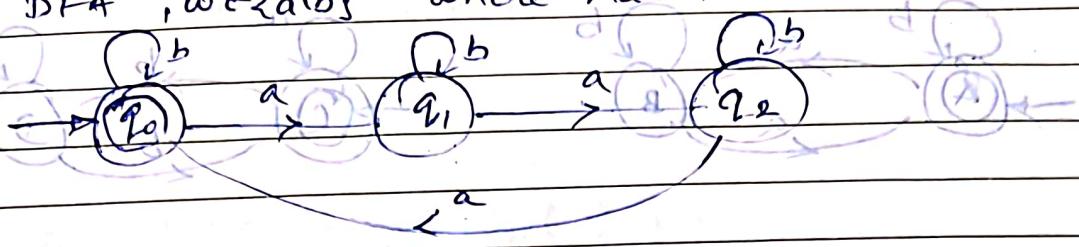
$$na(\omega) \equiv 0 \pmod{2}$$



* $na(\omega) \bmod 2 = 1$.

(GATE MARCH 2009) QUESTION

* DFA, $w \in \{a,b\}^*$ where $n_a(w) \bmod 3 = 0$



* DFA, $w \in \{a,b\}^*$, $n_a(w) \equiv 0 \pmod{2}$

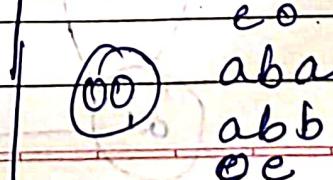
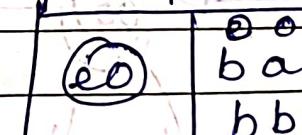
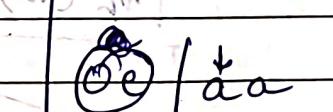
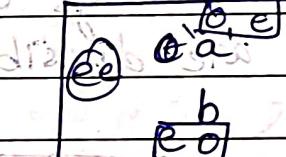
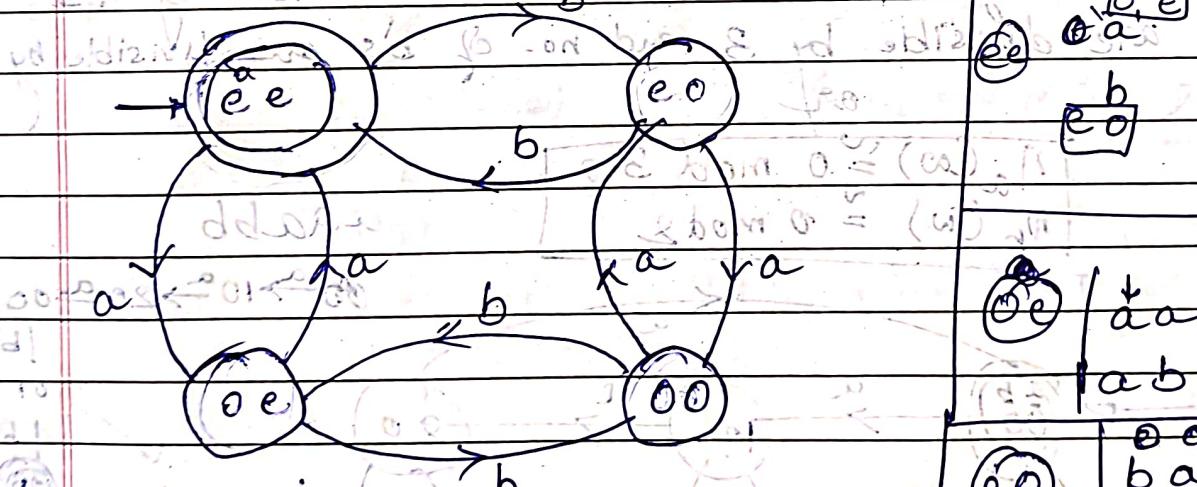
1st Method

$\Rightarrow \{ \text{aa, bb, aab, bab, babb, ...} \}$

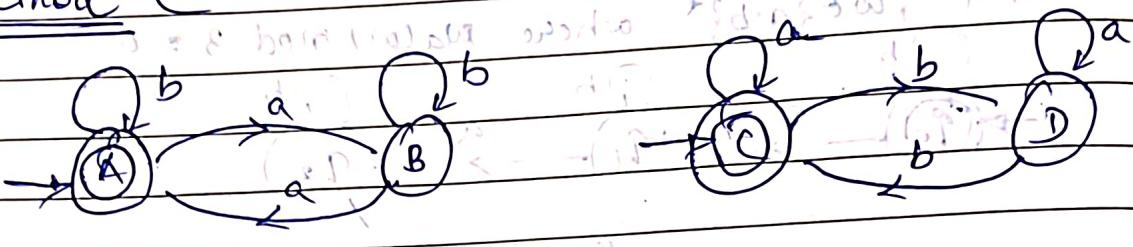
$n_a(w)$	$n_b(w)$	String
0	0	- aa, bb,
0	1	- aab
1	0	- aaabb
1	1	- ab

for two states no. of strings = 2ⁿ

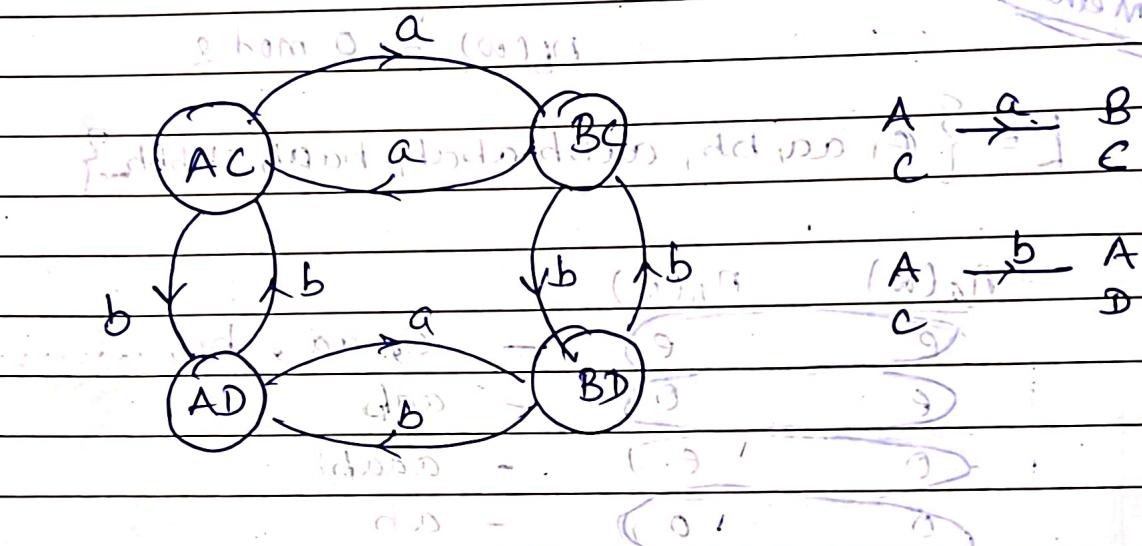
for 3 states no. of strings = 2ⁿ * 3ⁿ



2nd Method (CROSS PRODUCT METHOD)



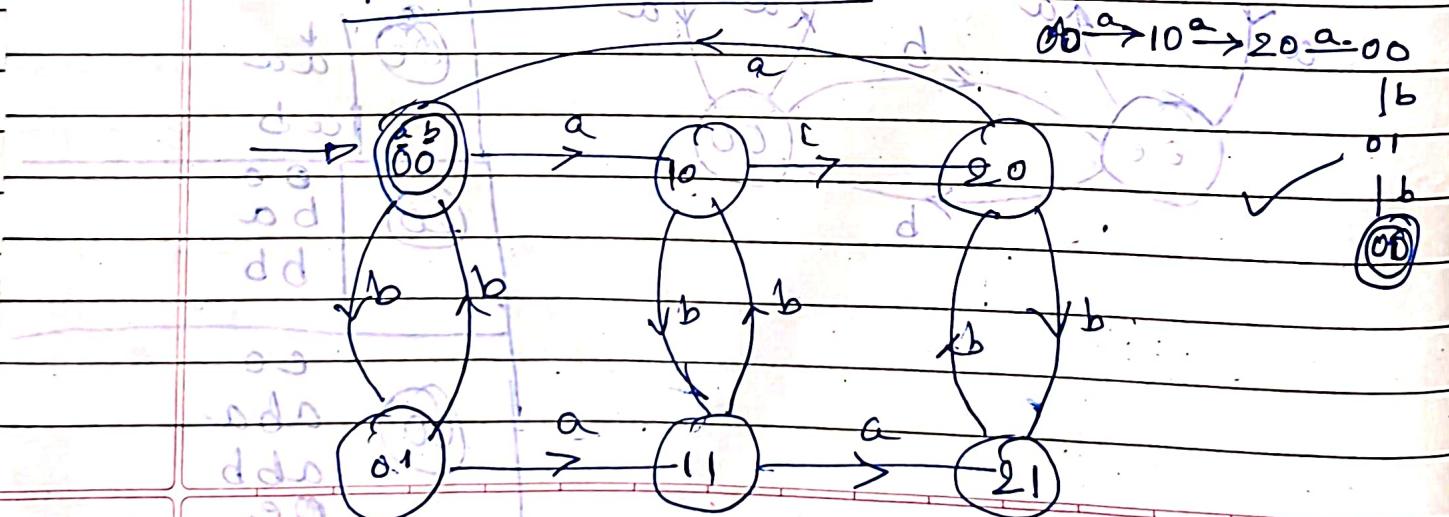
$$\{A, B\} \times \{C, D\} = \{AC, AD, BC, BD\}$$



* Construct a minimal DFA which accepts set of all strings over $\{a, b\}$ in which number of a 's are divisible by 3 and no. of b 's are divisible by 2.

$$n_a(w) \equiv 0 \pmod{3}$$

$$n_b(w) \equiv 0 \pmod{2}$$



* $n_a(\omega) \bmod 3 \geq n_b(\omega) \bmod 3$ (minima is 0 but 0s)

$$\checkmark = \{10, 20, 21, 11\}$$

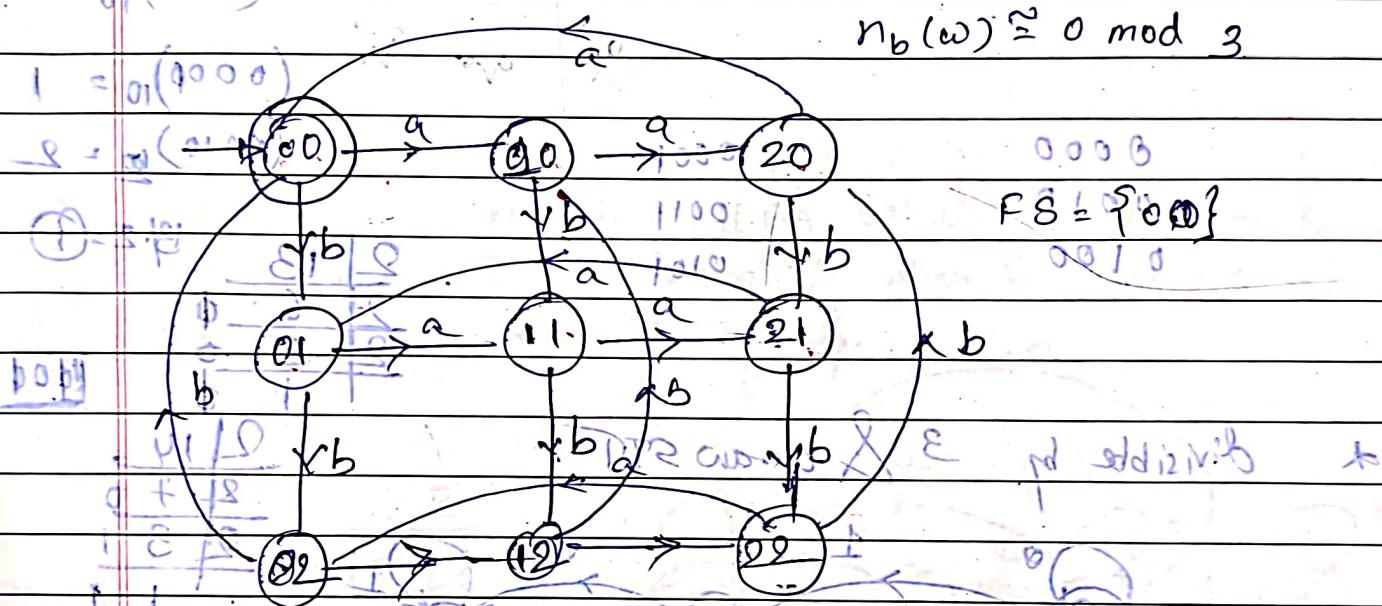
* $n_a(\omega) \bmod 3 = n_b(\omega) \bmod 2$

$$\checkmark = \{00, 11\}$$

$$\begin{aligned} n_a(\omega) \bmod 3 &= 0 \\ n_b(\omega) \bmod 3 &= 0 \end{aligned}$$

o * (Construct a DFA $\omega \in \{a, b\}^*$ $n_a(\omega) \equiv 0 \pmod{3}$

$$n_b(\omega) \equiv 0 \pmod{3}$$



$$n_a(\omega) \bmod 3 = 1 \quad \& \quad n_b(\omega) \bmod 3 = 2$$

$$S = \{10, 21\}$$

$$n_a(\omega) \bmod 3 > n_b(\omega) \bmod 3$$

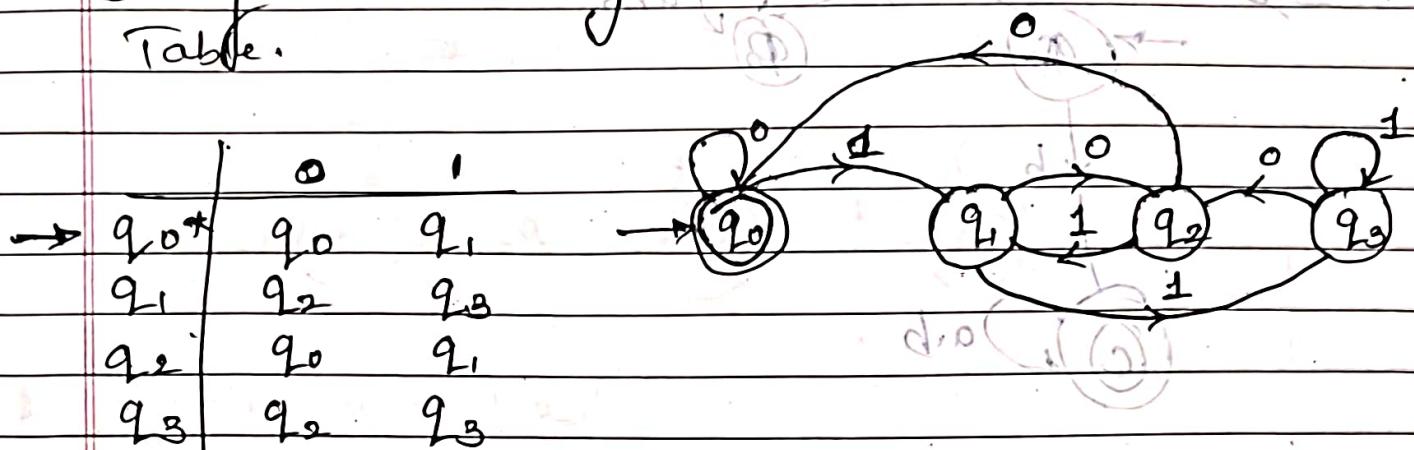
$$L = \{10, 20, 21\}$$



\overline{P}	\overline{Q}	\overline{R}

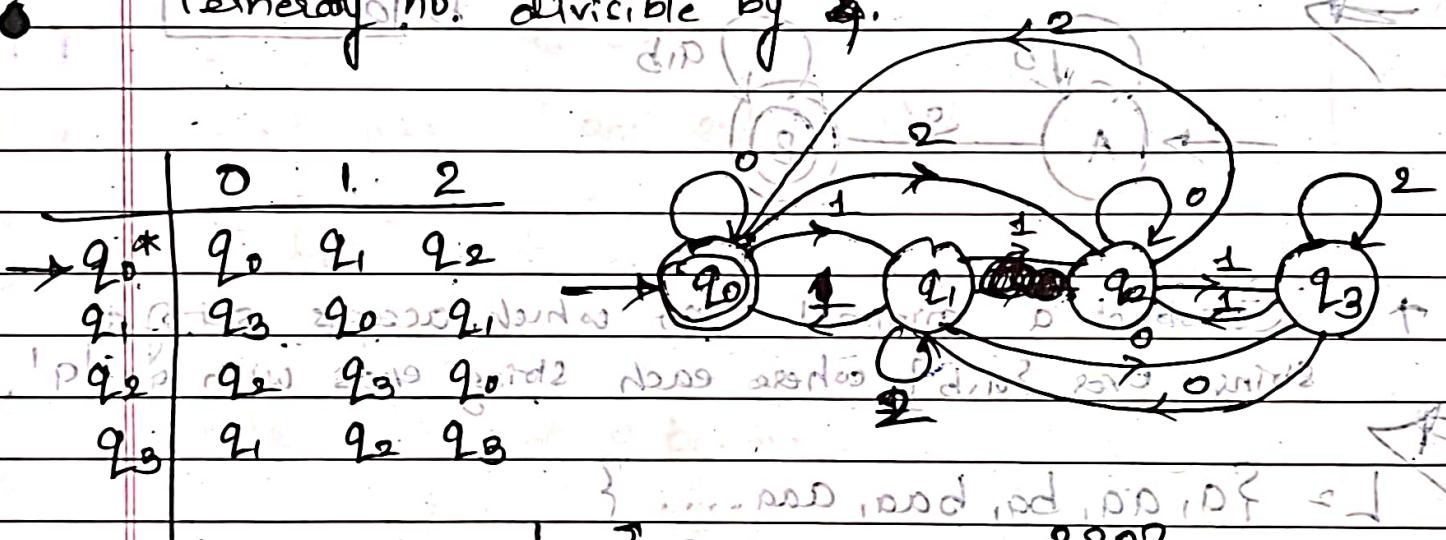
(PTB)

* Construct a minimal DFA which accepts set of all strings over $\{0, 1\}$ which when interpreted as binary no. divisible by 4. Also, draw a State Transition Table.



(to 2 classes NFA has minimum 3 states)
10² min. states for 2 class NFA
Comparing with 2 class NFA we get 2 states

* Construct a minimal DFA which accepts set of all strings over $\{0, 1, 2\}$ which when interpreted as Ternary no. divisible by 4.



$$L = \{0, 11, 22, 121, 202, \dots\}$$

$$(11)_3 =$$

$$(22)_3 =$$

$$\begin{aligned} (11)_3 &= 1 \times 3 + 1 \times 3 \\ (11)_2 &= 1 \times 2 + 1 \times 2 \\ (11)_2 &= 11 \end{aligned}$$

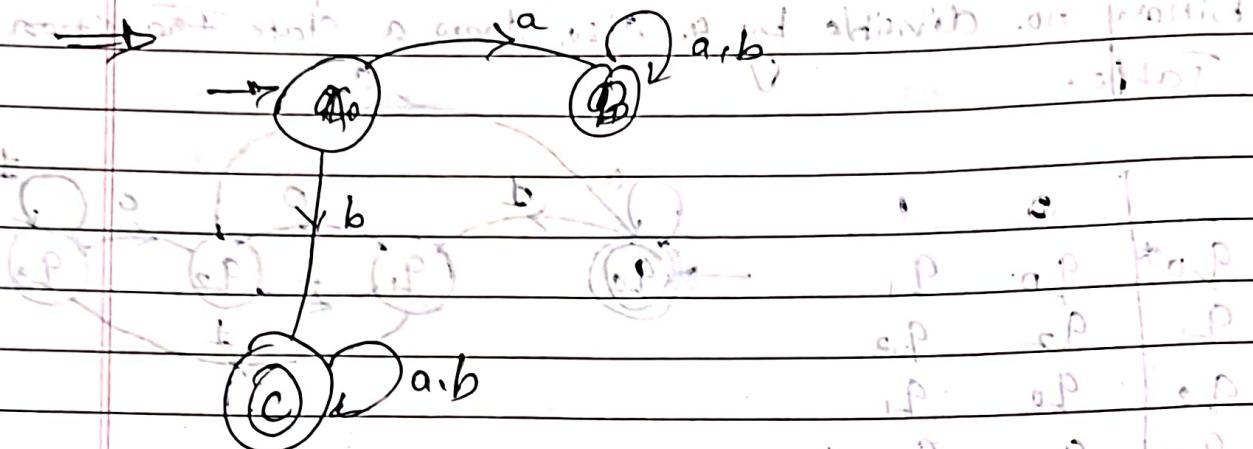
$$1 \times 3 + 1$$

$$3 + 1 = 4$$

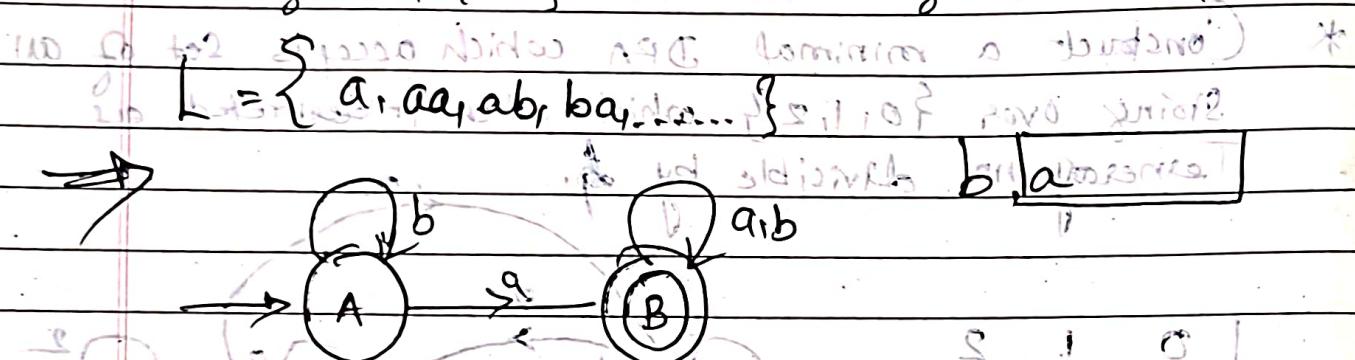
$$\begin{aligned} (22)_3 &= 2 \times 3 + 2 \\ 6 + 2 &= 8 \end{aligned}$$

$$\begin{aligned} 1 \times 3 + 2 \times 3 + 1 \times 3 &= 9 \\ 9 + 6 + 3 &= 18 \end{aligned}$$

* Construct a minimal DFA for set of all strings starting with a ; $\Sigma = \{a, b\}$.

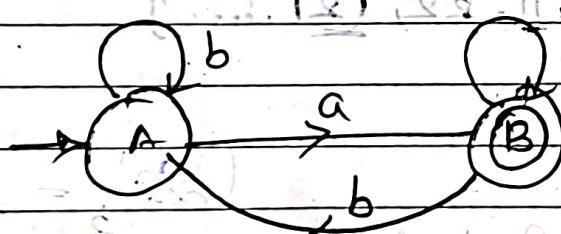


* Construct a minimal DFA which accepts set of all strings over $\{a, b\}$ where each string contains 'a'.



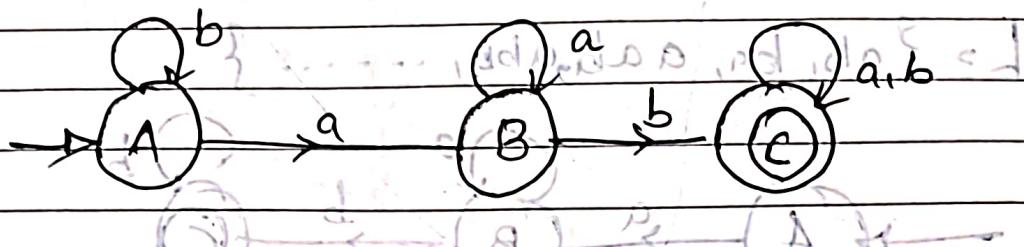
* Construct a minimal DFA which accepts set of all strings over $\{a, b\}$ where each string ends with 'aa'.

$$L = \{a, aa, ba, baa, aaa, \dots\}$$



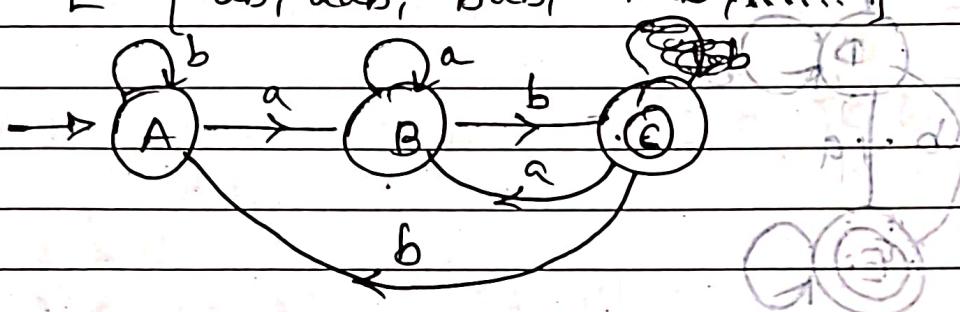
* Construct a DFA which accept set of all strings over $\{a, b\}$ which contains 'ab'.

$$\Rightarrow L = \{ab, aab, aba, bab, \dots\}$$



* Ending with "ab"

$$\Rightarrow L = \{ab, aab, bab, abab, \dots\}$$

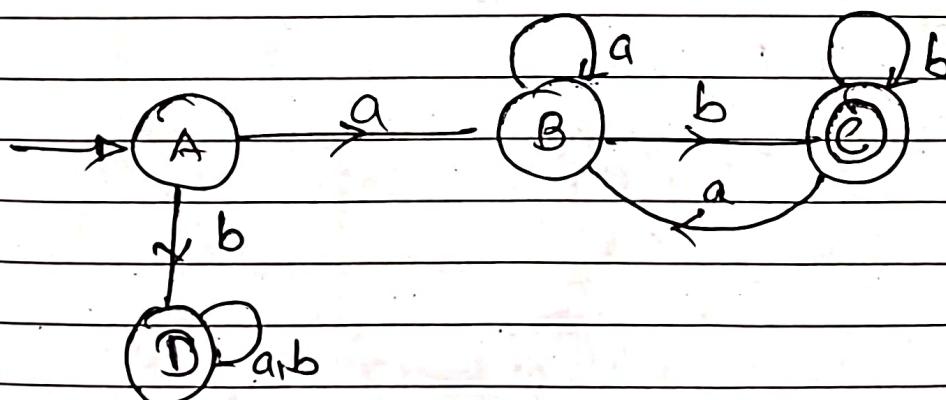


* Starts with 'a' and ends with 'b'

$$L_1 = \{a, ab, aa, aaa, \dots\}$$

$$L_2 = \{b, ab, bb, aab, \dots\}$$

$$L_1 L_2 = \{ab, aab, abb, \dots\}$$



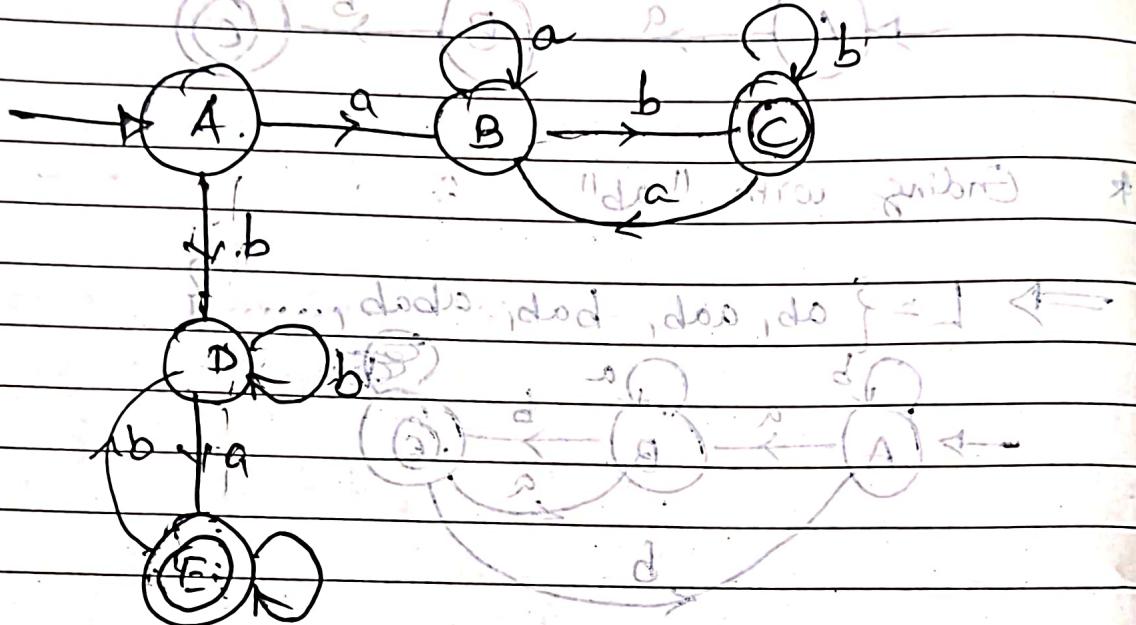
★

Starts and ends with different symbols

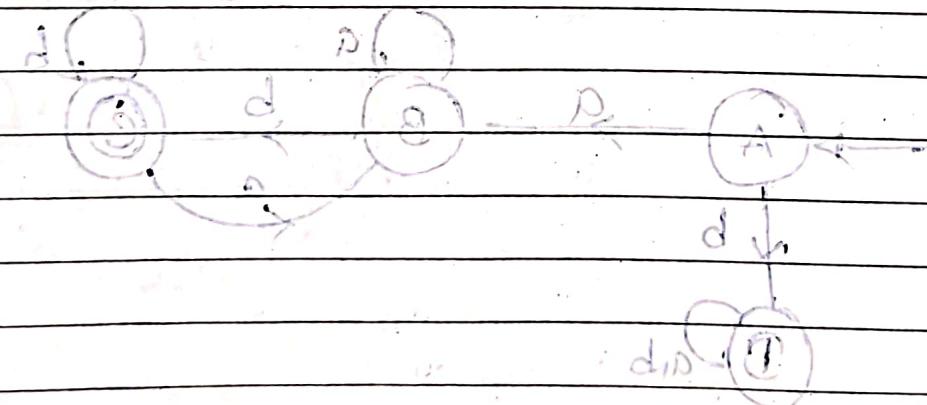
$$a \xrightarrow{ } b$$

$$b \xrightarrow{ } a$$

$$L = \{ab, ba, aab, abb, \dots\}$$



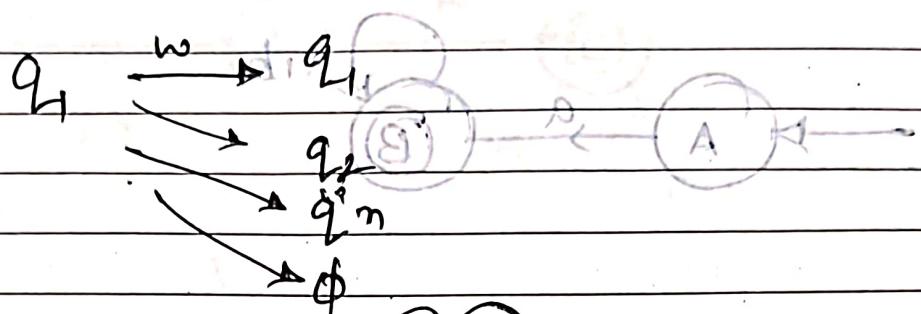
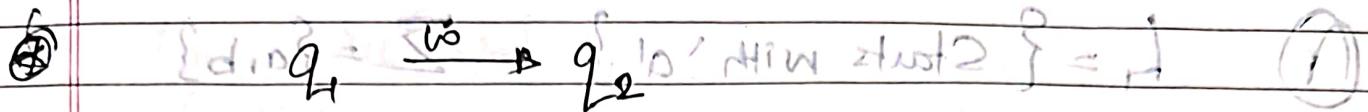
★ Starting and ending with same symbol



NFA

Non-Deterministic Finite Automata

PAGE No. / / /
DATE / / /

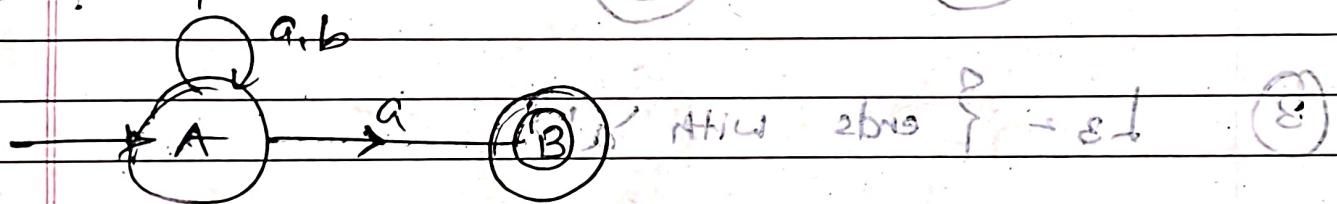


Computer Model

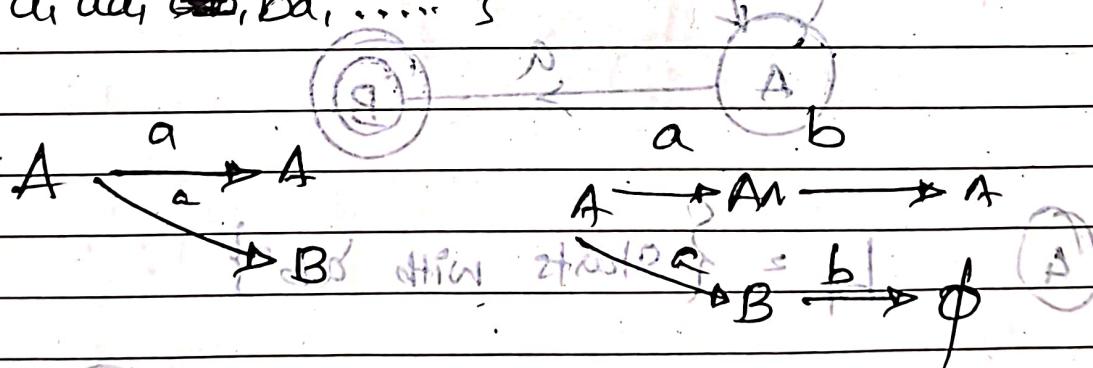
NFA $S: Q \times \Sigma \rightarrow 2^Q$ (2)

($Q, \Sigma, \delta, q_0, F$) DFA $\delta: Q \times \Sigma \rightarrow Q$

eg. $L = \{ \text{ends with 'a'} \}$ ($\{ \text{aa, ab, ba, } \dots \} \rightleftharpoons \Sigma = \{a, b\}$)

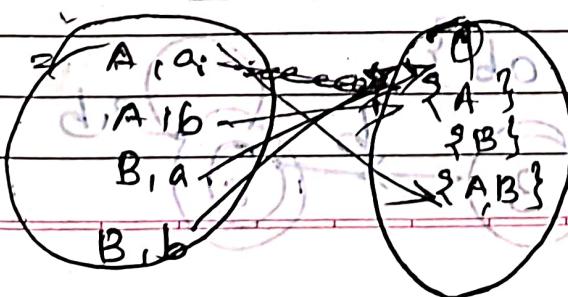


$L = \{ a, aa, ab, ba, \dots \}$



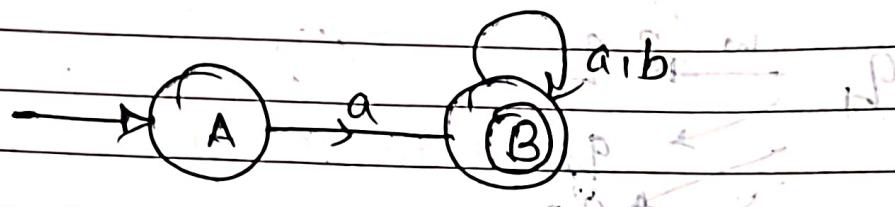
$Q = \{ A, B \}$ $\Sigma = \{ a, b \}$

$\leftarrow Q \times \Sigma$



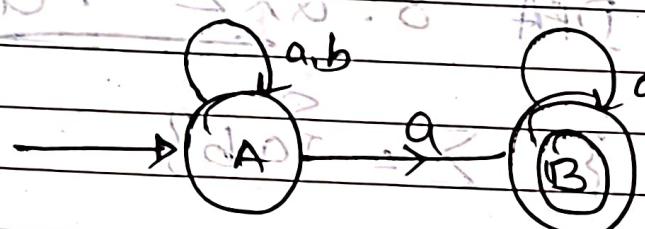
①

$$L_1 = \{ \text{Starts with 'a'} \} \quad \Sigma = \{a, b\}$$



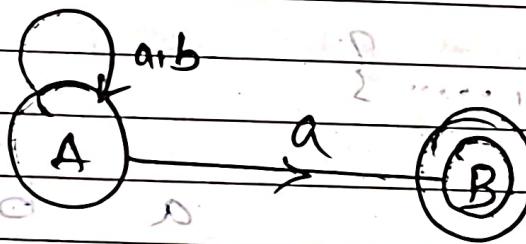
②

$$L_2 = \{ \text{Contains 'a'} \}$$



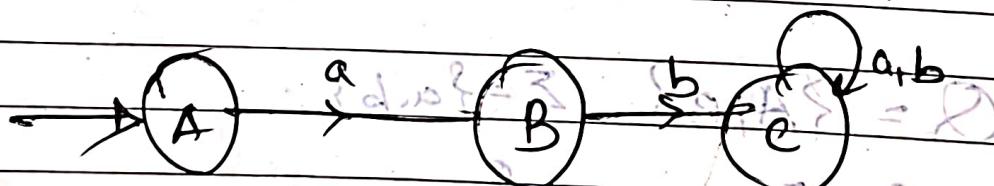
③

$$L_3 = \{ \text{ends with 'a'} \}$$



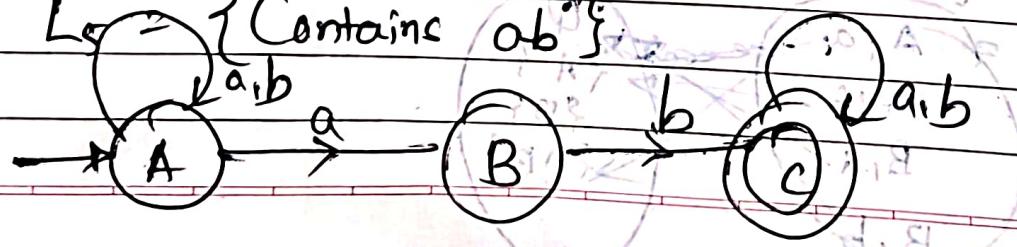
④

$$L_4 = \{ \text{Starts with 'ab'} \}$$



⑤

$$L_5 = \{ \text{Contains 'ab'} \}$$



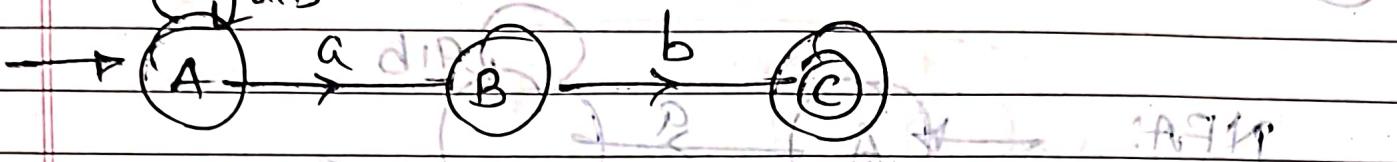
PAGE NO.
DATE

A9C et A9H pour l'application

(6)

Ende with Sabl

a,b



A9C pour T2

F.

a

A

d

o

G.

a

B

φ

A

C.

C

(I)

B

B

(1)

((8))

(A)

A9(1)

$$1 \otimes ((d, \{g, A\}) \otimes)$$

$$\{g, A\} = (d, A) \otimes$$

$$\{A\} = d(A) \otimes$$

$$0(d, A) \otimes = \{d, \{g, A\}\} \otimes$$

$$(d, g) \otimes \cap (d, A) \otimes \otimes \otimes = (d, \{g, A\}) \otimes$$

$$\{g, A\} =$$

$$0(d, A) \otimes = \{d, \{g, A\}\} \otimes$$

$$(d, g) \otimes \cup (d, A) \otimes = (d, \{g, A\}) \otimes$$

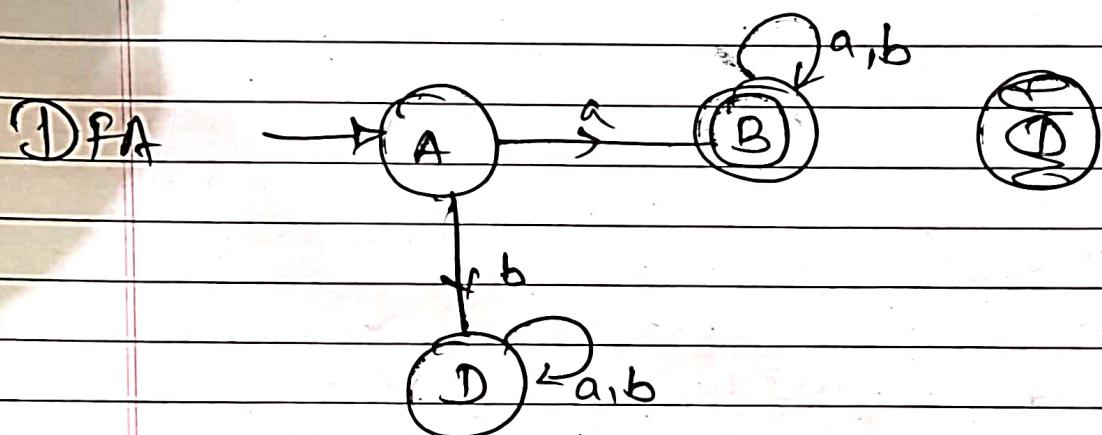
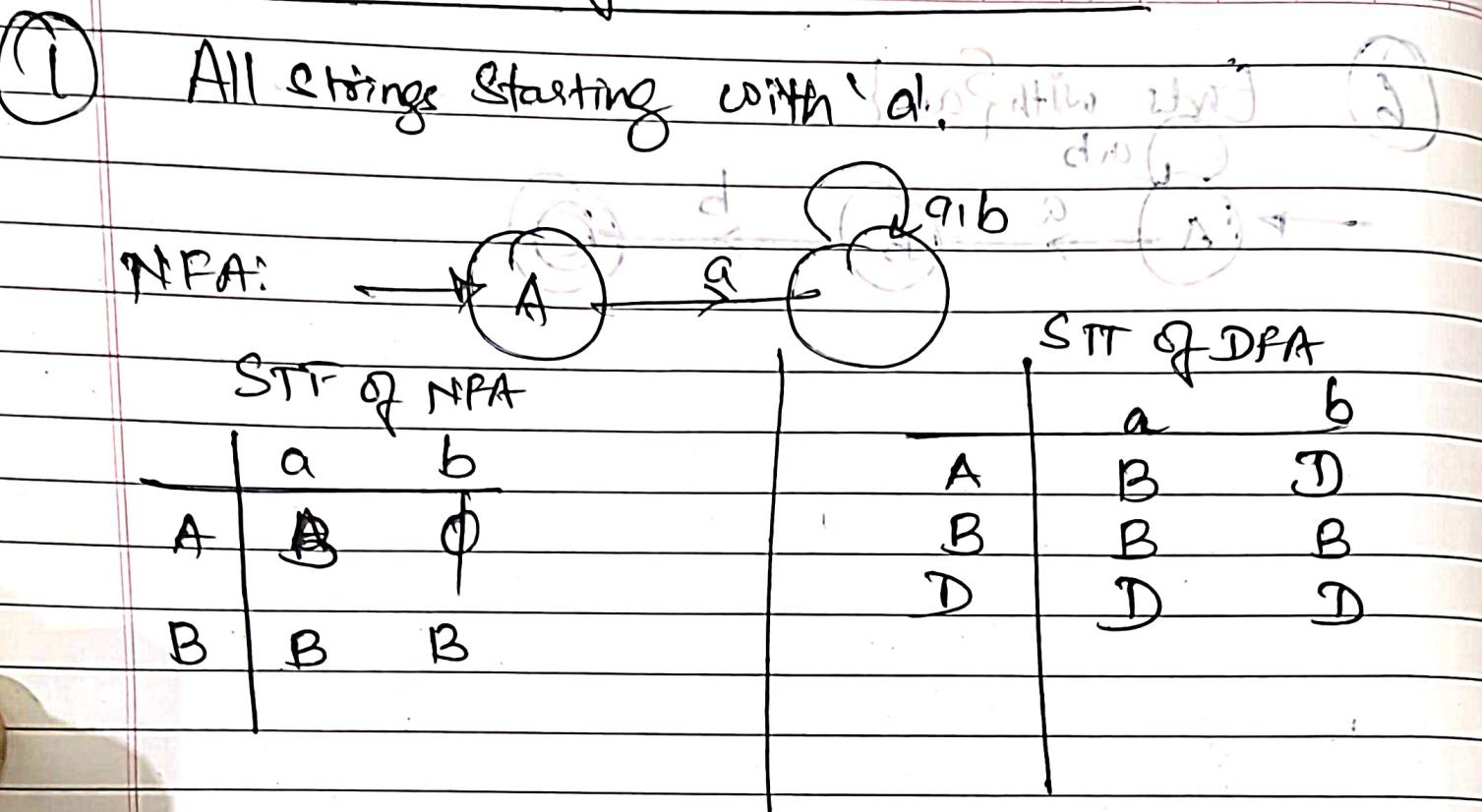
$$\{g, A\} =$$

$$(d, g) \otimes \cup (d, A) \otimes = (d, \{g, A\}) \otimes$$

$$\{g, A\} =$$

Conversion from NFA to DFA

PAGE No.



$$\delta(A, a) = \{A, B\}$$

$$\delta(A, b) = \{A\}$$

$$\delta(\{A, C\}, b) = \{A\}$$

$$\delta(\{A, B\}, a) = \delta(A, a) \cup \delta(B, a)$$

$$= \{A, B, C\}$$

$$\delta(\{A, B, C\}, a) = \delta(A, a) \cup$$

$$\delta(B, a) \cup \delta(C, a)$$

$$= \{A, B, C\}$$

$$\delta(\{A, B\}, b) = \delta(A, b) \cup \delta(B, b)$$

$$= \{A\} \cup \{C\}$$

$$\delta(\{A, B, C\}, b) = \delta(A, b) \cup$$

$$\delta(B, b) \cup \delta(C, b)$$

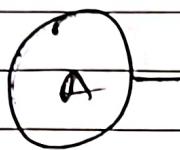
$$= \{A\}$$

$$\delta(\{A, C\}, a) = \delta(A, a) \cup \delta(C, a)$$

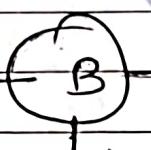
$$= \{A, B\} \cup \{A\} = \{A, B\}$$

(2)

"all strings in which second symbol from RHS is 'a'"



a, b



a



a, b



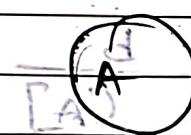
b

a, b

DPA

2nd symbol from LHS is 'a'

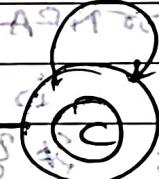
NPA



a, b

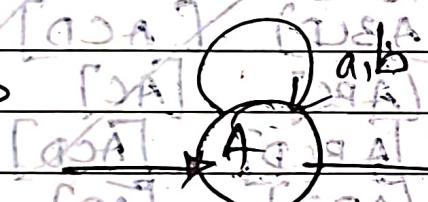


a

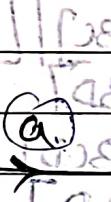


a, b

HRA



a, b



a



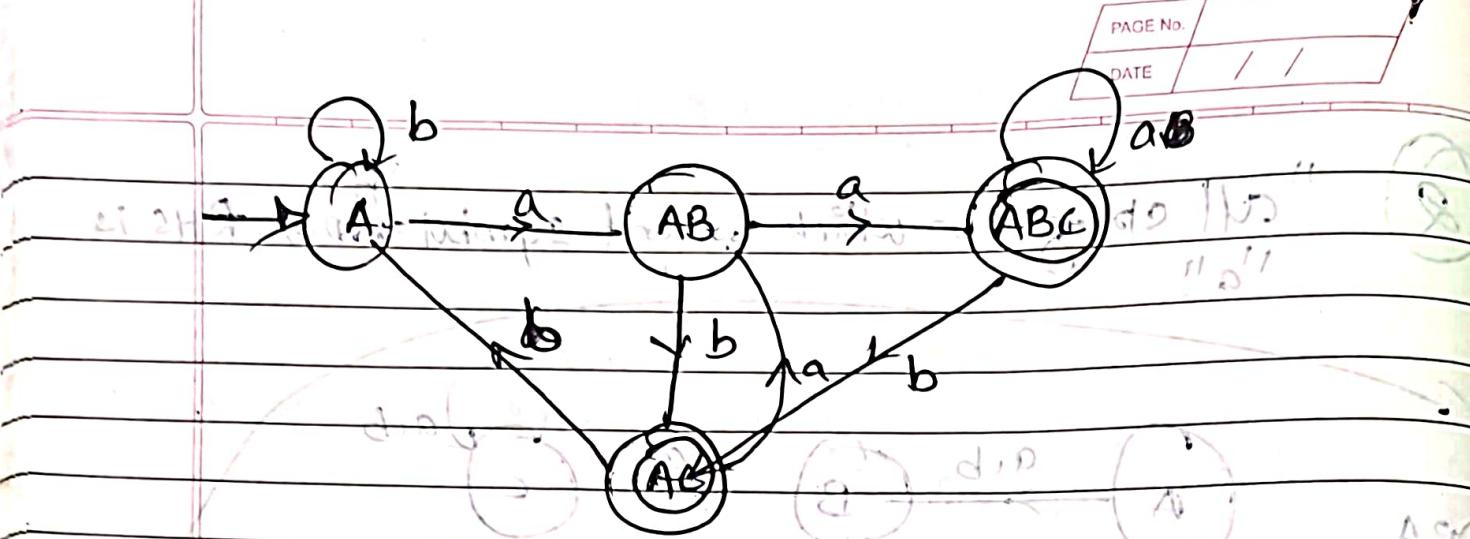
a, b

STT for NPA

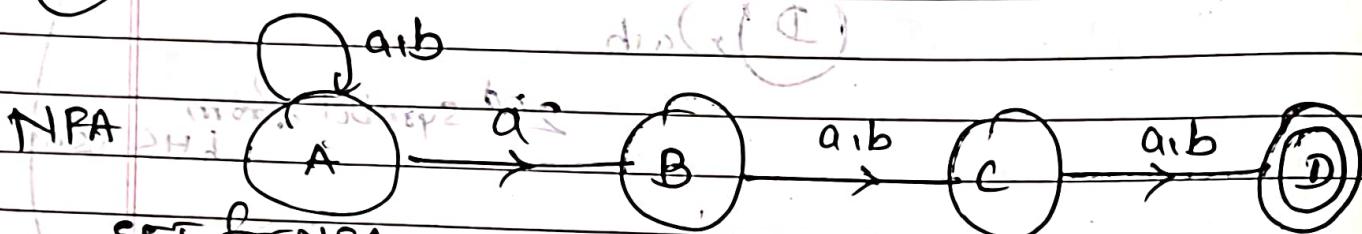
	a	b
a	$\{A, B\}$	$\{A\}$
b	$\{C\}$	$\{C\}$
*	$\{\cdot\}$	$\{\cdot\}$

STT for DPA

	a	b
a	$[AB]$	$[ABC]$
b	$[AC]$	$[CAB]$
*	$[ABC]$	$[CAB]$
		$[ac]$

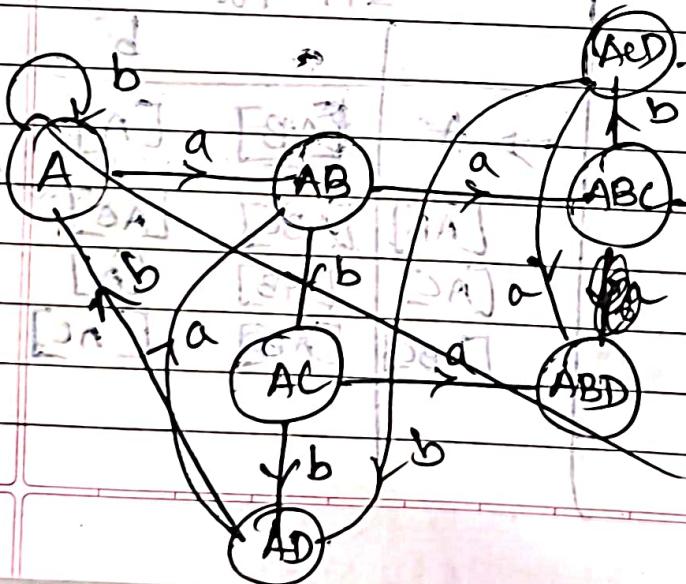


(b) 5th symbol from RHS is 'a'



	a	b
A:	{A,B}	{A}
B:	{C}	{C}
C:	{D}	{D}
D:	{E}	{}

	a	b
[A]	[PAB]	[A]
[AB]	[ABC]	[AC]
[AC]	[ABD]	[AD]
[AD]	[AB]	[A]
[ABC]	A.BCD	[ACD]
[ABD]	[ABC]	[AC]
[A BCD]	[ABCD]	[ACD]
[AeD]	[ABD]	[AD]



AGC (6 month summary)

	a	b
→ A	[AB]	[A]
[AB]	[ABC]	[Ac]
[AC]	[ABD]	[AD]
* [AD]	[AB]	[A]
• [ABC]	[ABCD]	[ACD]
← [ABD]	[ABC]	[AC]
↑ [ACD]	[ABD]	[AD]
← [ABCD]	[ABD]	[ABCD]

Analogous 0, $0 = |w|$ Analogous 1, $1 = |w|$ Analogous 2, $2 = |w|$ Analogous 3, $3 = |w|$

a	b	c
EP	IP	OP

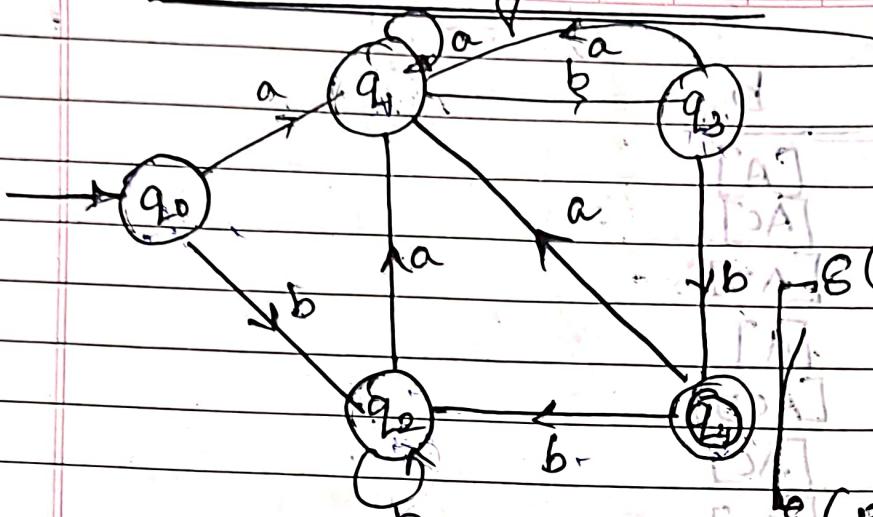
→ EP	EP, CP, IP, OP	Analogous 0
[EP]	[EP]	Analogous 1
[EP]	[EP]	Analogous 2
[EP]	[EP]	Analogous 3

signals out

Minimization of DFA

Initial

PAGE 2X
DATE / /



(P, q)

$\delta(P, w) \in F \Rightarrow \delta(q, w)$

$\delta(P, w) \notin F \Rightarrow \delta(q, w) \notin F$

$|w| = 0, \emptyset$ equivalent

$|w| = 1, 1$ equivalent

$|w| = 2, 2$ equivalent

$|w| = n, n$ equivalent

	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_3
q_2	q_1	q_2
q_3	q_1	q_4
$* q_4$	q_1	q_2

0 equivalent

$$[q_0 \ q_1 \ q_2 \ q_3] \quad [q_4] \leftarrow$$

1 equivalent

$$[q_0, q_1, q_2] \quad [q_3] \quad [q_4]$$

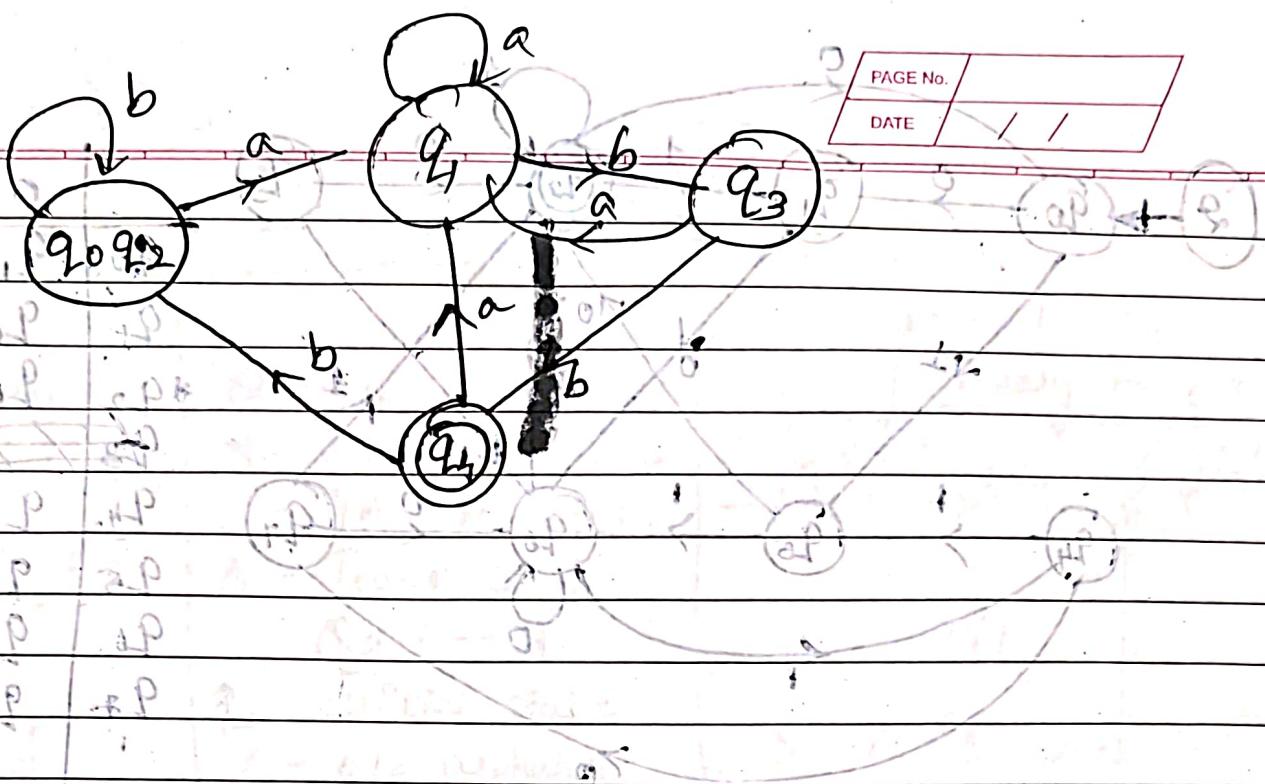
2 equivalent

$$[q_0, q_2] \quad [q_1] \quad [q_3] \quad [q_4]$$

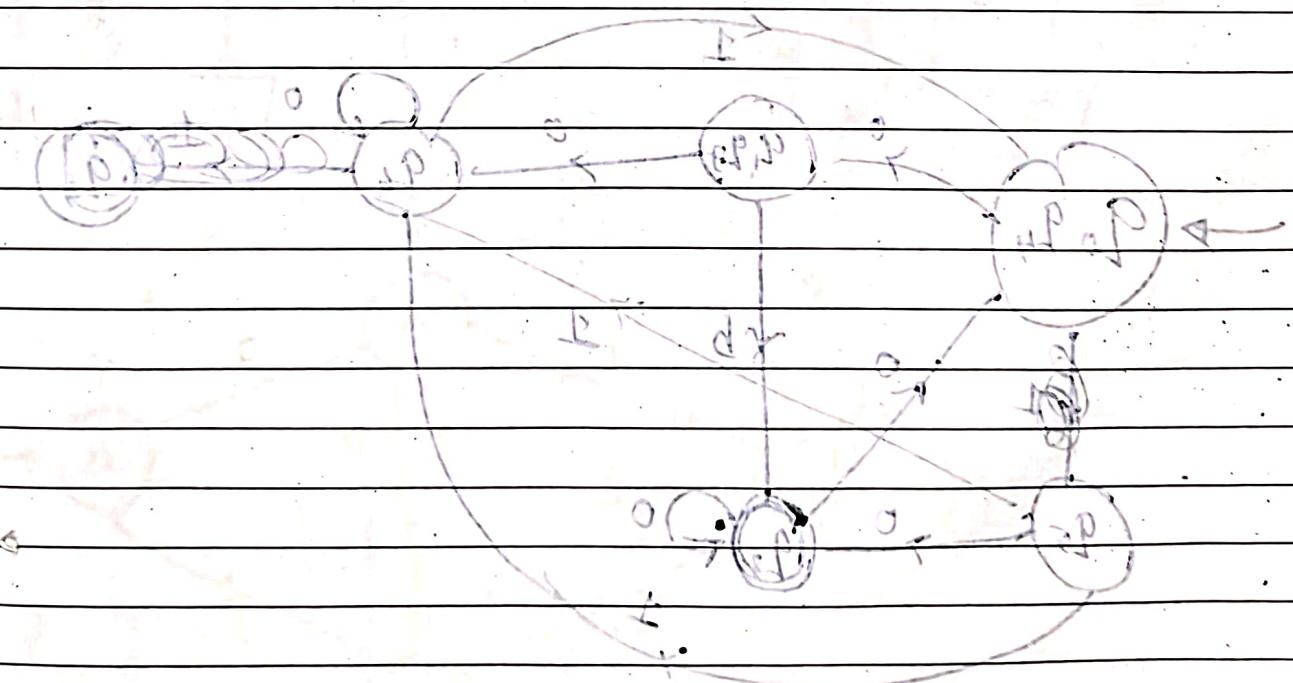
3 equivalent

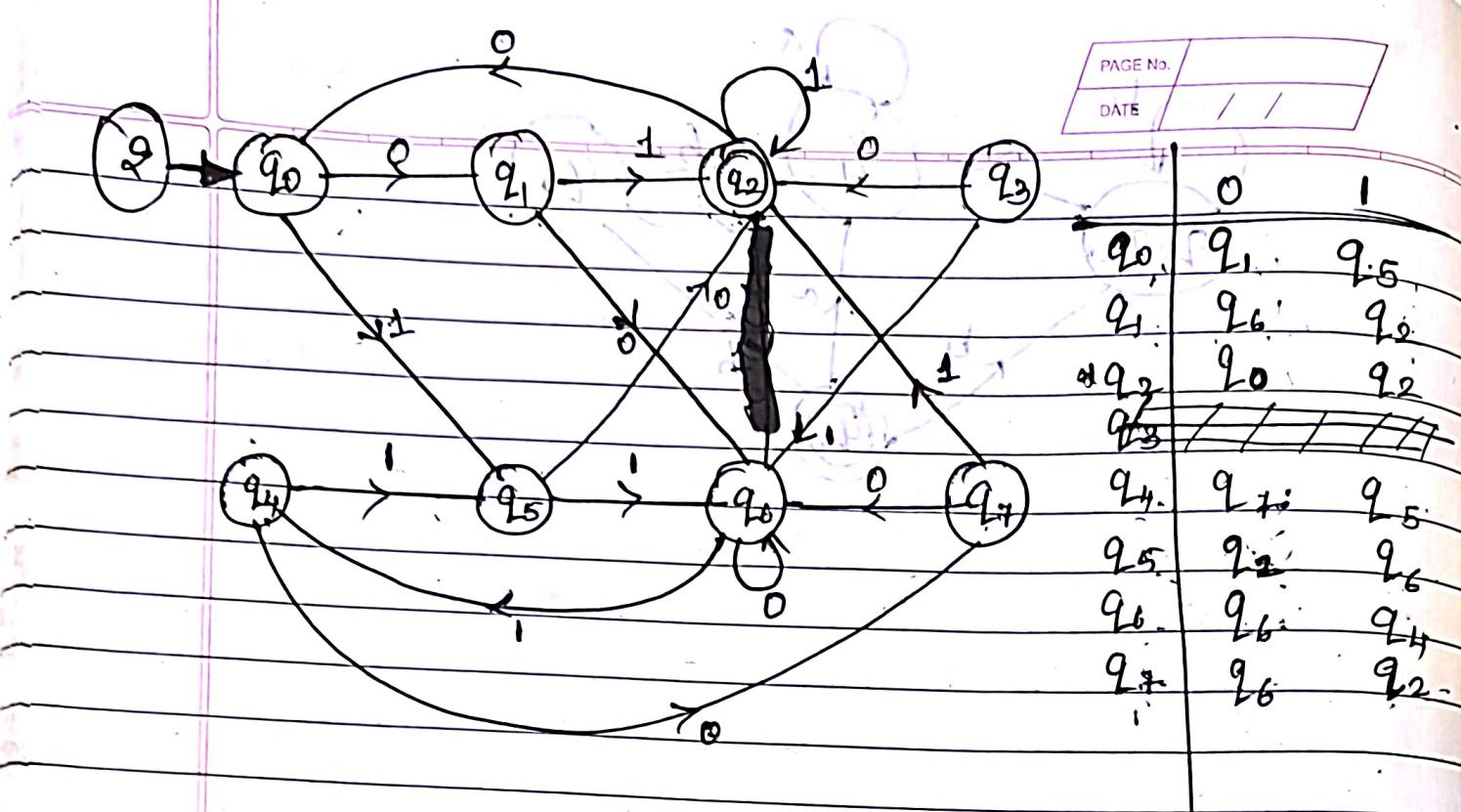
$$[q_0, q_2] \quad [q_1] \quad [q_3] \quad [q_4]$$

No changes.



$L = \{ \text{sp} | \text{fsp}, \text{isp}, \text{asp}, \text{nsip}, \text{rsip} \}$
 $\text{fsp} | \text{sp} | \text{fp}, \text{ip}, \text{ap}$
 $\text{isp} | \text{sp} | \text{fp}, \text{ip}$
 $\text{asp} | \text{sp} | \text{fp}, \text{ip}$
 $\text{nsip} | \text{sp} | \text{fp}, \text{ip}$
 $\text{rsip} | \text{sp} | \text{fp}, \text{ip}$





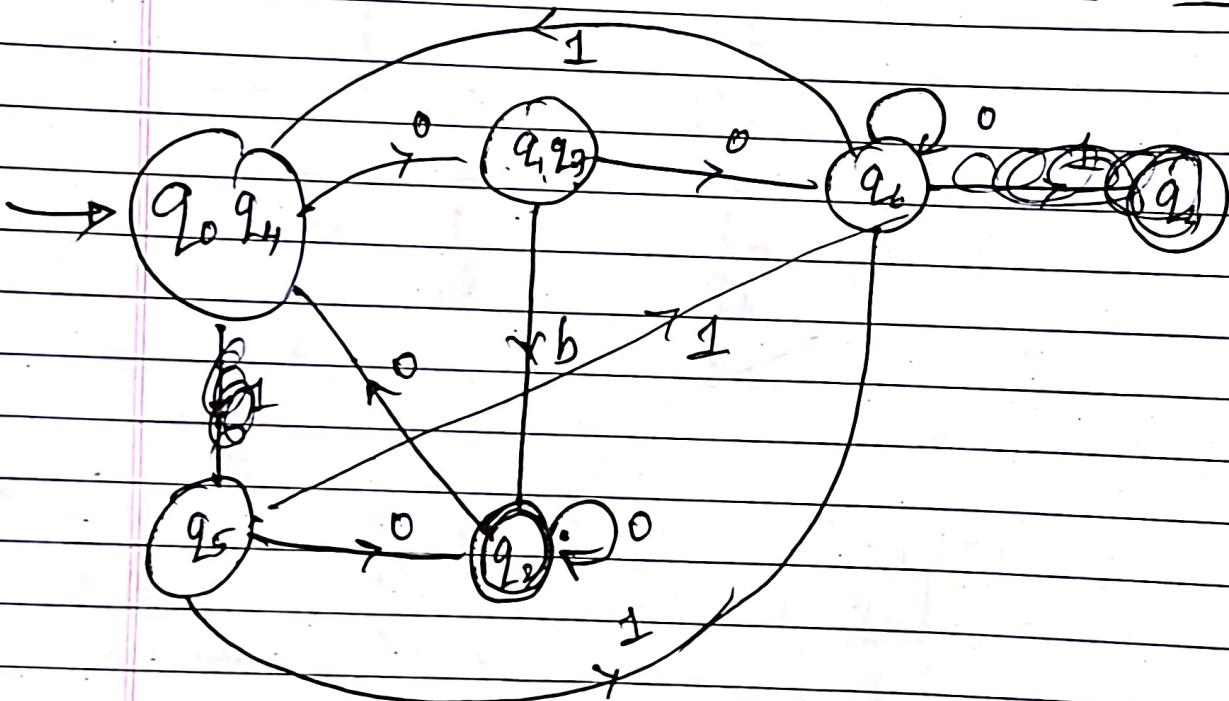
O equivalent

1 equivalent

9 équivalents

3 equivalent

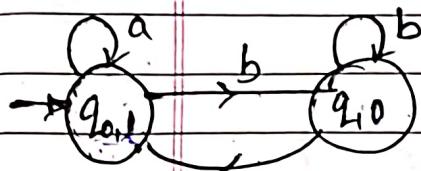
$$\begin{array}{c}
 \left[\begin{matrix} q_0 & q_1 & q_4 & q_5 & q_6 & q_7 \end{matrix} \right] \quad \left[\begin{matrix} q_2 \end{matrix} \right] \\
 \left[\begin{matrix} q_0 & q_4 & q_6 \end{matrix} \right] \quad \left[\begin{matrix} q_1 & q_5 \end{matrix} \right] \quad \left[\begin{matrix} q_3 \end{matrix} \right] \quad \left[\begin{matrix} q_2 \end{matrix} \right] \\
 \left[\begin{matrix} q_0 & q_4 \end{matrix} \right] \quad \left[\begin{matrix} q_6 \end{matrix} \right] \quad \left[\begin{matrix} q_1 & q_2 \end{matrix} \right] \quad \left[\begin{matrix} q_5 \end{matrix} \right] \quad \left[\begin{matrix} q_2 \end{matrix} \right] \\
 \left[\begin{matrix} q_0 & q_4 \end{matrix} \right] \quad \left[\begin{matrix} q_1 \end{matrix} \right] \quad \left[\begin{matrix} q_1 & q_7 \end{matrix} \right] \quad \left[\begin{matrix} q_5 \end{matrix} \right] \quad \left[\begin{matrix} q_1 \end{matrix} \right] \quad \left[\begin{matrix} q_2 \end{matrix} \right]
 \end{array}$$



FA with O/P

(D)

Modes m/c



$$\lambda: Q \rightarrow \Delta$$

$$q_0 \rightarrow 1 \quad q_1 \rightarrow 0$$

$(Q, \Sigma, \delta, q_0, \Delta, \lambda)$

Q - finite set of states

Σ - i/p alphabets

δ - Transition fcn

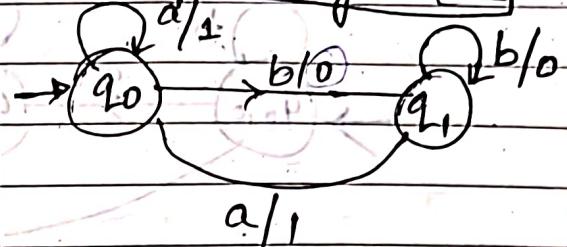
$$Q \times \Sigma \rightarrow Q$$

q_0 - initial state

Δ - o/p alphabets

λ - o/p function

Mealy m/c



$$\lambda: Q \times \Sigma \rightarrow \Delta$$

$$(q_0, a) \rightarrow 1$$

$$(q_0, b) \rightarrow 0$$

$$(q_1, b) \rightarrow 0$$

$$(q_1, a) \rightarrow 1$$

a, b, F, 0

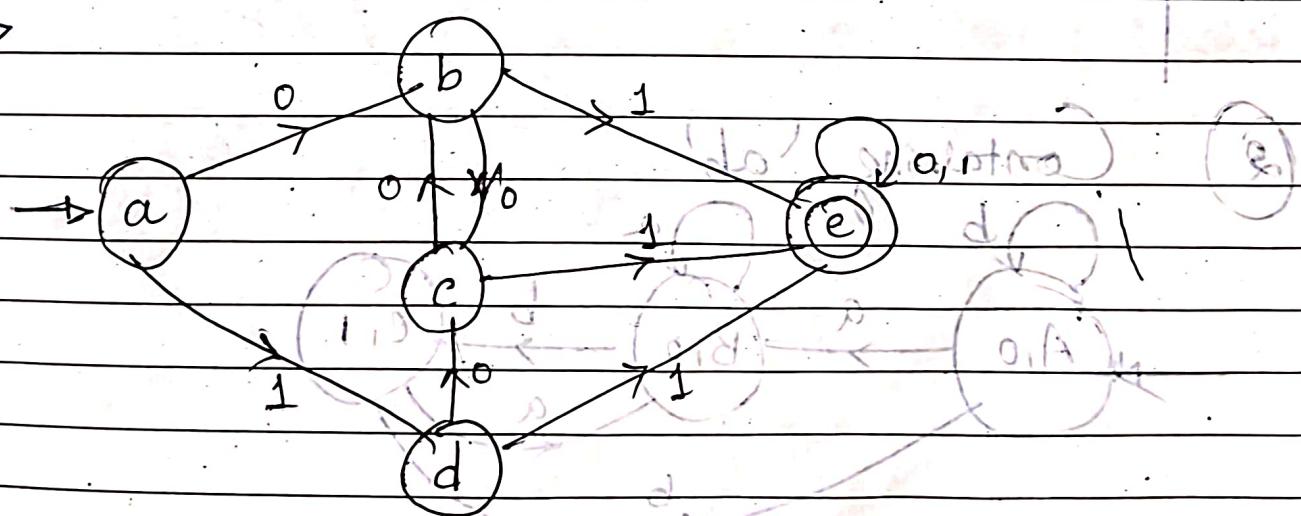
$n=2$

$(n+1)$ T/F 2/plane N

q_0	a	b	F	0
1	$(1, p)$	$(1, p)$	$1, p$	$0, p$
1	$(1, p)$	$(0, p)$	$0, p$	$1, p$

q_0	a	b	F	0
1	$(1, p)$	$(1, p)$	$1, p$	$0, p$
1	$(1, p)$	$(0, p)$	$0, p$	$1, p$

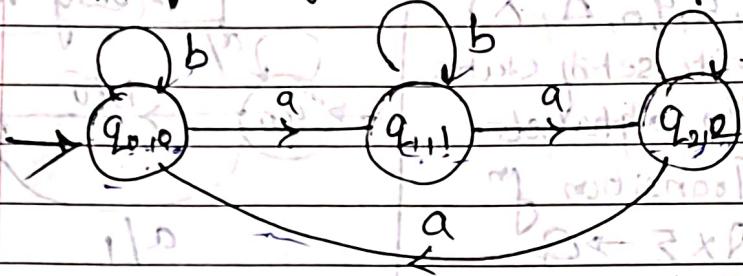
(B) \rightarrow



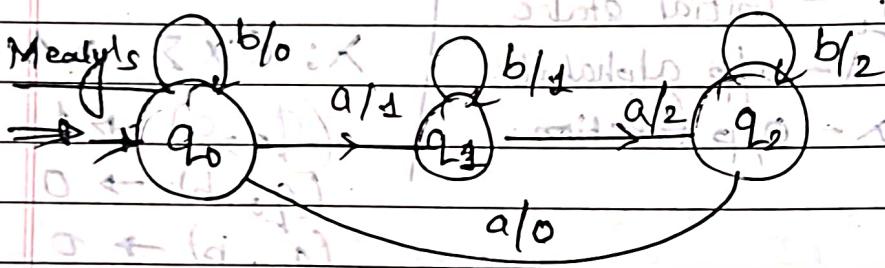
(1)

Conversion of Moore's machine to Mealy's machine

1) Count no. of stat. 3



Into Mealy's



Moore's STS

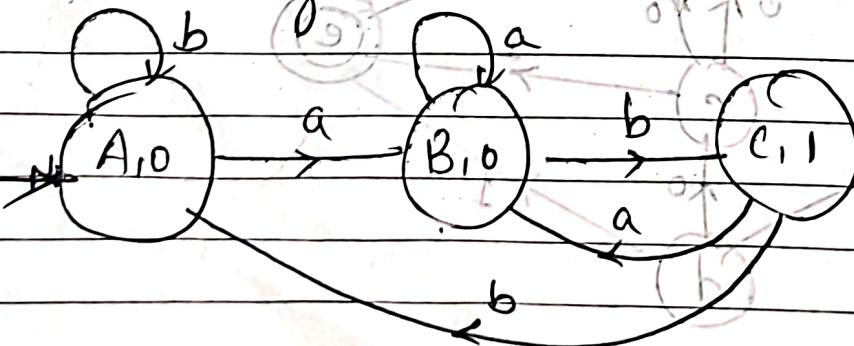
Mealy's STS

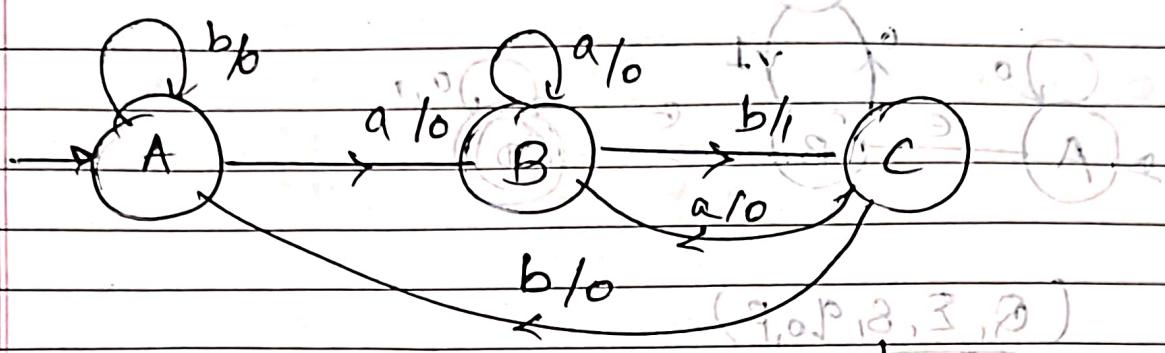
	a	b	Δ
q_0	q_1	q_0	0
q_1	q_2	q_1	1
q_2	q_0	q_2	2

	$a \rightarrow p$	$b \rightarrow p$
q_0	$(q_1, 1)$	$(q_0, 0)$
q_1	$(q_2, 2)$	$(q_1, 1)$
q_2	$(q_0, 0)$	$(q_2, 2)$

(2)

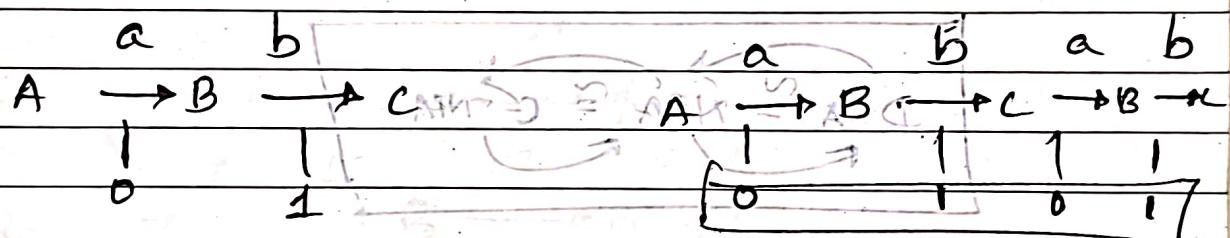
Containing 'ab'





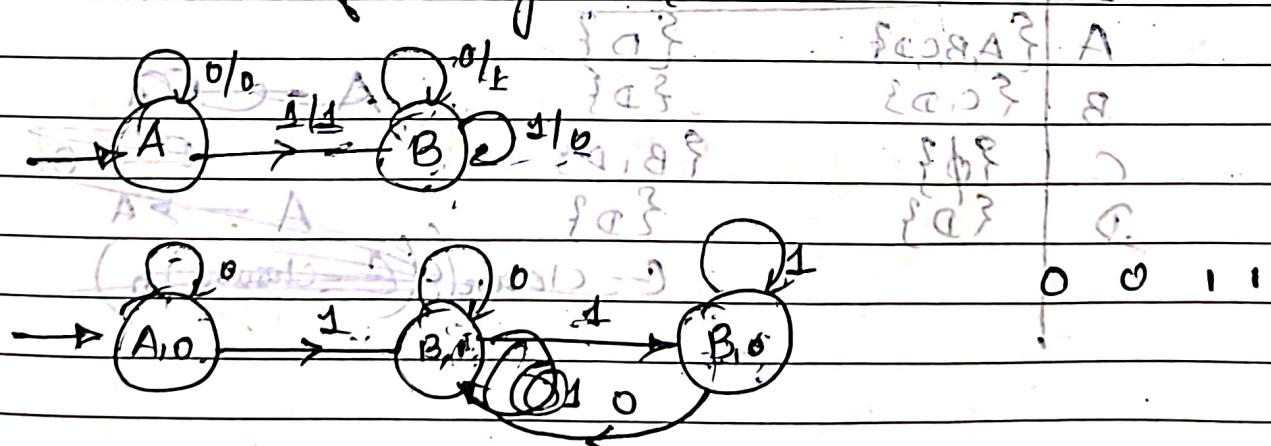
	a	b	Δ	$\{q_1, q_2, q_3\}$	a/0	b
A	B	A	0		(B, 0)	(A, 0)
B	B	C	0		(B, 0)	(C, 1)
C	B	{A, B, C}	1		(B, 0)	(A, 0)

$A \leftrightarrow A$

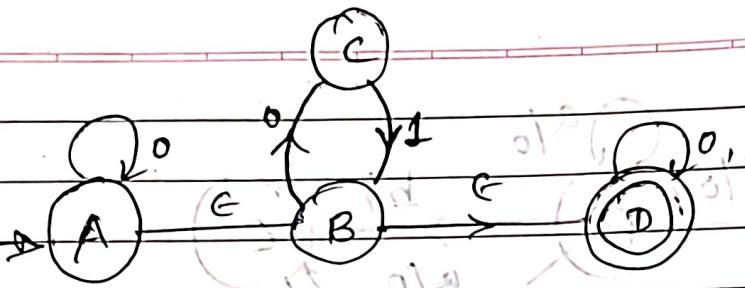


A911 ← A911-3

③ Conversion of Mealy's M/c to Moore's Machine



G-NFA



$(Q, \Sigma, \delta, q_0, F)$

$$S = \{Q \times \Sigma^* \cup \{q_0\} \rightarrow Q\}$$

(q_0, s)

(q_1, s)

(q_2, s)

(q_3, s)

(q_4, s)

(q_5, s)

(q_6, s)

(q_7, s)

(q_8, s)

(q_9, s)

(q_{10}, s)

(q_{11}, s)

(q_{12}, s)

(q_{13}, s)

(q_{14}, s)

(q_{15}, s)

(q_{16}, s)

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(q_{222}, s)

(q_{223}, s)

(q_{224}, s)

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(q_{226}, s)

(q_{227}, s)

(q_{228}, s)

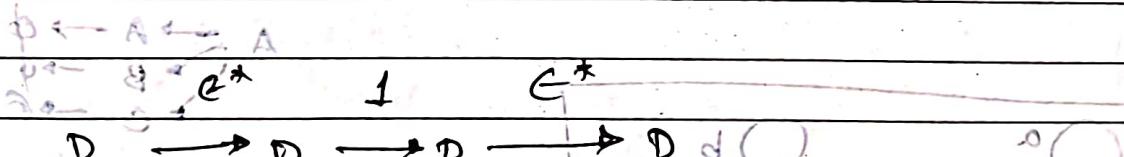
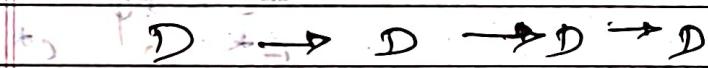
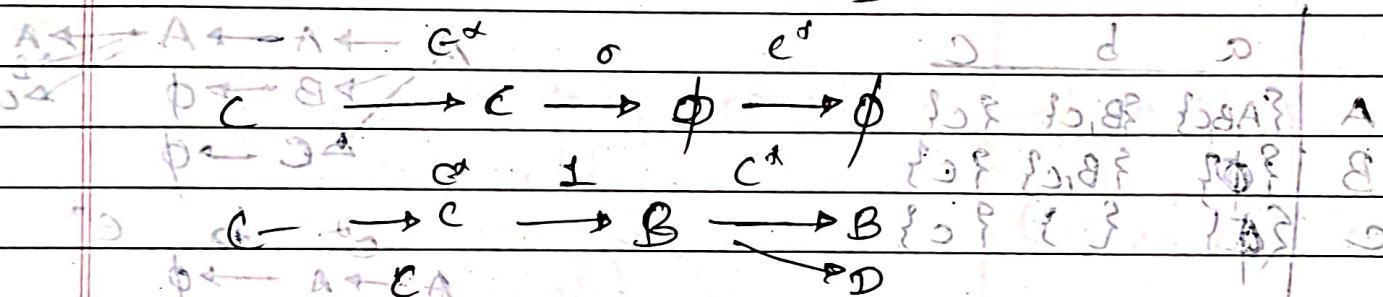
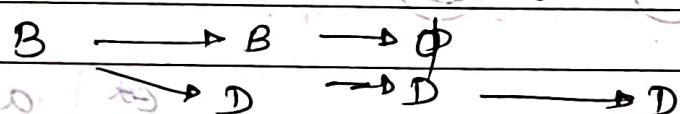
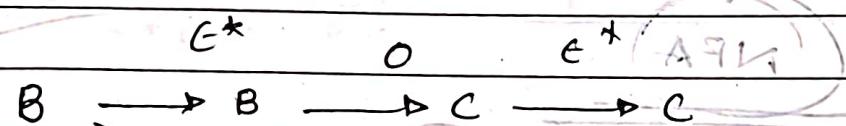
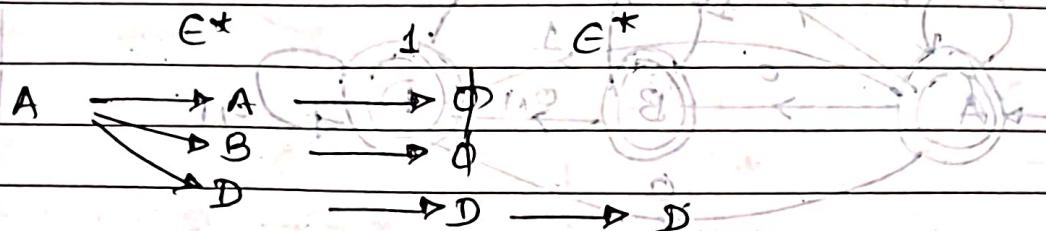
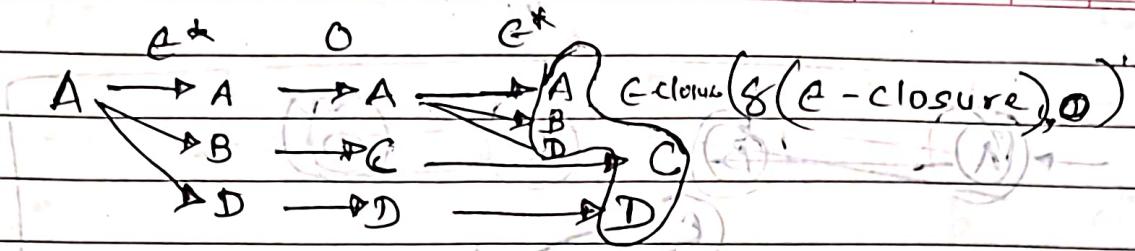
(q_{229}, s)

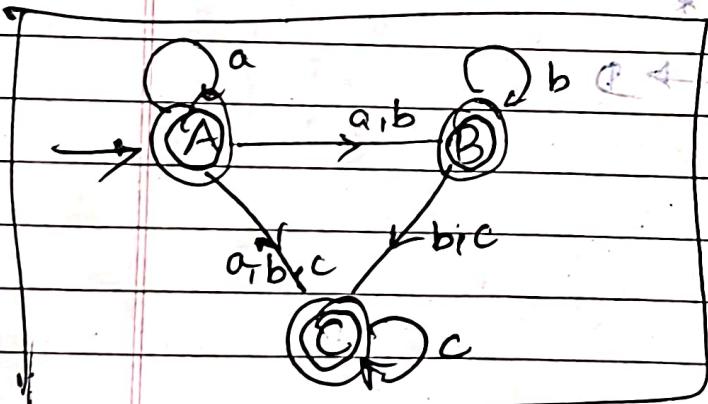
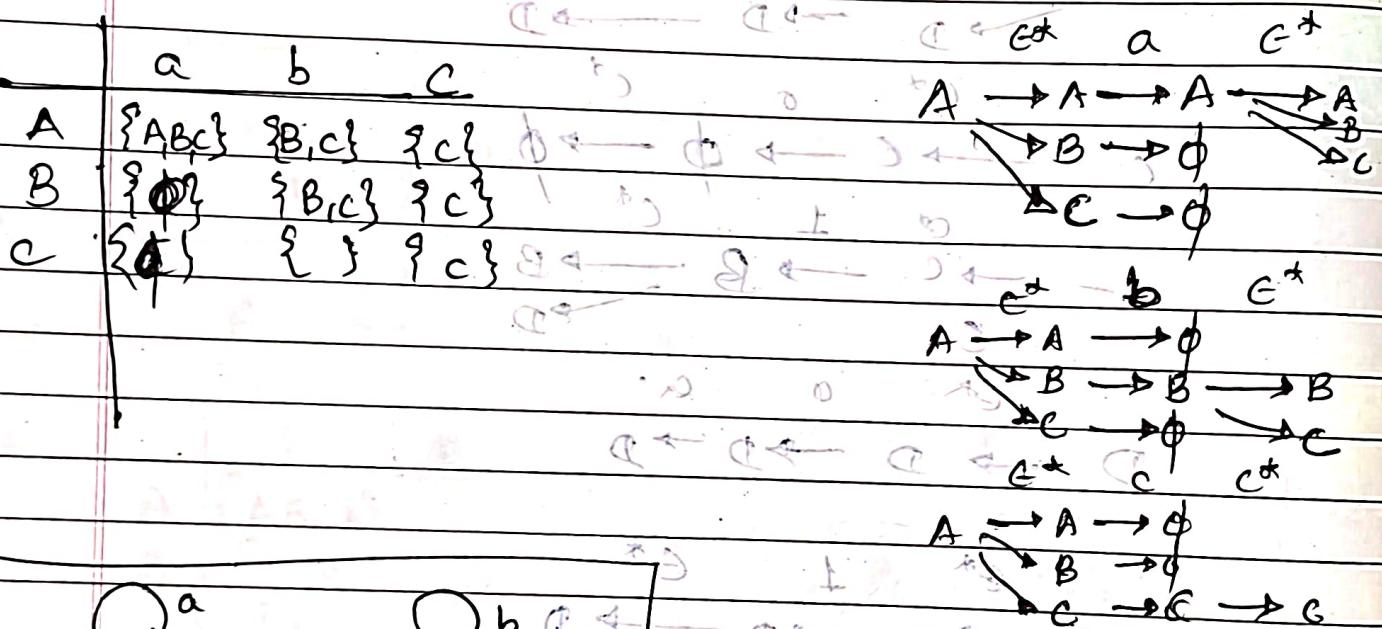
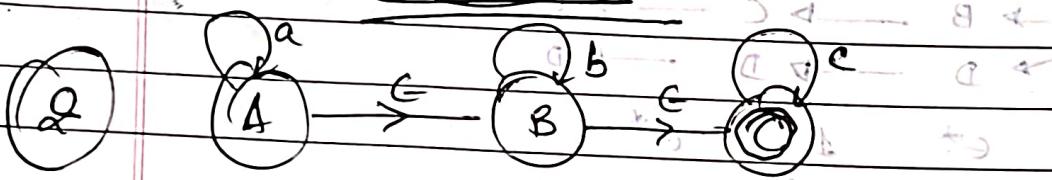
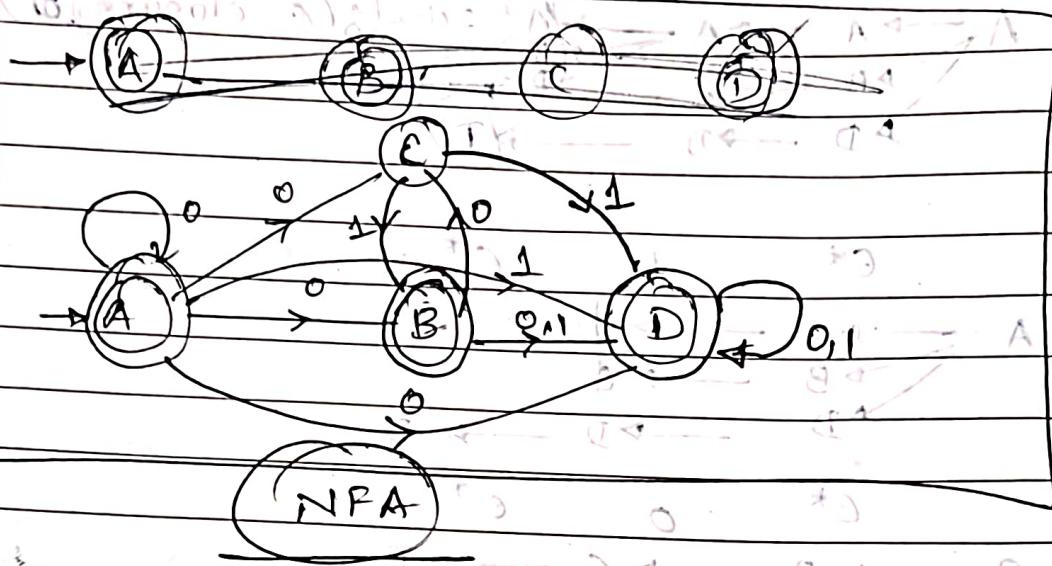
(q_{230}, s)

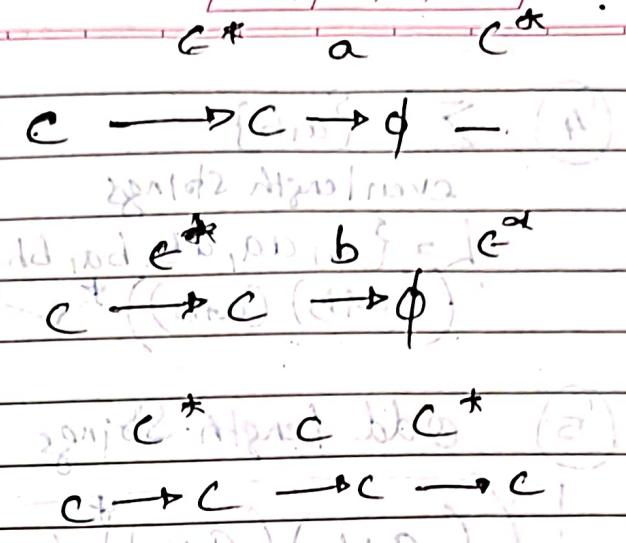
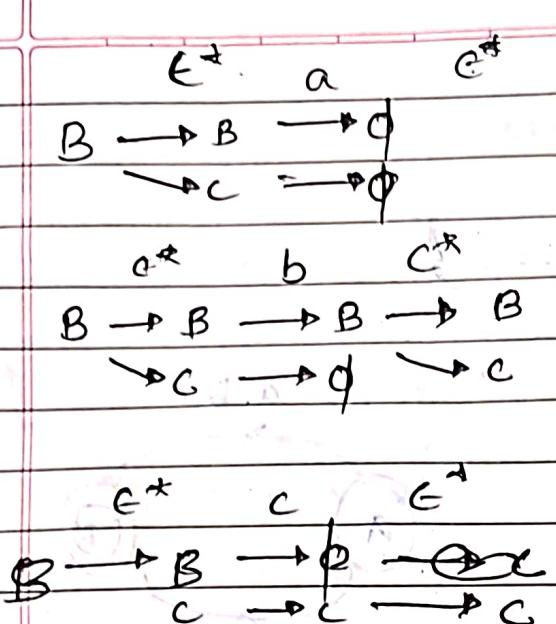
(q_{231}, s)

(q_{232}, s)

<p







Regular Expression

60

$$\Sigma = \{a, b\}$$

$L_1 = \text{length of all strings exactly } '2' \text{ in size}$ | length = 3

$$L = \{aa, ab, ba, bb\}$$

$$= aa + ab + ba + bb \quad ((a+b)(a+b)) = (a+b)(a+b)(a+b)$$

$$= a(a+b) + b(a+b)$$

$$= (a+b)(a+b) \quad \text{Ausdehnung} \quad \text{Simplifizierung} \quad \{d, B\} = 3$$

6

$$L_2 = \{aa, ab, ba, bb\}$$

Regular ex. length is at least 2

$$L = \{aa, ab, ba, bb, aba, \dots\} \quad \text{SFD: } 1.5 \quad \text{ZFD: } 1.5$$

$$(a+b)(a+b)(a+b)^*$$

(2)

Almost 2

$$L = \{e, a, b, ab, ba, bb\}$$

$$\cancel{(-a+b)} + \cancel{(a+b)} + (a+b)$$

$$= (at+bt+c)(at+bt+c) \quad \checkmark$$

(4) $\Sigma = \{a, b\}$

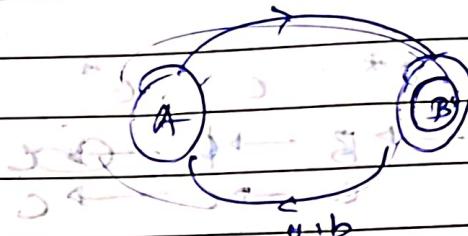
even length strings

$$L = \{ \epsilon, aa, ab, ba, bb, aaaa, \dots \}$$

$$(a+b)(a+b)^*$$

(5) Odd length strings

$$(a+b)(a+b)^*(a+b)$$



(6) String length divisible by 3.

$$k = 0, 3, 6, 9, 12, \dots$$

$$\Rightarrow ((a+b)(a+b)(a+b))^*$$

$$(a+b)((a+b)(a+b))^*$$

$$\{d \in \mathbb{N} : d \equiv 0 \pmod{3}\}$$

(7) String $L \equiv 2 \pmod{3}$ means string length is 2.

$$L \equiv 2 \pmod{3}$$

$$((a+b)(a+b)(a+b))^* (a+b)(a+b)$$

(8) $\Sigma = \{a, b\}$ strings in which 'a's are exactly 2.

$$\Rightarrow b^* a b^* a b^* \checkmark$$

$$\{dd, ad, da, aa\} \subset L$$

(9) Strings 'a's are atleast 2.

~~$b^* a b^* a (a+b)^*$~~

$$b^* a b^* a (a+b)^* \checkmark$$

$$\{(aa), aad, ada, daa\} \subset L$$

(10) No. of a's are atmost 2

$$b^* (a+a) b^* (a+a) b^*$$

Ques. 10. Some words are given below. Write all the words which have even number of 'a's.

(11)

No. of 'a's are even.

{ ad, odd, odd, odd, odd, d, o, e } = { }

$b^*(a^*)$
 $(b^* a b^* a b^*)^*$ $\xrightarrow{\text{After 2 roots}}$

*(odd) \rightarrow { } $\xrightarrow{\text{odd}}$ { ... odd, odd, odd, odd }

(12)

Starts with 'a'

$a (a+b)^* d$

$\xrightarrow{\text{d}}$ { ... -odd, odd, odd, odd }

$\xrightarrow{\text{d}}$ { ... even, even, even, even }

(13)

End with 'a' $\rightarrow a^*(d) d + a^*(d) + a^*(d)$

$(a+b)^* a (d+a) \xrightarrow{\text{H.O.P.}} (d+a)^* (d)$

$\xrightarrow{(d+a)^* (d) (d+a)} (d+a)^* d$

(14)

Containing 'a'

$(a+b)^* a (a+b)^*$

(15)

Starting & Ending with different symbols

$a (a+b)^* b + b (a+b)^* a$

$$R = \phi + R = R + \phi \quad \leftarrow$$

$$L = \phi \cdot R = R \cdot \phi \quad \leftarrow$$

$$R = S \cdot R = R \cdot S \quad \leftarrow$$

$$S = R \cdot L = L \cdot R \quad \leftarrow$$

$$G = R + S = R + L \quad \leftarrow$$

$$G + R = G \quad \leftarrow$$

(16)

S & E with same symbols.

~~a(a+b)^* a + b(a+b)^* b~~ $R = S \cdot R = R \cdot S \quad \leftarrow$

$L = \{ e, a, b, aa, bb, aba, bab, \dots \}$

$[a (a+b)^* a + b (a+b)^* b + (a+b)^* e]$

$$S = R \cdot L = L \cdot R \quad \leftarrow$$

$$G = R + S = R + L \quad \leftarrow$$

(17) No 2 'a's come together / 'aa' as substring

$L = \{ e, b, bb, a, ab, aba, abba, abab, ba, bab, baab, baba, \dots \}$

Building block - B, ab or ba

$(b+ab)^* + (b+ab)^* a$

$(b+ab)^* (e+a)$

(18) No 2 a's and no 2 b's should come together

$$L = \{ e, a, b, ab, ba, aba, bab, \dots \}$$

	Starts with	Ends with
$\{ a, aba, ababa, \dots \}$	a	$a^* (Tab)^*$ or $a^* (ba)^*$
$\{ ab, abab, ababab, \dots \}$	a	$b (ab)^*$ or $a (ba)^* b$
$\{ ba, baba, bababa, \dots \}$	b	$a^* (ba)^*$ or $b (ab)^* a$
$\{ b, bab, babab, \dots \}$	b	$b^* (bab)^*$ or $b (ab)^* b$ $(ba)^* b$

$$(ab)^* a + (ab)^* + b(ab)^* a + b(ab)^* b \\ (ab)^* (a+c) + b(ab)^* (b+c) \\ \cancel{(ab)^* (a+c)} [(e+b) (ab)^* (a+c)] \checkmark$$

Identities of NRE

$$\rightarrow \emptyset + R = R + \emptyset = R$$

$$\rightarrow \emptyset \cdot R = R \cdot \emptyset = \emptyset$$

$$\rightarrow G \cdot R = R \cdot G = R$$

$$\rightarrow G^* = G$$

$$\rightarrow \emptyset^* = G (a+d+ba) + d^* (b+d)$$

$$\rightarrow G + RR^* = R^* R + G = R^*$$