9 Surds and Indices

IMPORTANT FACTS AND FORMULAE

I. Laws of Indices:

(*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $\dfrac{a^m}{a^n} = a^{m-n}$ (*iii*) $(a^m)^n = a^{mn}$

(*iv*) $(ab)^n = a^n b^n$ (*v*) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ (*vi*) $a^0 = 1$

II. Surds: *Let a be a rational number and n be a positive integer such that* $a^{\frac{1}{n}} = \sqrt[n]{a}$ *is irrational. Then,* $\sqrt[n]{a}$ *is called a surd of order n.*

III. Laws of Surds:

(*i*) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (*ii*) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ (*iii*) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(*iv*) $(\sqrt[n]{a})^n = a$ (*v*) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ (*vi*) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

SOLVED EXAMPLES

Ex. 1. *Simplify :* (*i*) $(27)^{\frac{2}{3}}$ (*ii*) $(1024)^{-\frac{4}{5}}$ (*iii*) $\left(\dfrac{8}{125}\right)^{-\frac{4}{3}}$.

Sol. (*i*) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3 \times \frac{2}{3}\right)} = 3^2 = 9.$

(*ii*) $(1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5 \times \frac{(-4)}{5}\right\}} = 4^{-4} = \dfrac{1}{4^4} = \dfrac{1}{256}.$

(*iii*) $\left(\dfrac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\dfrac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\dfrac{2}{5}\right)^{\left\{3 \times \frac{(-4)}{3}\right\}} = \left(\dfrac{2}{5}\right)^{-4} = \left(\dfrac{5}{2}\right)^4 = \dfrac{5^4}{2^4} = \dfrac{625}{16}.$

Ex. 2. *What will come in place of both the question marks in the following question?* **(Bank Recruitment, 2010)**

$$(?)^{\frac{1}{4}} = \frac{48}{(?)^{\frac{3}{4}}}$$

Sol. Let $x^{\frac{1}{4}} = \dfrac{48}{x^{\frac{3}{4}}}$. Then, $x^{\frac{1}{4}} . x^{\frac{3}{4}} = 48 \Leftrightarrow x^{\left(\frac{1}{4}+\frac{3}{4}\right)} = 48 \Leftrightarrow x = 48.$

Ex. 3. *Evaluate :* (*i*) $(.00032)^{\frac{3}{5}}$ (*ii*) $(256)^{0.16} \times (16)^{0.18}$.

Sol. (*i*) $(0.00032)^{\frac{3}{5}} = \left(\dfrac{32}{100000}\right)^{\frac{3}{5}} = \left(\dfrac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\dfrac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\dfrac{1}{5}\right)^{\left(5 \times \frac{3}{5}\right)} = \left(\dfrac{1}{5}\right)^3 = \dfrac{1}{125}.$]

(*ii*) $(256)^{0.16} \times (16)^{0.18} = \{(16)^2\}^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$

$$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4.$$

Ex. 4. ***Solve :*** $\boldsymbol{9^{8.6} \times 8^{3.9} \times 72^{4.4} \times 9^{3.9} \times 8^{8.6} = 72^?}$**.** **(L.I.C., 2005)**

Sol. Let $9^{8.6} \times 8^{3.9} \times 72^{4.4} \times 9^{3.9} \times 8^{8.6} = 72^x$.

Then, $9^{(8.6 + 3.9)} \times 8^{(3.9 + 8.6)} \times 72^{4.4} = 72^x$

$\Leftrightarrow \quad 9^{12.5} \times 8^{12.5} \times 72^{4.4} = 72^x \Leftrightarrow (9 \times 8)^{12.5} \times 72^{4.4} = 72^x$

$\Leftrightarrow \quad 72^{12.5} \times 72^{4.4} = 72^x \Leftrightarrow 72^{(12.5 + 4.4)} = 72^x \Leftrightarrow 72^{16.9} = 72^x \Leftrightarrow x = 16.9.$

Ex. 5. ***Solve :*** $\boldsymbol{(0.064) \times (0.4)^7 = (0.4)^? \times (0.0256)^2}$**.** **(Bank P.O., 2010)**

Sol. Let $(0.064) \times (0.4)^7 = (0.4)^x \times (0.0256)^2$.

Then, $(0.4)^3 \times (0.4)^7 = (0.4)^x \times [(0.4)^4]^2$

$\Leftrightarrow \quad (0.4)^{(3 + 7)} = (0.4)^x \times (0.4)^8 \Leftrightarrow (0.4)^{10} = (0.4)^{x + 8} \Leftrightarrow x + 8 = 10 \Leftrightarrow x = 2.$

Ex. 6. ***What is the quotient when*** $\boldsymbol{(x^{-1} - 1)}$ ***is divided by*** $\boldsymbol{(x - 1)}$***?***

Sol. $$\frac{x^{-1} - 1}{x - 1} = \frac{\frac{1}{x} - 1}{x - 1} = \frac{(1 - x)}{x} \times \frac{1}{(x - 1)} = -\frac{1}{x}.$$

Hence, the required quotient is $-\frac{1}{x}$.

Ex. 7. ***If*** $\boldsymbol{2^{x-1} + 2^{x+1} = 1280}$***, then find the value of*** $\boldsymbol{x}$**.**

Sol. $2^{x-1} + 2^{x+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$$\Leftrightarrow \quad 2^{x-1} = \frac{1280}{5} = 256 = 2^8 \quad \Leftrightarrow \quad x - 1 = 8 \quad \Leftrightarrow \quad x = 9.$$

Hence, $x = 9$.

Ex. 8. ***Find the value of*** $\boldsymbol{\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}}$**.**

Sol. $$\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left\{(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{4}} = \left[5\left\{2^{\left(3 \times \frac{1}{3}\right)} + 3^{\left(3 \times \frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{4}}$$

$$= \{5\,(2 + 3)^3\}^{\frac{1}{4}} = (5 \times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4 \times \frac{1}{4}\right)} = 5^1 = 5.$$

Ex. 9. ***Find the value of*** $\boldsymbol{\left\{(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right\}}$**.**

Sol. $$\left[(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right] = \left[(4^2)^{\frac{3}{2}} + (4^2)^{-\frac{3}{2}}\right] = 4^{\left(2 \times \frac{3}{2}\right)} + 4^{\left\{2 \times \frac{(-3)}{2}\right\}}$$

$$= 4^3 + 4^{-3} = 4^3 + \frac{1}{4^3} = \left(64 + \frac{1}{64}\right) = \frac{4097}{64}.$$

Ex. 10. ***If*** $\boldsymbol{\left(\frac{1}{5}\right)^{3y} = 0.008}$***, then find the value of*** $\boldsymbol{(0.25)^y}$**.**

Sol. $$\left(\frac{1}{5}\right)^{3y} = 0.008 = \frac{8}{1000} = \frac{1}{125} = \left(\frac{1}{5}\right)^3 \quad \Leftrightarrow \quad 3y = 3 \quad \Leftrightarrow \quad y = 1.$$

$\therefore \ (0.25)^y = (0.25)^1 = 0.25.$

Ex. 11. *Simplify :* $\dfrac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$. **(S.S.C., 2005)**

Sol. Given expression $= \dfrac{\{(2.5)^2\}^{\frac{1}{2}} \times \{(0.12)^2\}^{\frac{1}{2}} + 1}{\{(0.3)^3\}^{\frac{1}{3}} \times (3^4)^{\frac{1}{4}}} = \dfrac{(2.5)^{\left(2\times\frac{1}{2}\right)} \times (0.12)^{\left(2\times\frac{1}{2}\right)} + 1}{(0.3)^{\left(3\times\frac{1}{3}\right)} \times 3^{\left(4\times\frac{1}{4}\right)}}$

$$= \frac{2.5 \times 0.12 + 1}{0.3 \times 3} = \frac{0.3+1}{0.9} = \frac{1.3}{0.9} = \frac{13}{9} = 1.444\ldots = 1.\bar{4}.$$

Ex. 12. *Find the value of* $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}} = \dfrac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{\left(5\times\frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \dfrac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$

$$= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$$

Ex. 13. *Find the value of* $\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right)$. **(N.I.F.T., 2003)**

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$$\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right) = (x-1)(x^3 + x^2 + x + 1)$$
$$= (x - 1)\,[x^2\,(x + 1) + (x + 1)]$$
$$= (x - 1)\,(x + 1)\,(x^2 + 1) = (x^2 - 1)\,(x^2 + 1)$$
$$= (x^4 - 1) = \left[\left(2^{\frac{1}{4}}\right)^4 - 1\right] = \left[2^{\left(\frac{1}{4}\times 4\right)} - 1\right] = (2-1) = 1.$$

Ex. 14. *Find the value of* $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol. $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \dfrac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(7\times\frac{1}{3}\right)}}{6^{\left(6\times\frac{1}{3}\right)}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}\right)}}{6^2}$

$$= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}-2\right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 6^1 = 6.$$

Ex. 15. *If* $\left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{q}{p}\right)^{px-q}$, *then find the value of x.*

Sol. $\left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{q}{p}\right)^{px-q} \Leftrightarrow \left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{p}{q}\right)^{-(px-q)}$

$$\Leftrightarrow rx - s = -(px - q) \Leftrightarrow rx - s = -px + q$$
$$\Leftrightarrow rx + px = q + s \Leftrightarrow x\,(p + r) = q + s$$
$$\Leftrightarrow x = \frac{q+s}{p+r}.$$

Ex. 16. ***If x = y^a, y = z^b and z = x^c, then find the value of abc.***

Sol. $z^1 = x^c = (y^a)^c$ $\quad [\because x = y^a]$

$= y^{(ac)} = (z^b)^{ac}$ $\quad [\because y = z^b]$

$= z^{b(ac)} = z^{abc}$

$\therefore abc = 1.$

Ex. 17. ***Simplify :*** $\left(\dfrac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\dfrac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\dfrac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$.

Sol. Given Expression $= \{x^{(a-b)}\}^{(a^2+b^2+ab)} \cdot \{x^{(b-c)}\}^{(b^2+c^2+bc)} \cdot \{x^{(c-a)}\}^{(c^2+a^2+ca)}$

$$= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$$

$$= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$$

Ex. 18. ***If $8^x \cdot 2^y = 512$ and $3^{3x+2y} = 9^6$, then what is the value of x and y?*** **(M.A.T., 2004)**

Sol. $8^x\, 2^y = 512 \Leftrightarrow (2^3)^x \cdot 2^y = 2^9 \Leftrightarrow 2^{3x+y} = 2^9 \Leftrightarrow 3x + y = 9$ ...(*i*)

And, $3^{3x+2y} = 9^6 \Leftrightarrow 3^{3x+2y} = (3^2)^6 = 3^{12} \Leftrightarrow 3x + 2y = 12$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get: $y = 3$.

Putting $y = 3$ in (*i*), we get: $3x = 6$ or $x = 2$.

Hence, $x = 2$ and $y = 3$.

Ex. 19. ***Find the largest from among $\sqrt[4]{6}$, $\sqrt{2}$ and $\sqrt[3]{4}$.***

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M. is 12. Changing each to a surd of order 12, we get :

$$\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = \left(6^{\frac{3}{12}}\right) = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}.$$

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = \left(2^{\frac{6}{12}}\right) = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}.$$

$$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = \left(4^{\frac{4}{12}}\right) = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}.$$

Clearly, $(256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

$\therefore$ Largest one is $(256)^{\frac{1}{12}}$ *i.e.*, $\sqrt[3]{4}$.

Ex. 20. ***Find the square root of $(3+\sqrt{5})$.*** **(L.I.C.A.A.O., 2007)**

Sol. $$\sqrt{3+\sqrt{5}} = \sqrt{3+2\sqrt{\frac{5}{4}}} = \sqrt{\frac{5}{2}+\frac{1}{2}+2\sqrt{\frac{5}{2}\times\frac{1}{2}}} = \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2+\left(\sqrt{\frac{1}{2}}\right)^2+2\sqrt{\frac{5}{2}}\sqrt{\frac{1}{2}}}$$

$$= \sqrt{\left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right)^2} = \left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right).$$

Ex. 21. ***If x = $3+2\sqrt{2}$, find the value of $\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)$.***

Sol. $$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}-2 = (3+2\sqrt{2})+\frac{1}{(3+2\sqrt{2})}-2$$

$$= (3+2\sqrt{2})+\frac{1}{(3+2\sqrt{2})}\times\frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}-2 = (3+2\sqrt{2})+(3-2\sqrt{2})-2 = 4.$$

$$\therefore \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right) = 2.$$

Ex. 22. *If $2^x = 3^y = 6^{-z}$, find the value of* $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$.

Sol. Let $2^x = 3^y = 6^{-z} = k$. Then, $2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}$ and $6 = k^{-\frac{1}{z}}$.

Now, $2 \times 3 = 6 \Leftrightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}} \Leftrightarrow k^{\left(\frac{1}{x}+\frac{1}{y}\right)} = k^{-\frac{1}{z}}$

$$\Leftrightarrow \frac{1}{x}+\frac{1}{y} = -\frac{1}{z} \Leftrightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0.$$

Ex. 23. *Given* $t = 2+\sqrt[3]{4}+\sqrt[3]{2}$, *determine the value of* $t^3 - 6t^2 + 6t - 2$. **(M.A.T., 2005)**

Sol. $t = 2+\sqrt[3]{4}+\sqrt[3]{2} \Rightarrow t-2 = \sqrt[3]{4}+\sqrt[3]{2} \Rightarrow (t-2)^3 = (\sqrt[3]{4}+\sqrt[3]{2})^3$

$\Rightarrow t^3 - 8 - 6t\,(t-2) = 4 + 2 + 3\,\sqrt[3]{8}\,(\sqrt[3]{4}+\sqrt[3]{2})$

$\Rightarrow t^3 - 8 - 6t^2 + 12t = 6 + 3 \times 2\,(t-2)$

$\Rightarrow t^3 - 6t^2 + 12t - 8 = 6 + 6t - 12 \Rightarrow t^3 - 6t^2 + 6t - 2 = 0.$

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer:*

1. $\sqrt[3]{5}$ is a surd of the order **(R.R.B., 2008)**
(a) $\frac{1}{3}$ (b) 1
(c) 2 (d) 3

2. $5^0 \times 8 =?$ **(R.R.B., 2006)**
(a) 0 (b) 8
(c) 40 (d) 200

3. Which of the following are equal in value?
I. 4^1 II. 1^4
III. 4^0 IV. 0^4
(a) I and II (b) II and III
(c) III and IV (d) I and IV

4. If $289 = 17^{\frac{1}{5}x}$, then x =? **(Bank P.O., 2009)**
(a) $\frac{2}{5}$ (b) 8
(c) 16 (d) 32
(e) None of these

5. $(81)^4 \div (9)^5 =?$ **(Agriculture Officers', 2009)**
(a) 9 (b) 81
(c) 729 (d) 6561
(e) None of these

6. The value of $\left(\frac{9^2 \times 18^4}{3^{16}}\right)$ is **(R.R.B., 2006)**
(a) $\frac{3}{2}$ (b) $\frac{4}{9}$
(c) $\frac{16}{81}$ (d) $\frac{32}{243}$

7. $[4^3 \times 5^4] \div 4^5 =?$ **(Bank Recruitment, 2008)**
(a) 29.0825 (b) 30.0925
(c) 35.6015 (d) 39.0625
(e) None of these

8. $9^3 \times 6^2 \div 3^3 =?$ **(L.I.C.A.D.O., 2007)**
(a) 948 (b) 972
(c) 984 (d) 1012
(e) None of these

9. $(19)^{12} \times (19)^8 \div (19)^4 = (19)^?$ **(Bank Recruitment, 2008)**
(a) 6 (b) 8
(c) 12 (d) 24
(e) None of these

10. $(64)^4 \div (8)^5 =?$ **(Agriculture Officer's, 2008)**
(a) $(8)^8$ (b) $(8)^2$
(c) $(8)^{12}$ (d) $(8)^4$
(e) None of these

11. $(1000)^{12} \div (10)^{30} =?$ **(Bank P.O., 2008)**
(a) $(1000)^2$ (b) 10
(c) 100 (d) $(100)^2$
(e) None of these

12. $(3)^8 \times (3)^4 =?$ **(Bank P.O., 2009)**
(a) $(27)^3$ (b) $(27)^5$
(c) $(729)^2$ (d) $(729)^3$
(e) None of these

13. $\frac{343 \times 49}{216 \times 16 \times 81} =?$ **(Bank P.O., 2010)**
(a) $\frac{7^5}{6^7}$ (b) $\frac{7^5}{6^8}$
(c) $\frac{7^6}{6^7}$ (d) $\frac{7^4}{6^8}$
(e) None of these

14. $\dfrac{16 \times 32}{9 \times 27 \times 81} = ?$ **(Bank P.O., 2009)**

(a) $\left(\frac{2}{3}\right)^9$ (b) $\left(\frac{2}{3}\right)^{11}$

(c) $\left(\frac{2}{3}\right)^{12}$ (d) $\left(\frac{2}{3}\right)^{13}$

(e) None of these

15. $9^3 \times (81)^2 \div (27)^3 = (3)^?$ **(Bank P.O., 2010)**

(a) 3 (b) 4

(c) 5 (d) 6

(e) None of these

16. $(6)^4 \div (36)^3 \times 216 = 6^{(?\,-\,5)}$ **(Bank Recruitment, 2010)**

(a) 1 (b) 4

(c) 6 (d) 7

(e) None of these

17. $(0.2)^2, \dfrac{1}{100}, (0.01)^{\frac{1}{2}}, (0.008)^{\frac{1}{3}}$. Of these, which one is the greatest? **(P.C.S., 2004)**

(a) $(0.008)^{\frac{1}{3}}$ (b) $(0.01)^{\frac{1}{2}}$

(c) $(0.2)^2$ (d) $\dfrac{1}{100}$

18. Which of the following expressions has the greatest value?

(a) $[(2^{-1})^0]^2$ (b) $\left[(4^0)^{-\frac{1}{2}}\right]^2$

(c) $[(2^{-2})^{-1}]^2$ (d) $[(2^{-1})^2]^2$

19. $(10)^{24} \times (10)^{-21} = ?$ **(Bank Recruitment, 2008)**

(a) 3 (b) 10

(c) 100 (d) 1000

(e) None of these

20. The value of $(256)^{\frac{5}{4}}$ is

(a) 512 (b) 984

(c) 1024 (d) 1032

21. The value of $(\sqrt{8})^{\frac{1}{3}}$ is

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) 8

22. The value of $\left(\dfrac{32}{243}\right)^{-\frac{4}{5}}$ is

(a) $\dfrac{4}{9}$ (b) $\dfrac{9}{4}$

(c) $\dfrac{16}{81}$ (d) $\dfrac{81}{16}$

23. The value of $\left(-\dfrac{1}{216}\right)^{-\frac{2}{3}}$ is

(a) 36 (b) – 36

(c) $\dfrac{1}{36}$ (d) $-\dfrac{1}{36}$

24. The value of $27^{-\frac{2}{3}}$ lies between **(C.D.S., 2002)**

(a) 0 and 1 (b) 1 and 2

(c) 2 and 3 (d) 3 and 4

25. The value of $\sqrt[3]{2^4\sqrt{2^{-5}\sqrt{2^6}}}$ is **(S.S.C., 2005)**

(a) 1 (b) 2

(c) $2^{\frac{5}{3}}$ (d) 2^5

26. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = ?$ **(R.R.B., 2007)**

(a) $2^{\frac{29}{31}}$ (b) $2^{\frac{31}{32}}$

(c) $2^{\frac{9}{2}}$ (d) $2^{\frac{11}{2}}$

27. The value of $(0.03125)^{-\frac{2}{5}}$ is **(R.R.B., 2006)**

(a) 4 (b) 9

(c) 12 (d) 31.25

28. $\left(\dfrac{1}{2}\right)^{-\frac{1}{2}}$ is equal to **(Section Officer's, 2005)**

(a) $\dfrac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$

(c) $-\sqrt{2}$ (d) $\sqrt{2}$

29. Simplified form of $\left[\left(\sqrt[5]{x^{-\frac{3}{5}}}\right)^{-\frac{5}{3}}\right]^5$ is **(S.S.C., 2010)**

(a) $\dfrac{1}{x}$ (b) x

(c) x^{-5} (d) x^5

30. What will come in place of both the question marks in the following question? **(Bank Recruitment, 2010)**

$$\frac{(?)^{\frac{2}{3}}}{42} = \frac{5}{(?)^{\frac{1}{3}}}$$

(a) 10 (b) $10\sqrt{2}$

(c) $\sqrt{20}$ (d) 20

(e) 210

31. The value of $5^{\frac{1}{4}} \times (125)^{0.25}$ is :

(a) $\sqrt{5}$ (b) 5

(c) $5\sqrt{5}$ (d) 25

32. The value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}}$ is **(M.B.A., 2003)**

(*a*) 102 (*b*) 105
(*c*) 107 (*d*) 109

33. $(2.4 \times 10^3) \div (8 \times 10^{-2})$ =?

(*a*) 3×10^{-5} (*b*) 3×10^4
(*c*) 3×10^5 (*d*) 30

34. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$

(*a*) $\frac{3}{4}$ (*b*) $\frac{2}{3}$
(*c*) $\frac{4}{9}$ (*d*) $\frac{1}{8}$

35. $(48)^{-\frac{2}{7}} \times (16)^{-\frac{5}{7}} \times (3)^{-\frac{5}{7}} = ?$ **(P.C.S., 2008)**

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{48}$
(*c*) 1 (*d*) 48

36. If $10^x = \frac{1}{2}$, then 10^{-8x} =? **(P.C.S., 2008)**

(*a*) $\frac{1}{256}$ (*b*) 16
(*c*) 80 (*d*) 256

37. If $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$, then x is equal to **(S.S.C., 2010)**

(*a*) – 2 (*b*) – 1
(*c*) 1 (*d*) 2

38. $49 \times 49 \times 49 \times 49 = 7^?$

(*a*) 4 (*b*) 7
(*c*) 8 (*d*) 16

39. The value of $(8^{-25} - 8^{-26})$ is

(*a*) 7×8^{-25} (*b*) 7×8^{-26}
(*c*) 8×8^{-26} (*d*) None of these

40. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}}$ =?

(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{8}$
(*c*) $\frac{1}{16}$ (*d*) $\frac{3}{16}$
(*e*) None of these

41. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is **(P.C.S., 2009)**

(*a*) $\frac{1}{2}$ (*b*) 1
(*c*) 2 (*d*) $\frac{7}{2}$

42. If $2^{2n-1} = \frac{1}{8^{n-3}}$, then the value of n is

(*a*) – 2 (*b*) 0
(*c*) 2 (*d*) 3

43. If $5^a = 3125$, then the value of $5^{(a-3)}$ is

(*a*) 25 (*b*) 125
(*c*) 625 (*d*) 1625

44. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is **(M.B.A., 2006)**

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 8

45. If $\sqrt{2^n} = 64$, then the value of n is

(*a*) 2 (*b*) 4
(*c*) 6 (*d*) 12

46. If $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then the value of n is

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

47. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then the value of n is

(*a*) 0 (*b*) 2
(*c*) 3 (*d*) 4

48. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of x is

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

49. If $2^x = \sqrt[3]{32}$, then x is equal to

(*a*) 5 (*b*) 3
(*c*) $\frac{3}{5}$ (*d*) $\frac{5}{3}$

50. If $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}}$, then x is equal to

(*a*) $\frac{1}{5}$ (*b*) $-\frac{1}{5}$
(*c*) $\frac{2}{5}$ (*d*) $-\frac{2}{5}$

51. If $5^{(x+3)} = 25^{(3x-4)}$, then the value of x is

(*a*) $\frac{5}{11}$ (*b*) $\frac{11}{5}$
(*c*) $\frac{11}{3}$ (*d*) $\frac{13}{5}$

52. $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$ when simplified is **(M.B.A., 2011)**

(*a*) $2^{n+1} - \frac{1}{8}$ (*b*) -2^{n+1}
(*c*) $1 - 2^n$ (*d*) $\frac{7}{8}$

53. Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is **(M.B.A., 2011)**

(a) a^4 (b) a^8
(c) a^{12} (d) a^{16}

54. $(256)^{0.16} \times (256)^{0.09} = ?$ **(S.S.C., 2004)**

(a) 4 (b) 16
(c) 64 (d) 256.25

55. $(0.04)^{-1.5} = ?$ **(Bank P.O., 2003)**

(a) 25 (b) 125
(c) 250 (d) 625

56. $(17)^{3.5} \times (17)^? = 17^8$ **(Bank P.O., 2003)**

(a) 2.29 (b) 2.75
(c) 4.25 (d) 4.5

57. $6^{1.2} \times 36^? \times 30^{2.4} \times 25^{1.3} = 30^5$ **(Specialist Officers', 2006)**

(a) 0.1 (b) 0.7
(c) 1.4 (d) 2.6
(e) None of these

58. $2^{3.6} \times 4^{3.6} \times 4^{3.6} \times (32)^{2.3} = (32)^?$ **(Specialist Officers', 2007)**

(a) 5.9 (b) 7.7
(c) 9.5 (d) 13.1
(e) None of these

59. $3^{3.5} \times 21^2 \times 42^{2.5} \div 2^{2.5} \times 7^{3.5} = 21^?$ **(Bank P.O., 2006)**

(a) 6.5 (b) 8
(c) 10 (d) 12.5
(e) None of these

60. $8^{0.4} \times 4^{1.6} \times 2^{1.6} = ?$ **(Agriculture Officers', 2009)**

(a) 48 (b) 52
(c) 64 (d) 76
(e) None of these

61. $8^7 \times 2^6 \div 8^{2.4} = 8^?$ **(Bank P.O., 2009)**

(a) 6.6 (b) 8.6
(c) 9.6 (d) 10.6
(e) None of these

62. $25^{2.7} \times 5^{4.2} \div 5^{5.4} = 25^?$ **(Bank Recruitment, 2010)**

(a) 1.6 (b) 1.7
(c) 3.2 (d) 3.6
(e) None of these

63. $8^{2.4} \times 2^{3.7} - (16)^{1.3} = 2^?$ **(Bank Recruitment, 2010)**

(a) 4.8 (b) 5.7
(c) 5.8 (d) 7.1
(e) None of these

64. $(0.04)^2 \div (0.008) \times (0.2)^6 = (0.2)^?$ **(Bank Recruitment, 2010)**

(a) 5 (b) 6
(c) 8 (d) 9
(e) None of these

65. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^?$ **(Bank P.O., 2003)**

(a) 3.5 (b) 4.5
(c) 6 (d) 7
(e) None of these

66. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^?$ **(Bank P.O., 2003)**

(a) 8.5 (b) 13
(c) 16 (d) 17.5
(e) None of these

67. The value of $\dfrac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$ is **(C.B.I., 2003)**

(a) $\dfrac{3}{7}$ (b) $\dfrac{7}{3}$
(c) $1\dfrac{3}{7}$ (d) $2\dfrac{2}{7}$

68. $(64x^3 \div 27\,a^{-3})^{-\frac{2}{3}} = ?$ **(R.R.B., 2006)**

(a) $\dfrac{9ax}{16}$ (b) $\dfrac{9}{16ax}$
(c) $\dfrac{9}{16x^2\,a^2}$ (d) $\dfrac{3}{4}x^{-2}\,a^{-2}$

69. If $2^{n+4} - 2^{n+2} = 3$, then n is equal to

(a) 0 (b) 2
(c) – 1 (d) – 2

70. If $2^{n-1} + 2^{n+1} = 320$, then n is equal to

(a) 6 (b) 8
(c) 5 (d) 7

71. If $3^x - 3^{x-1} = 18$, then the value of x^x is

(a) 3 (b) 8
(c) 27 (d) 216

72. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{(n+3)}} + 2^{-3}$ is equal to

(a) 2^{n+1} (b) $\left(\dfrac{9}{8} - 2^n\right)$
(c) $\left(-2^{n+1} + \dfrac{1}{8}\right)$ (d) 1

73. The value of $\dfrac{2^{3x+4} + 8^{x+1}}{8^{x+1} - 2^{3x+2}}$ is

(a) 3 (b) 4
(c) 5 (d) 6

74. The value of $\dfrac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}}$ is

(a) $-\dfrac{1}{36}$ (b) $\dfrac{2}{3}$
(c) $\dfrac{1}{13}$ (d) $\dfrac{5}{13}$

75. If $x = 5+2\sqrt{6}$, then $\sqrt{x}-\frac{1}{\sqrt{x}}$ is **(A.A.O. Exam, 2009)**

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$

(c) $\sqrt{3}+\sqrt{2}$ (d) $\sqrt{3}-\sqrt{2}$

76. $(4+\sqrt{7})$, expressed as a perfect square, is equal to **(Section Officers', 2005)**

(a) $(2+\sqrt{7})^2$ (b) $\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)^2$

(c) $\left\{\frac{1}{2}(\sqrt{7}+1)^2\right\}$ (d) $(\sqrt{3}+\sqrt{4})^2$

77. $\sqrt{8-2\sqrt{15}}$ is equal to **(C.P.O., 2007)**

(a) $3-\sqrt{5}$ (b) $\sqrt{5}-\sqrt{3}$

(c) $5-\sqrt{3}$ (d) $\sqrt{5}+\sqrt{3}$

78. $\sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}$ is equal to **(A.A.O. Exam, 2010)**

(a) $1-\sqrt{3}$ (b) $\sqrt{3}-1$

(c) $2(2-\sqrt{3})$ (d) $2(2+\sqrt{3})$

79. The value of $\frac{1}{\sqrt{12-\sqrt{140}}}-\frac{1}{\sqrt{8-\sqrt{60}}}-\frac{2}{\sqrt{10+\sqrt{84}}}$ is **(S.S.C., 2005)**

(a) 0 (b) 1

(c) 2 (d) 3

80. The value of the expression

$\sqrt{4+\sqrt{15}}+\sqrt{4-\sqrt{15}}-\sqrt{12-4\sqrt{5}}$ is

(a) an irrational number

(b) a negative integer

(c) a natural number

(d) a non-integer rational number

81. If $N=\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}$, then the value of N is **(A.A.O. Exam., 2009)**

(a) $2\sqrt{2}-1$ (b) 3

(c) 1 (d) 2

82. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to

(a) 1.45 (b) 1.88

(c) 2.9 (d) 3.7

83. If m and n are whole numbers suchthat $m^n = 121$, then the value of $(m-1)^{n+1}$ is **(S.S.C., 2001)**

(a) 1 (b) 10

(c) 121 (d) 1000

84. Number of prime factors in $(216)^{\frac{3}{5}}\times(2500)^{\frac{2}{5}}\times(300)^{\frac{1}{5}}$ is

(a) 6 (b) 7

(c) 8 (d) None of these

85. Number of prime factors in $\frac{6^{12}\times(35)^{28}\times(15)^{16}}{(14)^{12}\times(21)^{11}}$ is

(a) 56 (b) 66

(c) 112 (d) None of these

86. $1 + (3+1)(3^2+1)(3^4+1)(3^8+1)(3^{16}+1)(3^{32}+1)$ is equal to **(Section Officers', 2005)**

(a) $\frac{3^{64}-1}{2}$ (b) $\frac{3^{64}+1}{2}$

(c) $3^{64}-1$ (d) $3^{64}+1$

87. $\frac{1}{1+a^{(n-m)}}+\frac{1}{1+a^{(m-n)}}=?$ **(M.B.A., 2003; NMAT, 2006)**

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) a^{m+n}

88. If $a+b+c=0$, then the value of $(x^a)^{a^2-bc}\cdot(x^b)^{b^2-ca}\cdot(x^c)^{c^2-ab}$ is equal to

(a) -2 (b) -1

(c) 0 (d) 1

89. $\frac{1}{1+x^{(b-a)}+x^{(c-a)}}+\frac{1}{1+x^{(a-b)}+x^{(c-b)}}+\frac{1}{1+x^{(b-c)}+x^{(a-c)}}=?$ **(M.B.A., 2003)**

(a) 0 (b) 1

(c) x^{a-b-c} (d) None of these

90. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)}\cdot\left(\frac{x^c}{x^a}\right)^{(c+a-b)}\cdot\left(\frac{x^a}{x^b}\right)^{(a+b-c)}=?$ **(NMAT, 2005; L.I.C., 2003)**

(a) x^{abc} (b) 1

(c) $x^{ab+bc+ca}$ (d) x^{a+b+c}

91. $\left(\frac{x^a}{x^b}\right)^{(a+b)}\cdot\left(\frac{x^b}{x^c}\right)^{(b+c)}\cdot\left(\frac{x^c}{x^a}\right)^{(c+a)}=?$ **(M.B.A., 2006)**

(a) 0 (b) x^{abc}

(c) x^{a+b+c} (d) 1

92. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\cdot\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\cdot\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}=?$

(a) 1 (b) $x^{\frac{1}{abc}}$

(c) $x^{\frac{1}{(ab+bc+ca)}}$ (d) None of these

93. The expression $\dfrac{\left(x+\frac{1}{y}\right)^a \cdot \left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a \cdot \left(y-\frac{1}{x}\right)^b}$ reduces to

(*a*) $\left(\frac{x}{y}\right)^{a-b}$ (*b*) $\left(\frac{y}{x}\right)^{a-b}$

(*c*) $\left(\frac{x}{y}\right)^{a+b}$ (*d*) $\left(\frac{y}{x}\right)^{a+b}$

94. The value of $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} \cdot \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \cdot \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$ is

(*a*) 1 (*b*) *a*

(*c*) *b* (*d*) *c*

95. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and $xyz = 1$, then the value of $p + q + r$ would be **(M.B.A. 2008)**

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) a rational number

96. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals

(*a*) $\dfrac{xz}{x+z}$ (*b*) $\dfrac{xz}{2(x-z)}$

(*c*) $\dfrac{xz}{2(z-x)}$ (*d*) $\dfrac{2xz}{(x+z)}$

97. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is **(M.B.A., 2005; R.R.B., 2008)**

(*a*) 0 (*b*) 1

(*c*) $\dfrac{1}{abc}$ (*d*) *abc*

98. If $2^x = 4^y = 8^z$ and $\left(\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}\right) = \frac{24}{7}$, then the value of z is

(*a*) $\dfrac{7}{16}$ (*b*) $\dfrac{7}{32}$

(*c*) $\dfrac{7}{48}$ (*d*) $\dfrac{7}{64}$

99. Suppose $4^a = 5$, $5^b = 6$, $6^c = 7$, $7^d = 8$, then the value of *abcd* is **(A.A.O. Exam, 2009)**

(*a*) 1 (*b*) $\dfrac{3}{2}$

(*c*) 2 (*d*) $\dfrac{5}{2}$

100. If $abc = 1$, then $\left(\dfrac{1}{1+a+b^{-1}} + \dfrac{1}{1+b+c^{-1}} + \dfrac{1}{1+c+a^{-1}}\right) = ?$ **(C.D.S. 2004)**

(*a*) 0 (*b*) 1

(*c*) $\dfrac{1}{ab}$ (*d*) *ab*

101. If *a*, *b*, *c* are real numbers, then the value of $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$ is

(*a*) *abc* (*b*) $\sqrt{abc}$

(*c*) $\dfrac{1}{abc}$ (*d*) 1

102. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to **(R.R.B., 2003)**

(*a*) 0 (*b*) 2

(*c*) 4 (*d*) 6

103. If $x^y = y^x$, then $\left(\frac{x}{y}\right)^{\frac{x}{y}}$ is equal to **(M.C.A., 2005)**

(*a*) $x^{\frac{y}{x}}$ (*b*) $x^{\frac{x}{y}-1}$

(*c*) 1 (*d*) $x^{\frac{x}{y}}$

104. If $4^{x+y} = 1$ and $4^{x-y} = 4$, then the values of x and y respectively are

(*a*) $-\frac{1}{2}$ and $\frac{1}{2}$ (*b*) $-\frac{1}{2}$ and $-\frac{1}{2}$

(*c*) $\frac{1}{2}$ and $-\frac{1}{2}$ (*d*) $\frac{1}{2}$ and $\frac{1}{2}$

105. If $2^{2x-1} + 4^x = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}}$, then x equals

(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{2}{3}$

(*c*) $\dfrac{3}{2}$ (*d*) 1

106. If $3^{2x-y} = 3^{x+y} = \sqrt{27}$, the value of y is **(R.R.B., 2005)**

(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{1}{4}$

(*c*) $\dfrac{3}{2}$ (*d*) $\dfrac{3}{4}$

107. If $3^x = 5^y = 45^z$, then **(C.D.S., 2002)**

(*a*) $\dfrac{2}{z} = \dfrac{1}{y} - \dfrac{1}{x}$ (*b*) $\dfrac{2}{y} = \dfrac{1}{x} - \dfrac{1}{z}$

(*c*) $\dfrac{2}{x} = \dfrac{1}{z} - \dfrac{1}{y}$ (*d*) $x + y + z = 0$

108. Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$, the value of $x + y$ is **(M.B.A., 2010)**

(*a*) 18 (*b*) 21

(*c*) 24 (*d*) 27

109. What are the values of x and y that satisfy the equation $2^{0.7x} \cdot 3^{-1.25y} = \dfrac{8\sqrt{6}}{27}$? **(C.A.T., 2006)**

(a) $x = 2.5, y = 6$ (b) $x = 3, y = 5$
(c) $x = 3, y = 4$ (d) $x = 5, y = 2$

110. Let r be the result of doubling both thebase and the exponent of a^b, $b \neq 0$. If r equals the product of a^b by x^b, then x equals (M.B.A., 2010)
(a) 2 (b) 4
(c) $2a$ (d) $4a$

111. Which of the following is the greatest? (Section Officers', 2005)
(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$
(c) $\sqrt[4]{4}$ (d) $\sqrt[6]{6}$

112. The greatest of $\sqrt{2}, \sqrt[6]{3}, \sqrt[3]{4}, \sqrt[4]{5}$ is (S.S.C., 2005)
(a) $\sqrt{2}$ (b) $\sqrt[3]{4}$
(c) $\sqrt[4]{5}$ (d) $\sqrt[6]{3}$

113. The largest number in the sequence $1, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}, ..., n^{\frac{1}{n}}$ is (I.I.F.T., 2005)
(a) $2^{\frac{1}{2}}$ (b) $3^{\frac{1}{3}}$
(c) $5^{\frac{1}{5}}$ (d) $6^{\frac{1}{6}}$

114. If $x = 5 + 2\sqrt{6}$, then $\dfrac{(x-1)}{\sqrt{x}}$ is equal to
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{3}$ (d) $2\sqrt{3}$

115. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$, then $\{(x + y - z)^3 + 27xyz\}$ equals
(a) – 1 (b) 0
(c) 1 (d) 27

116. If $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$, then the value of $x^3 - 6x^2 + 6x$ is (R.R.B., 2008)
(a) 1 (b) 2
(c) 3 (d) None of these

117. Find the value of $(-2)^5 \times (2)^{-5} \times (3)^3$ [ESIC—UDC Exam, 2016]
(a) –108 (b) 27
(c) $(2)^{25} \times (3)^3$ (d) –27

118. The quotient when 10^{100} is divided by 5^{75} is [SSC—CHSL (10+2) Exam, 2015]
(a) $2^{25} \times 10^{75}$ (b) 10^{25}
(c) 2^{75} (d) $2^{75} \times 10^{25}$

119. The exponential form of $\sqrt{\sqrt{2} \times \sqrt{3}}$ is [SSC—CHSL (10+2) Exam, 2015]
(a) 6 (b) $6^{\frac{1}{2}}$
(c) $6^{\frac{1}{3}}$ (d) $6^{\frac{1}{4}}$

120. $21^{?} \times 21^{6.5} = 21^{12.4}$ [United India Insurance Co. Ltd., UIICL—Assistant (Online) Exam, 2015]
(a) 18.9 (b) 4.4
(c) 5.9 (d) 13.4

121. $\dfrac{5.4 \div 3 \times 16 + 2}{18 \div 5 \times 6 \div 3}$ [United India Insurance Co. Ltd., UIICL—Assistant (Online) Exam, 2015]
(a) 2 (b) 4
(c) 6 (d) 8

122. $\left(32 \times 10^{-5}\right)^{-2} \times 64 \div \left(2^{16} \times 10^{-4}\right) = 10^{?}$ [IBPS—RRB Office Assistant (Online) Exam, 2015]
(a) 6 (b) 10
(c) –8 (d) –6

ANSWERS

1. (d)	2. (b)	3. (b)	4. (e)	5. (c)	6. (c)	7. (d)	8. (b)	9. (e)	10. (e)
11. (a)	12. (c)	13. (a)	14. (a)	15. (c)	16. (c)	17. (a)	18. (c)	19. (d)	20. (c)
21. (c)	22. (d)	23. (a)	24. (a)	25. (b)	26. (b)	27. (a)	28. (d)	29. (b)	30. (e)
31. (b)	32. (a)	33. (b)	34. (c)	35. (b)	36. (d)	37. (b)	38. (c)	39. (b)	40. (c)
41. (c)	42. (c)	43. (a)	44. (a)	45. (d)	46. (d)	47. (c)	48. (d)	49. (d)	50. (d)
51. (b)	52. (d)	53. (a)	54. (a)	55. (b)	56. (d)	57. (b)	58. (a)	59. (b)	60. (c)
61. (a)	62. (e)	63. (b)	64. (e)	65. (d)	66. (b)	67. (a)	68. (c)	69. (d)	70. (d)
71. (c)	72. (d)	73. (d)	74. (a)	75. (a)	76. (c)	77. (b)	78. (b)	79. (a)	80. (a)
81. (c)	82. (c)	83. (d)	84. (b)	85. (b)	86. (b)	87. (c)	88. (d)	89. (b)	90. (b)
91. (d)	92. (a)	93. (c)	94. (a)	95. (a)	96. (d)	97. (b)	98. (c)	99. (b)	100. (b)
101. (d)	102. (c)	103. (b)	104. (c)	105. (a)	106. (a)	107. (c)	108. (d)	109. (d)	110. (d)
111. (b)	112. (b)	113. (b)	114. (b)	115. (b)	116. (b)	117. (d)	118. (d)	119. (d)	120. (c)
121. (a)	122. (d)								

SOLUTIONS

1. $\sqrt[n]{a}$ is called a surd of order n.

2. $5^0 \times 8 = 1 \times 8 = 8$. $[\because 5^0 = 1]$

3. I. $4^1 = 4$ II. $1^4 = 1$
III. $4^0 = 1$ IV. $0^4 = 0$

4. $289 = 17^{\frac{1}{5}x} \Rightarrow 17^2 = 17^{\frac{1}{5}x} \Rightarrow \frac{1}{5}x = 2 \Rightarrow x = 2\times 5 = 10.$

5. $(81)^4 \div (9)^5 = \frac{(9^2)^4}{(9^5)} = \frac{9^{(2\times 4)}}{9^5} = \frac{9^8}{9^5} = 9^{(8-5)} = 9^3 = 729.$

6. $$\left(\frac{9^2\times 18^4}{3^{16}}\right) = \frac{9^2\times(9\times 2)^4}{3^{16}} = \frac{(3^2)^2\times(3^2)^4\times 2^4}{3^{16}}$$
$$= \frac{3^4\times 3^8\times 2^4}{3^{16}} = \frac{3^{(4+8)}\times 2^4}{3^{16}} = \frac{3^{12}\times 2^4}{3^{16}}$$
$$= \frac{2^4}{3^{(16-12)}} = \frac{2^4}{3^4} = \frac{16}{81}.$$

7. $\frac{4^3\times 5^4}{4^5} = \frac{5^4}{4^{(5-3)}} = \frac{5^4}{4^2} = \frac{625}{16} = 39.0625.$

8. $$\frac{9^3\times 6^2}{3^3} = \frac{(3^2)^3\times(3\times 2)^2}{3^3} = \frac{3^{(2\times 3)}\times 3^2\times 2^2}{3^3} = \frac{3^{(6+2)}\times 2^2}{3^3}$$
$= 3^{(8-3)} \times 2^2 = 3^5 \times 2^2 = 243 \times 4 = 972.$

9. $\frac{(19)^{12}\times(19)^8}{(19)^4} = \frac{19^{(12+8)}}{(19)^4} = \frac{(19)^{20}}{(19)^4} = (19)^{(20-4)} = (19)^{16}.$

Hence, missing number = 16.

10. $(64)^4 \div (8)^5 = (8^2)^4 \div (8)^5 = (8)^{(2\times 4)} \div 8^5 = \frac{8^8}{8^5} = 8^{(8-5)} = 8^3.$

11. $(1000)^{12} \div (10)^{30} = \frac{(10^3)^{12}}{(10)^{30}} = \frac{(10)^{(3\times 12)}}{(10)^{30}} = \frac{(10)^{36}}{(10)^{30}} = (10)^{(36-30)}$
$= 10^6 = (10^3)^2 = (1000)^2.$

12. $3^8 \times 3^4 = 3^{(8+4)} = 3^{12} = (3^6)^2 = (729)^2.$

13. $$\frac{343\times 49}{216\times 16\times 81} = \frac{7^3\times 7^2}{6^3\times 2^4\times 3^4} = \frac{7^{(3+2)}}{6^3\times(2\times 3)^4}$$
$$= \frac{7^5}{6^3\times 6^4} = \frac{7^5}{6^{(3+4)}} = \frac{7^5}{6^7}.$$

14. $\frac{16\times 32}{9\times 27\times 81} = \frac{2^4\times 2^5}{3^2\times 3^3\times 3^4} = \frac{2^{(4+5)}}{3^{(2+3+4)}} = \frac{2^9}{3^9} = \left(\frac{2}{3}\right)^9.$

15. Let $9^3 \times (81)^2 \div (27)^3 = 3^x$. Then,
$$3^x = \frac{(3^2)^3\times(3^4)^2}{(3^3)^3} = \frac{3^{(2\times 3)}\times 3^{(4\times 2)}}{3^{(3\times 3)}} = \frac{3^6\times 3^8}{3^9} = \frac{3^{(6+8)}}{3^9}$$
$$\Rightarrow\quad 3^x = \frac{3^{14}}{3^9} = 3^{(14-9)} = 3^5 \Rightarrow x = 5.$$

16. Let $6^4 \div (36)^3 \times 216 = 6^{(x-5)}$.
Then, $6^{(x-5)} = 6^4 \div (6^2)^3 \times 6^3$
$= 6^4 \div 6^{(2\times 3)} \times 6^3 = 6^4 \div 6^6 \times 6^3 = 6^{(4-6+3)} = 6$
$\Rightarrow\quad x - 5 = 1 \Rightarrow x = 6.$

17. $(0.2)^2 = 0.2 \times 0.2 = 0.04;$
$\frac{1}{100} = 0.01;\quad (0.01)^{\frac12} = \left[(0.1)^2\right]^{\frac12} = (0.1)^{\left(2\times\frac12\right)} = 0.1;$
$(0.008)^{\frac13} = [(0.2)^3]^{\frac13} = (0.2)^{\left(3\times\frac13\right)} = 0.2.$
Clearly, $0.2 > 0.1 > 0.04 > 0.01$.

So, $(0.008)^{\frac13}$ is the greatest.

18. $[(2^{-1})^0]^2 = \left[\left(\frac12\right)^0\right]^2 = (1)^2 = 1.$

$\left[(4^0)^{-\frac12}\right]^2 = \left[(1)^{-\frac12}\right]^2 = (1)^2 = 1.$

$\left[(2^{-2})^{-1}\right]^2 = \left[\left(\frac{1}{2^2}\right)^{-1}\right]^2 = \left[\left(\frac14\right)^{-1}\right]^2 = (4)^2 = 16.$

$\left[(2^{-1})^2\right]^2 = \left[\left(\frac12\right)^2\right]^2 = \left(\frac14\right)^2 = \frac{1}{16}.$

19. $(10)^{24} \times (10)^{-21} = 10^{(24-21)} = 10^3 = 1000.$

20. $(256)^{\frac54} = (4^4)^{\frac54} = 4^{\left(4\times\frac54\right)} = 4^5 = 1024.$

21. $(\sqrt{8})^{\frac13} = \left(8^{\frac12}\right)^{\frac13} = 8^{\left(\frac12\times\frac13\right)}$

$= 8^{\frac16} = (2^3)^{\frac16} = 2^{\left(3\times\frac16\right)} = 2^{\frac12} = \sqrt{2}.$

22. $\left(\frac{32}{243}\right)^{-\frac45} = \left\{\left(\frac23\right)^5\right\}^{-\frac45} = \left(\frac23\right)^{5\times\frac{(-4)}{5}}$

$= \left(\frac23\right)^{(-4)} = \left(\frac32\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}.$

23. $\left(-\frac{1}{216}\right)^{-\frac23} = \left[\left(-\frac16\right)^3\right]^{-\frac23} = \left(-\frac16\right)^{3\times\frac{(-2)}{3}}$

$= \left(-\frac16\right)^{-2} = \frac{1}{\left(-\frac16\right)^2} = \frac{1}{\left(\frac{1}{36}\right)} = 36.$

24. $(27)^{-\frac23} = (3^3)^{-\frac23} = 3^{\left[3\times\left(-\frac23\right)\right]} = 3^{-2} = \frac{1}{3^2} = \frac19.$

Clearly, $0 < \frac19 < 1$.

25. $\sqrt[3]{2^4\sqrt{2^{-5}\sqrt{2^6}}} = \sqrt[3]{2^4\sqrt{2^{-5}(2^6)^{\frac12}}} = \sqrt[3]{2^4\sqrt{2^{-5}\,(2)^{\left(6\times\frac12\right)}}}$

$= \sqrt[3]{2^4\sqrt{2^{-5}\,.\,2^3}} = \sqrt[3]{2^4\sqrt{2^{(-5+3)}}}$

$= \sqrt[3]{2^4\sqrt{2^{(-2)}}} = \sqrt[3]{2^4\,.\,(2^{-2})^{\frac12}} = \sqrt[3]{2^4\,.\,2^{\left(-2\times\frac12\right)}}$

$$= \sqrt[3]{2^4 . 2^{(-1)}} = \sqrt[3]{2^{(4-1)}} = \sqrt[3]{2^3} = (2^3)^{\frac13}$$

$$= 2^{\left(3\times\frac13\right)} = 2.$$

26. $$\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt2}}}} = \sqrt{2\sqrt{2\sqrt{2\sqrt{2.2^{\frac12}}}}} = \sqrt{2\sqrt{2\sqrt{2\sqrt{2^{\left(1+\frac12\right)}}}}}$$

$$= \sqrt{2\sqrt{2\sqrt{2\sqrt{2^{\frac32}}}}} = \sqrt{2\sqrt{2\sqrt{2.\left(2^{\frac32}\right)^{\frac12}}}}$$

$$= \sqrt{2\sqrt{2\sqrt{2.2^{\left(\frac32\times\frac12\right)}}}} = \sqrt{2\sqrt{2\sqrt{2.2^{\frac34}}}}$$

$$= \sqrt{2\sqrt{2\sqrt{2^{\left(1+\frac34\right)}}}} = \sqrt{2\sqrt{2\sqrt{2^{\frac74}}}}$$

$$= \sqrt{2\sqrt{2.\left(2^{\frac74}\right)^{\frac12}}} = \sqrt{2\sqrt{2.2^{\left(\frac74\times\frac12\right)}}}$$

$$= \sqrt{2\sqrt{2.2^{\frac78}}} = \sqrt{2\sqrt{2^{\left(1+\frac78\right)}}} = \sqrt{2\sqrt{2^{\frac{15}{8}}}}$$

$$= \sqrt{2\left(2^{\frac{15}{8}}\right)^{\frac12}} = \sqrt{2.2^{\frac{15}{16}}} = \sqrt{2^{\left(1+\frac{15}{16}\right)}}$$

$$= \sqrt{2^{\frac{31}{16}}} = \left(2^{\frac{31}{16}}\right)^{\frac12} = 2^{\left(\frac{31}{16}\times\frac12\right)} = 2^{\frac{31}{32}}.$$

27. $$(0.03125)^{-\frac25} = [(0.5)^5]^{-\frac25} = 0.5^{\left[5\times\left(-\frac25\right)\right]} = (0.5)^{-2}$$

$$= \frac{1}{(0.5)^2} = \frac{1}{0.25} = 4.$$

28. $$\left(\frac12\right)^{-\frac12} = (2)^{\frac12} = \sqrt2.$$

29. $$\left[\left(\sqrt[5]{x^{-\frac35}}\right)^{-\frac53}\right]^5 = \left[\left\{\left(x^{-\frac35}\right)^{\frac15}\right\}^{-\frac53}\right]^5 = \left[\left(x^{\left\{\left(-\frac35\right)\times\frac15\right\}}\right)^{-\frac53}\right]^5$$

$$= \left[\left(x^{-\frac{3}{25}}\right)^{-\frac53}\right]^5 = \left[x^{\left\{\left(-\frac{3}{25}\right)\times\left(-\frac53\right)\right\}}\right]^5$$

$$= \left(x^{\frac15}\right)^5 = x^{\left(\frac15\times5\right)} = x.$$

30. Let $\frac{x^{\frac23}}{42} = \frac{5}{x^{\frac13}}$.

Then, $x^{\frac23}.x^{\frac13} = 42\times5 = 210 \Rightarrow x^{\left(\frac23+\frac13\right)} = 210 \Rightarrow x = 210.$

31. $$5^{\frac14}\times(125)^{0.25} = 5^{0.25}\times(5^3)^{0.25} = 5^{0.25}\times5^{(3\times0.25)}$$

$$= 5^{0.25}\times5^{0.75} = 5^{(0.25+0.75)} = 5^1 = 5.$$

32. $$\frac{1}{(216)^{-\frac23}} + \frac{1}{(256)^{-\frac34}} + \frac{1}{(32)^{-\frac15}}$$

$$= \frac{1}{(6^3)^{-\frac23}} + \frac{1}{(4^4)^{\left(-\frac34\right)}} + \frac{1}{(2^5)^{-\frac15}}$$

$$= \frac{1}{6^{3\times\frac{(-2)}{3}}} + \frac{1}{4^{4\times\frac{(-3)}{4}}} + \frac{1}{2^{5\times\frac{(-1)}{5}}} = \frac{1}{6^{-2}}+\frac{1}{4^{-3}}+\frac{1}{2^{-1}}$$

$$= (6^2+4^3+2^1) = (36+64+2) = 102.$$

33. $$(2.4\times10^3)\div(8\times10^{-2}) = \frac{2.4\times10^3}{8\times10^{-2}} = \frac{24\times10^2}{8\times10^{-2}} = (3\times10^4).$$

34. $$\left(\frac{1}{216}\right)^{-\frac23}\div\left(\frac{1}{27}\right)^{-\frac43} = (216)^{\frac23}\div(27)^{\frac43} = \frac{(216)^{\frac23}}{(27)^{\frac43}} = \frac{(6^3)^{\frac23}}{(3^3)^{\frac43}}$$

$$= \frac{6^{\left(3\times\frac23\right)}}{3^{\left(3\times\frac43\right)}} = \frac{6^2}{3^4} = \frac{36}{81} = \frac49.$$

35. $$(48)^{-\frac27}\times(16)^{-\frac57}\times(3)^{-\frac57} = (16\times3)^{-\frac27}\times(16)^{-\frac57}\times(3)^{-\frac57}$$

$$= (16)^{-\frac27}\times(3)^{-\frac27}\times(16)^{-\frac57}\times(3)^{-\frac57}$$

$$= (16)^{\left(-\frac27-\frac57\right)}\times(3)^{\left(-\frac27-\frac57\right)}$$

$$= (16)^{\left(-\frac77\right)}\times(3)^{\left(-\frac77\right)} = (16)^{-1}\times(3)^{-1}$$

$$= \frac1{16}\times\frac13 = \frac1{48}.$$

36. $$10^{-8x} = (10^x)^{-8} = \left(\frac12\right)^{-8} = 2^8 = 256.$$

37. $$\left(\frac35\right)^3\left(\frac35\right)^{-6} = \left(\frac35\right)^{2x-1} \Rightarrow \left(\frac35\right)^{(3-6)} = \left(\frac35\right)^{2x-1}$$

$$\Rightarrow \left(\frac35\right)^{-3} = \left(\frac35\right)^{2x-1}$$

$\Rightarrow 2x-1=-3 \Rightarrow 2x=-2 \Rightarrow x=-1.$

38. $49\times49\times49\times49 = 7^2\times7^2\times7^2\times7^2 = 7^{(2+2+2+2)} = 7^8.$

$\therefore$ Required number = 8.

39. $$8^{-25}-8^{-26} = \left(\frac{1}{8^{25}}-\frac{1}{8^{26}}\right) = \frac{(8-1)}{8^{26}} = 7\times8^{-26}.$$

40. $$(64)^{-\frac12}-(-32)^{-\frac45} = (8^2)^{-\frac12}-[(-2)^5]^{-\frac45}$$

$$= 8^{\left[2\times\left(\frac{-1}{2}\right)\right]} - (-2)^{\left[5\times\left(\frac{-4}{5}\right)\right]}$$

$$= 8^{-1}-(-2)^{-4} = \frac18-\frac{1}{(-2)^4}$$

$$= \left(\frac18-\frac1{16}\right) = \frac1{16}.$$

41. $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} \Leftrightarrow \left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-(x-3)} = \left(\frac{a}{b}\right)^{(3-x)}$

$\Leftrightarrow\ x - 1 = 3 - x \ \Leftrightarrow\ 2x = 4 \ \Leftrightarrow\ x = 2.$

42. $2^{2n-1} = \frac{1}{8^{n-3}} \Leftrightarrow 2^{2n-1} = \frac{1}{(2^3)^{n-3}}$

$= \frac{1}{2^{3(n-3)}} = \frac{1}{2^{(3n-9)}} = 2^{(9-3n)}$

$\Leftrightarrow\ 2n - 1 = 9 - 3n \ \Leftrightarrow\ 5n = 10$

$\Leftrightarrow\ n = 2.$

43. $5^a = 3125 = 5^5 \Rightarrow a = 5. \Rightarrow 5^{(a-3)} = 5^{(5-3)} = 5^2 = 25.$

44. $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2} \ \Leftrightarrow\ \frac{5 \times 5^{\frac{1}{2}} \times 5^3}{5^{-\frac{3}{2}}} = 5^{a+2}$

$\Leftrightarrow\ 5^{\left(1+\frac{1}{2}+3+\frac{3}{2}\right)} = 5^{a+2} \Leftrightarrow 5^6 = 5^{a+2}$

$\Leftrightarrow\ a + 2 = 6 \Leftrightarrow a = 4.$

45. $\sqrt{2^n} = 64 \ \Leftrightarrow\ (2^n)^{\frac{1}{2}} = 2^6 \ \Leftrightarrow\ 2^{\frac{n}{2}} = 2^6 \ \Leftrightarrow\ \frac{n}{2} = 6$

$\Leftrightarrow\ n = 12.$

46. $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3} \ \Leftrightarrow\ \left(3^{\frac{1}{2}}\right)^5 \times (3^2)^2 = 3^n \times 3 \times 3^{\frac{1}{2}}$

$\Leftrightarrow\ 3^{\left(\frac{1}{2}\times 5\right)} \times 3^{(2\times 2)} = 3^{\left(n+1+\frac{1}{2}\right)}$

$\Leftrightarrow\ 3^{\left(\frac{5}{2}+4\right)} = 3^{\left(n+\frac{3}{2}\right)} \Leftrightarrow n + \frac{3}{2} = \frac{13}{2}$

$\Leftrightarrow\ n = \left(\frac{13}{2} - \frac{3}{2}\right) = \frac{10}{2} = 5.$

47. $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27 \ \Leftrightarrow\ \frac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$

$\Leftrightarrow\ \frac{3^{2n} \times 3^5 \times 3^{(3\times 3)}}{3 \times 3^{(4\times 4)}} = 3^3$

$\Leftrightarrow\ \frac{3^{2n+5+9}}{3 \times 3^{16}} = 3^3 \Leftrightarrow \frac{3^{2n+14}}{3^{17}} = 3^3$

$\Leftrightarrow\ 3^{(2n+14-17)} = 3^3$

$\Leftrightarrow\ 3^{2n-3} = 3^3 \ \Leftrightarrow\ 2n - 3 = 3 \ \Leftrightarrow\ 2n = 6 \ \Leftrightarrow\ n = 3.$

48. $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3} \ \Leftrightarrow\ \frac{9^x}{4^x} \times \frac{8^{(x-1)}}{27^{(x-1)}} = \frac{2}{3}$

$\Leftrightarrow\ \frac{(3^2)^x}{(2^2)^x} \times \frac{(2^3)^{(x-1)}}{(3^3)^{(x-1)}} = \frac{2}{3} \ \Leftrightarrow\ \frac{3^{2x} \times 2^{3(x-1)}}{2^{2x} \times 3^{3(x-1)}} = \frac{2}{3}$

$\Leftrightarrow\ \frac{2^{(3x-3-2x)}}{3^{(3x-3-2x)}} = \frac{2}{3} \Leftrightarrow \frac{2^{(x-3)}}{3^{(x-3)}} = \frac{2}{3} \Leftrightarrow \left(\frac{2}{3}\right)^{(x-3)} = \left(\frac{2}{3}\right)^1$

$\Leftrightarrow\ x - 3 = 1 \ \Leftrightarrow x = 4.$

49. $2^x = \sqrt[3]{32} \ \Leftrightarrow\ 2^x = (32)^{\frac{1}{3}} = (2^5)^{\frac{1}{3}} = 2^{\frac{5}{3}} \ \Leftrightarrow\ x = \frac{5}{3}.$

50. $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}} \ \Leftrightarrow\ 2^x \times (2^3)^{\frac{1}{5}} = 2^{\frac{1}{5}} \ \Leftrightarrow\ 2^x \times 2^{\frac{3}{5}} = 2^{\frac{1}{5}}$

$\Leftrightarrow\ 2^{\left(x+\frac{3}{5}\right)} = 2^{\frac{1}{5}}$

$\Leftrightarrow\ x + \frac{3}{5} = \frac{1}{5} \ \Leftrightarrow\ x = \left(\frac{1}{5} - \frac{3}{5}\right) = \frac{-2}{5}.$

51. $5^{(x+3)} = 25^{(3x-4)} \Leftrightarrow 5^{(x+3)} = (5^2)^{(3x-4)} \Leftrightarrow 5^{(x+3)}$

$= 5^{2(3x-4)} \ \Leftrightarrow\ 5^{(x+3)} = 5^{(6x-8)}$

$\Leftrightarrow\ x + 3 = 6x - 8 \ \Leftrightarrow\ 5x = 11$

$\Leftrightarrow\ x = \frac{11}{5}.$

52. $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})} = \frac{2^{n+4} - 2^{n+1}}{2^{n+4}} = \frac{2^{n+4}}{2^{n+4}} - \frac{2^{n+1}}{2^{n+4}}$

$= 1 - 2^{n+1-(n+4)} = 1 - 2^{-3} = 1 - \frac{1}{8} = \frac{7}{8}.$

53. $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4 = \left[\left\{(a^9)^{\frac{1}{6}}\right\}^{\frac{1}{3}}\right]^4 . \left[\left\{(a^9)^{\frac{1}{3}}\right\}^{\frac{1}{6}}\right]^4$

$= a^{\left(9\times\frac{1}{6}\times\frac{1}{3}\times 4\right)} . a^{\left(9\times\frac{1}{3}\times\frac{1}{6}\times 4\right)}$

$= a^2 . a^2 = a^4.$

54. $(256)^{0.16} \times (256)^{0.09} = (256)^{(0.16+0.09)} = (256)^{0.25}$

$= (256)^{\left(\frac{25}{100}\right)}$

$= (256)^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4^{\left(4\times\frac{1}{4}\right)} = 4^1 = 4.$

55. $(0.04)^{-1.5} = \left(\frac{4}{100}\right)^{-1.5} = \left(\frac{1}{25}\right)^{-\frac{3}{2}} = (25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}}$

$= 5^{\left(2\times\frac{3}{2}\right)} = 5^3 = 125.$

56. Let $(17)^{3.5} \times (17)^x = 17^8$. Then, $(17)^{(3.5+x)} = (17)^8$.

$\therefore\ 3.5 + x = 8 \ \Leftrightarrow\ x = (8 - 3.5)$

$\Leftrightarrow\ x = 4.5.$

57. Let $6^{1.2} \times 36^x \times 30^{2.4} \times 25^{1.3} = 30^5$.

Then, $6^{1.2} \times (6^2)^x \times (6 \times 5)^{2.4} \times (5^2)^{1.3} = 30^5$

$\Leftrightarrow\ 6^{1.2} \times 6^{2x} \times 6^{2.4} \times 5^{2.4} \times 5^{2.6} = (6 \times 5)^5$

$\Leftrightarrow\ 6^{(1.2+2x+2.4)} \times 5^{(2.4+2.6)} = 6^5 \times 5^5$

$\Leftrightarrow\ 6^{(3.6+2x)} \times 5^5 = 6^5 \times 5^5 \Leftrightarrow 3.6 + 2x = 5 \Leftrightarrow 2x = 1.4$

$\Leftrightarrow\ x = 0.7.$

58. Let $2^{3.6} \times 4^{3.6} \times 4^{3.6} \times 32^{2.3} = 32^x$.

Then, $2^{3.6} \times (2^2)^{3.6} \times (2^2)^{3.6} \times (2^5)^{2.3} = (2^5)^x$

$\Leftrightarrow\ 2^{3.6} \times 2^{(2\times 3.6)} \times 2^{(2\times 3.6)} \times (2^5)^{2.3} = (2^5)^x$

$\Leftrightarrow\ 2^{(3.6+7.2+7.2)} \times (2^5)^{2.3} = (2^5)^x \Leftrightarrow 2^{18} \times (2^5)^{2.3} = (2^5)^x$

$\Leftrightarrow\ (2^5)^{3.6} \times (2^5)^{2.3} = (2^5)^x \ \Leftrightarrow\ (2^5)^{(3.6+2.3)} = (2^5)^x$

$\Leftrightarrow\ (2^5)^{5.9} = (2^5)^x \Leftrightarrow x = 5.9.$

59. Let $3^{3.5} \times (21)^2 \times (42)^{2.5} \div 2^{2.5} \times 7^{3.5} = 21^x$.

Then, $3^{3.5} \times 7^{3.5} \times (21)^2 \times (21 \times 2)^{2.5} \div 2^{2.5} = 21^x$

$\Leftrightarrow\ (21)^x = (3 \times 7)^{3.5} \times (21)^2 \times (21)^{2.5} \times 2^{2.5} \div 2^{2.5}$

$\Leftrightarrow\ (21)^x = (21)^{3.5} \times (21)^{(2+2.5)} \Leftrightarrow (21)^x = (21)^{3.5} \times (21)^{(4.5)}$

$\Leftrightarrow\ (21)^x = (21)^{(3.5+4.5)} = (21)^8$

$\Leftrightarrow\ x = 8.$

60. $8^{0.4} \times 4^{1.6} \times 2^{1.6} = (2^3)^{0.4} \times (2^2)^{1.6} \times 2^{1.6}$

$= 2^{(3 \times 0.4)} \times 2^{(2 \times 1.6)} \times 2^{1.6}$

$= 2^{1.2} \times 2^{3.2} \times 2^{1.6} = 2^{(1.2+3.2+1.6)} = 2^6 = 64.$

61. Let $8^7 \times 2^6 \div 8^{2.4} = 8^x$.

Then, $8^7 \times (2^3)^2 \div 8^{2.4} = 8^x$

$\Leftrightarrow\ 8^7 \times 8^2 \div 8^{2.4} = 8^x$

$\Leftrightarrow\ 8^x = 8^{(7+2-2.4)} \Leftrightarrow 8^x = 8^{6.6} \Leftrightarrow x = 6.6.$

62. Let $25^{2.7} \times 5^{4.2} \div 5^{5.4} = 25^x$.

Then, $(25)^{2.7} \times 5^{(4.2-5.4)} = 25^x$

$\Leftrightarrow\ (25)^{2.7} \times 5^{(-1.2)} = 25^x \Leftrightarrow (25)^{2.7} \times \dfrac{1}{5^{1.2}} = 25^x$

$\Leftrightarrow\ \dfrac{(25)^{2.7}}{(5^2)^{0.6}} = 25^x \Leftrightarrow \dfrac{(25)^{2.7}}{(25)^{0.6}} = 25^x$

$\Leftrightarrow\ 25^x = 25^{(2.7-0.6)} = 25^{2.1} \Leftrightarrow x = 2.1$

63. Let $8^{2.4} \times 2^{3.7} \div (16)^{1.3} = 2^x$.

Then, $(2^3)^{2.4} \times 2^{3.7} \div (2^4)^{1.3} = 2^x$

$\Leftrightarrow\ 2^{(3 \times 2.4)} \times 2^{3.7} \div 2^{(4 \times 1.3)} = 2^x$

$\Leftrightarrow\ 2^{7.2} \times 2^{3.7} \div 2^{5.2} = 2^x$

$\Leftrightarrow\ 2^x = 2^{(7.2+3.7-5.2)} = 2^{5.7} \Leftrightarrow x = 5.7.$

64. Let $(0.04)^2 \div (0.008) \times (0.2)^6 = (0.2)^x$.

Then, $(0.2)^x = [(0.2)^2]^2 \div (0.2)^3 \times (0.2)^6$

$\Leftrightarrow\ (0.2)^x = (0.2)^{(2 \times 2)} \div (0.2)^3 \times (0.2)^6$

$\Leftrightarrow\ (0.2)^x = (0.2)^4 \div (0.2)^3 \times (0.2)^6 = (0.2)^{(4-3+6)} = (0.2)^7$

$\Leftrightarrow\ x = 7.$

65. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^x$

$\Leftrightarrow\ (18)^{3.5} \times \dfrac{1}{(27)^{3.5}} \times 6^{3.5} = 2^x$

$\Leftrightarrow\ (3^2 \times 2)^{3.5} \times \dfrac{1}{(3^3)^{3.5}} \times (2 \times 3)^{3.5} = 2^x 11$

$\Leftrightarrow\ 3^{(2 \times 3.5)} \times 2^{3.5} \times \dfrac{1}{3^{(3 \times 3.5)}} \times 2^{3.5} \times 3^{3.5} = 2^x$

$\Leftrightarrow\ 3^7 \times 2^{3.5} \times \dfrac{1}{3^{10.5}} \times 2^{3.5} \times 3^{3.5} = 2^x$

$\Leftrightarrow\ \dfrac{3^{(7+3.5)}}{3^{10.5}} \times 2^{(3.5+3.5)} \Leftrightarrow 2^7 = 2^x \Leftrightarrow x = 7.$

66. Let $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^x$.

Then, $\dfrac{(5^2)^{7.5} \times (5)^{2.5}}{(5^3)^{1.5}} = 5^x \Leftrightarrow \dfrac{5^{(2 \times 7.5)} \times 5^{2.5}}{5^{(3 \times 1.5)}} = 5^x$

$\Leftrightarrow\ 1\dfrac{5^{15} \times 5^{2.5}}{5^{4.5}} = 5^x$

$\Leftrightarrow\ 5^x = 5^{(15+2.5-4.5)} = 5^{13} \Leftrightarrow x = 13.$

67. $\dfrac{(243)^{0.13} \times (243)^{0.07}}{7^{0.25} \times (49)^{0.075} \times (343)^{0.2}} = \dfrac{(243)^{(0.13+0.07)}}{7^{0.25} \times (7^2)^{0.075} \times (7^3)^{0.2}}$

$= \dfrac{(243)^{0.2}}{7^{0.25} \times 7^{(2 \times 0.075)} \times 7^{(3 \times 0.2)}}$

$= \dfrac{(3^5)^{0.2}}{7^{0.25} \times 7^{0.15} \times 7^{0.6}} = \dfrac{3^{(5 \times 0.2)}}{7^{(0.25+0.15+0.6)}} = \dfrac{3^1}{7^1} = \dfrac{3}{7}.$

68. $(64x^3 \div 27a^{-3})^{-\frac{2}{3}} = \left(\dfrac{64x^3}{27a^{-3}}\right)^{-\frac{2}{3}} = \left(\dfrac{4^3 \,.\, x^3}{3^3 \,.\, a^{-3}}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{4^3 \,.x^3 \,.a^3}{3^3}\right)^{-\frac{2}{3}}$

$= \dfrac{\{(4ax)^3\}^{-\frac{2}{3}}}{3^{3 \times \left(-\frac{2}{3}\right)}} = \dfrac{(4ax)^{3 \times \left(-\frac{2}{3}\right)}}{3^{-2}} = \dfrac{(4ax)^{-2}}{3^{-2}}$

$= \dfrac{3^2}{(4ax)^2} = \dfrac{9}{16a^2x^2}.$

69. $2^{n+4} - 2^{n+2} = 3 \Leftrightarrow 2^{n+2}(2^2 - 1) = 3$

$\Leftrightarrow\ 2^{n+2} = 1 = 2^0$

$\Leftrightarrow\ n + 2 = 0 \Leftrightarrow n = -2.$

70. $2^{n-1} + 2^{n+1} = 320 \Leftrightarrow 2^{n-1}(1 + 2^2) = 320$

$\Leftrightarrow\ 5 \times 2^{n-1} = 320$

$\Leftrightarrow\ 2^{n-1} = \dfrac{320}{5} = 64 = 2^6$

$\Leftrightarrow\ n - 1 = 6 \Leftrightarrow n = 7.$

71. $3^x - 3^{x-1} = 18 \Leftrightarrow 3^{x-1}(3 - 1) = 18$

$\Leftrightarrow\ 3^{x-1} = 9 = 3^2 \Leftrightarrow x - 1 = 2 \Leftrightarrow x = 3.$

$\therefore\ x^x = 3^3 = 27.$

72. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{n+3}} + 2^{-3} = \dfrac{2^{n+4} - 2^{n+1}}{2^{n+4}} + \dfrac{1}{2^3}$

$= \dfrac{2^{n+1}(2^3 - 1)}{2^{n+4}} + \dfrac{1}{2^3}$

$= \dfrac{2^{n+1} \times 7}{2^{n+1} \times 2^3} + \dfrac{1}{2^3} = \left(\dfrac{7}{8} + \dfrac{1}{8}\right) = \dfrac{8}{8} = 1.$

73. $\dfrac{2^{3x+4} + 8^{x+1}}{8^{x+1} - 2^{3x+2}} = \dfrac{2^{3x+4} + (2^3)^{x+1}}{(2^3)^{x+1} - 2^{3x+2}} = \dfrac{2^{3x+4} + 2^{3(x+1)}}{2^{3(x+1)} - 2^{3x+2}}$

$= \dfrac{2^{3x+4} + 2^{3x+3}}{2^{3x+3} - 2^{3x+2}} = \dfrac{2^{3x+3}(2+1)}{2^{3x+2}(2-1)}$

$= 3.2^{(3x+3)-(3x+2)} = 3.2^1$

$= 3 \times 2 = 6.$

74. $\dfrac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}} = \dfrac{2^{n-1}(1-2)}{2^{n+1}(2^3+1)} = \left(-\dfrac{1}{9}\right).2^{(n-1)-(n+1)}$

$= \left(-\dfrac{1}{9}\right).2^{-2} = \left(-\dfrac{1}{9}\right) \times \dfrac{1}{2^2} = \left(-\dfrac{1}{9}\right) \times \dfrac{1}{4}$

$= -\dfrac{1}{36}.$

75. $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2 = x + \dfrac{1}{x} - 2 = (5 + 2\sqrt{6}) + \dfrac{1}{(5 + 2\sqrt{6})} - 2$

$= (5 + 2\sqrt{6}) + \dfrac{1}{(5 + 2\sqrt{6})} \times \dfrac{(5 - 2\sqrt{6})}{(5 - 2\sqrt{6})} - 2$

$= (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) - 2 = 8.$

$\therefore \quad \left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)=\sqrt{8}=2\sqrt{2}.$

76. $4+\sqrt{7}=\dfrac{7}{2}+\dfrac{1}{2}+2\times\dfrac{\sqrt{7}}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}}$

$=\left(\dfrac{\sqrt{7}}{\sqrt{2}}\right)^2+\left(\dfrac{1}{\sqrt{2}}\right)^2+2\times\dfrac{\sqrt{7}}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}}$

$=\left(\dfrac{\sqrt{7}}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}\right)^2=\dfrac{1}{2}(\sqrt{7}+1)^2.$

77. $\sqrt{8-2\sqrt{15}}=\sqrt{5+3-2\times\sqrt{5}\times\sqrt{3}}$

$=\sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\times\sqrt{5}\times\sqrt{3}}$

$=\sqrt{(\sqrt{5}-\sqrt{3})^2}=(\sqrt{5}-\sqrt{3}).$

78. $\sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}=\sqrt{6-4\sqrt{3}+\sqrt{12+4-8\sqrt{3}}}$

$=\sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3})^2+(2)^2-2\times2\sqrt{3}\times2}}$

$=\sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3}-2)^2}}=\sqrt{6-4\sqrt{3}+2\sqrt{3}-2}$

$=\sqrt{(\sqrt{3})^2+(1)^2-2\times\sqrt{3}\times1}=\sqrt{(\sqrt{3}-1)^2}=\sqrt{3}-1.$

79. $\dfrac{1}{\sqrt{12-\sqrt{140}}}-\dfrac{1}{\sqrt{8-\sqrt{60}}}-\dfrac{2}{\sqrt{10+\sqrt{84}}}$

$=\dfrac{1}{\sqrt{12-\sqrt{4\times35}}}-\dfrac{1}{\sqrt{8-\sqrt{4\times15}}}-\dfrac{2}{\sqrt{10+\sqrt{4\times21}}}$

$=\dfrac{1}{\sqrt{12-2\sqrt{35}}}-\dfrac{1}{\sqrt{8-2\sqrt{15}}}-\dfrac{2}{\sqrt{10+2\sqrt{21}}}$

$=\dfrac{1}{\sqrt{7+5-2\sqrt{35}}}-\dfrac{1}{\sqrt{5+3-2\sqrt{15}}}-\dfrac{2}{\sqrt{7+3+2\sqrt{21}}}$

$=\dfrac{1}{\sqrt{(\sqrt{7})^2+(\sqrt{5})^2-2\times\sqrt{7}\times\sqrt{5}}}-\dfrac{1}{\sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\times\sqrt{5}\times\sqrt{3}}}-\dfrac{2}{\sqrt{(\sqrt{7})^2+(\sqrt{3})^2+2\times\sqrt{7}\times\sqrt{3}}}$

$=\dfrac{1}{\sqrt{(\sqrt{7}-\sqrt{5})^2}}-\dfrac{1}{\sqrt{(\sqrt{5}-\sqrt{3})^2}}-\dfrac{2}{\sqrt{(\sqrt{7}+\sqrt{3})^2}}$

$=\dfrac{1}{(\sqrt{7}-\sqrt{5})}-\dfrac{1}{(\sqrt{5}-\sqrt{3})}-\dfrac{2}{(\sqrt{7}+\sqrt{3})}$

$=\dfrac{1}{\sqrt{7}-\sqrt{5}}\times\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}-\dfrac{1}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}-\dfrac{2}{\sqrt{7}+\sqrt{3}}\times\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$=\dfrac{\sqrt{7}+\sqrt{5}}{7-5}-\dfrac{\sqrt{5}+\sqrt{3}}{5-3}-\dfrac{2(\sqrt{7}-\sqrt{3})}{7-3}$

$=\dfrac{(\sqrt{7}+\sqrt{5})}{2}-\dfrac{(\sqrt{5}+\sqrt{3})}{2}-\dfrac{(\sqrt{7}-\sqrt{3})}{2}$

$=\dfrac{\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{3}-\sqrt{7}+\sqrt{3}}{2}=0.$

80. $\sqrt{4+\sqrt{15}}=\sqrt{\dfrac{5}{2}+\dfrac{3}{2}+2\times\dfrac{\sqrt{5}}{\sqrt{2}}\times\dfrac{\sqrt{3}}{\sqrt{2}}}$

$=\sqrt{\left(\dfrac{\sqrt{5}}{\sqrt{2}}\right)^2+\left(\dfrac{\sqrt{3}}{\sqrt{2}}\right)^2+2\times\dfrac{\sqrt{5}}{\sqrt{2}}\times\dfrac{\sqrt{3}}{\sqrt{2}}}$

$=\sqrt{\left(\dfrac{\sqrt{5}}{\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}}\right)^2}=\dfrac{\sqrt{5}}{\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}}.$

Similarly, $\sqrt{4-\sqrt{15}}=\dfrac{\sqrt{5}}{\sqrt{2}}-\dfrac{\sqrt{3}}{\sqrt{2}}.$

$\sqrt{12-4\sqrt{5}}=\sqrt{10+2-2\times\sqrt{10}\times\sqrt{2}}$

$=\sqrt{(\sqrt{10})^2+(\sqrt{2})^2-2\times\sqrt{10}\times\sqrt{2}}$

$=\sqrt{(\sqrt{10}-\sqrt{2})^2}=(\sqrt{10}-\sqrt{2}).$

$\therefore$ Given expression

$=\left(\dfrac{\sqrt{5}}{\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}}\right)+\left(\dfrac{\sqrt{5}}{\sqrt{2}}-\dfrac{\sqrt{3}}{2}\right)-(\sqrt{10}-\sqrt{2})$

$=\dfrac{2\sqrt{5}}{\sqrt{2}}-\sqrt{10}+\sqrt{2}=\sqrt{10}-\sqrt{10}+\sqrt{2}=\sqrt{2},$

which is an irrational number.

81. Let $X=\dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}.$

Then $X^2=\dfrac{(\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2})^2}{(\sqrt{\sqrt{5}+1})^2}$

$=\dfrac{(\sqrt{5}+2)+(\sqrt{5}-2)+2\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)}}{(\sqrt{5}+1)}$

$=\dfrac{2\sqrt{5}+2\sqrt{(\sqrt{5})^2-(2)^2}}{\sqrt{5}+1}=\dfrac{2\sqrt{5}+2}{\sqrt{5}+1}$

$=\dfrac{2(\sqrt{5}+1)}{(\sqrt{5}+1)}=2$

$\Rightarrow \quad X=\sqrt{2}.$

$\therefore \quad N=\sqrt{2}-\sqrt{3-2\sqrt{2}}=\sqrt{2}-\sqrt{(\sqrt{2})^2+1^2-2\times\sqrt{2}\times1}$

$=\sqrt{2}-\sqrt{(\sqrt{2}-1)^2}=\sqrt{2}-(\sqrt{2}-1)=1.$

82. $x^z=y^2 \Leftrightarrow (10^{0.48})^z=(10^{0.70})^2$

$\Leftrightarrow 10^{(0.48z)}=10^{(2\times0.70)}=10^{1.40}$

$\Leftrightarrow 0.48z=1.40$

$\Leftrightarrow z=\dfrac{140}{48}=\dfrac{35}{12}=2.9$ (approx.).

83. We know that $11^2=121$.

Putting $m=11$ and $n=2$, we get :

$(m-1)^{n+1}=(11-1)^{(2+1)}=10^3=1000.$

84. $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}} = (3^3 \times 2^3)^{\frac{3}{5}} \times (5^4 \times 2^2)^{\frac{2}{5}} \times (5^2 \times 2^2 \times 3)^{\frac{1}{5}}$

$$= 3^{\left(3\times\frac{3}{5}\right)} \times 2^{\left(3\times\frac{3}{5}\right)} \times 5^{\left(4\times\frac{2}{5}\right)} \times 2^{\left(2\times\frac{2}{5}\right)} \times 5^{\left(2\times\frac{1}{5}\right)} \times 2^{\left(2\times\frac{1}{5}\right)} \times 3^{\frac{1}{5}}$$

$$= 3^{\frac{9}{5}} \times 2^{\frac{9}{5}} \times 5^{\frac{8}{5}} \times 2^{\frac{4}{5}} \times 5^{\frac{2}{5}} \times 2^{\frac{2}{5}} \times 3^{\frac{1}{5}}$$

$$= 3^{\left(\frac{9}{5}+\frac{1}{5}\right)} \times 2^{\left(\frac{9}{5}+\frac{4}{5}+\frac{2}{5}\right)} \times 5^{\left(\frac{8}{5}+\frac{2}{5}\right)} = 3^2 \times 2^3 \times 5^2.$$

Hence, the number of prime factors = (2 + 3 + 2) = 7.

85. $\dfrac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}} = \dfrac{(2\times 3)^{12} \times (5\times 7)^{28} \times (3\times 5)^{16}}{(2\times 7)^{12} \times (3\times 7)^{11}}$

$$= \frac{2^{12} \times 3^{12} \times 5^{28} \times 7^{28} \times 3^{16} \times 5^{16}}{2^{12} \times 7^{12} \times 3^{11} \times 7^{11}}$$

$$= 2^{(12-12)} \times 3^{(12+16-11)} \times 5^{(28+16)} \times 7^{(28-12-11)}$$

$$= 2^0 \times 3^{17} \times 5^{44} \times 7^5 = 3^{17} \times 5^{44} \times 7^5.$$

Number of prime factors = 17 + 44 + 5 = 66.

86. $1 + (3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)$

$$= 1 + \frac{1}{2}[(3 - 1)(3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}[(3^2 - 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}[(3^4 - 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}[(3^8 - 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}[(3^{16} - 1)(3^{16} + 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}[(3^{32} - 1)(3^{32} + 1)]$$

$$= 1 + \frac{1}{2}(3^{64} - 1) = \frac{2 + 3^{64} - 1}{2} = \frac{3^{64} + 1}{2}.$$

87. $\dfrac{1}{1 + a^{(n-m)}} + \dfrac{1}{1 + a^{(m-n)}} = \dfrac{1}{\left(1 + \dfrac{a^n}{a^m}\right)} + \dfrac{1}{\left(1 + \dfrac{a^m}{a^n}\right)}$

$$= \frac{a^m}{(a^m + a^n)} + \frac{a^n}{(a^m + a^n)} = \frac{(a^m + a^n)}{(a^m + a^n)} = 1.$$

88. $(x^a)^{a^2 - bc}\,(x^b)^{b^2 - ca}\cdot(x^c)^{c^2 - ab}$

$$= x^{[a(a^2 - bc)]}\cdot x^{[b(b^2 - ca)]}\cdot x^{[c(c^2 - ab)]}$$

$$= x^{(a^3 - abc)}\cdot x^{(b^3 - abc)}\cdot x^{(c^3 - abc)}$$

$$= x^{(a^3 - abc + b^3 - abc + c^3 - abc)} = x^{(a^3 + b^3 + c^3 - 3abc)}$$

$$= x^{(3abc - 3abc)} = x^0 = 1.$$

[$\therefore$ If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$]

89. Given Exp. $= \dfrac{1}{\left(1 + \dfrac{x^b}{x^a} + \dfrac{x^c}{x^a}\right)} + \dfrac{1}{\left(1 + \dfrac{x^a}{x^b} + \dfrac{x^c}{x^b}\right)} + \dfrac{1}{\left(1 + \dfrac{x^b}{x^c} + \dfrac{x^a}{x^c}\right)}$

$$= \frac{x^a}{(x^a + x^b + x^c)} + \frac{x^b}{(x^a + x^b + x^c)} + \frac{x^c}{(x^a + x^b + x^c)}$$

$$= \frac{(x^a + x^b + x^c)}{(x^a + x^b + x^c)} = 1.$$

90. Given Exp.

$$= x^{(b-c)(b+c-a)}\cdot x^{(c-a)(c+a-b)}\cdot x^{(a-b)(a+b-c)}$$

$$= x^{(b-c)(b+c)-a(b-c)}\cdot x^{(c-a)(c+a)-b(c-a)}\cdot x^{(a-b)(a+b)-c(a-b)}$$

$$= x^{(b^2 - c^2 + c^2 - a^2 + a^2 - b^2)}\cdot x^{-a(b-c)-b(c-a)-c(a-b)}$$

$$= (x^0 \times x^0) = (1 \times 1) = 1.$$

91. Given Exp. $= x^{(a-b)(a+b)}\cdot x^{(b-c)(b+c)}\cdot x^{(c-a)(c+a)}$

$$= x^{(a^2 - b^2)}\cdot x^{(b^2 - c^2)}\cdot x^{(c^2 - a^2)}$$

$$= x^{(a^2 - b^2 + b^2 - c^2 + c^2 - a^2)} = x^0 = 1.$$

92. Given Exp. $= \{x^{(a-b)}\}^{\frac{1}{ab}}\cdot\{x^{(b-c)}\}^{\frac{1}{bc}}\cdot\{x^{(c-a)}\}^{\frac{1}{ca}}$

$$= x^{\frac{(a-b)}{ab}}\cdot x^{\frac{(b-c)}{bc}}\cdot x^{\frac{(c-a)}{ca}}$$

$$= x^{\left\{\frac{(a-b)}{ab} + \frac{(b-c)}{bc} + \frac{(c-a)}{ca}\right\}}$$

$$= x^{\left(\frac{1}{b} - \frac{1}{a}\right) + \left(\frac{1}{c} - \frac{1}{b}\right) + \left(\frac{1}{a} - \frac{1}{c}\right)} = x^0 = 1.$$

93. Given Exp. $= \dfrac{\left(\dfrac{xy+1}{y}\right)^a\cdot\left(\dfrac{xy-1}{y}\right)^b}{\left(\dfrac{xy+1}{x}\right)^a\cdot\left(\dfrac{xy-1}{x}\right)^b}$

$$= \frac{(xy+1)^a\cdot(xy-1)^b\cdot x^a\cdot x^b}{(xy+1)^a\cdot(xy-1)^b\cdot y^a\cdot y^b}$$

$$= \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y}\right)^{a+b}.$$

94. Given Exp. $= x^{\frac{b+c}{(a-b)(c-a)}}\cdot x^{\frac{c+a}{(a-b)(b-c)}}\cdot x^{\frac{a+b}{(b-c)(c-a)}}$

$$= x^{\frac{(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)}{(a-b)(b-c)(c-a)}}$$

$$= x^{\frac{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)}{(a-b)(b-c)(c-a)}} = x^0 = 1.$$

95. Let $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}} = k$. Then, $x = k^p$, $y = k^q$, $z = k^r$.

$\therefore$ $xyz = 1 \Rightarrow k^p\cdot k^q\cdot k^r = 1 = k^0$

$\Rightarrow$ $k^{(p+q+r)} = k^0 \Rightarrow p + q + r = 0.$

96. Let $a^x = b^y = c^z = k$.

Then, $a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}$.

∴ $b^2 = ac \Leftrightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}} \Leftrightarrow k^{\left(\frac{2}{y}\right)} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$

∴ $\frac{2}{y} = \frac{(x+z)}{xz} \Leftrightarrow \frac{y}{2} = \frac{xz}{(x+z)} \Leftrightarrow y = \frac{2xz}{(x+z)}$.

97. $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz} \Rightarrow xyz = 1$.

98. $2^x = 4^y = 8^z \Leftrightarrow 2^x = 2^{2y} = 2^{3z} \Leftrightarrow x = 2y = 3z$.

∴ $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7} \Leftrightarrow \frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7}$

$\Leftrightarrow \frac{3}{6z} = \frac{24}{7} \Leftrightarrow z = \left(\frac{3}{6} \times \frac{7}{24}\right) = \frac{7}{48}$.

99. $8 = 7^d = (6^c)^d = 6^{cd} = (5^b)^{cd} = 5^{bcd} = (4^a)^{bcd} = 4^{abcd}$

$\Rightarrow 4^{abcd} = 8 \Rightarrow (2^2)^{abcd} = 2^3 \Rightarrow 2abcd = 3$

$\Rightarrow abcd = \frac{3}{2}$.

100. Given Exp. $= \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \frac{a}{a+ac+1}$

$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{1+b^{-1}+a} + \frac{a}{a+b^{-1}+1}$

$= \frac{1+a+b^{-1}}{1+a+b^{-1}} = 1.$

[∵ $abc = 1 \Rightarrow (bc)^{-1} = a \Rightarrow b^{-1}c^{-1} = a$ and $ac = b^{-1}$]

101. $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = (a^{-1})^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \cdot (b^{-1})^{\frac{1}{2}} \cdot c^{\frac{1}{2}} \cdot (c^{-1})^{\frac{1}{2}} \cdot a^{\frac{1}{2}}$

$= (a^{-1}a)^{\frac{1}{2}} \cdot (b \cdot b^{-1})^{\frac{1}{2}} \cdot (c \cdot c^{-1})^{\frac{1}{2}}$

$= (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} = (1 \times 1 \times 1) = 1.$

102. $3^{x-y} = 27 = 3^3$

$\Leftrightarrow x - y = 3$ ... (*i*)

$3^{x+y} = 243 = 3^5$

$\Leftrightarrow x + y = 5$ ... (*ii*)

On solving (*i*) and (*ii*), we get $x = 4$.

103. Let $x^y = y^x = k$.

Then, $x = k^{\frac{1}{y}}$ and $y = k^{\frac{1}{x}}$.

∴ $\left(\frac{x}{y}\right)^{\frac{x}{y}} = \left(\frac{k^{\frac{1}{y}}}{k^{\frac{1}{x}}}\right)^{\frac{x}{y}} = k^{\left[\left(\frac{1}{y}-\frac{1}{x}\right)\frac{x}{y}\right]} = k^{\left(\frac{x-y}{xy}\right)\frac{x}{y}} = k^{\left(\frac{x-y}{y^2}\right)}$

$= (x^y)^{\left(\frac{x-y}{y^2}\right)} = x^{\left(\frac{x-y}{y}\right)} = x^{\left(\frac{x}{y}-1\right)}$.

104. $4^{x+y} = 1 = 4^0 \Leftrightarrow x + y = 0$ ... (*i*)

$4^{x-y} = 4 = 4^1 \Rightarrow x - y = 1$ ... (*ii*)

Adding (*i*) and (*ii*), we get : $2x = 1$ or $x = \frac{1}{2}$.

Putting $x = \frac{1}{2}$ in (*i*), we get : $y = -\frac{1}{2}$.

105. $2^{2x-1} + 4^x = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}} \Leftrightarrow 2^{2x-1} + 2^{2x} = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}}$

$\Leftrightarrow 2^{(2x-1)}(1+2) = 2^{\left(x-\frac{1}{2}\right)}(1+2)$

$\Leftrightarrow 2^{(2x-1)} = 2^{\left(x-\frac{1}{2}\right)} \Leftrightarrow 2x - 1 = x - \frac{1}{2} \Leftrightarrow x = \frac{1}{2}$.

106. $3^{2x-y} = 3^{x+y} = \sqrt{3^3} = 3^{\frac{3}{2}} \Leftrightarrow 2x - y = \frac{3}{2}$ and $x + y = \frac{3}{2}$

$\Leftrightarrow 3x = \frac{3}{2} + \frac{3}{2} = 3 \Leftrightarrow x = 1$.

∴ $y = \left(\frac{3}{2} - 1\right) = \frac{1}{2}$.

107. Let $3^x = 5^y = 45^z = k$. Then, $3 = k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 45 = k^{\frac{1}{z}}$.

$45 = 3^2 \times 5$

$\Leftrightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^2 \cdot \left(k^{\frac{1}{y}}\right) = k^{\frac{2}{x}} \cdot k^{\frac{1}{y}} = k^{\left(\frac{2}{x}+\frac{1}{y}\right)}$

$\Leftrightarrow \frac{1}{z} = \frac{2}{x} + \frac{1}{y} \Leftrightarrow \frac{2}{x} = \frac{1}{z} - \frac{1}{y}$.

108. $2^x = 8^{y+1} \Leftrightarrow 2^x = (2^3)^{y+1} = 2^{(3y+3)}$

$\Leftrightarrow x = 3y + 3 \Leftrightarrow x - 3y = 3$ ... (*i*)

$9^y = 3^{x-9} \Leftrightarrow (3^2)^y = 3^{x-9}$

$\Leftrightarrow 2y = x - 9 \Leftrightarrow x - 2y = 9$ ... (*ii*)

Subtracting (*i*) from (*ii*), we get: $y = 6$. Putting $y = 6$ (*i*), we get $x = 21$.

∴ $x + y = 21 + 6 = 27$.

109. $2^{0.7x} \cdot 3^{-1.25y} = \frac{8\sqrt{6}}{27}$

$\Leftrightarrow \frac{2^{0.7x}}{3^{1.25y}} = \frac{2^3 \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{3^3} = \frac{2^{\left(3+\frac{1}{2}\right)}}{3^{\left(3-\frac{1}{2}\right)}} = \frac{2^{\frac{7}{2}}}{3^{\frac{5}{2}}} = \frac{2^{3.5}}{3^{2.5}}$.

∴ $0.7x = 3.5 \Rightarrow x = \frac{3.5}{0.7} = 5$ and $1.25y = 2.5$

$\Rightarrow y = \frac{2.5}{1.25} = 2$.

110. $r = (2a)^{2b} = 2^{2b} \times a^{2b} = (2^2)^b \times (a^b)^2 = 4^b \times (a^b)^2$.

Also, $r = a^b \times x^b$.

∴ $a^b \times x^b = 4^b \times (a^b)^2 \Leftrightarrow x^b = 4^b \times a^b = (4a)^b \Leftrightarrow x = 4a$.

111. L.C.M. of 2, 3, 4, 6 is 12.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}$.

$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 3^{\frac{4}{12}} = (3^4)^{\frac{1}{12}} = (81)^{\frac{1}{12}} = \sqrt[12]{81}$.

$\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 4^{\frac{3}{12}} = (4^3)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}.$

$\sqrt[6]{6} = 6^{\frac{1}{6}} = 6^{\left(\frac{1}{6}\times\frac{2}{2}\right)} = 6^{\frac{2}{12}} = (6^2)^{\frac{1}{12}} = (36)^{\frac{1}{12}} = \sqrt[12]{36}.$

Clearly, $\sqrt[12]{81}$ *i.e.*, $\sqrt[3]{3}$ is the greatest.

112. L.C.M of 2, 3, 4, 6 is 12.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}.$

$\sqrt[6]{3} = 3^{\frac{1}{6}} = 3^{\left(\frac{1}{6}\times\frac{2}{2}\right)} = 3^{\frac{2}{12}} = (3^2)^{\frac{1}{12}} = (9)^{\frac{1}{12}} = \sqrt[12]{9}.$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 4^{\frac{4}{12}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}} = \sqrt[12]{256}.$

$\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 5^{\frac{3}{12}} = (5^3)^{\frac{1}{12}} = (125)^{\frac{1}{12}} = \sqrt[12]{125}.$

Clearly, $\sqrt[12]{256}$ *i.e.*, $\sqrt[3]{4}$ is the greatest.

113. ....

114. $x = 5+2\sqrt{6} = 3+2+2\sqrt{6} = (\sqrt{3})^2+(\sqrt{2})^2+2\times\sqrt{3}\times\sqrt{2}$

$= (\sqrt{3}+\sqrt{2})^2.$

Also, $(x-1) = 4+2\sqrt{6} = 2\,(2+\sqrt{6}) = 2\sqrt{2}\,(\sqrt{2}+\sqrt{3}).$

$\therefore \quad \frac{(x-1)}{\sqrt{x}} = \frac{2\sqrt{2}\,(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = 2\sqrt{2}.$

115. $x^{\frac{1}{3}}+y^{\frac{1}{3}} = z^{\frac{1}{3}} \Rightarrow \left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right)^3 = \left(z^{\frac{1}{3}}\right)^3$

$\Rightarrow \quad x+y+3x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right) = z$

$\Rightarrow \quad x+y+3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} = z$

$\Rightarrow \quad x+y-z = -3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

$\Rightarrow \quad (x+y-z)^3 = \left(-3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}\right)^3$

$\Rightarrow \quad (x+y-z)^3 = -27xyz \Rightarrow (x+y-z)^3+27xyz = 0.$

116. $x = 2+2^{\frac{2}{3}}+2^{\frac{1}{3}} \Rightarrow (x-2) = 2^{\frac{2}{3}}+2^{\frac{1}{3}}$

$\Rightarrow \quad (x-2)^3 = \left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\right)^3$

$= 2^2+2+3.2^{\frac{2}{3}}\cdot2^{\frac{1}{3}}\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\right)$

$\Rightarrow \quad (x-2)^3 = 6+6\,(x-2) = 6+6x-12$

$\Rightarrow \quad (x-2)^3 = 6x-6 \Rightarrow x^3-8-6x\,(x-2) = 6x-6$

$\Rightarrow \quad x^3-8-6x^2+12x = 6x-6 \Rightarrow x^3-6x^2+6x = 2.$

117. Given expression $(-2)^5\times(2)^{-5}\times(3)^3$

$\frac{(-2)^5}{2^5}\times(3)^3 \qquad \left\{\because a^{-m} = \frac{1}{a^m}\right\}$

$\frac{(-1)^5(2)^5\times(3)^3}{(2^5)} = (-3)^3 = -27$

118. Expression $= \frac{(10)^{100}}{(5)^{75}}$

$= \frac{(2\times5)^{100}}{(5)^{75}} = \frac{(2)^{100}\times(5)^{100}}{(5)^{75}} = 2^{100}\times\frac{5^{100}}{5^{75}} = 2^{100}\times5^{(100-75)}$

$\left\{\because \frac{a^m}{a^n} = a^{m-n}\right\}$

$= 2^{100}\times5^{25}$

$= 2^{25}\times5^{25}\times2^{75} \qquad \{\because a^m\times a^n = a^{m+n}\}$

$= (10)^{25}\times2^{75} \qquad \{\because a^m\times b^m = ab^m\}$

119. Expression $= \sqrt{\sqrt{2}\times\sqrt{3}}$

$= \left(\sqrt{2}\times\sqrt{3}\right)^{\frac{1}{2}} = \left(2^{\frac{1}{2}}\times3^{\frac{1}{2}}\right)^{\frac{1}{2}} \qquad \{\because a^m\times b^m = ab^m\}$

$= (6)^{\frac{1}{2}\times\frac{1}{2}} = (6)^{\frac{1}{4}} \qquad \left\{\because \left(a^m\right)^n = a^{mn}\right\}$

120. $21^?\times21^{6.5} = 21^{12.4} \qquad \{\because a^m\times a^n = a^{m+n}\}$

$\Rightarrow \quad 21^{?+6.5} = 21^{12.4}$

$\Rightarrow$? + 6.5 = 12.4

$\Rightarrow$? = 12.4 − 6.5 = 5.9

121. $\frac{5.4\div3\times16\div2}{18\div5\times6\div3}$

$= \frac{\frac{5.4}{3}\times\frac{16}{2}}{\frac{18}{5}\times\frac{6}{3}} = \frac{1.8\times8}{3.6\times2} = 2$

122. $\left(32\times10^{-5}\right)^2\times64\div\left(2^{16}\times10^{-4}\right) = 10^?$

$\Rightarrow \quad \left(2^5\times10^{-5}\right)^2\times2^6\div\left(2^{16}\times10^{-4}\right) = 10^? \qquad \left\{\because \left(a^m\right)^n = a^{mn}\right\}$

$\Rightarrow \quad \frac{2^{10}\times10^{-10}\times2^6}{2^{16}\times10^{-4}} = 10^? \qquad \{\because a^m\times a^n = a^{m+n}\}$

$\Rightarrow \quad \frac{2^{16}\times10^4}{2^{16}\times10^{10}} = 10^? \qquad \left\{\because a^{-m} = \frac{1}{a^m}\right\}$

$\Rightarrow \quad 10^{4-10} = 10^?$

$\Rightarrow \quad 10^{-6} = 10^?$

$\Rightarrow$? = − 6