Unit – 2 Random Variable Probability Distribution

1. A Discrete Random Variable X has the following probability mass function.

X	-2	-1	0	1	2	3
P(x=X)	0.1	k	0.2	2k	0.3	3k

Find (i) k, (ii) $p(X \geq 2)$, (iii) $p(-2 < X < 2)$, (iv) $p(X \geq -1)$.

2. A Discrete Random Variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P(x=X)	0	2a	3a	7a	8a	16a	6a	9a

Find (i) a, (ii) $p(X < 3)$, (iii) $p(2 \leq X < 7)$.

3. A Discrete Random Variable X has the following probability distribution.

X	0	1	2
P(x=X)	K	2k	3k

Find (i) k, (ii) $p(X < 2)$, (iii) $p(X \leq 2)$, (iv) $p(0 < X < 2)$, (v) Cumulative Distribution Function

4. Let X be Random Variable and $p(x = X)$ is given Probability Mass Function given by

X	0	1	2	3	4	5	6	7
P(x=X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find (i) k, (ii) $p(X < 2)$, (iii) $p(X \leq 6)$, (iv) $p(3 < X \leq 6)$, (v) Cumulative Distribution Function

5. Show that the function f(x) defined by

$$\begin{aligned} f(x) &= \frac{1}{7} \quad if\ 1 < x < 8 \\ &= 0 \quad otherwise \end{aligned}$$

is a probability density function for a Random Variable. Hence, find $p(3 < X < 10)$.

6. Find the constant k such that the function

$$\begin{aligned} f(x) &= kx^2 \quad if\ 0 < x < 3 \\ &= 0 \quad otherwise \end{aligned}$$

Is a probability density function and compute (i) $P(1 < X < 2)$, (ii) $P(X < 2)$, and (iii) $P(X \geq 2)$.

7. If X is a continuous Random Variable with Probability Density Function

$$\begin{aligned} f(x) &= x^2 \quad if\ 0 \leq x \leq 1 \\ &= 0 \quad otherwise \end{aligned}$$

If $P(a \leq X \leq 1) = \frac{19}{81}$, find the value of a.

8. Let X be a continuous Random Variable with Probability Density Function

$$f(x) = kx(1-x) \quad if\ 0 \le x \le 1$$
$$= 0 \qquad otherwise$$

Find k and determine a number b such that P(X ≤ b) = P(X ≥ b).

9. Find the Raw moment and Central moment for the given probability distribution.

X	-2	-1	0	1	2
P(x=X)	0.2	0.1	0.3	0.3	0.1

10. Find the Raw moment and Central moment for the given probability distribution

$$f(x) = 3x^2 \quad if\ 0 < x < 1$$
$$= 0 \qquad otherwise$$

14. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find *P(X ≥ 1).*

15. The mean and variance of a binomial distribution are 3 and 1.2 respectively. Find *n, p*, and *P(X < 4)*.

16. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 such men now at 60 at least 7 will live up to 70?

17. If 10% of the screws produced by a machine are defective, find the probability that out of 5 screws chosen at random, *(i)* none is defective, *(ii)* one is defective, and *(iii)* at most two are defective.

18. A multiple-choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced die and checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4, and the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative making, what is the probability that the student secures a distinction?

19. The probability of a man hitting a target is 1/3. *(i)* If he fires 5 times, what is the probability of his hitting the target at least twice? *(ii)* How many times must he fire so that the probability of his hitting the target at least once is more than 90%?

20. If the variance of a Poisson variate is 3, find the probability that $(i) X = 0\ (ii)\ 0 < X \le 3\ and\ (iii)\ 1 \le X < 4.$

21. If a Poisson distribution is such that $\frac{3}{2}P(X = 1) = P(X = 3)$,
Find $(i) P(X \ge 1)\ \ (ii)\ P(X \le 3)\ and\ (iii)\ P(2 \le X \le 5).$

22.If a Poisson distribution is such that, $3P(X = 4) = \frac{1}{2}P(X = 2) + P(X = 0)$,
Find $(i) the\ mean\ of\ X\ (ii)\ P(X \le 2).$

23. A manufacturer of cotter pins knows that 5% of his products are defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?

24. Six coins are tossed 6400 times. Using the Poisson distribution, what is the approximate probability of getting six heads 10 times?

25. If 2% of lightbulbs are defective, find the probability that *(i)* at least one is defective, and *(ii)* exactly 7 are defective. Also, find $P(1 < X < 8)$ in a sample of 100.

26.X is normally distributed and the mean of *X* is 12 and the SD is 4. Find out the probability of the following:

$(i) X \geq 20 \quad (ii)\ X \leq 20\ and\ (iii)\ 0 \leq X \leq 12.$

27.If *X* is normally distributed with a mean of 2 and an SD of 0.1, find $P(|X - 2| \geq 0.01)$?

28.The lifetime of a certain kind of batteries has a mean life of 400 hours and the standard deviation as 45 hours. Assuming the distribution of lifetime to be normal, find *(i)* the percentage of batteries with a lifetime of at least 470 hours, *(ii)* the proportion of batteries with a lifetime between 385 and 415 hours, and *(iii)* the minimum life of the best 5% of batteries.

29. If the weights of 300 students are normally distributed with a mean of 68 kg and a standard deviation of 3 kg, how many students have weights *(i)* greater than 72 kg? *(ii)* less than or equal to 64 kg? *(iii)* between 65 kg and 71 kg inclusive?

30. The mean yield for a one-acre plot is 662 kg with an SD of 32 kg. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to have yields *(i)* over 700 kg? *(ii)* below 650 kg? *(iii)* What is the lowest yield of the best 100 plots?

31. The average time it takes to serve a customer at a petrol pump is 6 minutes. The service time follows exponential distribution. Calculate the probability that
    i. A customer will take less than 2 minutes to complete the service.
    ii. A customer will take between 4 and 5 minutes to get the service.
    iii. A customer will take more than 10 minutes for his service.

32. The length of time *X* to complete a job is exponentially distributed with $E(X) = 10\ hours$.
    i. Compute the probability of job completion between two consecutive jobs exceeding 20 hours.
    ii. The cost of job completion is given by $C = 4 + 2X + 2X^2$. Find the expected value of C.