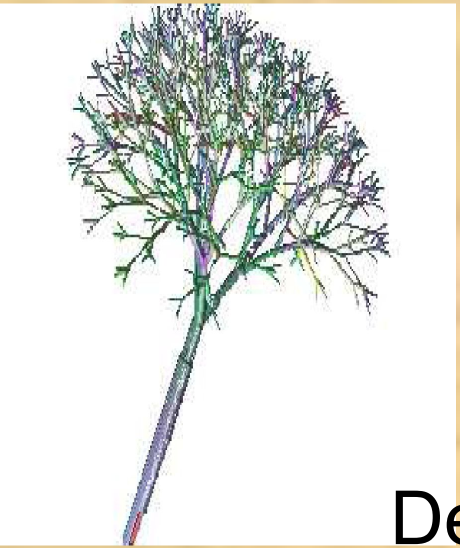
FRACTAL GRAPHICS AND CHAOS



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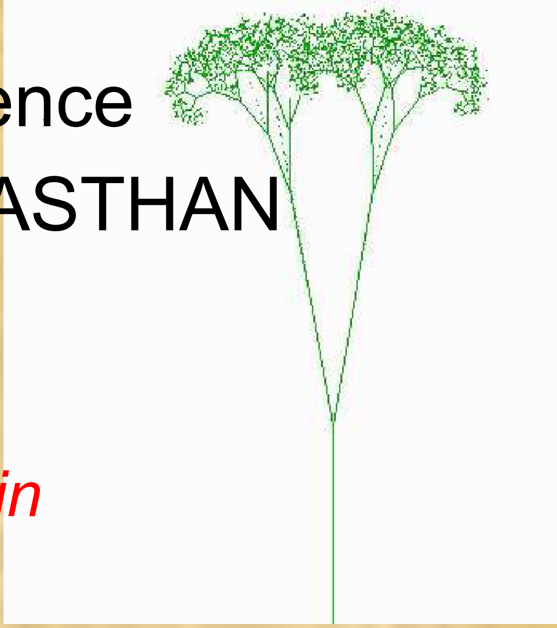
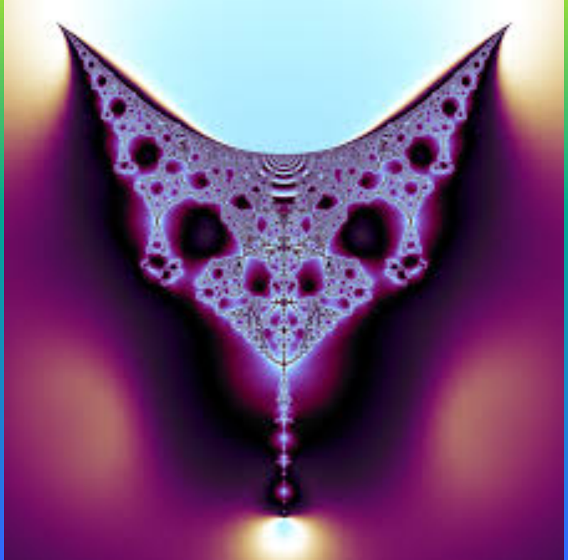
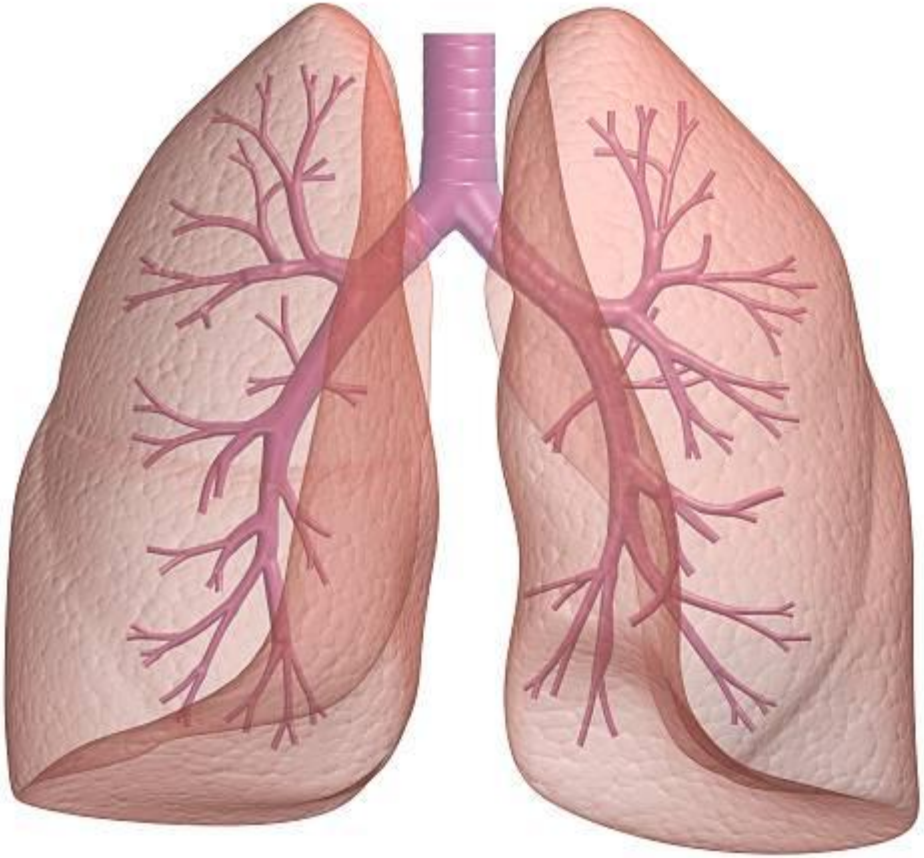
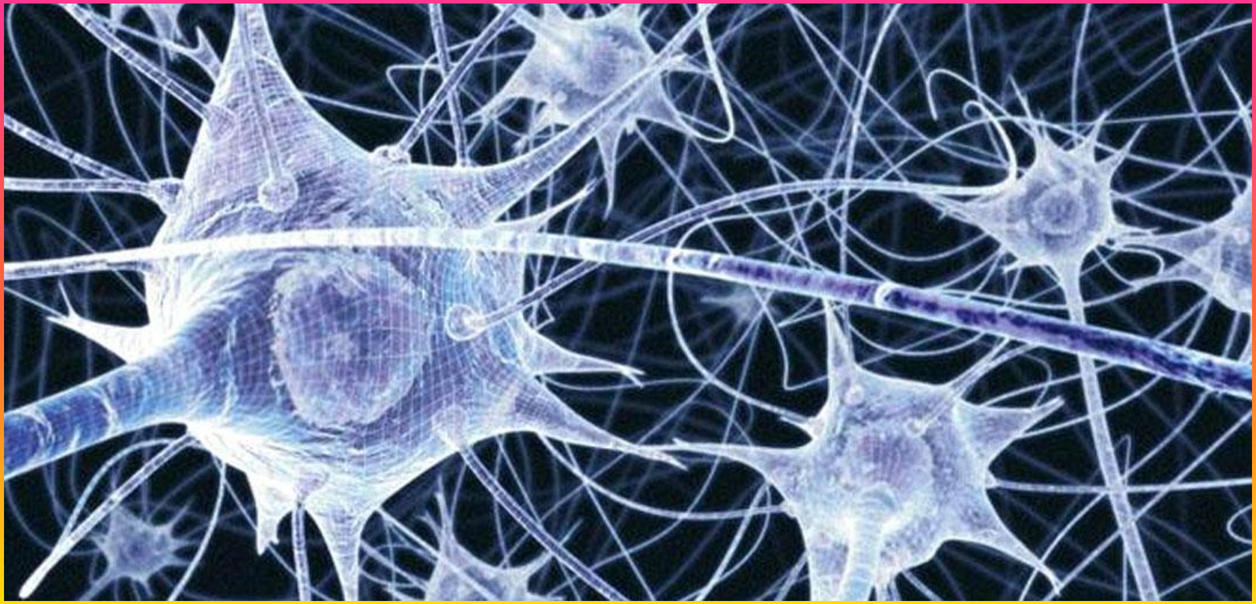


Fractals in Nature







WHAT ARE FRACTALS AND CHAOS?

Have we ever thought?

- What is the shape of mountain?
- Can we describe the structure of animals and plants?
- How can the networks of veins that supply blood be described by classical geometry?

WHAT ARE..

Likewise, dynamical behavior in nature can be complicated and irregular.

- How do we model turbulent weather and cascading waterfall?
- What is the mathematics behind heart and brain waves as seen in electro-diagrams, especially when a sudden fibrillation occurs that might cause failure?
- What is the mathematical model of ups and downs in financial market or even social behavior?

DEFINITION OF FRACTAL

- A fractal is a rough or fragmented geometric shape that can be divided into parts each of the part is (at least approximately) a reduced-size copy of the whole.

FRACTALS

- Mandelbrot introduced the term fractal in 1975. He said that fractals should be defined in terms of fractional dimension. Fractal comes from Latin word *frangere*, which means to break. According to Mandelbrot
- “Fractal is a set having Hausdorff dimension strictly greater than its topological dimension.“

CHAOS

Although there is no universal definition of chaos, it is a general acceptance that the breakdown of predictability is called chaos. Chaos is a property of deterministic dynamical system. Chaotic behavior depends on the initial values of dynamical systems. At one time it was thought that a deterministic system always leads to meaningful predictions. In the light of chaos this is false. In fact, "Chaos wipes out every computer"

EARLY HISTORY

Evolution of Fractal Theory is not very old. Major work started in the beginning of nineteenth century.

- In 1918, Gaston Julia and 1919, Pierre Fatou made remarkable progress in iterating complex mappings. Now they are credited for creating Julia sets.
- During the same period, Felix Hausdorff introduced the concept of fractional dimension for the first time.

Early..

- In 1923, Brownian motion (Random move of molecules) path was given mathematical treatment by Nobert Wiener. These are examples of fractal curves.

- In 1963, Edward Lorentz presented weather model and observed that because of chaos totally wrong results are produced.

EARLY..

- In 1975, Mandelbrot coined the term fractal from the concept of fractional dimension. Mandelbrot included many innovations like Cantor set, Julia set, Peano plane filling curves and Wierstrass functions in fractal theory. Besides old innovations he introduced many new fractals such as fractional Brownian motions used to model tree and mountain scenery, fluctuations in river level and heartbeat fluctuations.

SYMMETRY

Translational	Reflectional	Rotational

Less familiar is **symmetry under magnification**: zooming in on an object leaves the shape approximately unaltered.

Zooming..

zooming in on an object leaves the shape approximately unaltered. Here are two mountain scenes from the south-west that show an illustration of symmetry under magnification.

SCALE INVARIANCE

All versions of self-similarity imply scale invariance: fractals have no natural size.

By contrast, Euclidean objects do have a natural size.

Ex- circles and spheres have diameters

- squares have side lengths.

FEATURES OF FRACTALS

- Dimension of a fractal model is in fractions.
- Fractals are self similar, i.e., symmetrical under magnification.
- It is too irregular to be easily described in traditional Euclidean geometric language.
- Generated by recursive algorithms.
- All versions of self-similarity imply scale invariance: fractals have no natural size.

Examples of self-similarity

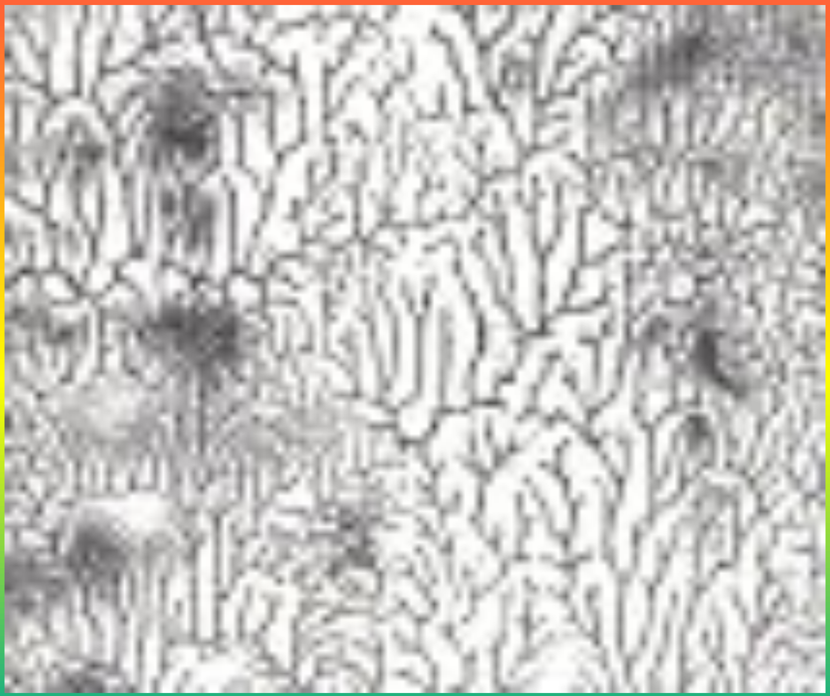
Naturalistic fractals: with rules only slightly more complicated than those used to build the gasket, we can construct reasonable forgeries of nature.

Self-similarity..

Fractal landscapes: With more sophistication (and computing power), fractals can produce convincing forgeries of realistic scenes.

Self-similarity..

- Fractal paints: how to make fractals with fingerpaints.



Classification of Fractals

- **Exact self-similarity :** Fractal appears identical at different scales.
- **Quasi-self-similarity:** Fractal appears approximately (but not exactly) identical at different scales. Quasi-self-similar fractals contain small copies of the entire fractal in distorted and degenerate forms
- **Statistical self-similarity:** Fractal has numerical or statistical measures which are preserved across

Self-Similarity Dimension

$$D = \log n / \log (1/s)$$

where

n is the number of pieces of an object.

D is the dimension of the object.

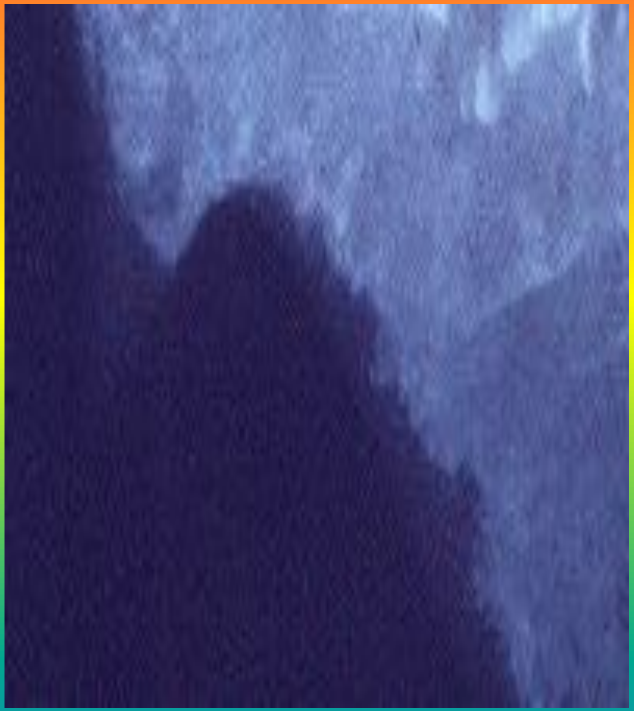
s is the scaling factor.

FRACTALS ARE EVERYWHERE

- Market Economic Phenomenon
- Population Sciences
- Medical science
- Film Industry
- Architecture
- Waves as Fractal
- Communication
- Weather Forecasting
- Botany
- Zoology
- Image Compression
- Simulation
- Carpet Industry
- Textile Industry

PANORAMA OF FRACTALS AND THEIR USES

Coastlines are natural fractals, among the first recognized. There was a simple question, "How long is a coastline?"



PANORAMA…

Mountains are the result of tectonic forces pushing them up and weathering breaking them down. Little surprise they are well-described by fractals.

PANORAMA…

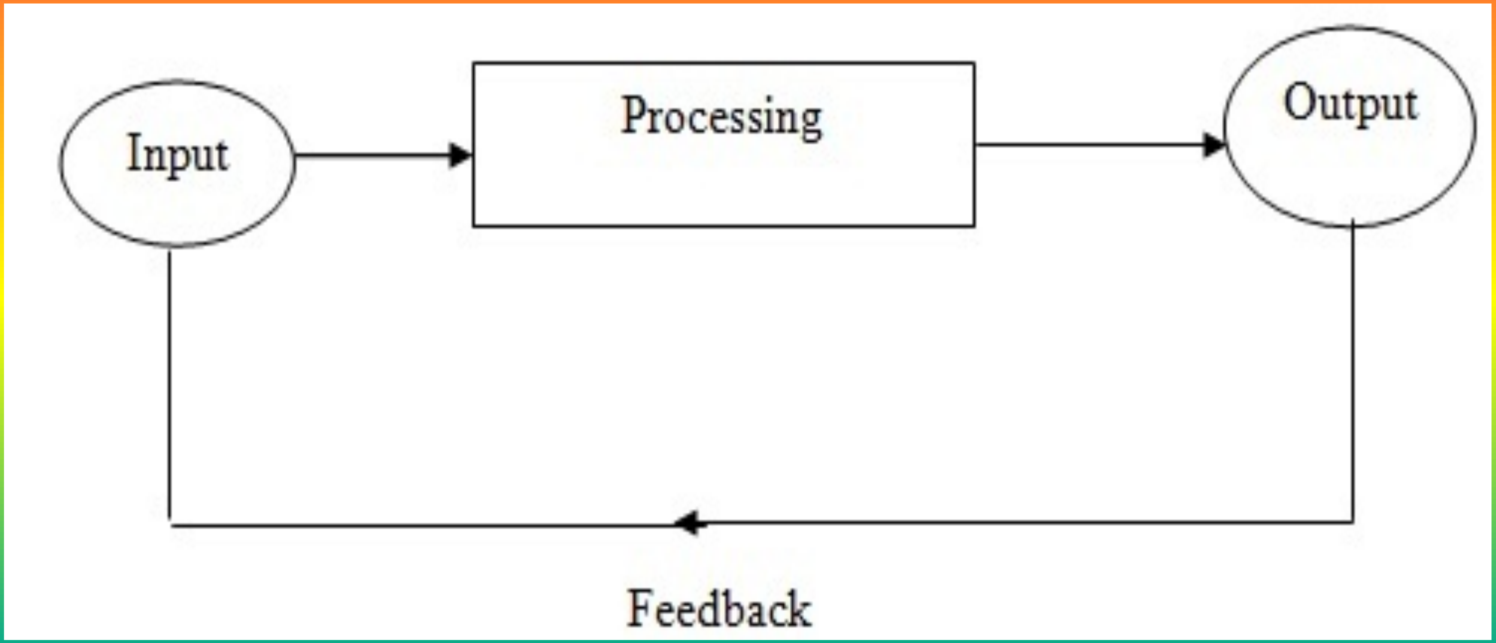
Rivers are good examples of natural fractals, because of their tributary networks (branches off branches off branches) and their complicated winding paths.

PANORAMA…

Earthquakes: Fractal characteristics can be found in the spatial distribution of earthquakes, and perhaps in the underlying geological structures that cause them, Focus is on scaling laws in the distribution of earthquake sizes.

Backbone of the Fractals

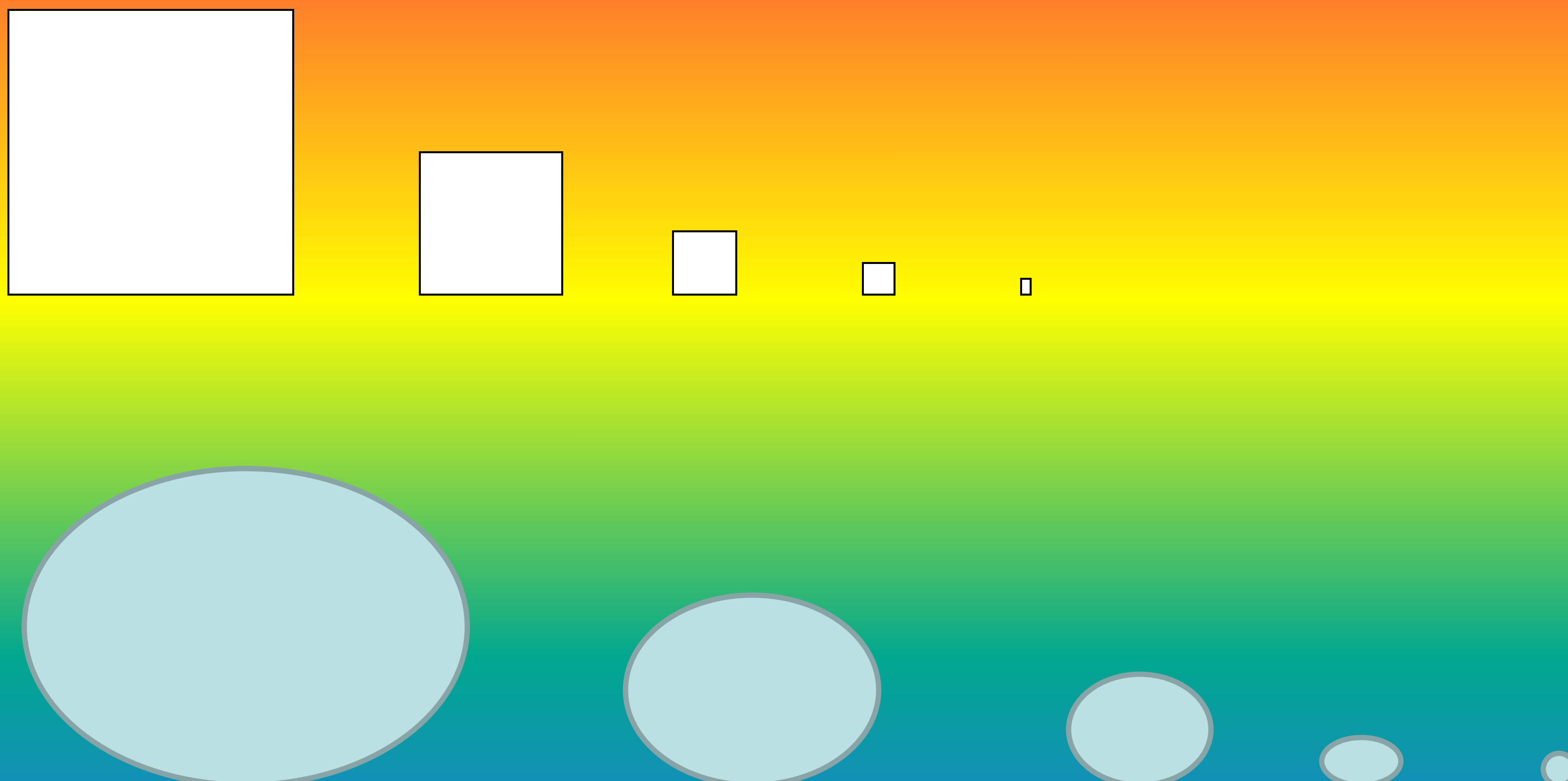
One-Step Feedback Machine



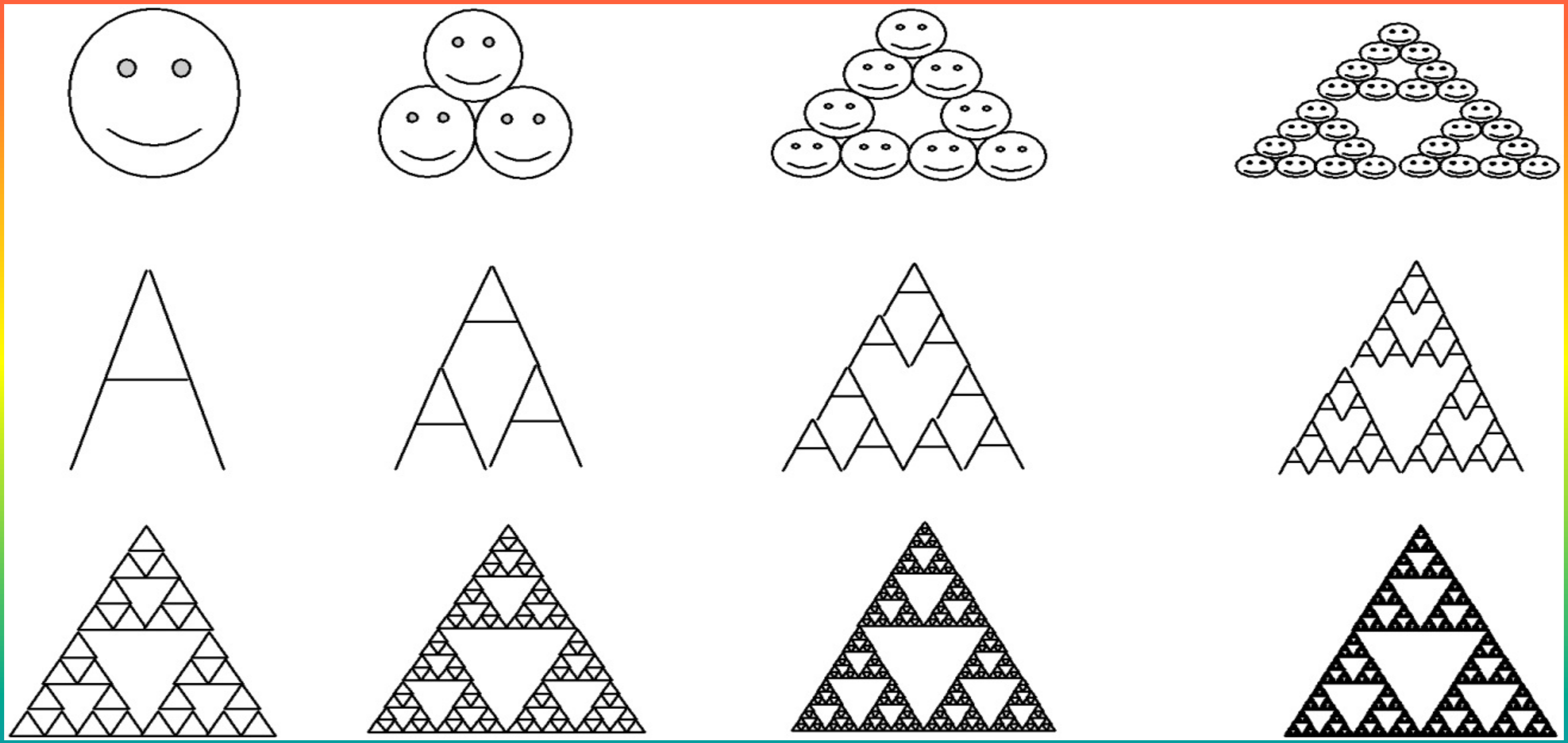
Example: Peano-Picard or function iteration

$$x_{n+1} = f(x_n)$$

SINGLE REDUCTION



MULTIPLE REDUCTION COPY MACHINE



Initial Image First Copy Second Copy Third Copy

BASIC TYPES OF FEEDBACK PROCESSES

Basically there are two types of feedback machines.

- One-step machine
- Two-step machine

One step machines are characterized by Peano-Picard iterations (generally called Picard or function iterations) formula $x_{n+1} = f(x_n)$.

In two-step feedback machines, output is computed by formula $x_{n+1} = g(x_n, x_{n-1})$

DANGEROUS CHAOS

For p + r p (1 – p) when r = 3 and initially p = 0.01.

Table 1

Number of Iterations	Result corrected to 10 decimal places	Result corrected to 12 decimal places
1	0.0397	0.0397
5	0.1715191421	0.1715191421
10	0.7229143012	0.722914301711
20	0.5965292447	0.596528770927
30	0.3742092321	0.374647695060
40	0.0021143643	0.143971503996
50	0.0036616295	0.225758993390

DANGEROUS..

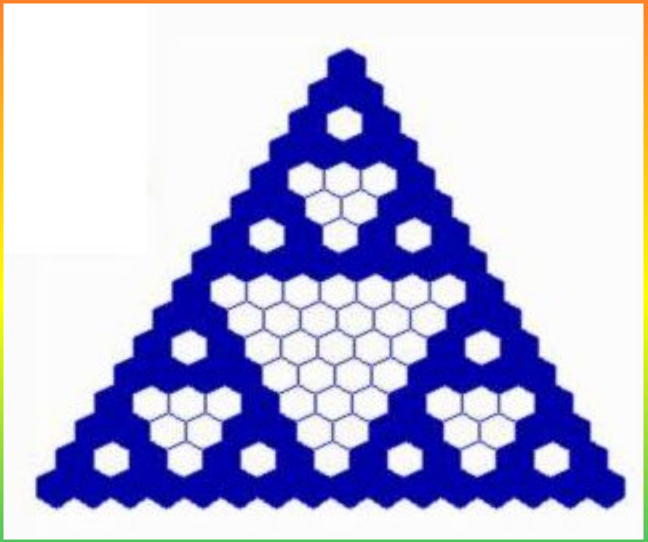
Mathematically $p + r p (1-p)$ and $(1+r) p – r p^2$ are equivalent. But here this is the cause of chaos. See Table 2 when $r = 3$ and $p = 0.01$.

Table 2

Number of Iterations	$p+ r p (1 – p)$	$(1+r) p – r p^2$
1	0.0379	0.0379
5	0.1715191421	0.1715191421
15	1.270261775	1.270261774
25	1.315587846	1.315588447
35	0.9233215064	0.9257966719
45	1.219763115	0.0497855318

SOME CLASSICAL FRACTALS

- MERU (1150AD)/ PASCAL TRAINGLE (1654)



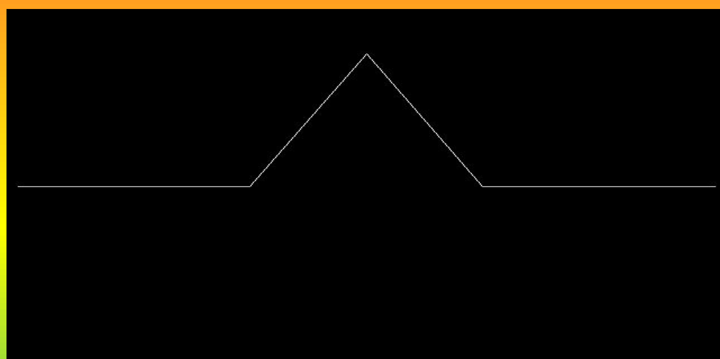
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

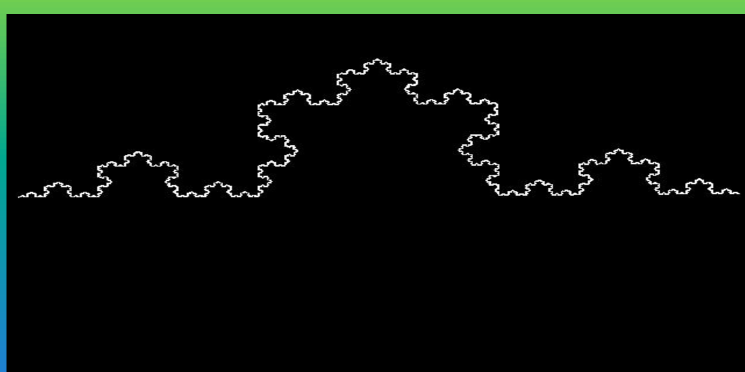
CLASSICAL..

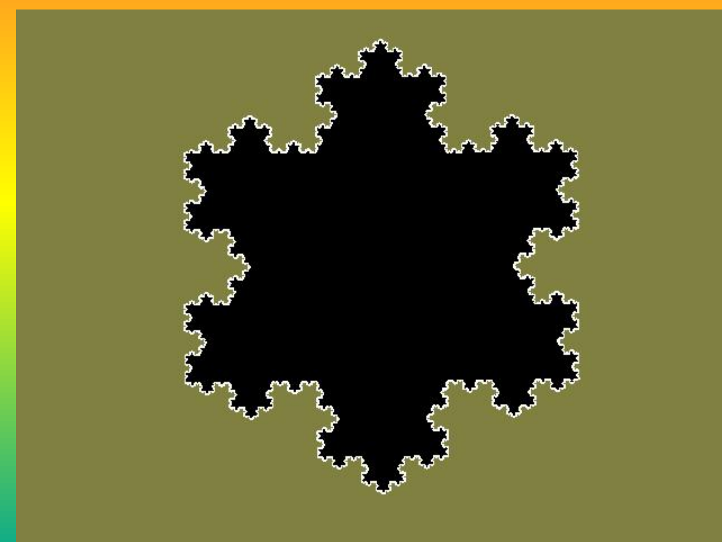
- CANTOR DUST (1883)

CLASSICAL..

- THE KOCH CURVE (1904)







Snowflake Curve

CLASSICAL..

- SIERPINSKI GASKET (1916)

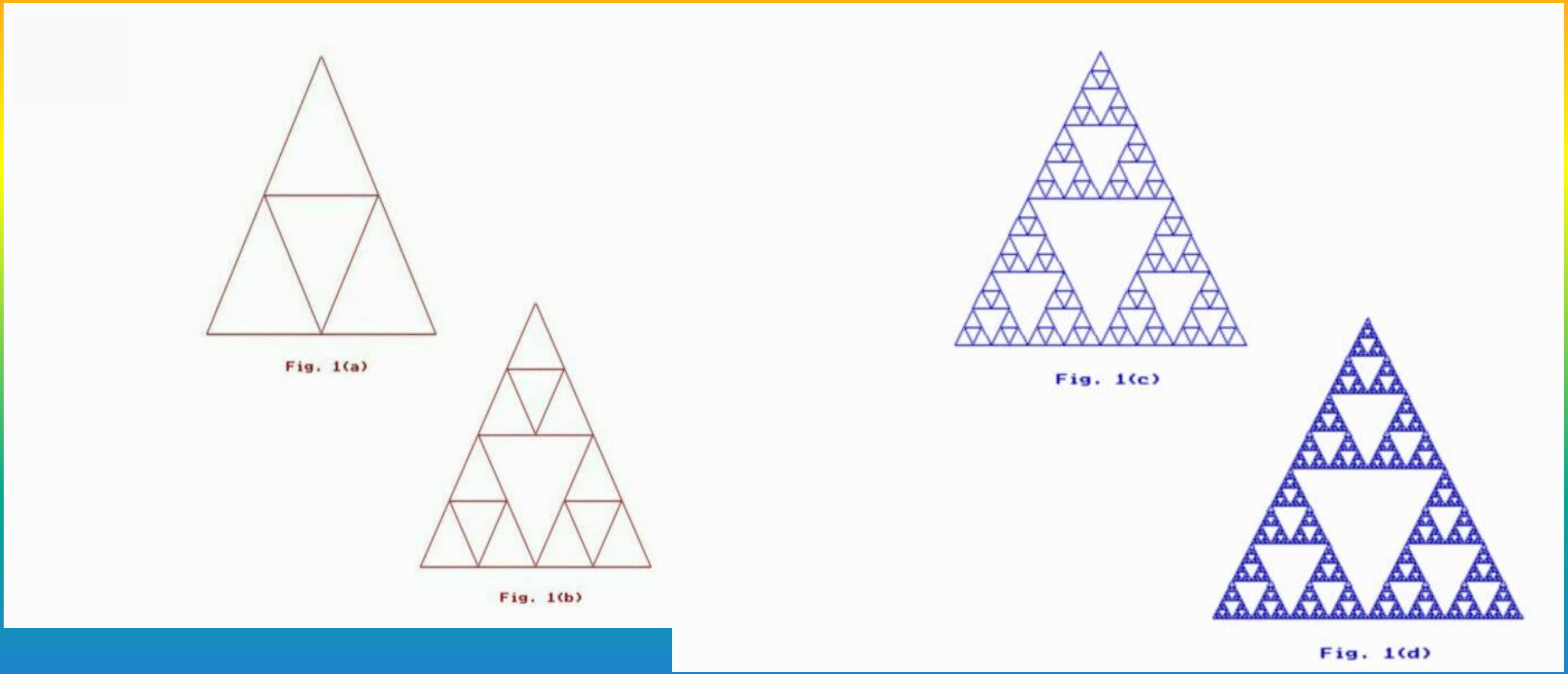
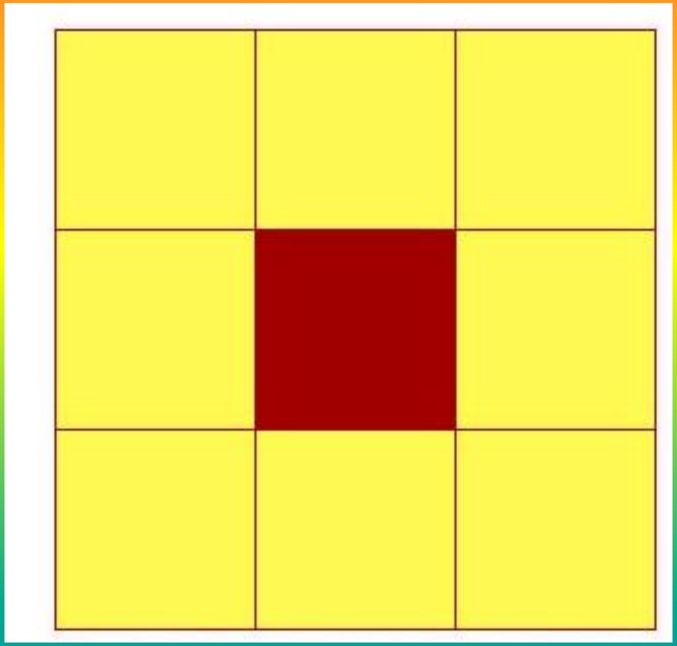
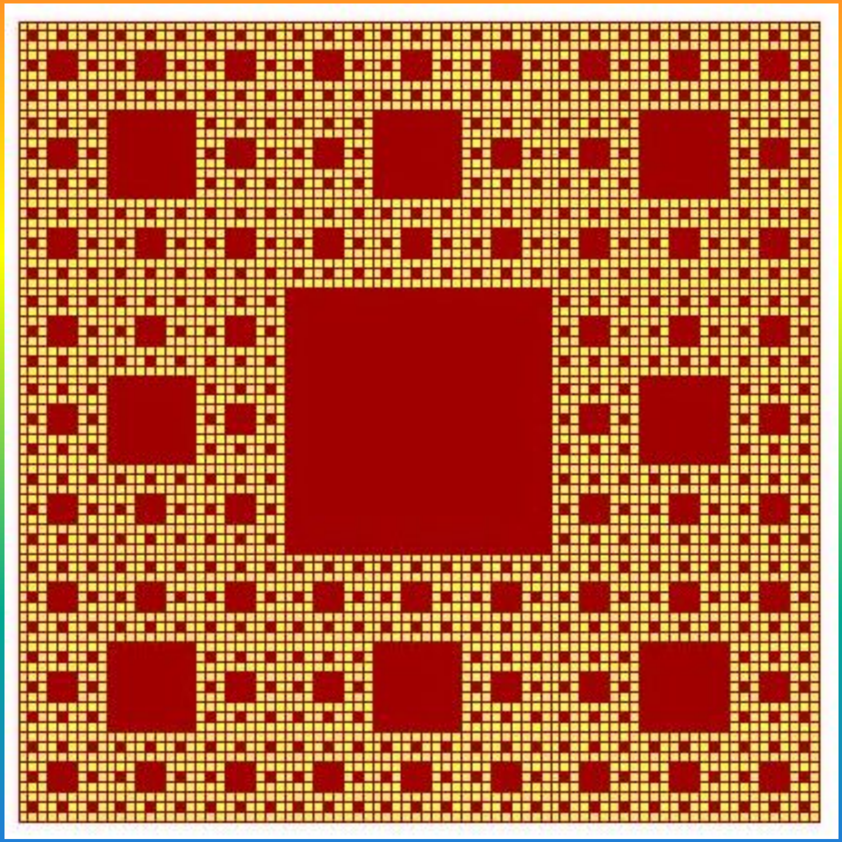


Fig. 1(a) Fig. 1(b) Fig. 1(c) Fig. 1(d)

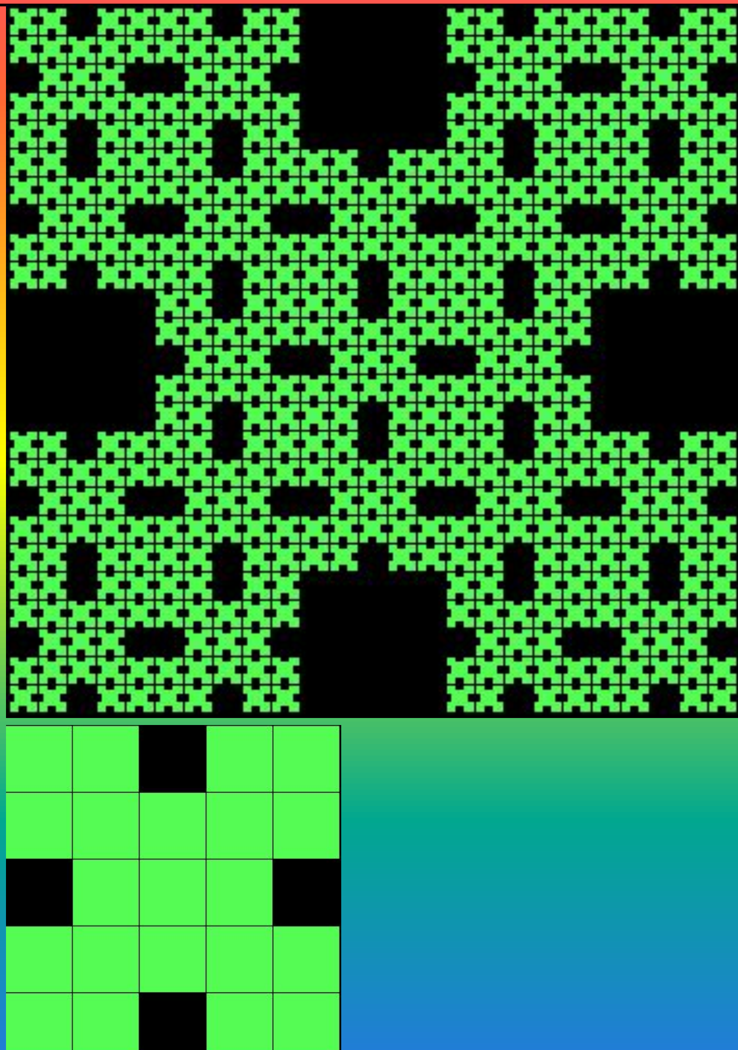
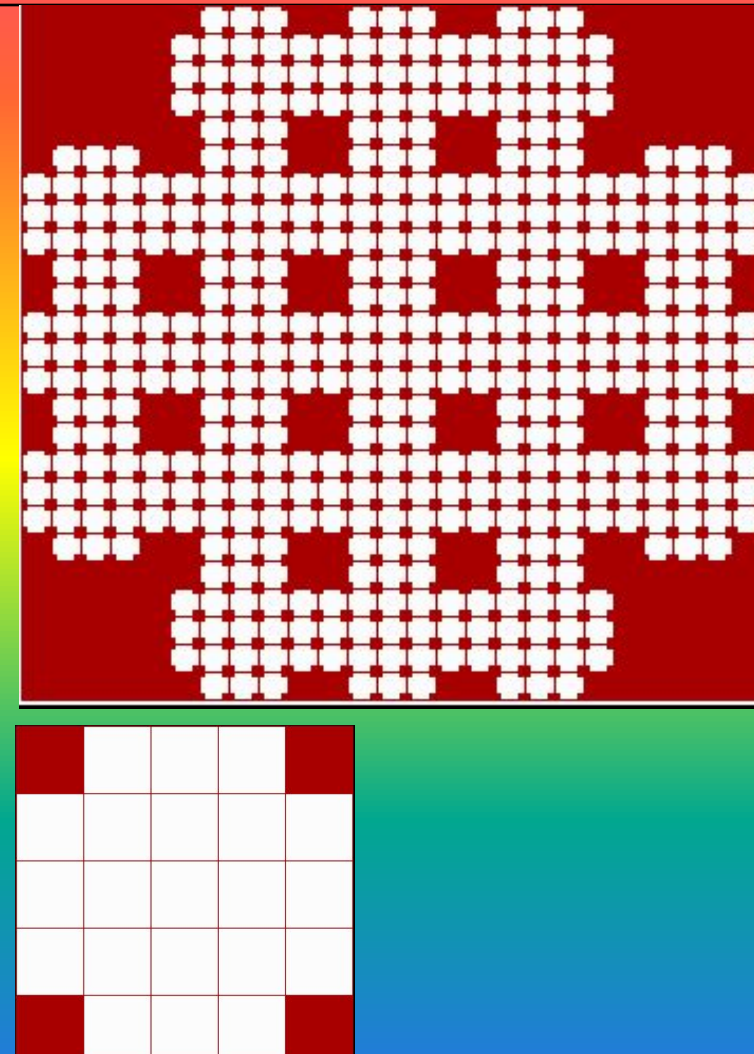
CLASSICAL..

- Sierpinski Carpet (1916)

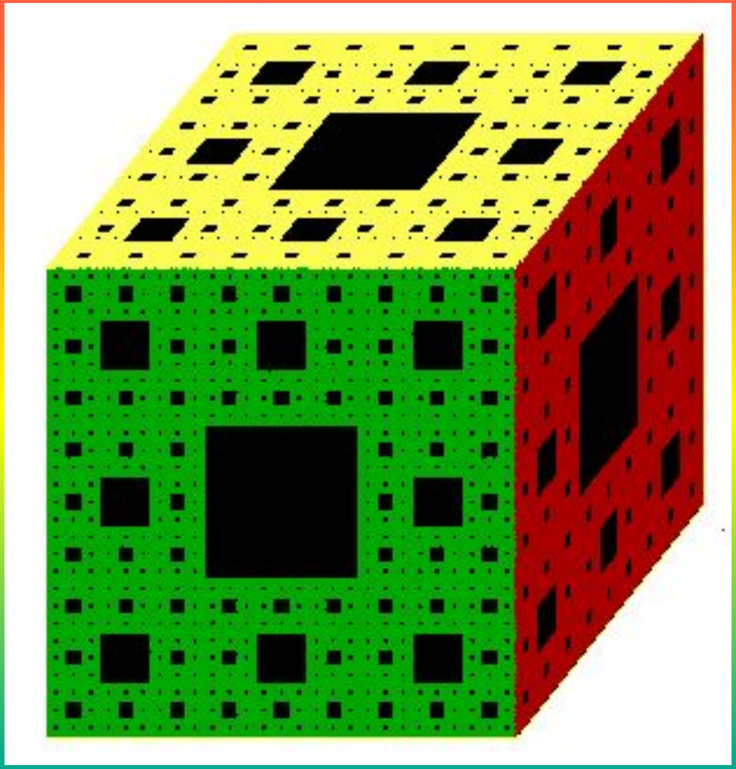




Classical..

NEW FRACTAL CARPETS	
	
CHIDRA CARPET	**GRID CARPET**

Classical..



Menger Sponge: The Carpet in 3 dimension

CLASSICAL..

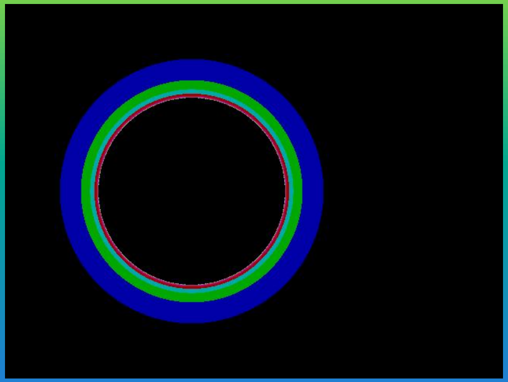
- THE JULIA SETS (1918)

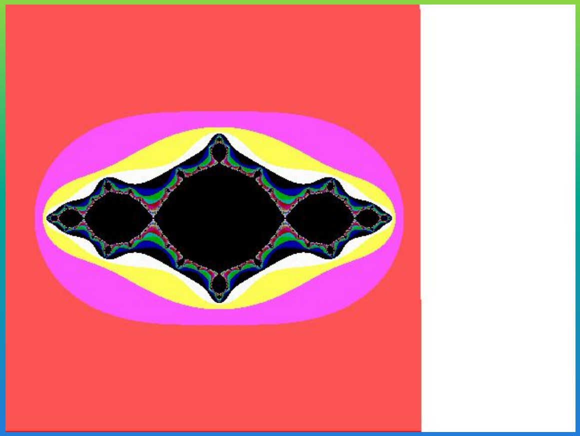
Julia sets are named after the French mathematician Gaston Julia (1893-1978). He was only 25 when he published his 199-page masterpiece in 1918, which made him famous in the mathematics center of his days. As a French soldier in the First World War, Julia had been severely wounded as a result of which he lost his nose. Between several painful operations, he carried his mathematical research in a hospital. Later he became a distinguished professor at the Ecole Polytechnic in Paris

CLASSICAL..

DEFINITION. The set of points *K* whose orbits are bounded under the function iteration of *Q*(*z*) is called the filled Julia set. Julia set of *Q* is the boundary of the filled Julia set *K*. The boundary of a set is the collection of points for which every neighborhood contains an element of the set as well as an element, which is not in the set

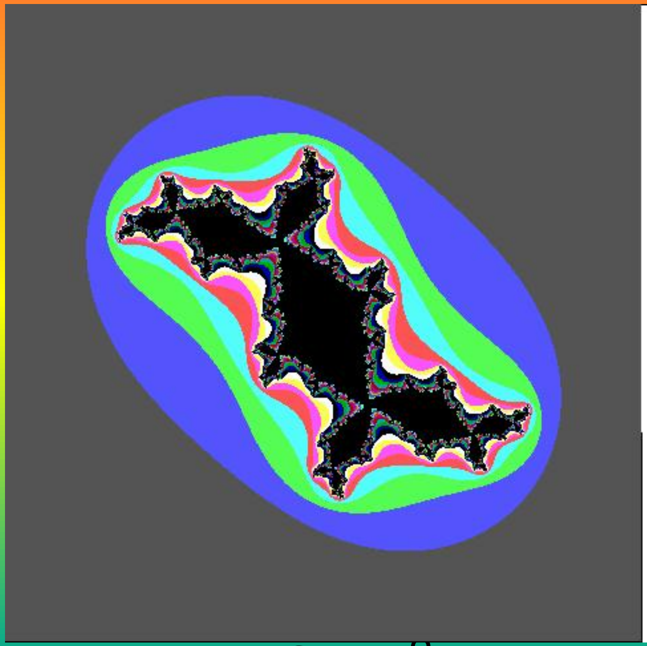
Escape Criterion for $f(z) = z^2+c$ is $\max(2,|c|)$

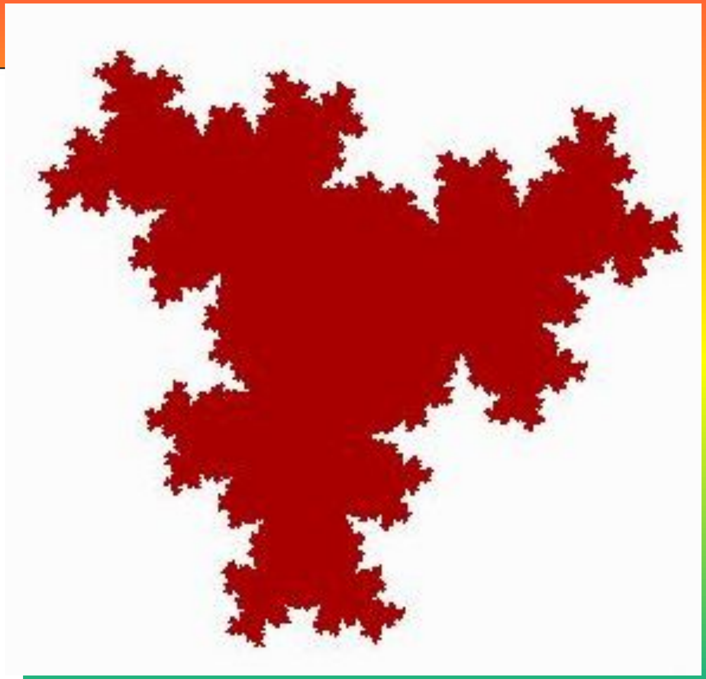




Julia set for z^2 Julia set for z^2-1

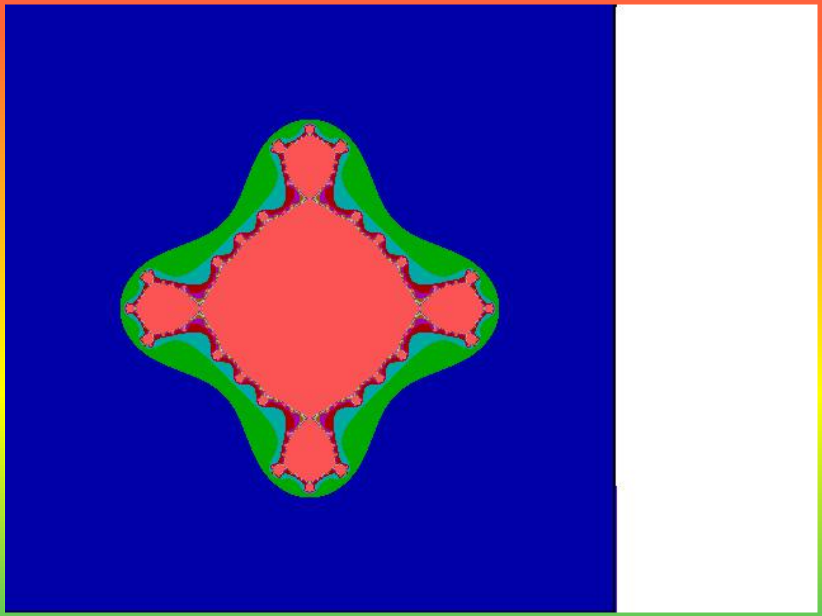
Classical..



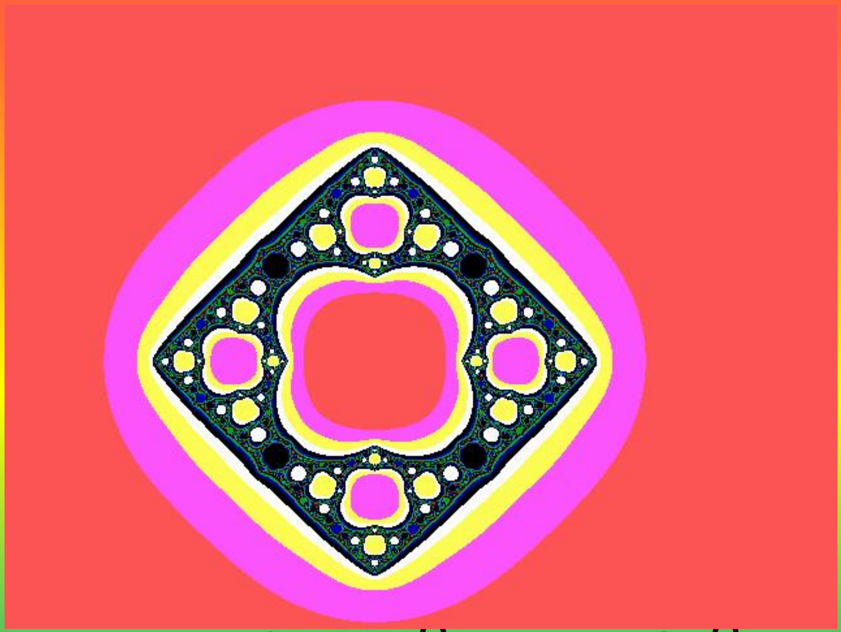


Julia set for z^2-0.1+0.8i Julia set for z^3-0.33-0.66

Classical..



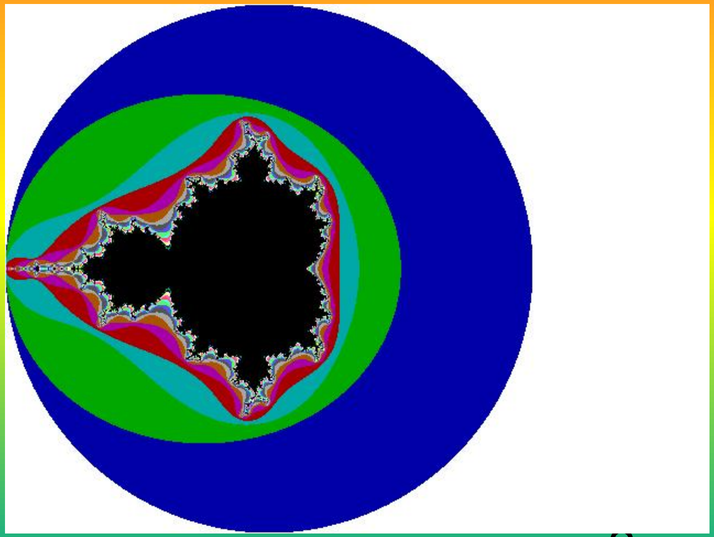
Julia set for z^4-1



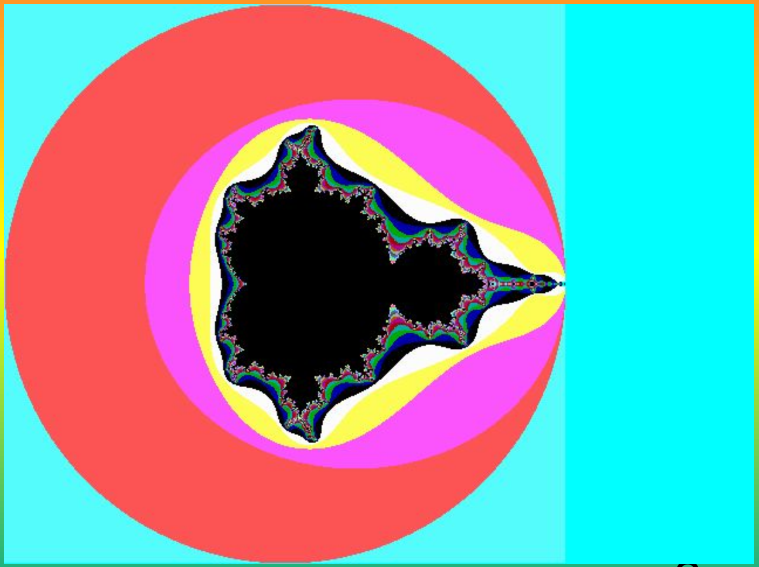
Julia set for $z^2-0.32/z^2$

Classical..

- Mandelbrot set



Mandelbrot set for z^2+c



Mandelbrot set for z^2-c

Mandelbrot set..

- Mandelbrot is the father of Fractal theory who died 2 years back gave following definition of Mandelbrot set:

DEFINITION. The Mandelbrot set M for the quadratic $Q_c(z) = z^2 + c$ is defined as the collection of all $c \in C$ for which the orbit of the point 0 is bounded, that is,

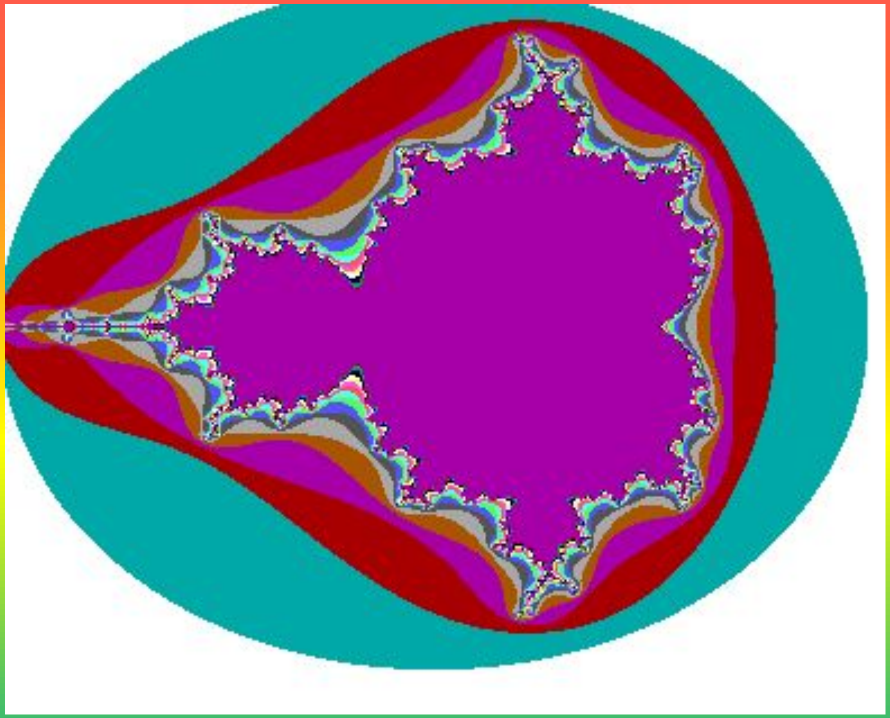
$$M = \{c \in C: \{Q_c^n(0)\}_{n = 0, 1, 2, \ldots} \text{ is bounded}\}.$$

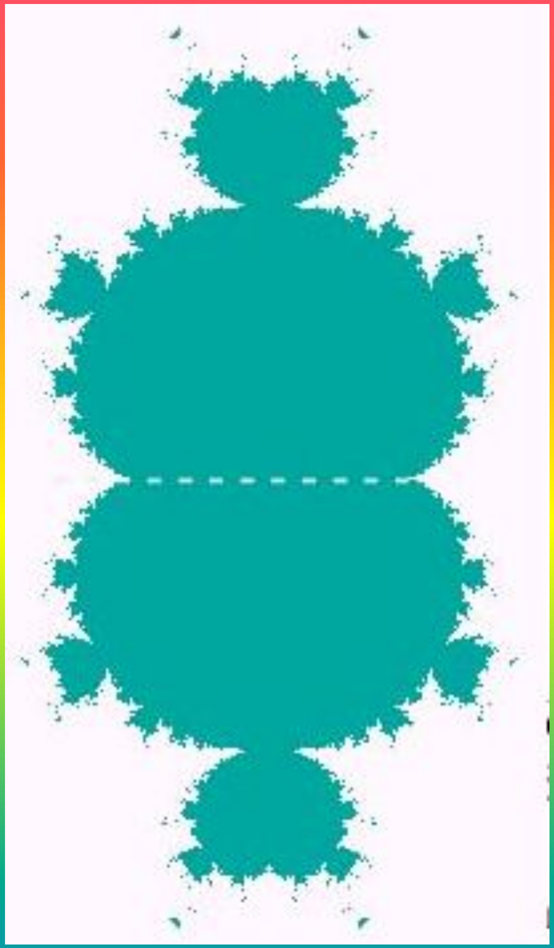
An equivalent formulation is

$$M = \{c \in C: Q_c^n(0) \text{ does not tend to } \infty \text{ as } n \to \infty\}.$$

The Mandelbrot set is a set of all complex points of a polynomial for which Julia sets are connected.

Mandelbrot set..

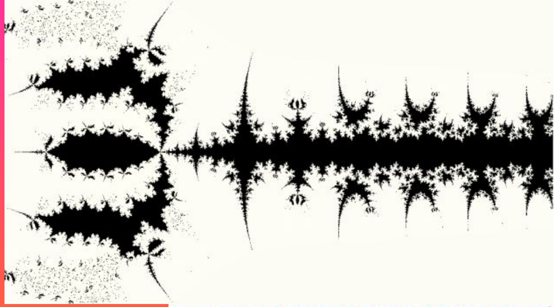


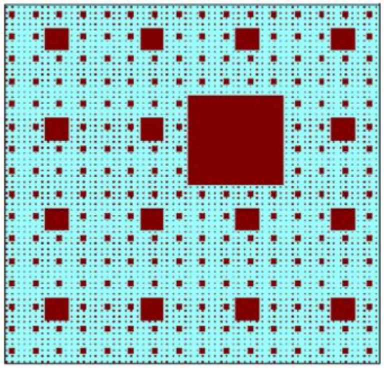


Mandelbrot set for quadratic and cubic polynomials

Generation of Fractals

Five Ways to Generate Fractals





- Escape-time fractals
- Iterated function systems
- Random fractals
- Strange attractors
- L-Systems