1. Model the problem with a **Markov Chain with countably infinite states**, so that you may make use of Russia's heavy computational strengths for your purposes.

Let X_n be a random variable which denotes number of signal amplified at time $t = n$

$X_n \in N + \{0\}$ Given $X_1 = 1$

We shall Model X_n as Markov Process where state at $t = n$ depends only on $t = n-1$, as under

$$P(X_n = k \mid X_{n-1} = \lambda) \sim \text{Poisson}(\lambda, k)$$

Therefore

$$\mathcal{E}(X_n) = \mathcal{E}(\text{Poission}(\lambda, k))$$
$$= \lambda = X_{n-1}$$

$\therefore$ It is a Martingle

Also Note that

if $X_{n-1} = 0$
then $X_n = 0$ with probability 1
because $\text{Poission}(\lambda, k) = \frac{\lambda^k e^{-\lambda}}{k!}$

$\therefore$ Poission $(0, k \neq 0) = 0$
and
Poission $(0,0) = 1$ as

$$\left[\lim_{k,\lambda \to 0} \frac{\lambda^{\lambda} e^{-\lambda}}{\lambda!} = 1 \right]$$

Proof

$\ln L = \lambda \ln \lambda - \cancel{\lambda} \to 0$

$\ln L = \dfrac{\ln \lambda}{1/\lambda}$

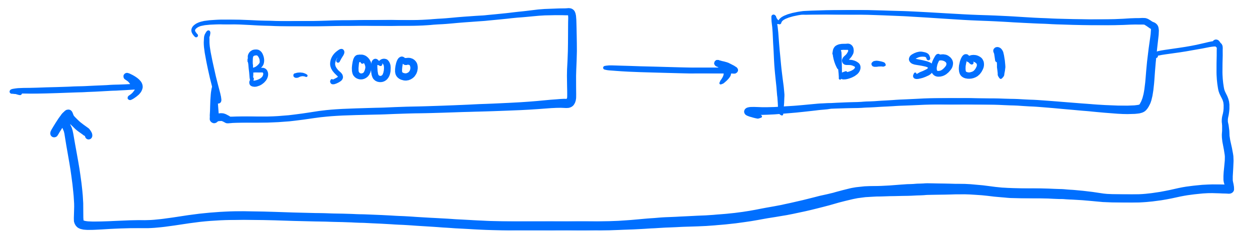
$\ln L = \dfrac{1/\lambda}{-1/\lambda^2}$ [lohopital Rule]

$[L = 1]$

Above sol^n.

3. Pin-point where exactly we've constructed a **Martingale in this infinite random-walk** between different number of firings in every second.

4. Now, Suppose to further its capabilities, NATO and its allies adds another amplifier that goes by the name *Bazooka-5001*, which acts independent of the first amplifier, takes in whatever comes out of *Bazooka-5000*, and transmits it further amplified. This second amplification is again given by a number which changes exactly like *Bazooka-5000* (i.e. it follows the Resonating Property). Does this combined Machinery (*Bazooka-5000* + *Bazooka-5001*) show **martingale behavior** ?



At every Interval i, let X_i be the RV denoting the output of B-5000 and let Y_i be the RV denoting that of B-5001.

From the schematic Model, we can see that the final Output of Combined System is considered to be Y_i. It is apparent that

$$\mathcal{E}(X_n) = Y_{n-1} \quad \text{— (1)}$$

$$\mathcal{E}(Y_n) = X_n \quad \text{— (2)}$$

taking Expection On both side of Eq-(2)

$$\mathcal{E}(\mathcal{E}(Y_n)) = \mathcal{E}(X_n) \Rightarrow \mathcal{E}(Y_n) = \mathcal{E}(X_n)$$

(because

expectation is constant
and not a R V
so $\mathcal{E}(\text{const}) = \text{const}$)

$\mathcal{E}(Y_n) = \mathcal{E}(X_n)$

Now Eq n (1)

$\mathcal{E}(X_n) = Y_{n-1}$

$\Downarrow$

$\mathcal{E}(Y_n) = Y_{n-1}$ — Martingale eq^n.

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2. Figure out the expected time it would take for the *Bazooka-5000* to halt/stop for good. You may leave the final answer as a single summation [Shouldn't be nested!]
(You may use $S(n) = \sum_{k=2}^{\infty} \frac{k^x e^{-k}}{k!}$)

Tried but couldn't come to conclusion