Poission Process

1. Arrival Epoch
2. Interarrival time
3. Counting variable

~~Arrival~~ Arrival Process

$0 < S_1 < S_2 \dots$ where $S_{i-1} < S_i$

$X_i = S_i - S_{i-1} \quad i \geq 2$

inter arrival Time

S_i arrival epochs

$\{S_n \leq t\} = \{N(t) \geq n\}$ for all $n \geq 1$, $t > 0$

If $S_n = \tau$ for some $\tau \leq t$, then $N(\tau) = n$ and $N(t) \geq n$

Poission Process

- Memoryless

$\Pr\{X > t + x\} = \Pr\{X > t\} \Pr\{X > x\}$

$\Pr\{X > t + x \mid X > t\} = \Pr\{X > x\}$

Heavy tailed Distribution

- Generally Queues at Airport.

$S_{N(t)}$ - last arrival up to and including t

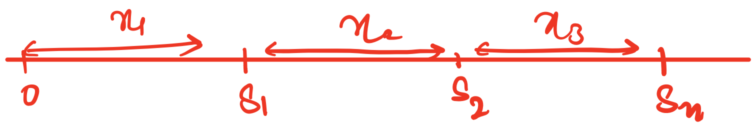
$N(t') - N(t)$ for $t' > t$

$N(t'-t) = N(t') - N(t)$

Staitonary & Independent Increment.

$f_{S_n} = \lambda^n \; t^{n-1} \exp(-\lambda t)$

Erlang Distribution $\nearrow$ $\frac{\quad\cdots\quad}{(n-1)!}$



$$P_{N(t)}(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Poisson Processes

- Sum of 2 poisson is Poisson.

Poisson Process

$$P(N(t, t+\delta)) = \begin{cases} 1 - \lambda\delta + o(\delta) \to 0 & n = 0 \\ \lambda\delta + o(\delta) \to 0 & n = 1 \\ o(\delta) \to 0 & n \geq 2 \end{cases}$$

$$\frac{o(\delta)}{\delta} \to 0$$

Combining Independent Poisson Process. $(N_1(t) + N_2(t))$

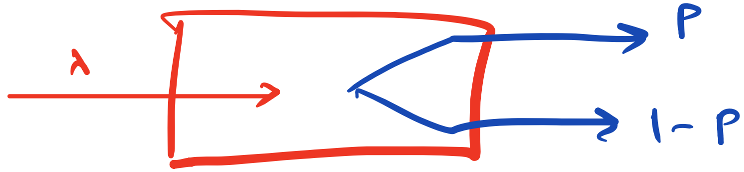
$\lambda = \lambda_1 + \lambda_2$

Convolution

$$\Big[P_{\bar{N}}(t, t+\delta)(1) = P_{\bar{N}_1}(t, t+\delta)(1)\, P_{\bar{N}_2}(t, t+\delta)(0) + P_{\bar{N}_1}(t, t+\delta)(0) + P_{\bar{N}_2}(t, t+\delta) \Big]$$

$= \delta(\lambda_1+\lambda_2)$

Spliting a Poission Process



rate $\lambda_1 = \lambda p$

rate $\lambda_2 = \lambda(1-p)$

Last Come First Service

arrival rate λ
service time μ

Probability Intrupt $= \dfrac{\lambda}{\lambda+\mu}$

Conditional Arrival Densities

$$f_{S(N)\,|\,N(t)}(s^{(n)}\,|\,n) = \frac{n!}{t^n}$$

Due to Ordering

uniform n dimensial property

ParaDox

Mean Interarrival time : $1/\lambda$