Bayes Theoreum.

$P(A_i)$ intial belief

$P(B|A_i) =$

$P(A_i|B) =$ revised belief

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{P(B)}$$

defined.

new belief

old belief

$\Rightarrow$ $Var(X+Y) = Var(X) + Var(Y)$
when X, Y are independent.

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

$y = ax + b$

$$Cov(X,Y) = E((X - E(X))(Y - E(Y)))$$

$$\Longrightarrow Var\left(\sum_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} Var(x_i) + \sum_{i \neq j} Cov(x_i, x_j)$$

Independence $\Rightarrow$ Covariance 0

not the other way around

Correlation : $\dfrac{Cov(X,Y)}{\sigma_x \sigma_y}$

Law of Total Variance

$$\Rightarrow Var(X) = \underbrace{E[Var(X|Y)]}_{\text{average variability between range}} + \underbrace{Var(E(X|Y))}_{\text{Variability between Section}}$$

$Y = X_1 + X_2 + X_3 - \cdots X_n$

$E(Y) = E(N)E(X)$

$\Rightarrow$ $Var(Y) = E[Var(Y|N)] + Var(E[Y|N])$

$Var(Y) = E(N)Var(X) + E(X)^2 var(X)$

- If you buy a lottery ticket every day, what is the distribution of the length of the first string of losing days?

$\Rightarrow$

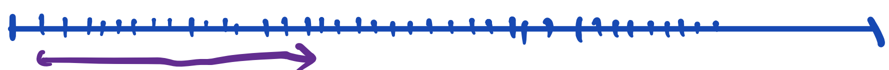
this time if I start watching, so previous there must be a foresight.

first string of losing days

$L =$ Geometric after first failure.

$L+1 =$ Geometric. → would be false as

Poisson Eqn. → can be visualised as



$p = \lambda\delta$

$\lambda\tau = np$

$n = \frac{t}{\delta}$

$\Rightarrow$ $P(K, T) = \frac{(\lambda t)^K e^{-\lambda T}}{K!}$

number of time event happened

Var $= \lambda t$.
Mean $= \lambda t$

Used no Good / Bad than 'New.

Exponential Variable - first Part of Poisson Variable

Light bulb example

- Each light bulb has independent exponential(λ) lifetime
- Install three light bulbs. Find expected time until last light bulb dies out.

$\Rightarrow$ would be a sum of

$$\frac{1}{3\lambda} + \frac{1}{2\lambda} + \frac{1}{\lambda}$$

$\frac{1}{\lambda}$ → Only 1 is switched on

$\frac{1}{2\lambda}$ → when only 2 are onn

$\frac{1}{3\lambda}$ → when all 3 are on and first arrival

Random Incidence for Poisson.

Bus arrival at 5 min and 10 min

E(Time for bus arrival) = 7.5

E(when person arrived, the next bus) = $\frac{1}{3}(5) + \frac{2}{3}(10)$

Markov Process

- given current state, the past doesn't matter.

$$\left[\, r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1)\, P_{kj} \,\right]$$

State i is recurrent if starting from i, wherever you want to go there is a way to return.

$$\pi_j = \sum \pi_k P_{kj}$$

$$\sum \pi_j = 1$$

$$\pi_i p_i = \pi_{i+1} q_{i+1}$$

Special Case

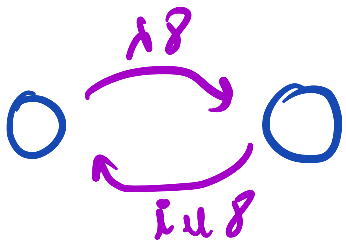
p_i = constant for all i

q_i = constant for all

$$\pi_i = \pi_0 \rho^i \qquad \rho = \frac{p_i}{q_{i+1}}$$

Phone Company Problem

Call originates as Poisson Process



$$\pi_i = \frac{\pi_0 \lambda^i}{\mu^i \, i!}$$

Bayesian Inferences

$$\left[P_{\theta|x}(\theta|x) = \frac{P_\theta(\theta) \cdot P_{x|\theta}(x|\theta)}{P(x)} \right]$$

Posterior Distribution