

Mathematical Model
- Make sure how it is well posed.

Wikipedia

Stochastic Process

- Special type of Probability function

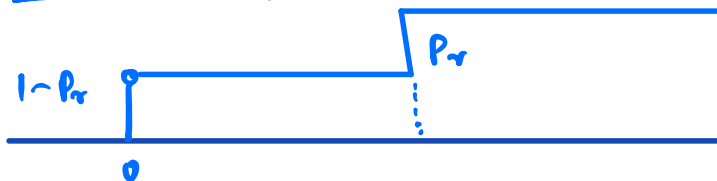
1. Counting Process : Sequencing Model
2. Poisson Process :
3. Renewable Process :
→ Markov chains
4. Random Walk and Nightingale

Expectations

$$E(x) = \int F_x^c(n) \xrightarrow{\text{Riemann Sum}} \int n f(n) dn$$

↓
Monotonic
Increasing
with x .

Indicator Random Variable



Stochastically Independent between each-other.

$$\frac{S_n}{n} = \frac{x_1 + x_2 + \dots + x_n}{n} \rightarrow \text{becomes deterministic when } n \rightarrow \infty$$

Finest Grain of Detail

$$\Pr \left\{ \frac{S_n - n\bar{x}}{\sqrt{n}\sigma} \leq y \right\} = \int_{-\infty}^y \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} dt$$

$$\frac{P_{S_n(k+1)}}{P_{S_n(k)}} = \frac{n-k}{k+1} \frac{p}{q} \rightarrow \text{strictly decreasing.}$$

Central limit Theorem.

$k = np + i$ \rightarrow Putting in the value

$$= \frac{n - (np + i)}{(np + i + 1)} \left(\frac{p}{q} \right)$$

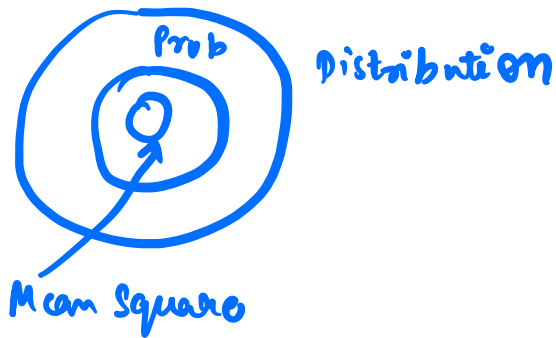
$1 + 2 + 3 + \dots$

$$= \ln\left(1 - \frac{i}{np}\right) - \ln\left(1 - \frac{i+1}{np}\right)$$

$$= \sum \dots \approx -\frac{j^2}{2npq}$$

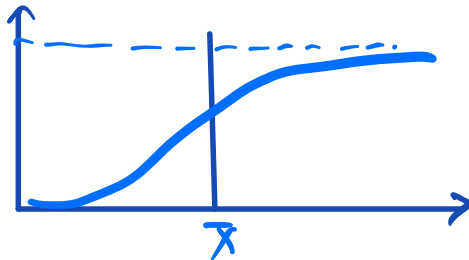
$$P_{Sn}(pn+j) = P_{Sn}(pn) \exp\left[-\frac{j^2}{2npq}\right] \rightarrow \text{PMF}$$

Convergence in Probability \Rightarrow Convergence in Distribution.



Weak Law of Large Numbers
 \hookrightarrow less powerful but
 More general

Central Limit Theorem



$$\{ \lim_{n \rightarrow \infty} Z_n(\omega) = \underline{Z(\omega)} \} = 1$$

\nwarrow maps into sample path
 might converge to something else (ω).

Strong Law of Large Numbers \rightarrow