Recap

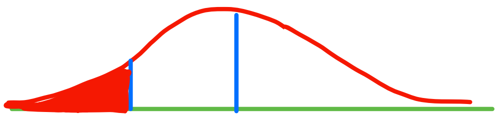
Hypothesis Testing

H_0, H_a (Assumption) ← Hypothesis Testing → Data

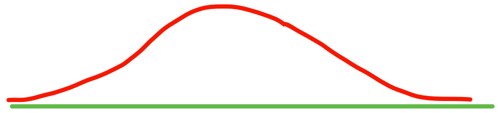
Steps involved in Hypothesis Testing

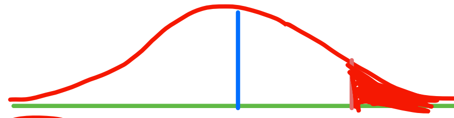
1. Setup Null and Alternate Hypothesis
2. Choose Distribution (To Be Covered Today)
   Test Statistic and α (significance level)
3. Select Left / Right / Two-Tailed
4. Compute P- Value
5. Compare P-Value to α :
   if P-val $< \alpha$; Reject Null Hypo
   else : Fail to Reject Null Hypothesis

Tailed Tests



Left Tailed Test





Right tailed Test

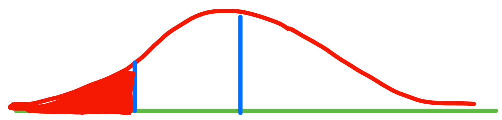
Burger Example

> Suppose there is a burger place that claims that all their burgers are **200 grams**. A customer who consumed their burger is still hungry after eating, and wants to prove that their burgers are lighter, and not as much as promised.

$H_0: \mu \geq 200G$

$H_a: \mu < 200G$

$\alpha = 5\%$



0.05 200Gms

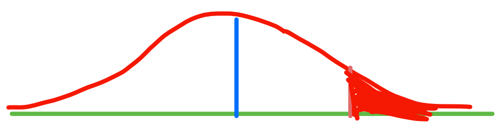
P-val

Consider the example of the Legacy Model, which had an accuracy of 90%. You want to claim that your new model is better.

H_0: Legacy ≤ 90

H_a: Acc $> 90\%$

$\alpha = 5\%$



0.95 P-value

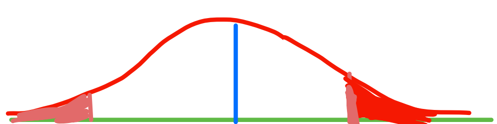
Right tailed Test

Suppose you are looking at the height of people in India. It is believed that the average height of Indians is 65 inches. You want to find out if that holds true for the people of your state. Are they taller or shorter?

H_0: Height $= 65$

H_a: Height $\neq 65$

$\alpha = 5\%$



0.025 65 0.975

Types of Errors

Judge Says \ Actual	Innocent (H_0)	Guilty (H_a)
Innocent	TN	FN
Guilty	FP	TP

$H_0 \Rightarrow$ Innocent (Negative)

> **Case 1:** We decide that the accused is innocent, and he is actually innocent (i.e. **True Negative**)
>
> **Case 2:** We decide that the accused is guilty, but he is actually innocent (Type-1) FP
>
> **Case 3:** We decide that the accused is innocent, but he is actually guilty (Type-2) FN
>
> **Case 4:** We decide that the accused is guilty, and he is actually guilty (i.e. **True Positive**)

① Type - 1

② $\alpha \rightarrow 5\%$ or 1%

CL $\Rightarrow$ 95%

③ FP

④ Incorrect Rejection of Null Hypothesis

Ex: Innocent person sent to Death Sentence

① Type - 2

② β

③ FN

④ Fail to Reject Incorrect Null Hypothesis

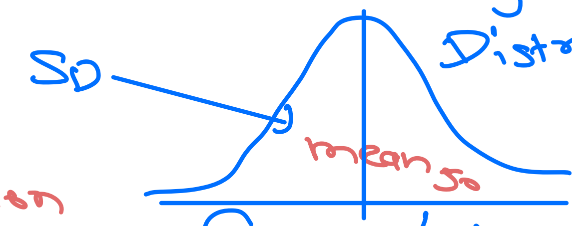
Ex: Guilty person set free

μ Population $= 65$ inch

$\sigma_{pop} = 2.5$ inches

Sample (50) ⇒ $mean_{50}$ $\bar{X}_{50}$

mean value of Distribution



mean of means ⇒ Population Mean

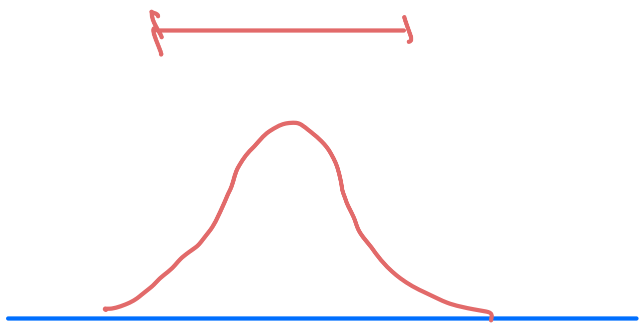
Gaussian / Normal / Z - distribut

S

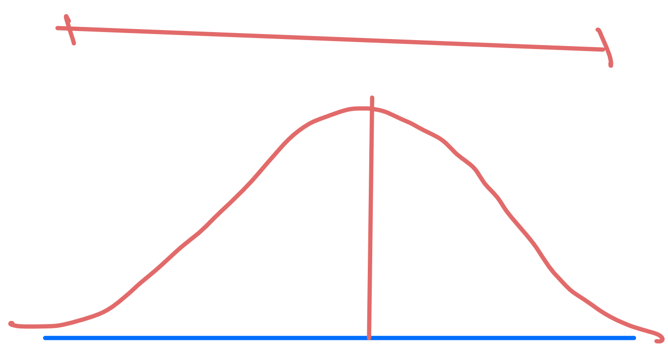
$$SE \Rightarrow \frac{2.5}{\sqrt{50}}$$

$SE_{50} < SE_5$

$SE_5 > SE$



Sample-size 50



Sample Size 5

One-Sample Z-test

Suppose there is a Retail Store Chain that sells Shampoo bottles

This chain has **2000 stores** across India.

The parameters for weekly sales of the shampoo bottle were reported as:

- Mean: 1800
- Standard deviation 100

This was calculated by analyzing a lot of historical data

As a Manager / Owner / Data Scientist, you want to increase these sales, to generate more revenue.

You decide to do this experiment with 2 competing marketing firms

- **Firm A**
    - Worked on **50 stores**
    - Sold an **average 1850** bottles of shampoo
- **Firm B**
    - Worked on **5 stores**
    - Sold an **average 1900** bottles of shampoo

> **Which firm gave better results?**

Clearly the sales are more for Firm B, but it seems that the number of stores under them were siginficantly less than Firm A.

It is possible that this increase by Firm B is just a chance factor because the standard deviation of the population was 100.

> **How do we quantify this and determine if it is just by chance or if it is actually statistically significant?**

When we talk about statistical significance, the word significance level pops into mind.

Since this is a big decision that would affect revenue, you want to be very very sure (99% confidence) about your decision, i.e. $\alpha = 0.01$

So, we need to employ the framework we saw and conduct hypothesis testing to see which firm's results are more significant.

H_0 : No improvement by marketing firm A

H_a : Sales improved by Firm A

Changed

Step 1

$H_0: \mu \leq 1800$

$H_a: \mu > 1800$

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Step 2: $\alpha = 0.05$

Normal Distribution

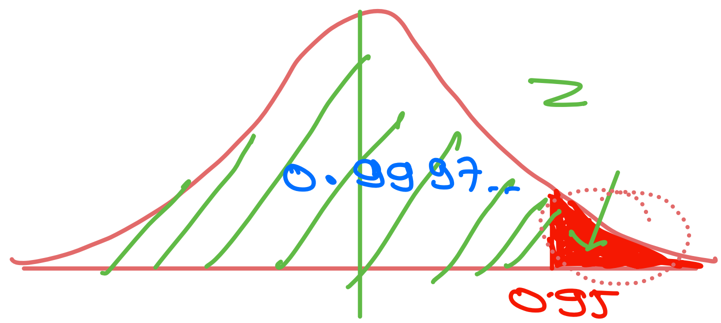
Firm A	Firm B
↓	↓
E @ 1800	E @ 1800
SE @ $\frac{100}{\sqrt{50}}$	SE @ $\frac{100}{\sqrt{5}}$

Step 3: Select the Tail

Right Tailed Test

Step 4:

Firm A



Observation ⇒ 1850

Z-score ⇒ $\dfrac{1850-1800}{SE}$

$\mu = 1800$

$SE \Rightarrow 100/\sqrt{50}$

$P = 1 - cdf(Z)$

$P \Rightarrow 0.003$

Risk of Type I Error

$\alpha = 5\% \Rightarrow 0.05$ $\alpha = 1\% \Rightarrow 0.01$

Reject Null Hypothesis
Since p-val $< \alpha$

Firm B : p-val = 0.012

$\alpha \Rightarrow 5\%$ $\alpha \Rightarrow 1\%$

Reject Null Hypothes — Fail to Reject Hypothesis

Firm ⇒ $P(m > 1900) \mid H_0$

(Test-Statistic ⇒ Z Statistic)

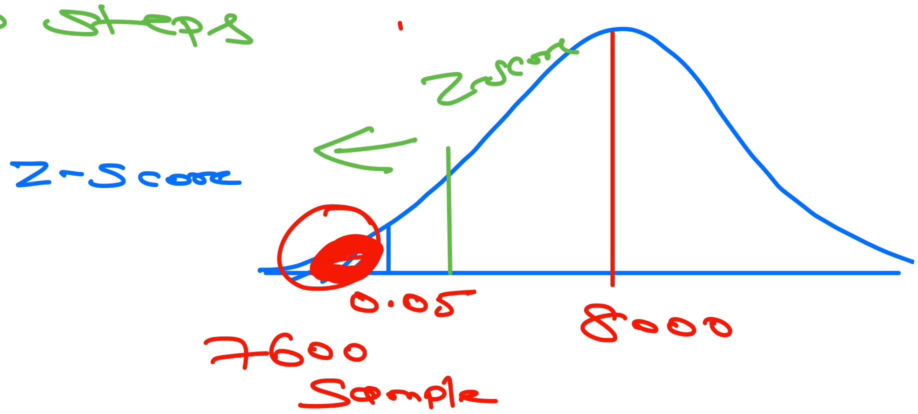
Fitness → 8000 Steps

Sample → 7600
30

Left tailed Test

$\alpha = 5\%$

SD = 1200 steps



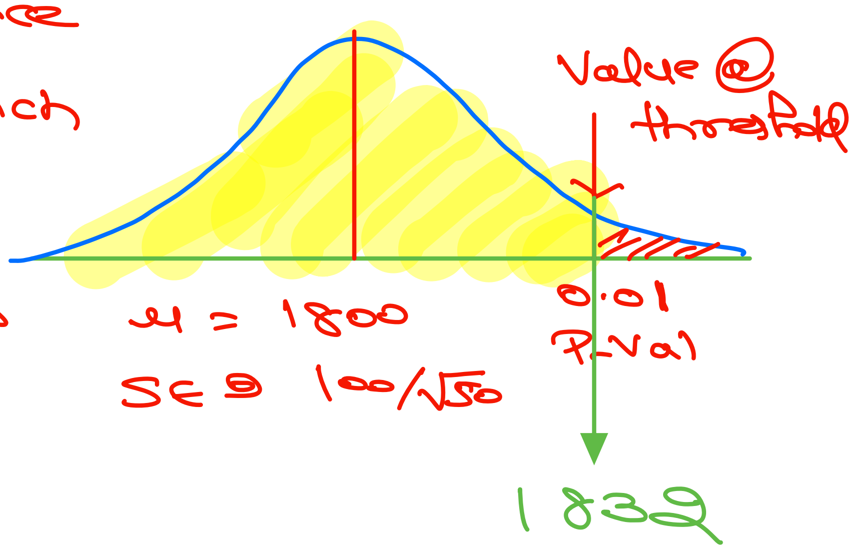
p-value ≈ 0.0339, Reject the Null Hypothesis.

A fitness App claims that its users walk an average of 8,000 steps per day. A random sample of 30 users showed an average of 7,600 steps per day with a standard deviation of 1,200 steps. Conduct a left-tailed Z-test at a 5% significance level to determine if the App's claim is supported. What is the p-value?

Critical Value

Firm A

Value @ Significance Level Beyond which we can reject Null Hypothesis



α = 0.01 ⟶ 1832 (Critical Value)

α ⟶ 0.01 = 1850
S = 50

Q Can i Use 1832 as Critical Level for firm B as well?

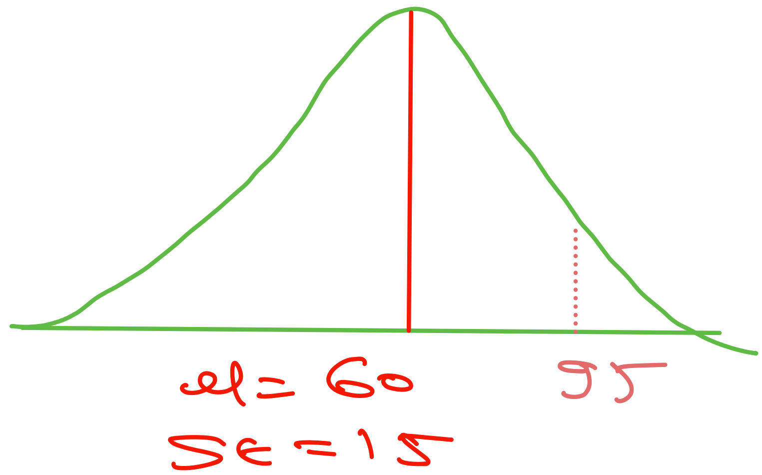
α = 0.01 ✓

S = 5 ✗

H.W: Calculate Critical Value for Firm B mean-score = 60

SE ⇒ 15

Critical value @ 95 CL



$Z \Rightarrow$ ppf(0.95)

$X \Rightarrow Z * SE + \text{mean}$

$\Rightarrow 84.67$

$\boxed{\text{Confidence Interval}}$

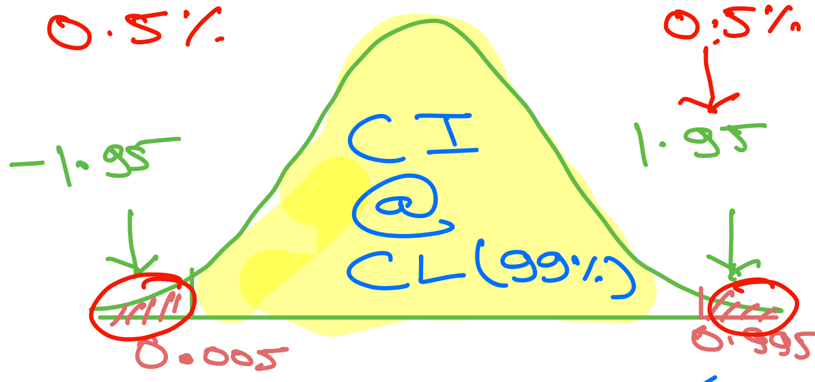
Let's consider firm A's Example:

Hypothesis:

H_0 : Firm A has No Effect
i.e. $\mu = 1800$

H_a : Firm A has effect on Sale i.e. $\mu \neq 1800$

$\alpha = 0.01$



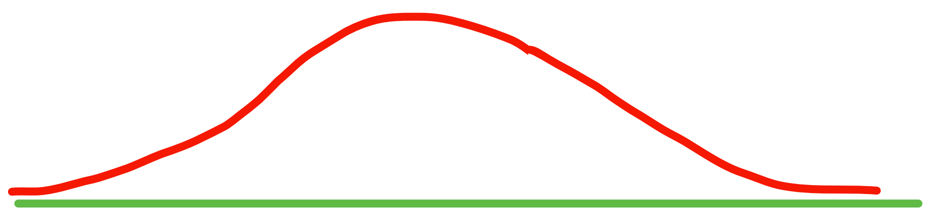
Z @ 1.95

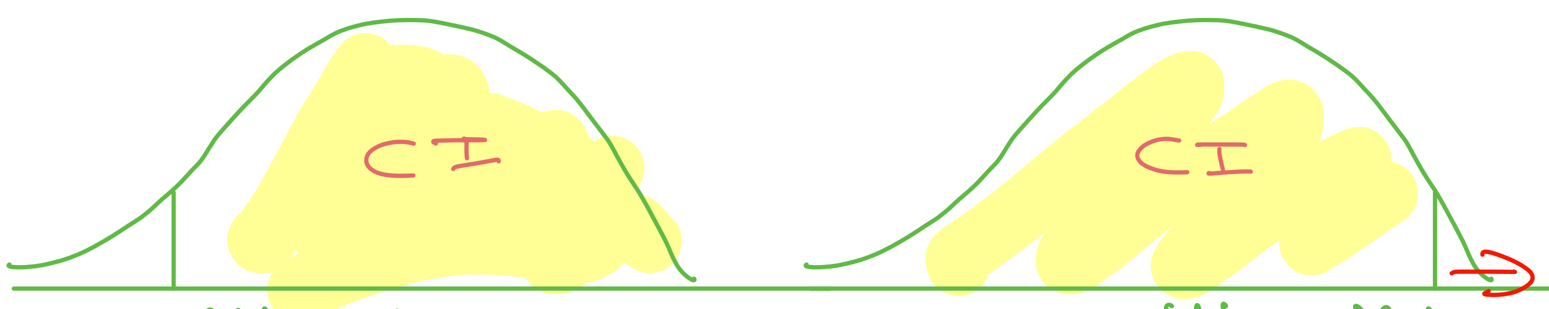
reject Null Hypothesis

margin of Error

CI @ $\left[\bar{x} - Z * \frac{\sigma}{\sqrt{n}} , \bar{x} + Z * \frac{\sigma}{\sqrt{n}}\right]$

CI @ $(\bar{x} - ME , \bar{x} + ME)$





Critical Value
Left
$[CV , \infty]$

Critical Value
Right
$[-\infty , CV]$

To Find CI associated with a CL

Step 1: Identify type of Test and area of interest for CI

Step 2: Calculate Z-score corresponding to CL

Step 3: Calculate Margin of Error

$$Z * \sigma/\sqrt{n}$$

Step 4: Calculate CI

$$CI = \bar{x} \pm (Z * \sigma/\sqrt{n})$$

β : Error 2

(we will cover this in ~~detail~~ detail Next Session)

Power of Test : $1 - \beta$

Q- Cohen's

$\boxed{\text{Power of Test}}$

What is Type-2 Error?

Effect Size

Let's explore this with an example scenario. Imagine you're following a specific diet plan.

Diet Plan A:

- Individuals on Diet Plan A may experience weight changes, but the degree of weight loss or gain varies widely among participants.
- It's like having a group where some individuals may lose a small amount of weight, while others may experience a significant change.