

# Agenda

- 1 Sample T-test
- 2 Sample T-Test
- Paired T-test

## 1 Sample T-test

- ⊛ In most scenarios population std deviation ( $\sigma$ ) is unknown, hence we cannot calculate z-score / z-statistic.
- ⊛ However given the sample data we can compute sample-standard deviation ( $S$ ), and use that as Estimate for ( $\sigma$ ).
- 2 This results in modified framework: T-test and the statistic is known as T-statistic
- 2 T-statistic follows T-Distribution
- 2 When dealing with smaller samples ( $< 30$ ), t-test is more appropriate as it accounts for increased Uncertainty

Example: 1 Sample Z-test to 1 Sample Test

IQ Increase with pill

data = 110, 105, 98, 102, 99, 104, 115, 95

$H_0 : \mu = 100$  (pill has no effect)

$H_a : \mu \neq 100$  (pill has effect)

$$\mu = 100$$

$$\bar{x} = 103.5 \quad \text{population SE} = \frac{\sigma}{\sqrt{n}} \quad \times$$

$$\sigma = \times$$

(cannot calculate)

$$n = 8$$

$$\text{sample SE} = \frac{s}{\sqrt{n}} \quad \checkmark$$

$s$  = can be calculated from data, hence:

$$z\text{-statistic} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\sigma \rightarrow s$

$$t\text{-statistic} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$s \rightarrow$  Sample S.D (Bessel's correction)  
 $\downarrow$   
 $sse \rightarrow \frac{s}{\sqrt{n}}$

## 2 Sample T-Test

School Example : Two Sample Test

①

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2 \quad (\text{two tailed})$$

②

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 > \mu_2 \quad (\text{Right tailed})$$

③

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 < \mu_2 \quad (\text{Left tailed})$$

For the above test, we can use `ttest_ind` function from `scipy.stats`

ind stands for independent

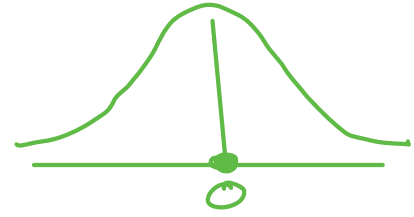
$$H_a : \mu_1 > \mu_2$$

$$H_a : \mu_1 < \mu_2$$

for two Sample t-test:

$Z \sim N(0,1)$   
 $t \rightarrow t(0,1)$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$



## Paired T-test

\* It is used primarily to compare means of two related Groups (Before and after)

- This typically occurs when you're studying the impact of a treatment, intervention, or change within the same subjects over time or in some paired way.
- In your case, you're comparing "Before" and "After" measurements on an individual basis. For each person, you have two measurements:
  - **Person 1: Before and After**
  - **Person 2: Before and After**
- The paired t-test takes into account the paired nature of the data.
- It calculates the mean difference ( $\mu_{after} - \mu_{before}$ ) of the paired measurements and then assesses whether it is statistically significant or not.
- This helps you determine if there's a significant change between the "Before" and "After" measurements within each pair.

Data

id	Before	After
1	val1	val'1
2	val2	val'2
3	„	„
„		

$$H_0: \mu_{\text{Before}} = \mu_{\text{after}}$$

## Null and Alternate hypothesis

- Null Hypothesis ( $H_0$ ): Problem-solving has no effect on the test scores.
  - In other words, the mean test scores before (test\_1) and after (test\_2) problem-solving are equal.
- Alternative Hypothesis ( $H_a$ ): Problem-solving had an effect on the test scores.
  - This implies that the mean test scores before and after problem-solving are not equal.

+ - statistic

$$t = \frac{\bar{D}}{\frac{S_D}{\sqrt{n}}}$$

$\bar{D}$  : mean of Differences

$S_D$  : SD of Differences

For the above test, we can use `ttest_rel` function from `scipy.stats`

id	Before	After	$D_i$
1	Val 1	Val' 1	$V_1 - V'_1$
2	Val 2	Val' 2	$V_2 - V'_2$
3	,,	,,	
..			

$\bar{D}$  : mean of  $D_i$

$S_D$  : SD of  $D_i$

## Conclusion

We covered

1) 1 Sample T-test

2) 2 Sample Independent T-test

3) Paired T-test

4) T-test is used for Testing Relationship b/w

Categorical  $\rightarrow$  Continuous

### Note about T-test:

- If sample size  $n > 30$ , then the sample standard deviation will be very close to the value of standard error, i.e.  $\frac{s}{\sqrt{n}} \approx \frac{\sigma}{\sqrt{n}}$
- i.e. T-test and Z-test become essentially the same.
- If the number of samples is low, you go for a T-test
- If it is high, you can use either T test or Z test
- That's why, scipy does not even have an implementation for Z-test.

num vs Cat  $\rightarrow$  Z-test  
 $\rightarrow$  T-test

Cat vs Cat  $\rightarrow$  Chi Square

num vs num  $\rightarrow$  Correlation

# DOF

One week of Movie Night

7 different Movies on 7 days

One movies on 1st Day  $\rightarrow$   
freedom to pick any of  
the 7

One movies on 2nd Day  $\rightarrow$   
freedom to pick any of  
the 6

⋮

One movies on 7th Day  $\rightarrow$   
0

Ex: 1st person = 35 lakh

2<sup>nd</sup> person = 36 lakh

3<sup>rd</sup> person = ?

Avg salary of 35 lakh

3<sup>rd</sup> person = ?

Can we find 3<sup>rd</sup> person Salary = ?

Ex: 1<sup>st</sup> person = 35 lakh

2<sup>nd</sup> person = 36 lakh

3<sup>rd</sup> person = ?

4<sup>th</sup> person = 55 lakh

Avg salary is 35 lakh

Can we find 3<sup>rd</sup> person Salary = ?