All Formula's

1. Conditional probability: $P[A|B] = \frac{P[A \cap B]}{P[B]}$
2. From conditional probability, we get,

$P[A \cap B] = P[A|B] \; * \; P[B]$, which is known as Multiplication Rule.

3. Bayes Theorem: $P[A|B] = \frac{P[B|A] * P[A]}{P[B]}$
4. Law of total probability = $P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$
5. Independent Events: $\boxed{P[A \cap B] = P[A] \; * \; P[B]}$

Claim:

If A and B are mutually Exclusive then they cannot B independent

① Mutually Exclusive

$$A \cap B = \{ \}$$

$$P(A \cap B) = 0$$

② Independent Events

$$P(A \cap B) = P(A) \times P(B)$$

Both statement 1 and statement conflict with each other

Q. Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language processing (NLP).
Also, it is seen that 20 have taken both CV and NLP.
Given that a person has taken NLP, what is the probability that he has also taken CV?

1

$$P(CV) = \frac{60}{100} = 0.6$$

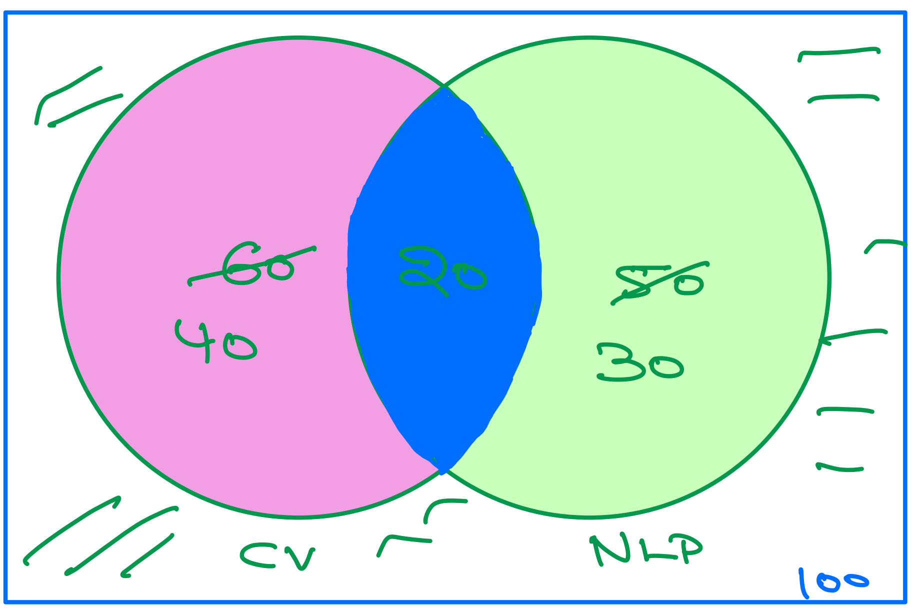
$$P(NLP) = \frac{50}{100} = 0.5$$

$$P(CV \cap NLP) = \frac{20}{100} = 0.2$$

$$P(CV \mid NLP) = \frac{P(CV \cap NLP)}{P(NLP)}$$

$$= \frac{0.2}{0.5} \times \frac{10}{10} = \frac{2}{5}$$

2



$$P(CV \mid NLP)$$

$$\frac{20}{50}$$

In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD

- What is the probability that a randomly chosen faculty member is a female and has PHD? (1) $P(PHD \cap F)$
- What is the probability that a randomly chosen faculty member is a male and has PHD? (2) $P(PHD \cap M)$
- What is the probability that a randomly chosen faculty member has a PHD? (3) $P(PHD)$
- What is the probability that a randomly chosen PHD holder is female? (4) $P(F \mid PHD)$

$P(F) = 30\% = 0.3$

$P(M) = 70\% = 0.7$

$P(phd \mid F) = 0.6$

$P(PHD \mid M) \Rightarrow 0.4$

Q (1) $P(F \cap PHD) = P(PHD \mid F) \times P(F)$

$\Rightarrow 0.6 \times 0.3$

$\Rightarrow 0.18\%$

Q-2 $P(M \cap PHD) = P(PHD \mid M) \times P(M)$

$\Rightarrow 0.7 \times 0.4$

$\Rightarrow 0.28$

Q-3 $P(PHD)$

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) \times P B_i$$

$\Rightarrow$

$$P(PHD) = \sum_{G \to M}^{F} P(PHD|G) \times P_G$$

$$P(PHD) = \begin{array}{c} P(PHD|M) \times P(M) \\ + \\ P(PHD|F) \times P(F) \end{array}$$

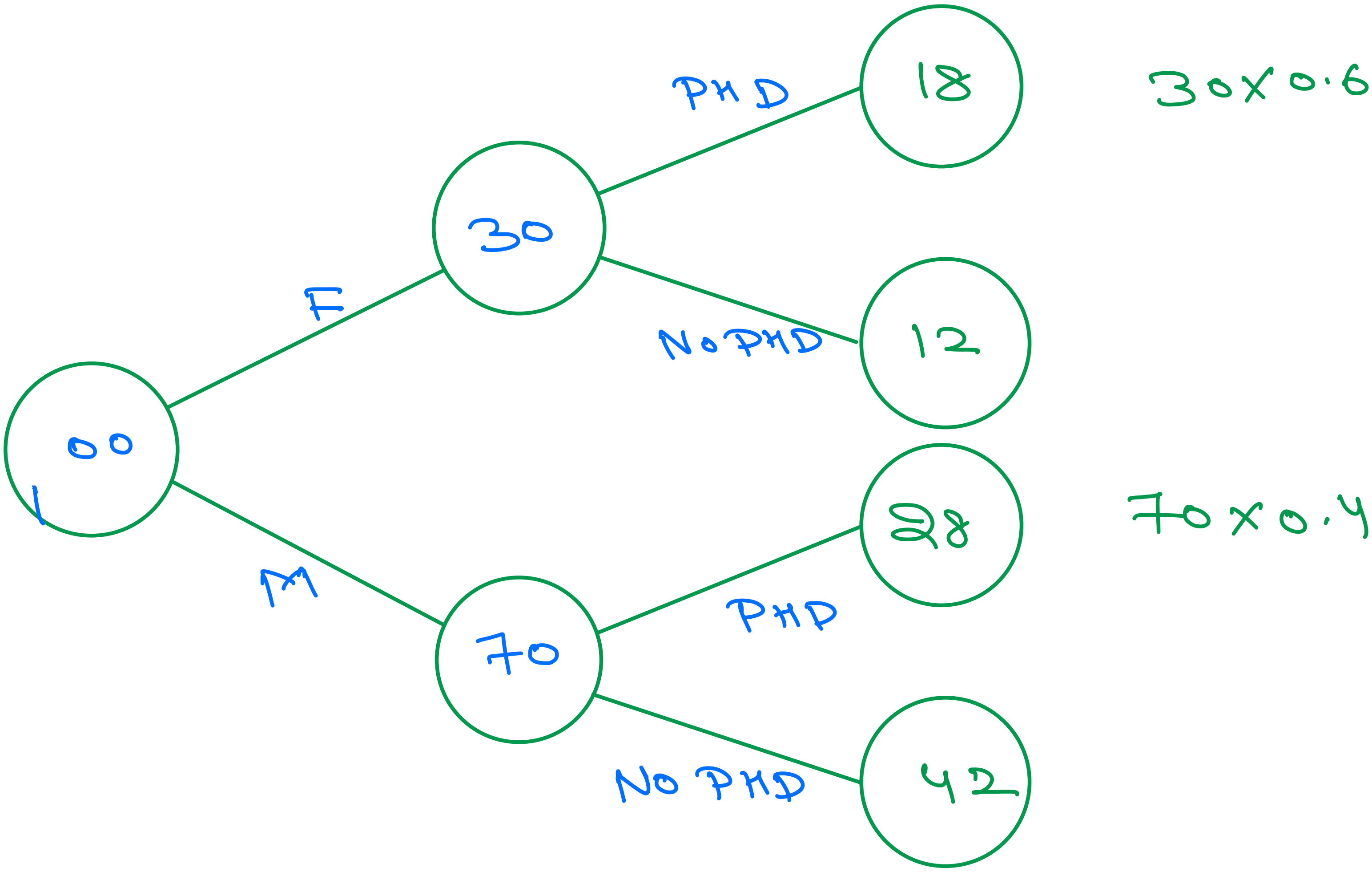
Q-4 $$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$$P(F|PHD) = \frac{P(PHD|F) \times P(F)}{P(PHD)}$$

$P(PHD) \downarrow$ Law of Total P

$$= \frac{0.6 \times 0.3}{0.46}$$

Q 5: What is P of randomly chosen PHD member is MALE?



$P(\text{PhD} \mid F) = 0.6$

$P(\text{PHD} \mid M) = 0.4$

① $P(F \cap \text{PHD}) = \dfrac{18}{100} = 0.18$

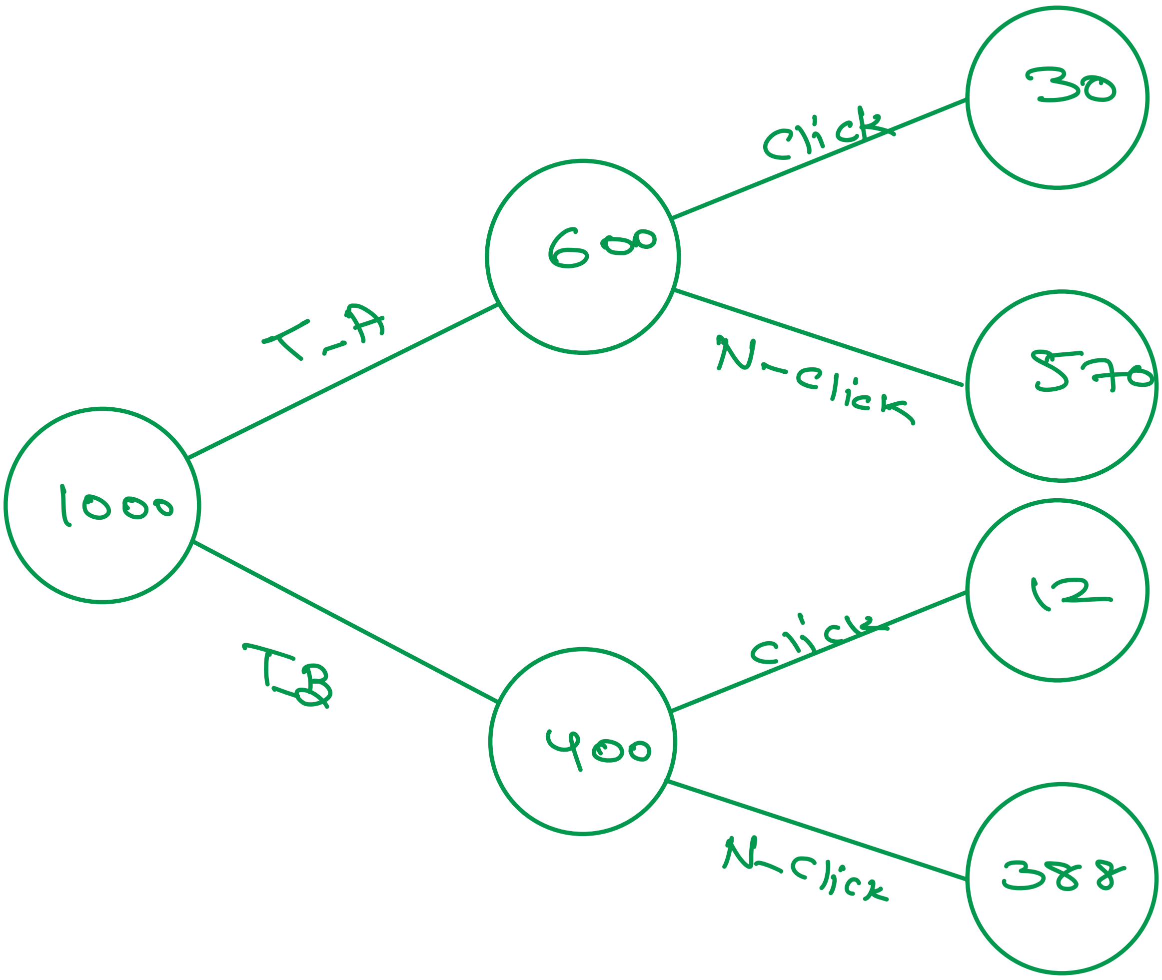
② $P(M \cap \text{PHD}) = \dfrac{28}{100} = 0.28$

③ $P(\text{PHD}) = \dfrac{46}{100} = 0.46$

③ $P(F \mid \text{PHD})$ $\dfrac{18}{28+18} = \dfrac{18}{46}$

Given PHD member what is P(F)

An website shows two types of ads:
60% of the visitors see Type A ads, and 40% visitors see Type B ads.
The click-through rate for A is 5%, and for B is 3%.



Q:=

A visitor to the website does not click the ad.
What is the probability that he saw Type A ad?

$P(T\_A \mid N\_click)$ $\rightarrow$ $\dfrac{P(T\_A \cap N\_Click)}{P(N\_Click)}$

$\dfrac{570}{570+388}$

Q. A website has noticed the following stats. Among those who saw the ad, 70% saw it on Youtube, 50% saw it on Amazon, 35% saw it on both.

A random person who saw the ad on Amazon is chosen. What is the probability that he also saw the ad on Youtube?

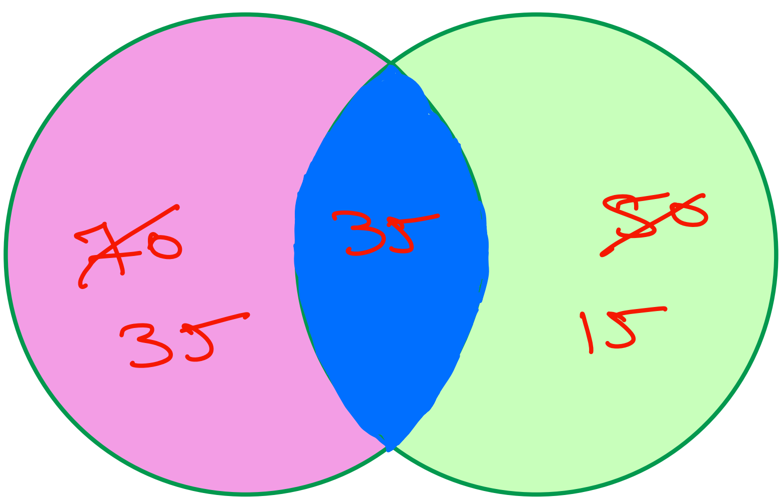
$P(Y) = 0.7$

$P(A) = 0.5$

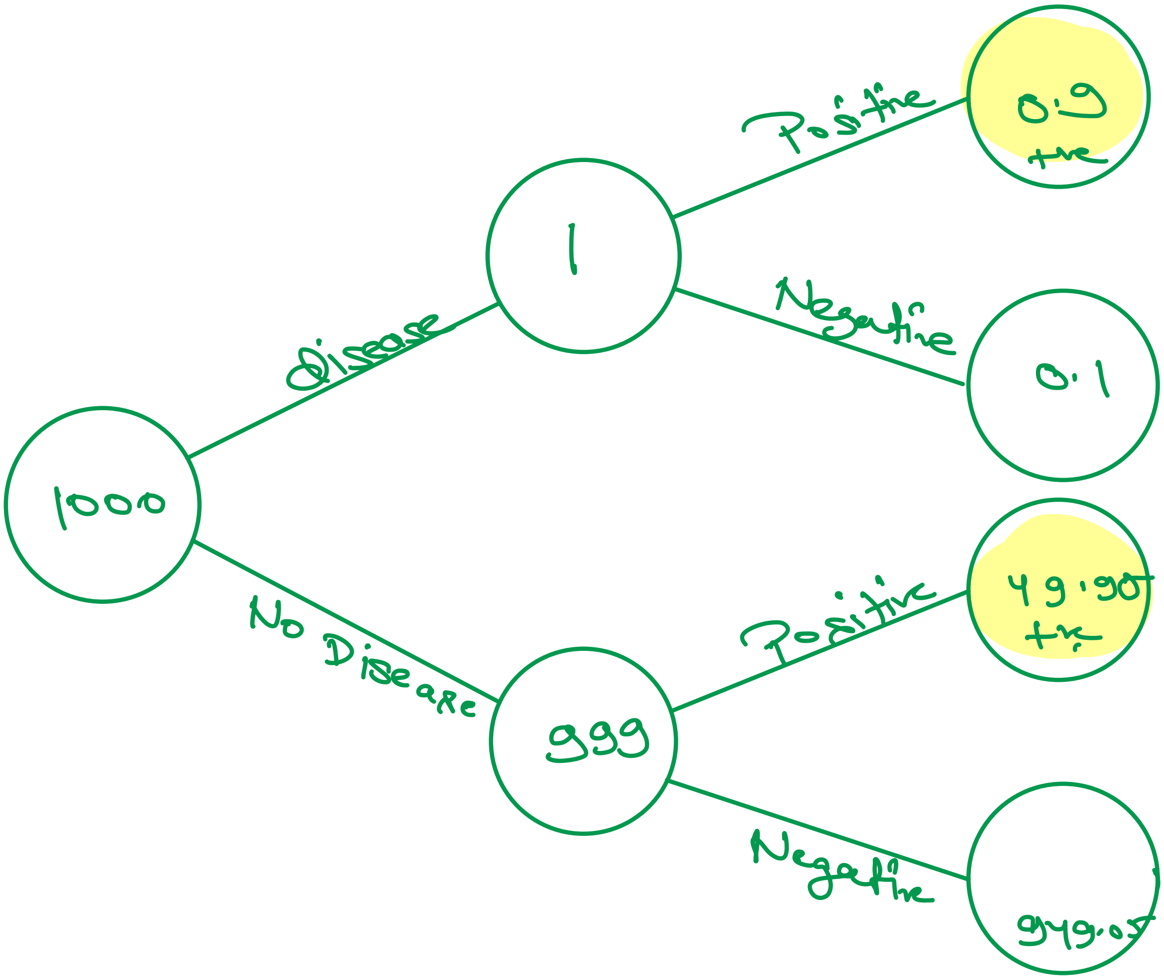
$P(Y \cap A) = 0.35$

1) $P(A \mid Y) \rightarrow \dfrac{P(A \cap Y)}{P(Y)} = \dfrac{0.35}{0.7}$

2) $P(Y \mid A) = \dfrac{0.35}{0.50}$



$= \dfrac{35}{50}$



999×0.05

$$\frac{}{49.95 + 0.9} \Rightarrow \frac{0.9}{50.85}$$

T_1	T_2
H	T
H	H
T	T
T	H