Disclaimer: Please note that any topics that are not covered in today's lecture will be covered in the next lecture.

Content

- Combinatorics
- Permutations
- Permutation : Generic Approach
- Combinations

Suppose we have 2 True/False questions. In how many ways can they be solved?

For question 1, we have two possible outcomes:

- True
- False

Similarly, for question 2 as well.

Since we need to solve both questions, will we add or multiply their number of possible outcomes?

We will multiply since we have to solve question 1 AND 2

Instead, if we had to solve question 1 OR 2, we would've added.

This is because, when considering Event 1 AND Event 2, we are talking about two independent events.

- Solving question 1 is independent from solving question 2
- Hence, we need to multiply to consider their combined effect.

Therefore, we can solve them in 2 * 2 = 4 ways:

- True, True
- True, False
- · False, True
- · False, False

Permutation and Combination

What is a permutation?

- When talking about permutations, we mean arrangement of objects.
- Therefore, as with arranging objects, the most important thing is **order** is which they are arranged.
 - This means that $(i, j) \neq (j, i)$

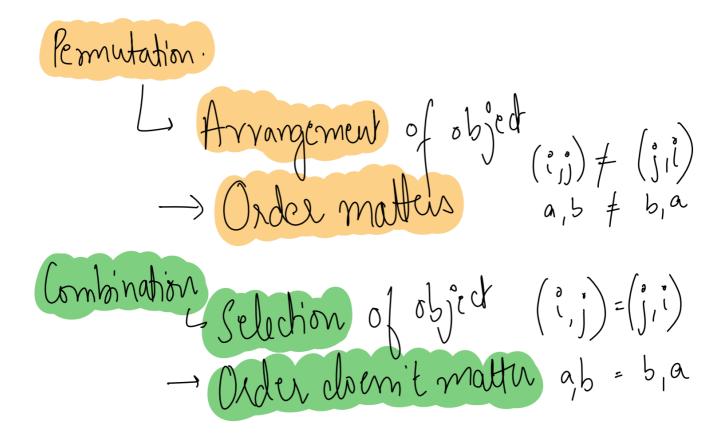
Formal Definition: A permutation is an arrangement of items or elements in a specific order, where the order of the arrangement matters.

The second aspect is **Combinations**

What is a combination?

- Combination is Selection of objects.
- Over here, the order of objects does not matter.
 - This means that (i, j) = (j, i)

Formal Definition: A combination is selection of items or elements where the order of the arrangement does not matter.



Permutation : Generic Formula

Q1. How would we arrange N object, given that there only 3 slots?

Since there are 3 slots for N objects, the no. of ways in which we can arrange them is ${}^{N}P_{3}$

i.e.
$$^{N}P_{3}=N.(N-1).(N-2)$$

Q2. How would we arrange N object, given that there only 4 slots?

$$^{N}P_{4} = N.(N-1).(N-2).(N-3)$$

We can observe a pattern between the no. of slots/blanks, and the last term of the above expressions

Q3. Then how would we arrange N object, given that there are k slots available?

This can be found using:

Let's re-write this equation by multiplying and dividing by same expression, as:

As we know, we can write this in the form of factorial as: ${}^{N}P_{k}=\frac{N!}{(N-k)!}$

Combinations

- Combinations, in simple terms, are all the different ways you can choose a certain number of items from a group, where the order in which you pick them doesn't matter.
- It's like making a sandwich with different ingredients the combination is the unique mix of ingredients you choose, regardless of the order you add them.

In the language of combinatorics, this number of ways of **selecting** is known as **Combination**.

Similarly, we can write the **general formula** for combinations in terms of permutations as:

$${}^{n}C_{k} = \frac{{}^{n}P_{k}}{k!}$$

We can further expand it as: ${}^{n}C_{k} = \frac{n!}{k!(n-k)!}$