**Disclaimer: Please note that any topics that are not covered in today's lecture will be covered in the next lecture**.

## Content

- **Combinatorics**
- **Permutations**
- **Permutation : Generic Approach**
- **Combinations**

> **Suppose we have 2 True/False questions. In how many ways can they be solved?**

For question 1, we have two possible outcomes:

- True
- False

Similarly, for question 2 as well.

> **Since we need to solve both questions, will we add or multiply their number of possible outcomes?**

We will multiply since we have to solve question **1 AND 2**

Instead, if we had to solve question **1 OR 2**, we would've added.

This is because, when considering `Event 1 AND Event 2`, we are talking about two **independent events**.

- Solving question 1 is independent from solving question 2
- Hence, we need to multiply to consider their combined effect.

Therefore, we can solve them in $2 * 2 = 4$ ways:

- True, True
- True, False
- False, True
- False, False

## Permutation and Combination

> **What is a permutation?**

- When talking about permutations, we mean **arrangement of objects**.
- Therefore, as with arranging objects, the most important thing is **order** is which they are arranged.
    - This means that $(i, j) \neq (j, i)$

**Formal Definition:** `A permutation is an arrangement of items or elements in a specific order, where the order of the arrangement matters.`

The second aspect is **Combinations**

> **What is a combination?**

- Combination is **Selection of objects**.
- Over here, the order of objects **does not matter**.
    - This means that $(i, j) = (j, i)$

    **Formal Definition:** `A combination is selection of items or elements where the order of the arrangement does not matter.`

Permutation.
↳ Arrangement of object $(i,j) \neq (j,i)$
→ Order matters $a,b \neq b,a$

Combination
↳ Selection of object $(i,j) = (j,i)$
→ Order doesn't matter $a,b = b,a$

## Permutation : Generic Formula

> **Q1. How would we arrange N object, given that there only 3 slots?**

Since there are 3 slots for `N` objects, the no. of ways in which we can arrange them is ${}^{N}P_3$

i.e. ${}^{N}P_3 = N.(N-1).(N-2)$

> **Q2. How would we arrange N object, given that there only 4 slots?**

$${}^{N}P_4 = N.(N-1).(N-2).(N-3)$$

We can observe a pattern between the no. of slots/blanks, and the last term of the above expressions

> **Q3. Then how would we arrange N object, given that there are `k` slots available?**

This can be found using:

$${}^{N}P_k = N(N-1)(N-2)(N-3)....(N-(k-1)) = N(N-1)(N-2)(N-3)....($$

Let's re-write this equation by multiplying and dividing by same expression, as:

$${}^{N}P_k = N(N-1)(N-2)(N-3)....(N-(k-1)) = N(N-1)(N-2)(N-3)....($$

As we know, we can write this in the form of factorial as: ${}^{N}P_k = \frac{N!}{(N-k)!}$

## Combinations

- Combinations, in simple terms, are all the different ways you can choose a certain number of items from a group, where the order in which you pick them doesn't matter.
- It's like making a sandwich with different ingredients – the combination is the unique mix of ingredients you choose, regardless of the order you add them.

In the language of combinatorics, this number of ways of **selecting** is known as **Combination**.

Similarly, we can write the **general formula** for combinations in terms of permutations as:
${}^{n}C_k = \frac{{}^{n}P_k}{k!}$

We can further expand it as: ${}^{n}C_k = \frac{n!}{k!(n-k)!}$

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