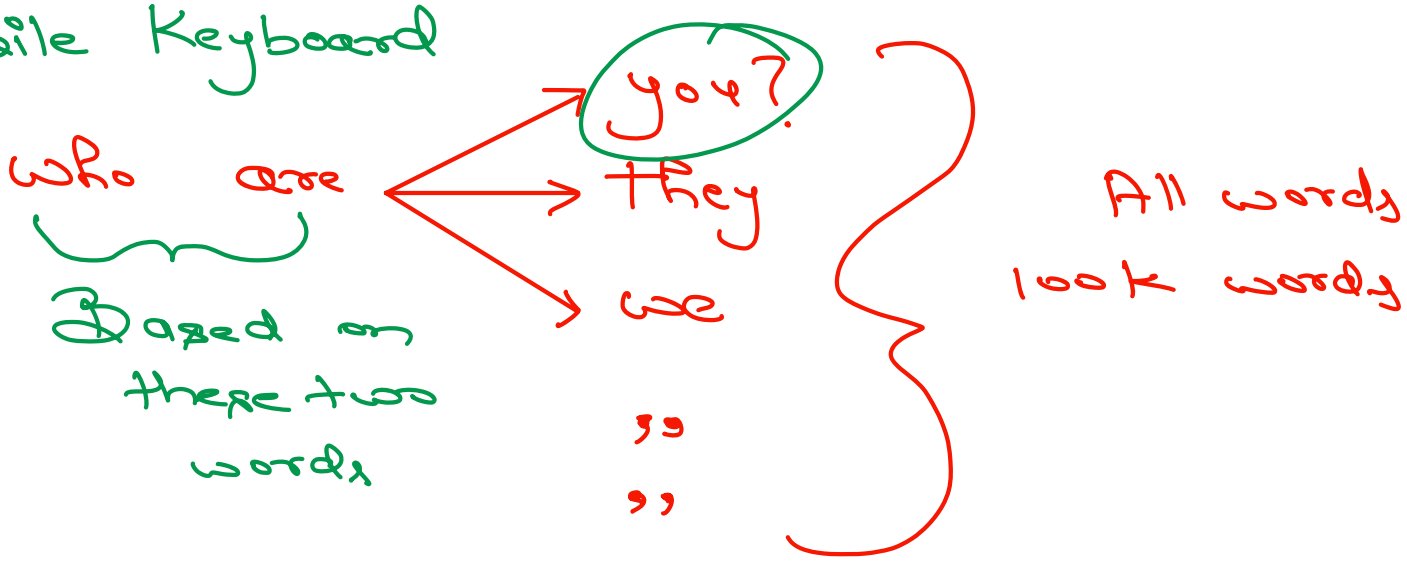
Agenda

- Conditional Probability
- Multiplication Rule
- Marginal and Joint probability
- Tree Diagram Approach
- Law of Total probability
- Baye's Theorem

Conditional Probability

Mobile Keyboard



$P(x = you \mid w_1 = who \text{ and } w_2 = are)$

$P(x = they \mid w_1 = who \text{ and } w_2 = are)$

$P(x = we \mid w_1 = who \text{ and } w_2 = are)$

,,

(0, 1)

$P_{you \mid who\ and\ are} = 0.8$ ②

$P_{they \mid who\ and\ are} = 0.7$ ③

$P_{we \mid who\ and\ are} = 0.9$ ①

↓ Selected you

$P_{Rahul \mid who\ and\ are\ and\ you}$

Experiment

$D_1 + D_2$	D_2: 1	2	3	4	5	6
D_1: 1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(D_1 = 2) = \frac{6}{36}$

$P(D_1 + D_2 \leq 5) = \frac{10}{36}$

$P(D_1 = 2 \cap D_1 + D_2 \leq 5)$

↑ intersection

$\frac{3}{36}$

⇒ What is $P(D_1 = 2)$ given that $D_1 + D_2 \leq 5$

(A) (B)

$P(D = 2 \mid D_1 + D_2 \leq 5) \Rightarrow \frac{3}{10}$

↓

$$\frac{P(D_1 = 2 \cap D_1 + D_2 \leq 5)}{P(D_1 + D_2 \leq 5)} \Rightarrow \frac{\frac{3}{36}}{\frac{10}{36}}$$

Multiplication Rule

$$P(B) \times P_{A|B} = \frac{P(A \cap B)}{P(B)} \times P(B)$$

important

$$P(A \cap B) = P(A/B) \times P(B)$$

(Product / multiplicate) Rule

$$P(B \cap A) = P(B/A) \times P(A)$$

Q $P(A/B)$ same as $P(B/A)$?

X

Questions

Marginal and Joint probability

* Marginal probability (Unconditional Probs)

Ex1:

$P(D_1 = 2)$

$P(D_2 = 3)$

Ex2: $P_{Century}$

$P_{India\ win}$

* Joint probability

→ P of multiple Events occurring together

Ex: $P(D = 2 \cap D_1 + D_2 \leq 5)$

A ∩ B

Ex2:

$P_{C \cap \omega}$

$P_{C^c \cap \omega}$

* Conditional

Ex2: $P_{C|\omega}$ $P_{\omega|C}$ $P_{L|C}$

Tree Diagram Approach

Questions

Email Spam System

- Let's say 30% of all Emails are Spam
- 70% are Non-Spam
- 80% of all spam Emails contain word "purchase"
- 10% of Non-spam contains purchase
- Overall what % of Email will have word "purchase"

Answers

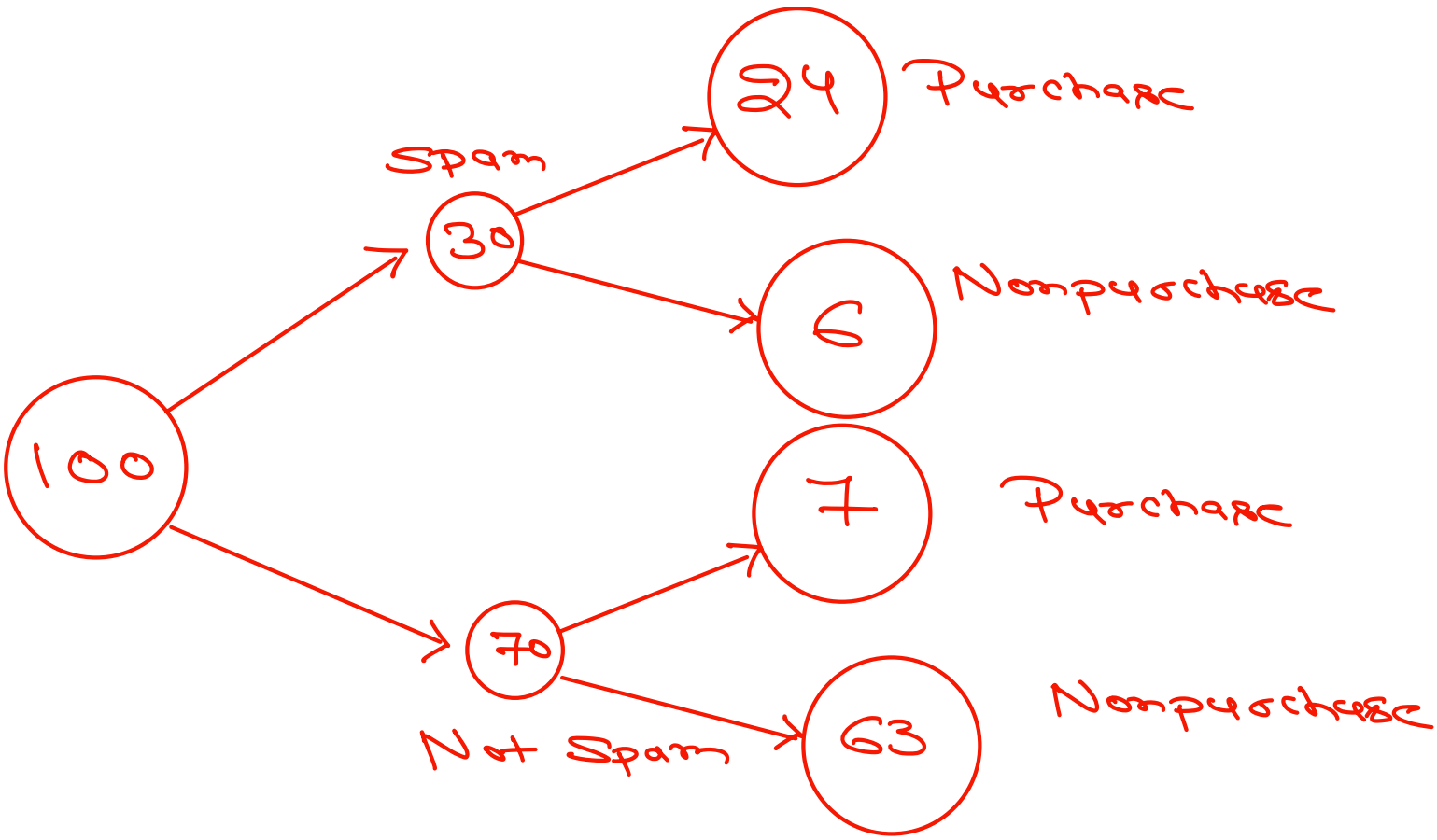
$$P_{Spam} = 0.3$$

$$P_{not\ spam} = P^{C}_{Spam} = 0.7$$

$$P_{purchase \mid Spam} = 0.8$$

$$P_{purchase \mid \overset{Not}{Spam}} = 0.1$$

$$P(purchase) \ ?$$



$P_{purchase} = \frac{24+7}{100} = 0.31 = 31\%$

$P_{purchase}$ (Pur) $=$ $\frac{24}{100} + \frac{7}{100}$

$P(purchase \cap Spam)$

$+$

$P(purchase \cap Non\text{-}Spam)$

$$P(A \cap B) = P(A/B) \times P(B)$$

$$P_{pyr} = P_{pyr/Spam} \times P_{Spam} + P_{pyr/NonSpam} \times P_{NonSpam}$$

Type $\begin{cases} \text{Not Spam} \\ \text{Spam} \end{cases}$ $\begin{cases} \text{Type 1} \\ \text{Type 2} \end{cases}$

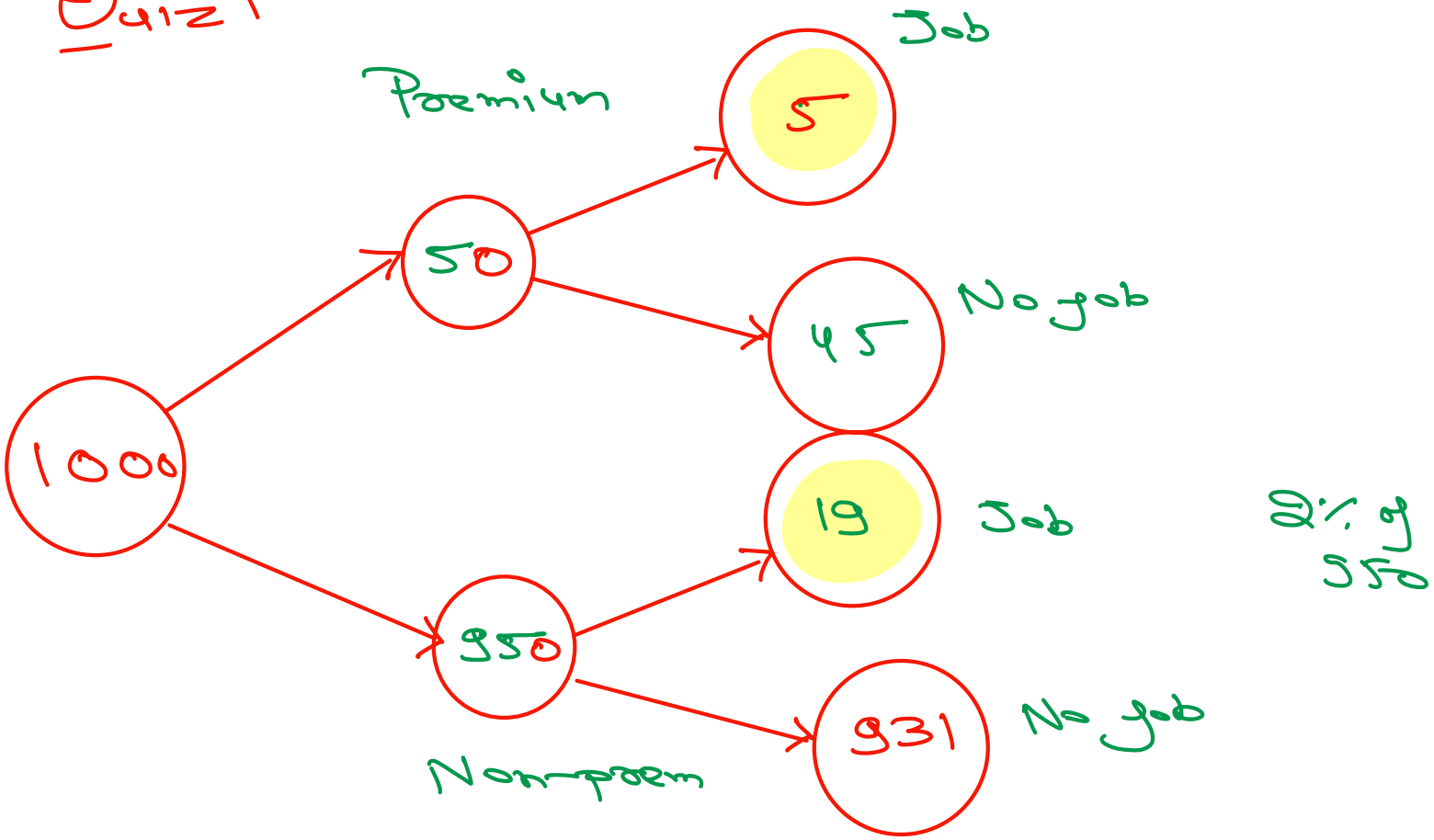
$$P_{pyr} \Rightarrow \sum_{i=1}^{2} P_{pyr/Type_i} \times Type_i$$

Law of Total probability

$$P(A) = \sum_{i=1}^{n} P(A/B_i) \times P(B_i)$$

It is known that -
5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities.
Overall, what percentage of people are actively seeking new job opportunities

Quiz 1



$\frac{24}{1000} = 2.4\%$

Questions : Solve above Question with help of Law of Total prob

1. Conditional Probability:

- $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

2. Multiplication Rule:

- $P(A \cap B) = P(A \mid B) \cdot P(B)$

3) Law of Total Probability:

- $P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$

Before starting a new topic, let's solve a few quizzes first.

Q Quiz 2

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A disease affects 10% of the population.
Among those who have the disease, 80% get “positive” test result
Among those who don’t have the disease, 5% get “positive” test result.
What is P(+ve | Disease) ?

a) 0.1
b) 0.8
c) 0.05
d) 0.85
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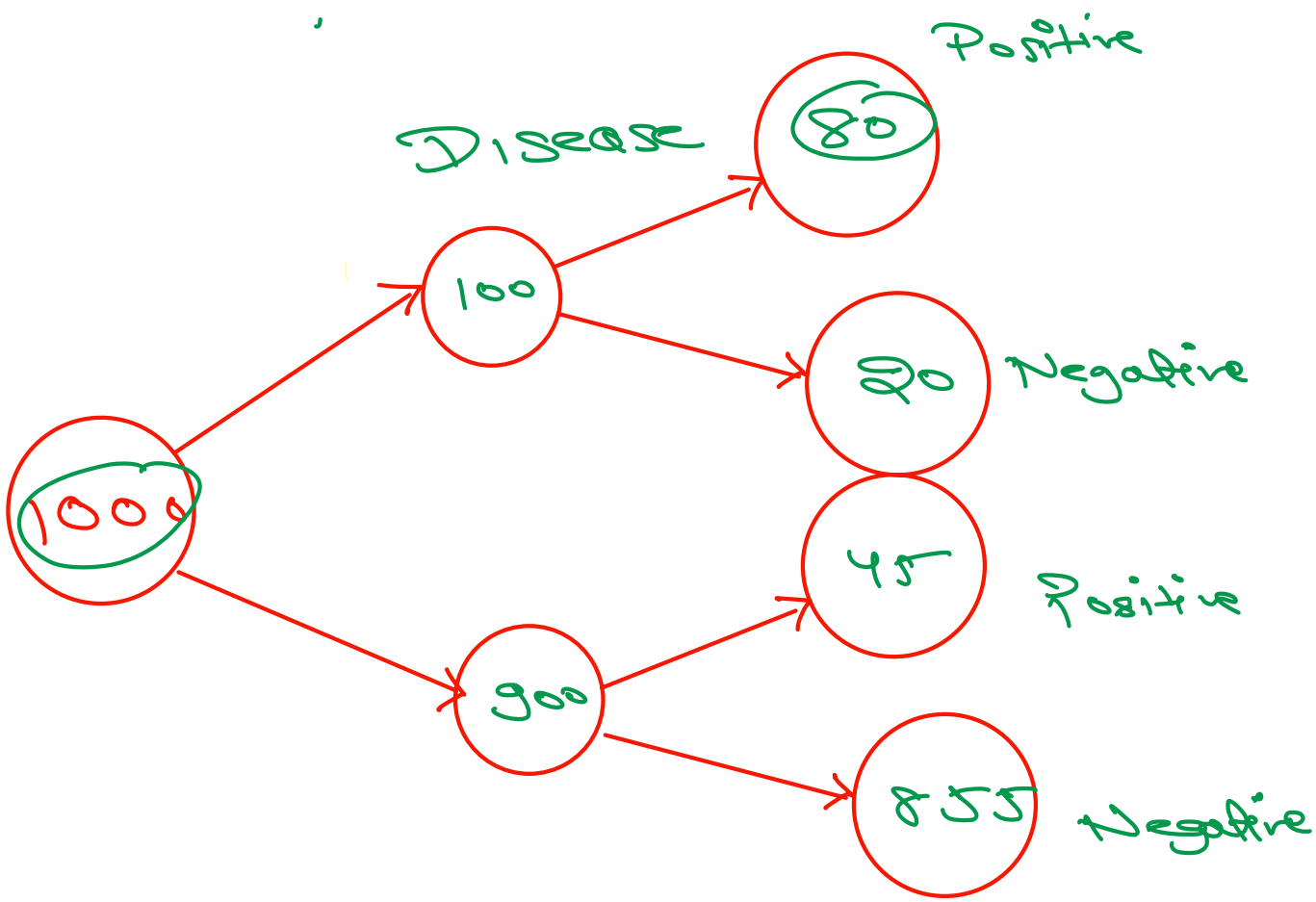

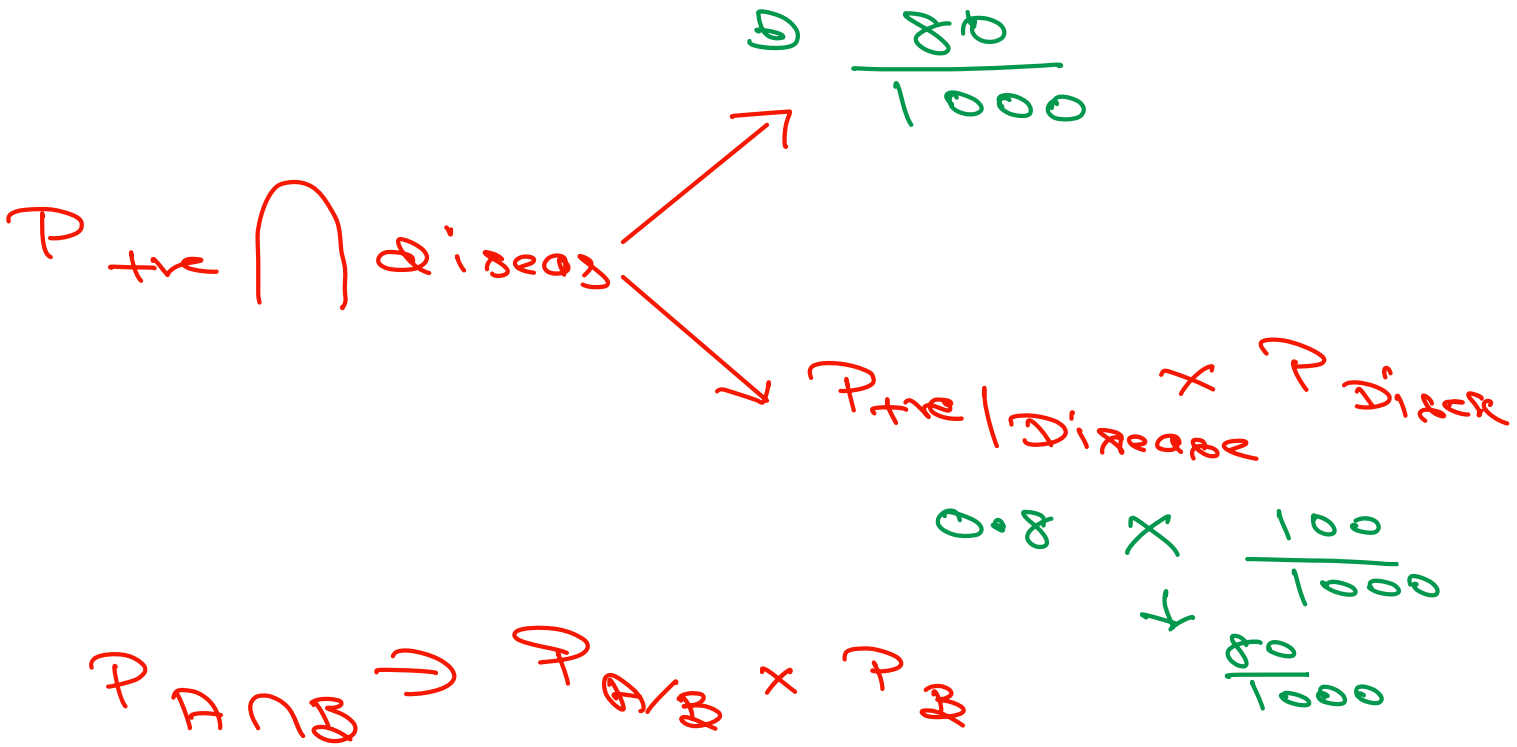

$P_{A \cap B} \Rightarrow P_{A/B} \times P_B$

Baye's Theorem

$$P(A|B) \Rightarrow \frac{P(B|A) \times P(A)}{P(B)}$$

Questions : Derive Abov formula

Hint :

$$P(A \cap B) = P(A/B) \times P(B)$$

$$P(B \cap A) = P(B/A) \times P(A)$$