Agenda

- → Conditional Probability
- → Multiplication Rule
- → Marginal and Joint probability
- → Tree Diagram Approach
- → Law of Total probability
- → Baye's Theorem

$\boxed{\text{Conditional Probability}}$

Mobile Keyboard

$\underbrace{\text{who are}}_{\text{Based on these two words}}$ → you? / they / we / ,, / ,,

All words

100k words

$$\left.\begin{array}{l} P(x = \text{you} \mid w_1 = \text{who and } w_2 = \text{are}) \\ P(x = \text{they} \mid w_1 = \text{who and } w_2 = \text{are}) \\ P(x = \text{we} \mid w_1 = \text{who and } w_2 = \text{are}) \\ \quad ,, \end{array}\right\} (0,1)$$

$P_{you \mid who\ and\ are} = 0.8$ ②

$P_{they \mid who\ and\ are} = 0.7$ ③

$P_{we \mid who\ and\ are} = 0.9$ ①

↓ selected you

$P_{next \mid who\ and\ are\ and\ you}$

$\boxed{\text{Experiment}}$

D_1+D_2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

D_2 →

D_1 ↓

$P(D_1=2) = \frac{6}{36}$

$P(D_1+D_2 \leq 5) = \frac{10}{36}$

$P(D_1=2 \cap D_1+D_2 \leq 5$

↑ intersection

$\frac{3}{36}$

Q) What is $P(D_1=2)$ given that $D_1+D_2 \leq 5$

$$P(\underset{(A)}{D=2} \mid \underset{(B)}{D_1+D_2 \leq 5}) \Rightarrow \frac{3}{10}$$

↓

$$\frac{P(D_1=2 \cap D_1+D_2 \leq 5)}{P(D_1+D_2 \leq 5)} \Rightarrow \frac{\frac{3}{36}}{\frac{10}{36}}$$

Multiplication Rule

$$P(B) \times P_{A|B} = \frac{P(A \cap B)}{\cancel{P(B)}} \times \cancel{P(B)}$$

$$\downarrow$$

important

$$\boxed{P(A \cap B) = P(A/B) \times P(B)}$$

(Product / Multiplicate Rule)

$$P(B \cap A) = P(B/A) \times P(A)$$

Q $P(A/B)$ same as $P(B/A)$?

$\times$

Questions

Marginal and Joint probability

* Marginal probability (Unconditional Probs)

Ex1:

$P(D_1 = 2)$

$P(D_2 = 3)$

Ex2:

$P_{century}$

$P_{India\ win}$

* Joint probability

=> P of multiple Events occuring together

Ex1: $P(D = 2 \cap D_1 + D_2 \leq 5)$

$$\underbrace{D=2}_{A} \cap \underbrace{D_1+D_2 \leq 5}_{B}$$

Ex2:

$P_{c \cap w}$

$P_{c^c \cap w}$

* Conditional

Ex2: $P_{c|w}$ $P_{w|c}$ $P_{L|c}$

Tree Diagram Approach

Question: Email Spam System

- Let's say 30% of all Emails are Spam
- 70% are Non-Spam
- 80% of all spam Emails contain word 'purchase'
- 10% of Non-spam contains purchase
- Overall what % of Email will have word 'purchase'

Answer:

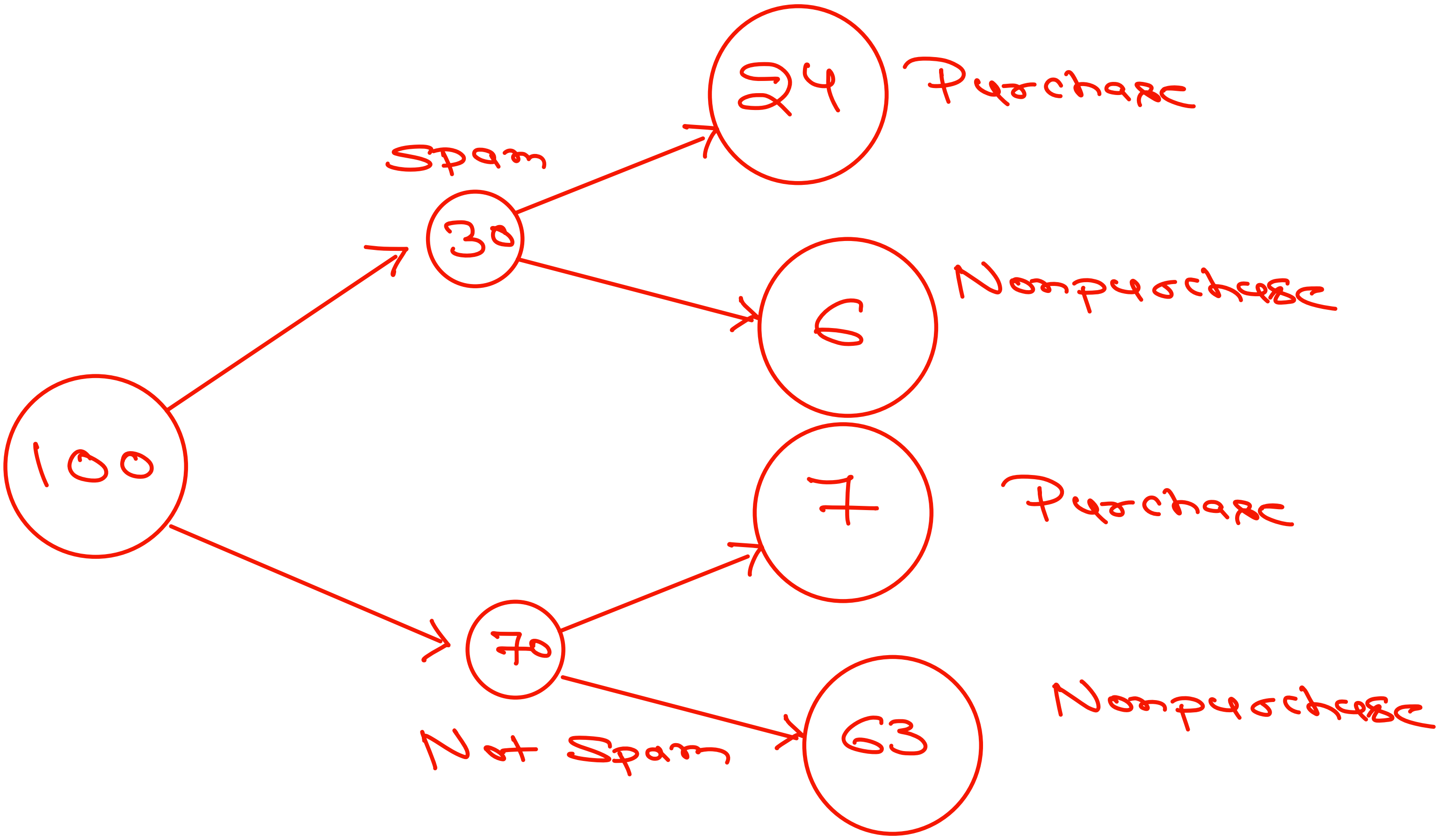
$$P_{Spam} = 0.3$$

$$P_{not\,spam} = P^{C}_{Spam} = 0.7$$

$$P_{purchase \cap spam} = 0.8$$

$$P_{purchase \cap Not\,Spam} = 0.1$$

$$P(\text{purchase}) \; ?$$



$$P_{purchase} \Rightarrow \frac{24+7}{100} \Rightarrow 0.31 \Rightarrow 31\%$$

$$\frac{24}{100} + \frac{7}{100}$$

$$P_{purchase}\ (Pur) \Rightarrow P(\text{purchase} \cap \text{Spam}) + P(\text{purchase} \cap \text{Non-spam})$$

$$\boxed{P(A \cap B) = P(A/B) \times P(B)}$$

$$P_{pyr} = P_{pyr/Spam} \times P_{Spam} + P_{pyr/Nonspam} \times P_{nonspam}$$

Type < Not Spam ← Type 1
Type < Spam ← Type 2

$$P_{pyr} = \sum_{i=1}^{2} P_{pyr/Type_i} \times Type_i$$

Law of Total probability

$$P(A) = \sum_{i=1}^{n} P(A/B_i) \times P(B_i)$$

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It is known that -
5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities.
Overall, what percentage of people are actively seeking new job opportunitie
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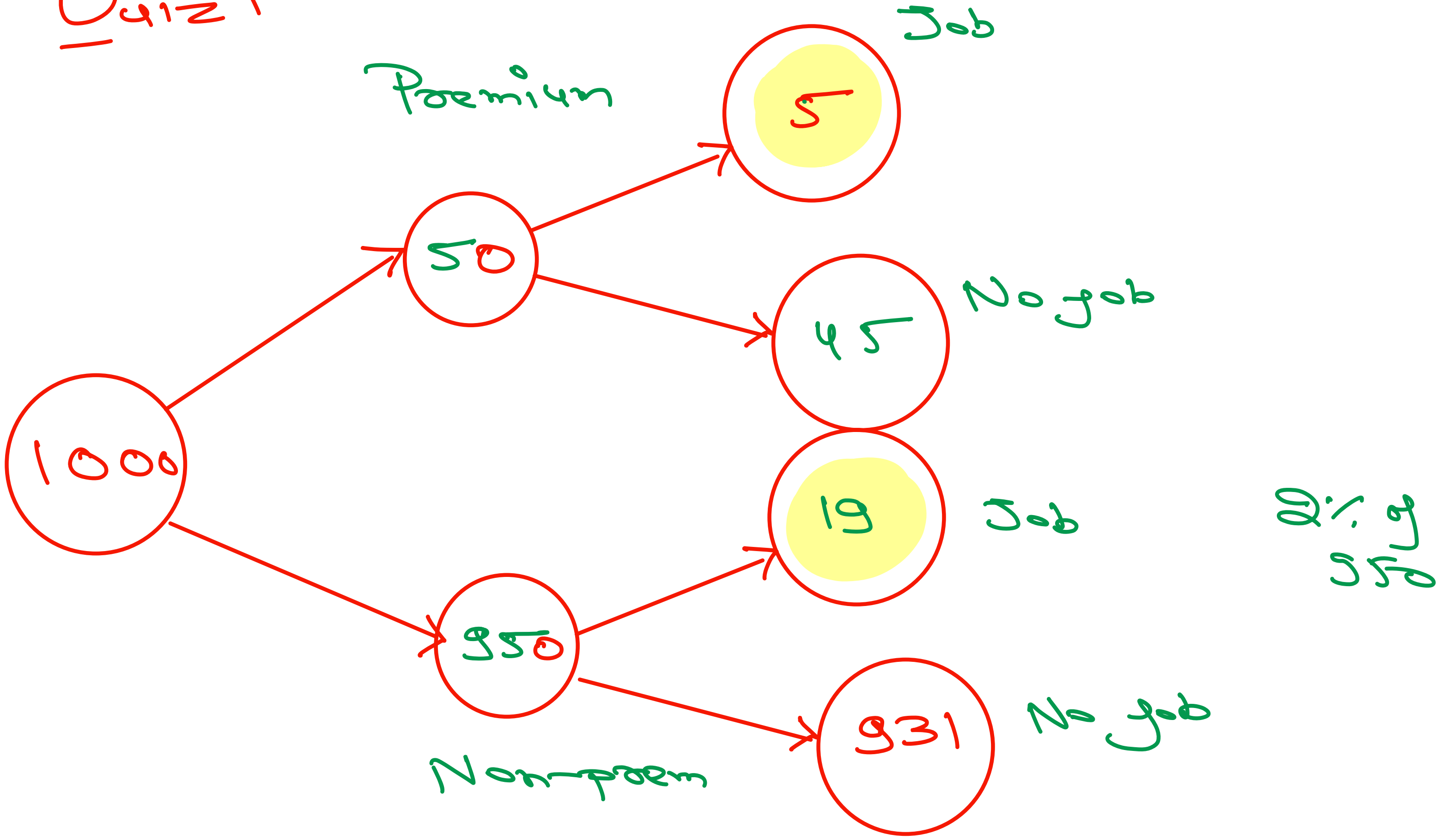
Quiz 1



$$\frac{24}{1000} = 2.4\%$$

Questions : Solve above Questions with help of Law of Total prob

1. Conditional Probability:

- $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

2. Multiplication Rule:

- $P(A \cap B) = P(A \mid B) \cdot P(B)$

3) Law of Total Probability:

- $P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$

Before starting a new topic, let's solve a few quizzes first.

Quiz

```
A disease affects 10% of the population.
Among those who have the disease, 80% get "positive" test result
Among those who don't have the disease, 5% get "positive" test result.
What is P(+ve | Disease) ?

a) 0.1
b) 0.8
c) 0.05
d) 0.85
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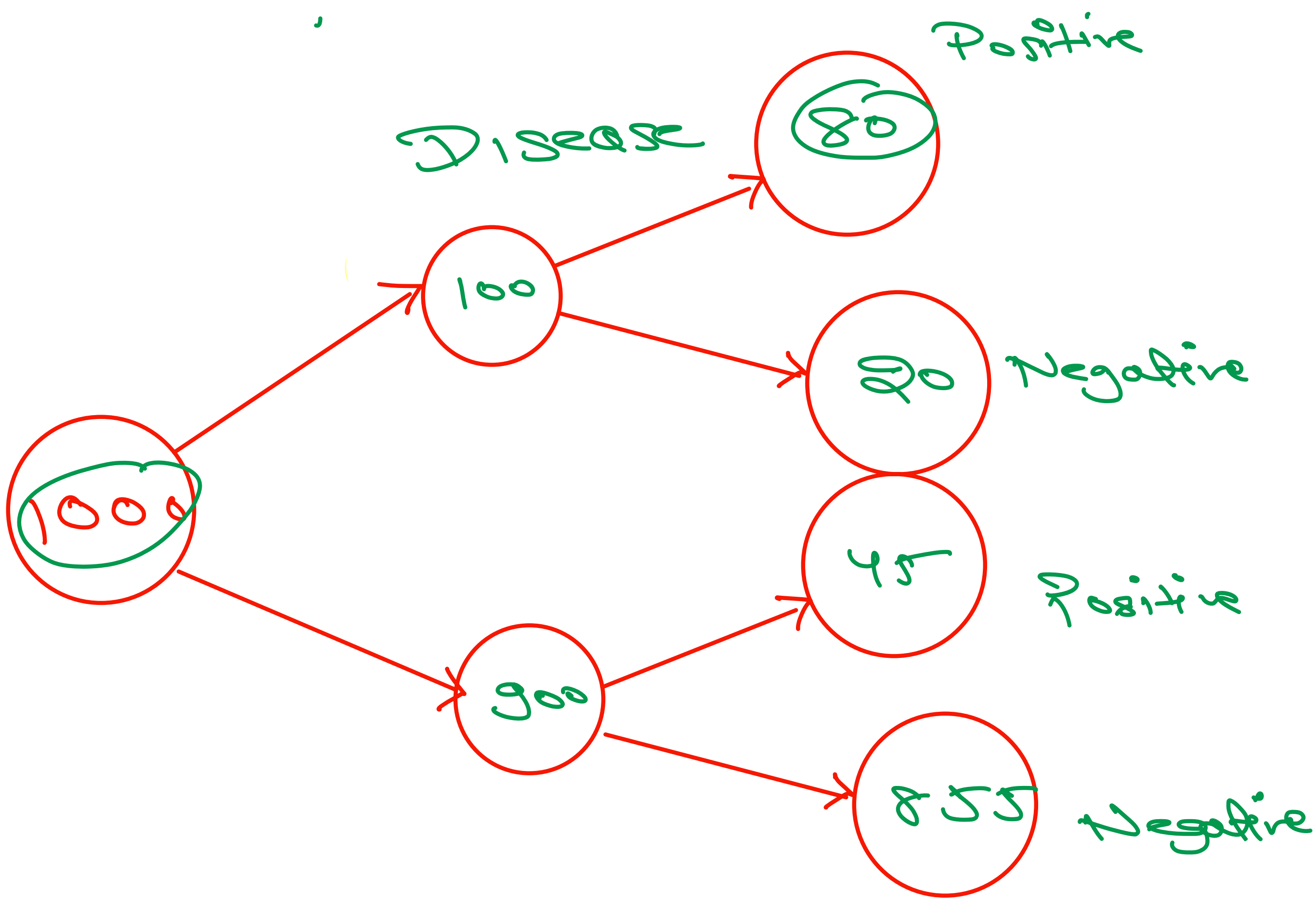

$$P_{+ve \cap Disease}$$

$$\Rightarrow \frac{80}{1000}$$

$$P_{+ve | Disease} \times P_{Disease}$$

$$0.8 \times \frac{100}{1000}$$

$$\downarrow$$

$$\frac{80}{1000}$$

$$P_{A \cap B} \Rightarrow P_{A/B} \times P_B$$

Baye's Theorem

$$P(A|B) \Rightarrow \frac{P(B|A) \times P(A)}{P(B)}$$

Questions : Derive Above formula

Hint :
$$P(A \cap B) = P(A/B) \times P(B)$$
$$P(B \cap A) = P(B/A) \times P(A)$$