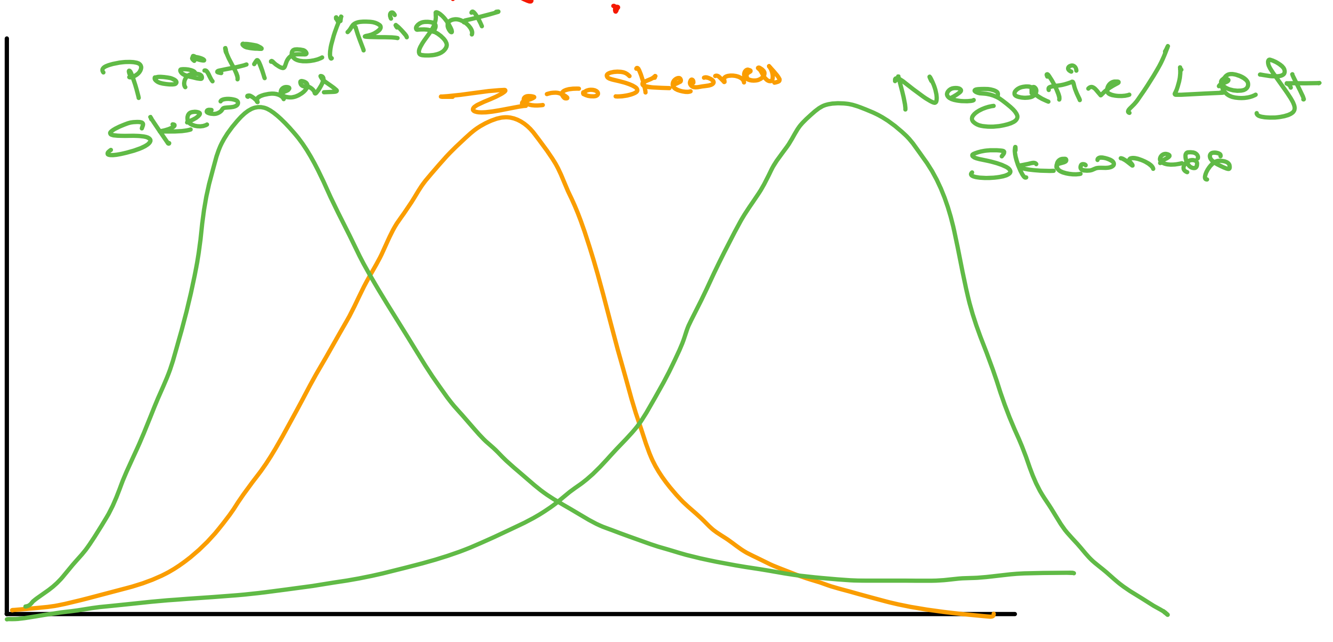
Log Normal Distributions

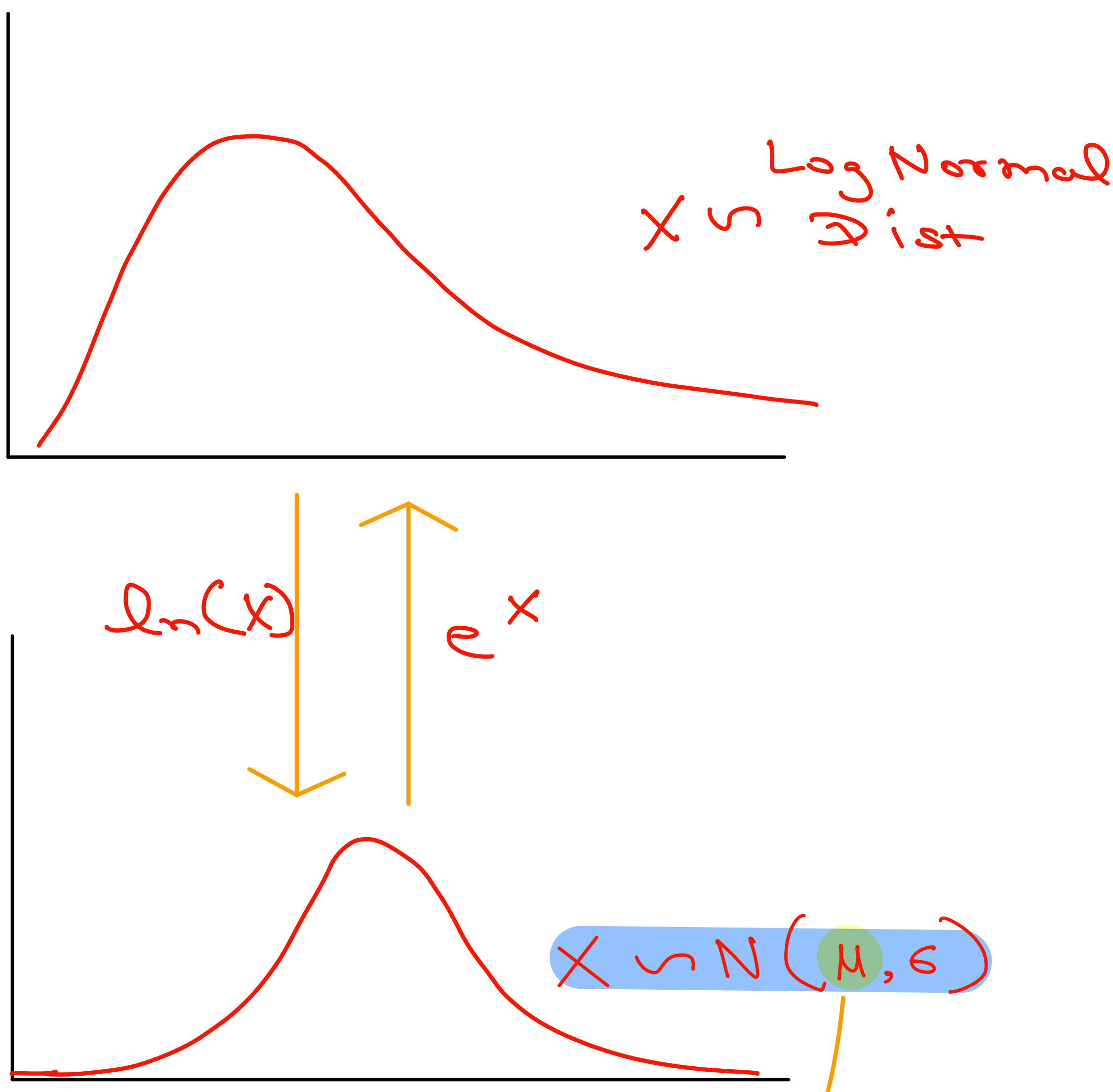
Case: You are collecting wait-time before the food parcel gets delivered to customer.

The LogNormal Distribution is a continuous probability Distribution that models the Right Skewed Data

Q: What is Skewness ?



Log Normal ⇄ Normal Dist



Log Normal Parameters

mean $= e^{\left(\mu + \frac{\sigma^2}{2}\right)}$

Var $= \left(e^{\sigma^2} - 1\right) e^{2\mu + \sigma^2}$

Key properties of Log Normal:

① Positive

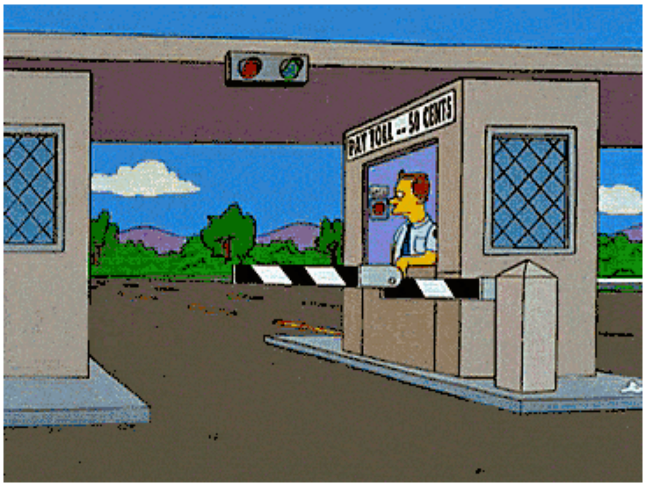
② Right Skewed

③ Multiplicative process

$X \rightarrow$ depends on multiple factors (Traffic, availability, parcel and prep-time)

$\boxed{\text{Poisson Distributions}}$

You are at a Toll booth Observe the number of vehicles passed for One hour.



λ = 30 vehicles per hour

Q What is P 40 vehicle will pass in Next One hour?

Q What is P more than 40 vehicle will pass in Next One hour?

Poisson Distribution models the number of Events that Occur in fixed interval of Time and Space

$\lambda \Rightarrow$ Rate or mean of successes during that specific interval

Scenario $\Rightarrow$ $\lambda = 30V/hr$

Q P(25 vehicles in 30 min)

Can we use λ as it is?

X

1 hr 60 mins $\Rightarrow$ $\frac{30}{2}$ $\Rightarrow$ 15 V/30 min

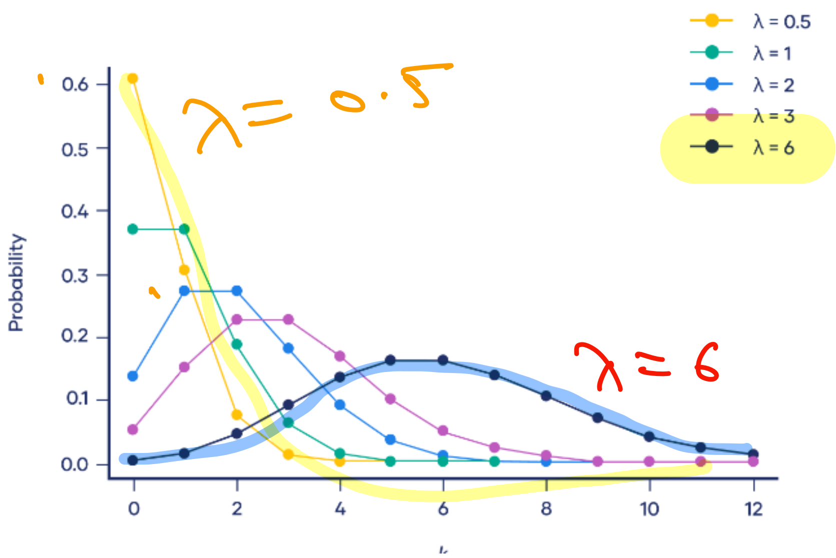
Poisson Distribution Formula

$$\boxed{PMF(X=k) \Rightarrow \frac{\lambda^{k} * e^{-\lambda}}{k!}}$$

k : Number of Successes

λ : Rate/mean of Successes per time

e : 2.71828 ...



Rules of Poisson Distribution

① Counting : Suitable for counting Discrete number of Event in fixed Time interval

② Independence : Every Event must be independent

③ Rate (λ or μ) : Fixed/Constant during Specific interval

④ No simultaneous Events: Two events must Not occur together. They would be counted as One Event.

Questions

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suppose a particular hospital experiences an average of 2 births per hour.
We can use the formula above to determine
the probability of experiencing 0, 1, 2, 3 births, etc. in a given hour:
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$\lambda = 2$ per hour

$$\frac{\lambda^k * e^{-\lambda}}{k!}$$

$$P(X=0) \Rightarrow \frac{2^0 * e^{-2}}{0!} \Rightarrow 0.135$$

$k=0$

$P(X=1) \Rightarrow 0.2707$

$P(X=2) \Rightarrow 0.2707$

$P(X=3) \Rightarrow 0.1804$

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Let "X" be the number of typos in a page in a printed book, with mean of 3 typos per page
What is the probability that a randomly selected page has atmost 1 typo?
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$P[X \leq 1] \Rightarrow P(X=0) + P(X=1)$

$CDF(X=1)$

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The shop is open for 8 hours. The average number of customers is 74 - assume Poisson
distributed.
(a) What is the probability that in 2 hours, there will be at most 15 customers?
(b)What is the probability that in 2 hours, there will be at least 7 customers?
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Resume @ 8:35

$\frac{74}{8} \times 2$

$\underline{Q}$ is $\lambda = 74$

ⓐ $\lambda = 18.5 \mid 2hr$ $\quad P(X \leq 15)$

ⓑ $\lambda = 18.5 \mid 2hr$ $\quad P(X \geq 7)$

$1 - P(X \leq 6)$

It is known that a certain website makes 10 sales per hour. In a given hour, what is the probability that the site makes exactly 8 sales?

$\longrightarrow \lambda = 10$

$\longrightarrow k = 8$

PMF

It is known that a certain hospital experience 4 births per hour.
In a given hour, what is the probability that 4 or less births occur?

$\rightarrow \lambda = 4$

$\rightarrow k \leq 4$

CDF

Poisson Distribution Approximation to Binomial Distribution

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There are 80 students in a kinder garden class.
Each one of them has 0.015 probability of forgetting their lunch on any given day.

(a) What is the average or expected number of students who forgot lunch in the class?
(b) What is the probability that exactly 3 of them will forget their lunch today?
```

Given

$p = 0.015$

ⓐ Expected Value $\Rightarrow \sum_{i=1}^{n} n_i \times p_i \Rightarrow 80 \times 0.015$
$\Rightarrow 1.2$

ⓑ $P(X=3)$

Let's Try Binomial Dist

$n \Rightarrow 80$

$k \Rightarrow 3$

$p \Rightarrow 0.015$

$$^{n}C_{k}\, p^{k}\, (1-p)^{n-k}$$

$$^{80}C_{3}(0.015)^{3}\,(1-0.015)^{80-3}$$

$\Rightarrow 0.08$

Poisson:

$\lambda = ?$

Can we use Expected Value as Rate?

Yes if it satisfies some Condn

$$\lambda = np$$

$$\frac{(np)^{3} * e^{-(np)}}{k} \qquad \frac{\lambda^{k} * e^{-\lambda}}{k!}$$

Poisson Distribution can be used as approximation for Binomial Distribution with $\lambda = n \times p$

Give following Condition:

Any of these 3 must be True:

- This approximation is good if $n \geq 20$ and $p \leq 0.05$ such that $np \leq 1$,
- or if $n > 50$ and $p < 0.1$ such that $np < 5$,
- or if $n \geq 100$ and $np \leq 10$.

$n = 80$

$p \Rightarrow 0.015$

$np \Rightarrow 1.2$

Estimation with Bootstrap?