Methodology

This section describes the systematic process by which the AI-driven budget optimization model is developed and evaluated. We proceed in five stages: (1) Data Collection & Preprocessing; (2) Regression Analysis for Lever Identification; (3) Markov Decision Process (MDP) Formulation; (4) Policy Computation; and (5) Monte Carlo Stress-Testing.

1. Data Collection & Preprocessing

- **Scope & Sources:** Annual budget records over the past 5–10 years, including Approved Budget B^{app}_t, Released Budget B^{rel}_t, and Actual Expenditure B^{act}_t for each sector.
- **Macroeconomic indicators:** GDP (G_t), inflation rate (π_t), and sector-specific performance measures (e.g., enrolment, health outcomes).

- **Cleaning & Imputation:** Identify missing entries in B^{rel}_t or B^{act}_t. Impute using a time-series–aware approach (e.g., linear interpolation or Kalman filter). Flag imputed values and record an indicator variable $\delta^{imp}_t \in \{0,1\}$.

- **Feature Engineering:** Compute Derived Metrics and normalize variables:

```
E_t = B_t^act / B_t^app
E^GDP_t = B_t^act / G_t
ΔG_t = (G_t - G_{t-1}) / G_{t-1}
Normalize variables to zero mean and unit variance for regression stability.
```

2. Regression Analysis for Lever Identification

Model Specification: We model the key outcome $Y_t = E^{GDP}_t$ as a function of candidate drivers $X_t = [\Delta G_t, E_{t-1}, \delta^{imp}_t, \ldots]$.

```
Y_t = β_0 + Σ_i β_i X_{t,i} + ε_t,   ε_t ~ N(0, σ^2)
```

Estimation & Selection: Fit an Elastic Net regression (L1 + L2 penalties):

```
min_β (1/T) Σ_{t=1}^T (Y_t - β^T X_t)^2 + λ_1 ||β||_1 + λ_2 ||β||_2^2
Select significant coefficients {β_i ≠ 0} as decision levers a_i.
```

3. Markov Decision Process Formulation

Define the tuple <S, A, T, R, γ>:

- **States (S):** Discrete fiscal states {Under■Spend, On■Track, Over■Commit}.
- **Actions (A):** Budget adjustments from regression levers (e.g., Increase/Decrease by 5%).

- **Transition (T):** Use regression to estimate ■_{t+1} and map to next state, $T(s'|s,a)$.
- **Reward (R):** $R(s,a) = w_1 E_{t+1} - w_2$ ■$\{E_{t+1}<\alpha\}$.
- **Discount (γ):** Discount factor (e.g., $\gamma=0.95$).

4. Policy Computation

Solve for the optimal policy $\pi^*: S \to A$ via Value Iteration:

```
Initialize V_0(s) = 0 for all s ∈ S
Repeat until convergence:
  V_{k+1}(s) = max_a [ R(s,a) + γ Σ_{s'} T(s'|s,a) V_k(s') ]
π*(s) = argmax_a [ R(s,a) + γ Σ_{s'} T(s'|s,a) V*(s') ]
Convergence: ||V_{k+1} - V_k||_∞ < ε.
```

5. Monte Carlo Stress-Testing

Uncertainty Characterization: Fit distributions to regression residuals ε_t.

Simulation Protocol: For N runs (e.g., 10,000):

- Sample random shocks ε■ each period.
- Begin from state s_0 and apply $\pi^*(s)$.
- Propagate via $T(s'|s,a)$ with sampled shocks; record $\{E^{GDP}_t\}$.

Outcome Analysis: Compute mean execution, Value-at-Risk (VaR), expected shortfall; identify policy failure scenarios ($E_t < \alpha$).