6.3 Gas Exchange in the Lungs

Learning Objectives

- Distinguish between respiratory quotient and respiratory exchange ratio
- List the three steps in the uptake of oxygen by the blood
- Write the Ideal Gas Equation
- Define partial pressure of a gas
- Define vapor pressure
- Give the vapor pressure of water at body temperature
- Write Henry's law for dissolution of gas in aqueous solutions
- Define what is meant by STPD
- Be able to convert partial pressures and volumes at STPD to BTPS
- List the sequential barriers to diffusion that are present between air and blood in the lungs
- List the factors that make gas exchange in the lungs rapid
- Describe the driving force for gas movement
- Define the diffusing capacity of a gas
- Define the anatomic dead space
- Distinguish between alveolar ventilation and pulmonary ventilation
- Write the alveolar ventilation equation
- Recognize the alveolar gas equation and identify its variables
- Explain why alveolar gas composition is relatively constant
- Describe the time course of gas equilibration across the alveoli

THE RESPIRATORY SYSTEM EXCHANGES BLOOD GASES WITH ATMOSPHERIC GASES

As pointed out in Chapter 6.1, cellular metabolism of carbohydrates and fats entails their complete oxidation. The overall reactions are written as

$$C_6H_{12}O_6 + 6O_2 \Rightarrow 6CO_2 + 6H_2O + \text{heat}$$

$$C_{51}H_{98}O_6 + 72.5O_2 \Rightarrow 51CO_2 + 49H_2O + \text{heat}$$

During constant metabolic conditions, there is a constant production of CO_2 and constant consumption of O_2. The ratio of these two is given a special name, the **respiratory quotient**. It is defined as

$$[6.3.1] \qquad R = \frac{Q_{CO_2}}{Q_{O_2}}$$

where Q_{CO_2} is the rate of production of CO_2, in moles per unit time or proportional units, and Q_{O_2} is the rate of consumption of O_2, in the same units as Q_{CO_2}. The value of R depends on the type of materials being oxidized for energy. When carbohydrate is the only fuel being oxidized for energy, the respiratory quotient is 1.0: there is a molecule of CO_2 produced for every molecule of oxygen consumed. When fats alone are being oxidized, the respiratory quotient is $51/72.5 = 0.70$. Protein oxidation produces a respiratory quotient intermediate between these extremes. Normally the body consumes a mixture of carbohydrates, fats, and proteins, and a typical value for R is 0.80. Its range is $0.7 < R < 1.0$.

The **respiratory exchange ratio** is the ratio of the rate of CO_2 to O_2 exchanged between the body and the atmosphere. At steady state, this is the same as the ratio of gas produced and consumed by the tissues, the respiratory quotient. However, the transient state between one metabolic state and another involves storage or depletion of gases in the tissues and blood and for short times the respiratory exchange ratio can be different from the respiratory quotient.

An understanding of gas exchange requires understanding the behavior of gases both in the gas phase and dissolved in the aqueous phase. We begin with a review of these principles.

THE PARTIAL PRESSURE OF A GAS IS ITS MOLE FRACTION TIMES THE TOTAL PRESSURE

A gas contains an assortment of molecules that expands to fill any container. The gas molecules move rapidly in random directions until they collide with each other or with the walls of the container. When they hit the sides of the wall they rebound, thereby undergoing a change in momentum. This change in momentum corresponds to force exerted on the molecule *by* the wall, which is equal but opposite in direction to the force of the molecule *on* the wall. When many molecules collide with the

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TABLE 6.3.1 Components of Air and Partial Pressure at STPD (1 atm pressure, 0°C with 0% Humidity)

Component of Air	Content in Volume Percent	Content in Mole Fraction	Partial Pressure (mmHg)
N_2	78.08	0.7808	593.5
O_2	20.95	0.2095	159.2
Ar	0.93	0.0093	7.1
CO_2	0.03	0.0003	0.2
H_2O	0.00	0.00	0.0
Total	100.00	1.0000	760.0

wall, they produce an average force distributed over an area. This is the pressure, the force per unit area. The velocity, and hence the momentum, of the molecules increases with the temperature. These ideas are subsumed into the **Ideal Gas Equation**:

$$PV = nRT \tag{6.3.2}$$

Here P is the pressure in atmospheres or mmHg or pascals ($1\ \text{Pa} = 1\ \text{N m}^{-2}$), V is the volume in L, n is the number of moles of gas, R is the gas constant ($0.082\ \text{L atm mol}^{-1}\ \text{K}^{-1}$; $1.986\ \text{cal mol}^{-1}\ \text{K}^{-1}$; $8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$), and T is the temperature in K. This can be rewritten easily as

$$P = \frac{n}{V}RT \tag{6.3.3}$$

where n/V is the concentration of the gas. The **Ideal Gas Equation** given here is approximately true for a variety of gases, including O_2, N_2, He, CO_2, and many other gases. These gases are not ideal and the corrections for nonideality depend on the kind of gas.

If a gas consists of only one kind of molecule, adding successive increments of gas adds incremental pressure according to the number of moles in the increment. Thus if pressure P_1 is produced by n_1 moles of the gas, then adding n_2 moles to the same volume adds pressure P_2:

$$\begin{aligned} P &= \frac{n}{V}RT \\ &= \frac{n_1 + n_2}{V}RT \\ &= \frac{n_1}{V}RT + \frac{n_2}{V}RT \\ &= P_1 + P_2 \end{aligned} \tag{6.3.4}$$

Thus we can partition the total pressure in a gas according to the number of moles of gas that contributes to the total pressure. In a mixture of gases, we ought to expect the same additive properties of the pressure. Suppose now that we have a mixture of n_A moles of gas A and n_C moles of gas C. According to Eqn [6.3.4], we would have a total pressure given as

$$\begin{aligned} P &= \frac{n}{V}RT \\ &= \frac{n_A}{V}RT + \frac{n_C}{V}RT \\ &= P_A + P_C \\ &= \frac{n_A}{n}\frac{n}{V}RT + \frac{n_C}{n}\frac{n}{V}RT \end{aligned} \tag{6.3.5}$$

Here the pressure contributed by each gas is called the **partial pressure**. Each gas in a mixture contributes to the total pressure according to its concentration. Because the concentrations of all gases at the same P and T are proportional to the number of molecules, within the limits of ideal behavior, the concentrations in a mixture are proportional to the **mole fractions**. This is evident from the last expression in Eqn [6.3.5], from which we identify the partial pressure of A as

$$P_A = \frac{n_A}{n}P_B = f_A P_B \tag{6.3.6}$$

where f_A is the **mole fraction** of gas A, defined as the ratio of the number of moles of A to the total number of moles in the mixture, and P_B is the total or barometric pressure. Thus from Eqn [6.3.5], we see that the total pressure is the sum of the partial pressures which, by Eqn [6.3.6], is proportional to the mole fractions. The mole fraction and partial pressure of each kind of gas in the atmosphere at sea level (atmospheric pressure = 1 atm = 760 mmHg) are given in Table 6.3.1.

Because N_2 gas is inert with respect to body metabolism, the partial pressures of all gases other than H_2O vapor, O_2, and CO_2 are typically included in the contribution made by N_2. Although air contains Ar and a variety of other gases including He, H_2, Ne, Kr, CH_4, and NO_2, we will lump the partial pressures of all these gases into that made by N_2. Thus we write

$$P_{\text{total}} = P_{CO_2} + P_{O_2} + P_{N_2} + P_{H_2O} \tag{6.3.7}$$

THE VAPOR PRESSURE OF WATER IS THE PARTIAL PRESSURE OF WATER IN THE GAS PHASE THAT IS IN EQUILIBRIUM WITH LIQUID WATER

Water molecules in the gas phase exert a pressure just like all other molecules. If a gas is placed in contact with liquid water in a closed container, water will evaporate until an equilibrium is reached in which the rate of water evaporation is equal to its rate of condensation. The gas phase at this equilibrium will be **saturated** with water vapor. The partial pressure of water in the gas phase under these conditions is *defined* as the vapor pressure (see Figure 6.3.1). Water can evaporate into a gas phase that is not in a closed container. In this case, the gas phase may or may not be saturated with water vapor. If there is any water in the gas phase, its partial pressure will be determined by its mole fraction as described in Eqn [6.3.6].

THE VAPOR PRESSURE OF WATER AT BODY TEMPERATURE IS 47 MMHG

At 37°C = 310 K, the vapor pressure of water is 47 mmHg. If we inspire dry air, the mucus membranes lining the nasal passages and airways will moisten the air by adding water. Generally, the air that reaches the alveoli has already become saturated with water vapor at body temperature. The added water expands the gas phase and dilutes all other gaseous components. If we assume that the inspired air is completely dry, then the partial pressures of gases in moist tracheal air will be given by

$$P_A = f_A(P_B - P_{H_2O}) \quad [6.3.8]$$

where P_B is the barometric pressure. It is important to recognize that this equation converts the mole fraction in dry air, f_A, to the partial pressure in water-saturated air, P_A, in this equation. Thus humidifying the inspired air reduces the partial pressures of all of the gas components. The mole fractions and partial pressures of moist tracheal air are shown in Table 6.3.2 along with these values for dry atmospheric air. The values for P_{N_2} differ from those in Table 6.3.1 because it also includes the partial pressure of argon and other rare atmospheric gases.

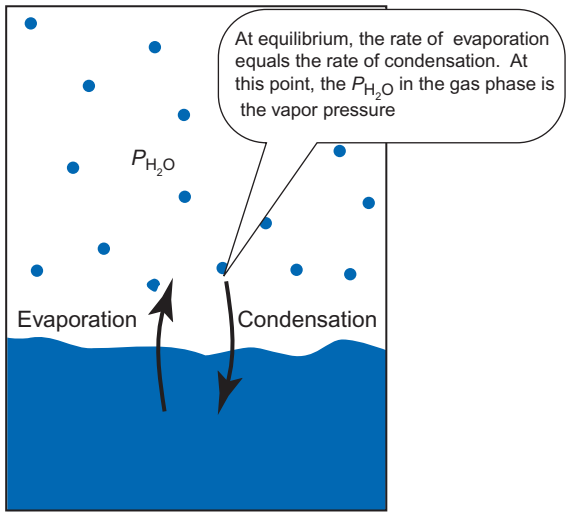

FIGURE 6.3.1 Water vapor pressure. Liquid water in contact with the gas phase will evaporate (form water vapor) until there is a dynamic equilibrium between the rate of evaporation and the rate of condensation. At this point of equilibrium, the partial pressure of water in the gas phase is equal to the vapor pressure. Heating the water increases the rate of evaporation and so raises the vapor pressure. The boiling point of water is the temperature at which the vapor pressure is equal to the atmospheric pressure.

The **relative humidity** is a common measure of the amount of water in the gaseous phase. It is defined as the ratio of the measured partial pressure of water to the vapor pressure of water.

HENRY'S LAW DESCRIBES THE DISSOLUTION OF GASES IN WATER

Henry's law, discovered in 1803, relates the mole fraction of gas that is dissolved in the liquid, watery phase to the partial pressure:

$$x_A = \beta_A P_A \quad [6.3.9]$$

where x_A is the mole fraction of gas A in the aqueous phase, β_A is its solubility, and P_A is the partial pressure of the gas in the gaseous phase in equilibrium with the aqueous phase, in atmospheres. Physiologists typically convert partial pressures to units of mmHg. The mole fraction is simply the ratio of the moles of gas to the total number of moles in the mixture. For dilute solutions of gases, when n_A, the number of moles of gas, is very much less than n_W, the number of moles of water, the mole fraction is linearly related to the concentration of gas:

$$x_A = \frac{n_A}{n_A + n_W} \approx \frac{n_A}{n_W} = \frac{n_A}{V/\overline{V}_W} = \overline{V}_W \frac{n_A}{V} \quad [6.3.10]$$

where $\overline{V}_W$ is the volume of water per mole (its partial molar volume) and n_A and n_W are the moles of gas A and water, respectively, in the mixture. Inserting this result into Eqn [6.3.9], we get

$$\overline{V}_W \frac{n_A}{V} = \beta_A P_A$$

$$[A] = \frac{\beta_A}{\overline{V}_W} P_A \quad [6.3.11]$$

$$[A] = \alpha_A P_A$$

TABLE 6.3.2 Composition and Partial Pressure of Gases in Dry Air and Moist Tracheal Air

Gas	Dry Inspired Atmospheric Air		Moist Tracheal Air	
	Mole Fraction	Partial Pressure (mmHg)	Mole Fraction	Partial Pressure (mmHg)
N_2 (with Ar, etc.)	0.7901	600.6	0.7412	563.3
O_2	0.2095	159.2	0.1966	149.4
CO_2	0.0003	0.2	0.0003	0.2
H_2O	0.0	0	0.0618	47.0
Total	1.0	760.0	1.0	760.0

TABLE 6.3.3 Solubilities of Gases of Physiological Interest at 37°C and STPD

Gas	β (mole fraction atm^{-1})	α (mL gas mL $water^{-1}$ atm^{-1})	α (mL gas dL $water^{-1}$ $mmHg^{-1}$)
N_2	1.026×10^{-5}	0.0127	0.00169
O_2	1.932×10^{-5}	0.0241	0.00317
CO_2	4.560×10^{-4}	0.567	0.0747
CO	1.524×10^{-5}	0.01897	0.00250

Solubilities were calculated by extrapolation of β versus temperature or from formulas of $\beta(T)$ given in *Handbook of Chemistry and Physics*, 73rd ed., CRC Press Inc., Baton Rouge, 1992.

From Eqn [6.3.11], we can relate the molar concentration of gas to its partial pressure. However, physiologists use a particular unit of concentration which is mL of gas per unit volume of solution, and the volume of solution is usually expressed in deciliters (dL = 100 mL = 0.1 L). Because the volume of a gas depends on the pressure and temperature and the water content, the volumes of gases used in respiratory physiology are often expressed as the volume the gas would occupy under standard conditions. These standard conditions are referred to as **STPD**, meaning **s**tandard **t**emperature and **p**ressure, **d**ry. STPD refers to the conditions of 0°C = 273.16 K, 1 atm pressure = 760 mmHg and zero water vapor. Under these conditions, the molar volume of the ideal gas is 22.4 L mol^{-1}. Because of these choices of units, the final solubility factors that we will use are in units of mL dL^{-1} $mmHg^{-1}$. The solubility of gases depends strongly on the temperature. The values for the solubilities given in Table 6.3.3 are for water at 37°C = 310.2 K, with gas volumes at STPD.

CONVERSION OF PARTIAL PRESSURES AND VOLUMES AT STPD TO THOSE AT BTPS

As noted above, the rates of O_2 consumption and CO_2 production are expressed in units of volume at STPD or standard temperature and pressure, dry. The lung volumes and rates of movement of lung volumes, on the other hand, are measured at BTPS or body temperature and pressure, saturated. In addition, sometimes measurements are made at ATPS—ambient temperature and pressure, saturated. This is the case for the measurements made with the spirometer. Conversion of these volumes follows the principles described in Appendix 6.3.A2.

GASES DIFFUSE ACROSS THE ALVEOLAR MEMBRANE PASSIVELY

There are no carriers or pumps for gases. Until recently, the model of membrane permeation by gases was by dissolution within the membrane and diffusion through the lipid core. However, recent discoveries suggest that cholesterol reduces CO_2 permeability of lipid bilayers and suggest that proteins may mediate CO_2 diffusion. These proteins likely act as gas channels. AQP1 is one likely candidate for a CO_2 channel. At the present time there is no evidence for channels for O_2. Regardless of the path, the passive flux can be described by

$$J_s = p\Delta C \quad [6.3.12]$$

where p is the permeability, J_s is the flux of solute s, which in this case is a gas such as O_2 or CO_2, and ΔC is the concentration difference across the alveolar membrane. This equation was developed to describe passive transport across a microporous membrane or a lipid bilayer membrane (see Eqn [2.5.9]). It describes diffusion kinetics: the overall flux depends linearly on the concentration difference and shows no saturation. This equation applies to diffusion of gas through any single phase, but it must be modified when diffusion occurs across phases due to the different solubility of gases within those phases. The result (see Eqn [6.3.14]) shows that **the partial pressure of gas drives diffusion**.

Eqn [6.3.11] states that the concentration of gas in the aqueous phase is linearly related to the partial pressure of gas in equilibrium with it. Thus the ΔC in Eqn [6.3.12] is linearly related to ΔP_s, the difference in partial pressure across the alveolar membrane.

In traveling from the alveolar air to the blood, O_2 encounters a variety of barriers. These are shown schematically in Figure 6.3.2. These barriers include:

- the alveolar lining, extracellular to the alveolar cell;
- alveolar cell, consisting of two plasma membranes and intervening cytosol;
- interstitial fluid;
- endothelial cell, consisting of two plasma membranes and intervening cytosol;
- plasma;
- red blood cell membrane.

The barriers include the watery lining of the alveoli that contains surfactant, the alveolar cell, the interstitial fluid, the endothelial cell, the plasma, and the red blood cell membrane.

Figure 6.3.2 illustrates how the diffusional barrier between air and blood is made up of many sequential layers. Each of these layers is characterized by an equation like Eqn [6.3.12]. The permeabilities of each layer depend on a variety of factors. We may model diffusion of gas across each layer similarly to the penetration of lipid soluble solutes through lipid bilayer membranes

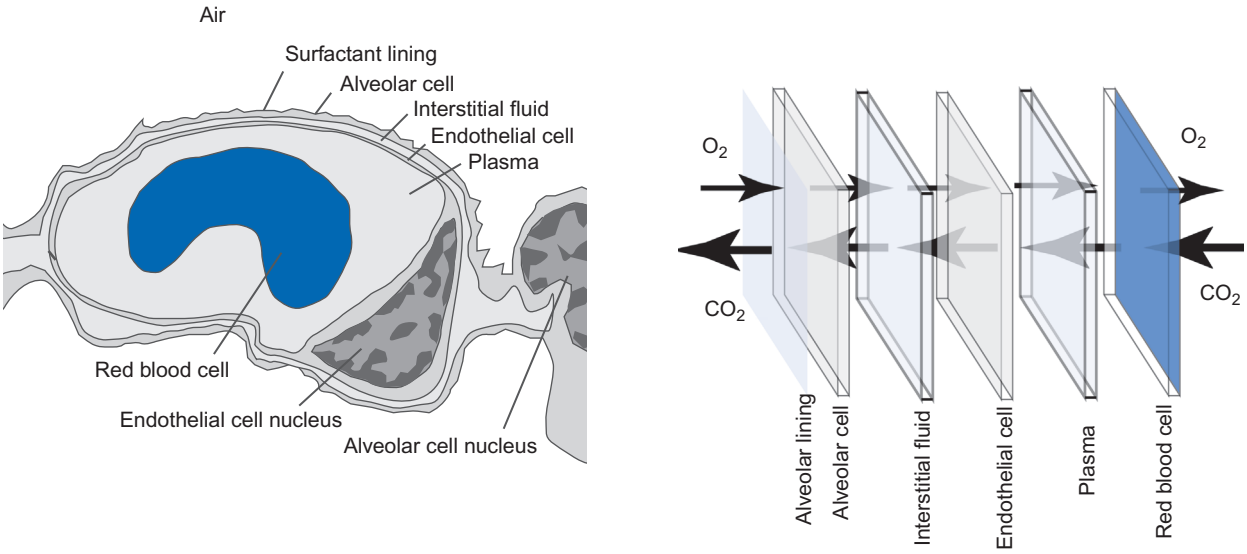

FIGURE 6.3.2 Alveolar membrane with successive barriers to diffusional transfer of gases from air to blood.

(see Chapter 2.5). In this case, the permeability is given by

$$[6.3.13] \qquad p = \frac{k_s D_s}{\delta}$$

where k_s is the partition coefficient, D_s is the diffusion coefficient, and δ is the thickness of the barrier. In the case of gas penetration through the alveolar membrane, the partition coefficient is related to the solubility. The diffusion coefficient that we use in Eqn [6.3.13] must be the one that pertains through the barrier. In the case of the respiratory gases, it is the diffusion coefficient either through the membrane lipids or through the watery phase making up the interstitial fluid, or the cytosol of the alveolar cell, and so on. It is not the diffusion coefficient in the gaseous phase. The permeability is enhanced through each layer by:

- *High solubility*. Gases that dissolve rapidly and to high concentrations (have large k_s) produce larger concentrations of materials that enhance the flux.
- *Large diffusion coefficients*, D_s. These are fixed by physical characteristics of the gases and are not physiological variables.
- *Short diffusion distances*, δ. The overall separation of the air from the blood is amazingly tiny. Typically the entire alveolar membrane is only 0.5×10^{-4} cm = 0.5 μm thick.

The flux of gas across a single barrier is given by Eqn [6.3.12]. This flux is the amount of gas that diffuses per unit time per unit area of membrane. This is an intensive property that does not depend on how large the surface is. The flow of gas, on the other hand, is an extensive property given by

$$[6.3.14] \qquad Q_s = \frac{A k_s \alpha_s D_s}{\delta} \Delta P_s$$

Here the terms $k_s\alpha_s$ enable us to express the flow in terms of the observable partial pressure of the gas. Each of the terms in Eqn [6.3.14] affects the total flow. The system is adapted for rapid flow by the magnitude of these factors:

- Because of the large number of alveoli, the total surface area for gas exchange is about 70 m^2. **This large surface area increases the flow proportionate to the area**.
- **The diffusion distance, δ, is small**. As mentioned for the flux across each layer, short diffusion distances increase the flux. They do the same here for the flow.
- **The physical behavior of gases and interaction with alveolar components sets the solubility and partition coefficients.** Because CO_2 has a higher solubility than O_2 (see Table 6.3.3), the flow of CO_2 for a given partial pressure difference is 20 times faster than the flow of O_2.
- **The pressure gradients drive flow**. Even though CO_2 can diffuse more rapidly per unit pressure difference, larger pressure differences of O_2 can assure the same rate of flow. Because the rate of O_2 consumption is nearly equal to the rate of CO_2 production (the respiratory quotient, R, 0.8), the exchange of O_2 is in fact about the same as that of CO_2.

THE *DIFFUSING CAPACITY* IS THE FLOW PER UNIT PARTIAL PRESSURE

The permeability of a complex sandwich of barriers is an algebraic function of all of the individual permeabilities. The permeabilities act like conductances. In a series of barriers such as occurs in the alveolar membranes, the resistances add; thus the total permeability for a series of six barriers that comprise the alveolar membrane becomes a function of all of the permeabilities. It is not practical to measure all of the terms in Eqn [6.3.14] that help determine Q_s. Instead, these terms can be lumped into a single term, the diffusing capacity. It is given by

$$[6.3.15] \qquad Q_s = D_L \Delta P$$

where D_L is the diffusing capacity. Its units are in mL of gas min^{-1} $mmHg^{-1}$, and it is calculated as $Q_s/\Delta P$. Each gas has its own diffusing capacity because the solubility and diffusion coefficients for each gas are different. The diffusing capacity is not constant because the effective surface area for gas exchange can be altered by increasing the blood perfusion of underperfused areas of the lung. At rest the diffusing capacity of O_2 is about 21 mL min^{-1} $mmHg^{-1}$ and that of CO_2 is about 20 times higher. The greater diffusing capacity of CO_2 derives from its much higher solubility. During exercise, $D_{L_{O_2}}$ increases to as much as 65 mL min^{-1} $mmHg^{-1}$.

THE *ANATOMIC DEAD SPACE* REDUCES THE VOLUME OF INSPIRED AIR THAT EXCHANGES WITH THE BLOOD

As described in Chapter 6.2, each breath at rest moves a tidal volume of air into the lungs. Some of this volume replaces air within the trachea and upper parts of the tracheobronchial tree that do not exchange with the blood. The volume of gas that does not exchange is said to occupy the **anatomic dead space**. The **alveolar ventilation** can be calculated as

$$[6.3.16] \qquad Q_A = \nu_R(V_T - V_D)$$

where Q_A is the alveolar ventilation, in units of L min^{-1}, ν_R is the respiration rate, V_T is the tidal volume, and V_D is the volume of the anatomic dead space. The anatomic dead space is typically about 150 mL. Typical alveolar ventilation at 12 breaths min^{-1} with $V_T = 500$ mL and $V_D = 150$ mL is 12 min^{-1} $(0.5-0.15\ L) = 4.2$ L min^{-1}. The alveolar ventilation differs from the pulmonary ventilation, which is $\nu_R V_T$, by an amount equal to the dead space ventilation.

Recall from Chapter 6.2 that the functional residual capacity is the volume of air left in the lungs after a normal expiration. The FRC is the sum of the expiratory reserve volume (ERV) and the residual volume (RV). Although these volumes vary individually, typical values are 1.1 L for ERV and 1.2 L for RV, and the FRC is about 2.3 L. Since $V_T = 0.5$ L and $V_D = 0.15$ L, each breath adds 0.35 L of new atmospheric air to 2.3 L of alveolar air. Because each breath renews only a small fraction of alveolar air, the composition of alveolar air remains fairly constant. This idea is presented schematically in Figure 6.3.3.

The consequence of the functional residual capacity is that expired air is not devoid of O_2 nor is the CO_2 levels in expired air as high as that in alveolar air. As a result, rescue breathing ("mouth-to-mouth rescuscitation") is possible. These facts have led to revision of the standards for CPR, cardiopulmonary resuscitation. Since alveolar air is only gradually altered by ventilation, it is more important in CPR to focus on the cardiac part than the pulmonary part, and emphasis is placed on external stimulation of circulation with fewer rescue breaths. The current recommendation is 30 chest pumps to 2 rescue breaths.

PHYSIOLOGIC DEAD SPACE IS LARGER THAN THE ANATOMIC DEAD SPACE

The rate of CO_2 elimination in the expired air is the rate of CO_2 expiration minus the rate of CO_2 inspired. Since the expired air originates in part from the alveolar air, we can write

$$[6.3.17] \qquad Q_E f_{E_{CO_2}} - Q_I f_{I_{CO_2}} = Q_A f_{A_{CO_2}} - Q_A^* f_{I_{CO_2}}$$

where Q indicates flow of gas, in L min^{-1}, and f indicates a mole fraction that is proportional to the partial pressure of gas. Here Q_E is the flow of expired gas, Q_I is the flow of inspired gas, Q_A is the flow of *expired* air that exchanges with blood, Q_A^* is the flow of *inspired* air that exchanges, and the various values of f correspond. Since $f_{I_{CO_2}}$ is near zero, we can eliminate its terms from Eqn [6.3.17]. The flow of expired air is just the respiratory rate times the volume of expired air per breath: $Q_E = \nu_R V_T$, where V_T is the tidal volume. The alveolar ventilation, Q_A, is similarly given in Eqn [6.3.16] as $\nu_R(V_T - V_D)$. Thus Eqn [6.3.17] becomes

$$[6.3.18] \qquad V_T f_{E_{CO_2}} = (V_T - V_D) f_{A_{CO_2}}$$

We can rearrange this equation to solve for V_D, the **physiologic dead space**:

$$[6.3.19] \qquad V_D = V_T \frac{(f_{A_{CO_2}} - f_{E_{CO_2}})}{f_{A_{CO_2}}} = V_T \left[1 - \frac{f_{E_{CO_2}}}{f_{A_{CO_2}}}\right]$$

In the practical determination of V_D, the mole fractions are replaced by the partial pressures of CO_2 in their respective volumes. The physiologic dead space defined by Eqn [6.3.19] is always larger than the anatomic dead space and reflects areas of the lungs that are underperfused with blood. Gas exchange is complete in these areas, but additional blood flow would result in additional gas exchange, lower $P_{A_{O_2}}$ and higher $P_{A_{CO_2}}$. The "wasted volume" is the difference between the physiological dead space and the anatomic dead space. Thus the anatomic dead space is the volume of ventilation

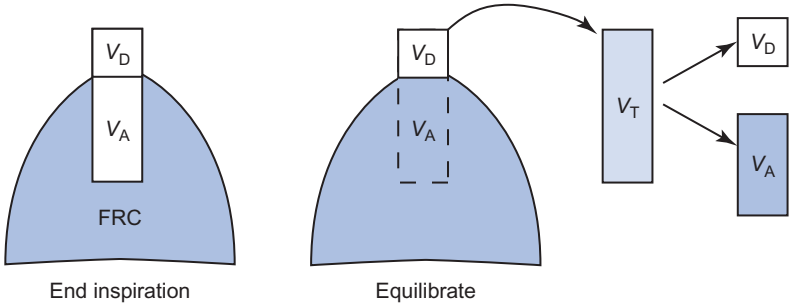

FIGURE 6.3.3 Tidal volume and the anatomic dead space. At the end of a normal inspiration, about 2.3 L of air remains in the lungs. About 0.5 L of tidal air is inspired, but only about 0.35 L reaches the alveoli because about 0.15 L is in the anatomic dead space that does not exchange gas with the blood. The expired air contains a mixture of dead space air and alveolar air. Thus the mole fraction of CO_2 in the expired air is the volume-weighted average of the mole fractions in alveolar air and inspired air.

where gas exchange cannot occur (because the diffusion distances to the blood are too great and anatomic structures intervene); the physiological dead space is the virtual or equivalent volume of ventilation where gas exchange does not occur; and the wasted volume = physiological dead space − anatomic dead space is the volume where gas exchange could occur but does not because of underperfusion with blood.

THE RATE OF CO_2 PRODUCTION ALLOWS CALCULATION OF ALVEOLAR VENTILATION

At steady state, the net rate of CO_2 production by the body is equal to the net rate of CO_2 elimination through the lungs. This is simply expressed as the difference between the rate of CO_2 input and CO_2 output:

$$[6.3.20] \qquad Q_{CO_2} = Q_A f_{A_{CO_2}} - Q_A^* f_{I_{CO_2}}$$

where Q_{CO_2} is the rate of CO_2 elimination, in mL min^{-1}, Q_A is the flow of expired air that exchanges gas, Q_A^* is the flow of inspired air that exchanges, and f refers to the mole fraction of gas in the expired or inspired alveolar gas. Since the mole fraction of CO_2 in inspired air is very low, we can neglect this and rearrange Eqn [6.3.20] to obtain the **alveolar ventilation equation**:

$$[6.3.21] \qquad Q_A = \frac{Q_{CO_2}}{P_{A_{CO_2}}}(P_B - 47)$$

where P_B is in mmHg and 47 is the vapor pressure of water at body temperature, in mmHg.

A full derivation of this equation is given in Appendix 6.3.A1. It is important to remember that **all of the flows are measured in volumes of gases at STPD**. The alveolar ventilation equation illustrates the inverse relationship between alveolar ventilation and $P_{A_{CO_2}}$, the partial pressure of CO_2 in alveolar air. If Q_{CO_2}, the production of CO_2 by the body, is constant, then **increases in alveolar ventilation will decrease** $P_{A_{CO_2}}$. Conversely, **decreases in Q_A will increase** $P_{A_{CO_2}}$. This inverse relation between alveolar ventilation and $P_{A_{CO_2}}$ holds for the steady state at any given metabolic rate. The alveolar ventilation equation can be rearranged to solve for $P_{A_{CO_2}}$.

$$[6.3.22] \qquad P_{A_{CO_2}} = \frac{Q_{CO_2}}{Q_A}(P_B - 47)$$

THE ALVEOLAR GAS EQUATION ALLOWS CALCULATION OF $P_{A_{O_2}}$

By considering that the net consumption of O_2 at steady state is the difference between inspired O_2 content and expired O_2 content, it is possible to derive an equation that relates $P_{A_{O_2}}$ to the inspired P_{O_2}, $P_{A_{CO_2}}$ and the respiratory quotient, R. The alveolar gas equation is derived in Appendix 6.3.1 and is given as

$$[6.3.23] \qquad P_{A_{O_2}} = P_{I_{O_2}} - \frac{1}{R}P_{A_{O_2}} + f_{I_{O_2}}\left(\frac{1-R}{R}\right)P_{A_{O_2}}$$

Because the last term usually gives a slight correction, this is often approximated as

$$[6.3.24] \qquad P_{A_{O_2}} = f_{I_{O_2}}(P_B - 47) - \frac{1}{R}P_{A_{O_2}}$$

The value of $P_{A_{O_2}}$ at rest is approximately 100 mmHg.

BLOOD IS IN THE LUNGS FOR LESS THAN A SECOND—BUT THAT IS LONG ENOUGH TO EQUILIBRATE THE GASES

The total amount of blood in the lung capillaries is normally about 70 mL, approximately one stroke volume of the heart. At a heart rate of 72 bpm, the blood is in the lungs only 60 s min^{-1}/72 min^{-1} = 0.83 s. During this short time, venous blood entering the lungs equilibrates completely with the alveolar air so that the exiting blood has the same P_{CO_2} and P_{O_2} as alveolar air. Blood actually equilibrates faster than its dwell time, indicating that there is some reserve in the diffusing capacity to accommodate the increased cardiac output and higher needs for gas exchange during exercise. Figure 6.3.4 illustrates the time course of blood P_{O_2} and P_{CO_2} during its brief course through the alveolar capillaries. The relative roles of diffusion and perfusion in capillary exchange are discussed in Chapter 5.10. Since, during normal conditions, P_{CO2} and P_{O2} equilibrate before the end of the capillary, this exchange is perfusion limited: if more blood flowed through, the transit time would be shorter yet the blood would still equilibrate, meaning more gas would be transferred into the blood. The transfer is perfusion limited.

EXAMPLE 6.3.1 Calculate $P_{A_{CO_2}}$ at Rest

At rest, the rate of CO_2 production is about 200 mL min^{-1}, STPD; tidal volume is 0.51 L, anatomic dead space is about 150 mL, and respiratory rate is about 12 min^{-1}.

The alveolar ventilation, Q_A, is calculated as 12 min^{-1} × (510 − 150 mL) = 4320 mL min^{-1}. This volume is measured at body temperature, saturated. It must be converted to STPD. The conversion (see Appendix 6.3.A2) is given as

$Q_{A_{STPD}} = Q_{A_{BTPS}}/1.2104 = 4320$ mL min^{-1}/1.2104 = 3570 mL min^{-1}

We can then calculate $P_{A_{CO_2}}$ = 200 mL min^{-1}/3570 mL min^{-1} × (760 mmHg − 47 mmHg)

$$P_{A_{CO_2}} = \mathbf{40\ mmHg}$$

BLOOD FLOW TO THE LUNG VARIES WITH POSITION WITH RESPECT TO GRAVITY

As described in Chapter 5.8, the pulmonary circulation is a low-pressure system that is influenced by gravity much more strongly than the systemic circulation. The pulmonary arterial pressure averages about 15 mmHg and in the left atrium it averages about 5 mmHg. The total pulmonary vascular resistance (PVR) can be calculated to be about 2 mmHg L^{-1} min, which is about a tenth of the systemic total peripheral resistance. The relatively small driving force for blood flow is affected by gravity: a 1 cm vertical ascent in the lungs reduces the pressure by 0.77 mmHg. Thus the pulmonary arterial pressure 10 cm above the heart will be reduced by 7.7 mmHg. Traditionally, the lung has been divided into three zones, each about one-third of the lungs. In the upper third, blood flow is reduced because the arteriolar blood pressure, P_a, is less than the alveolar gas pressure, P_A, and so blood flow is limited. In the middle third, $P_a > P_A$, but $P_A > P_v$, the blood pressure on the venous side. This partially collapses the capillaries and dams up the blood in the circulation. In the lower third, $P_a > P_A$ and $P_v > P_A$ and blood flow is maximal. This situation is illustrated in Figure 6.3.5.

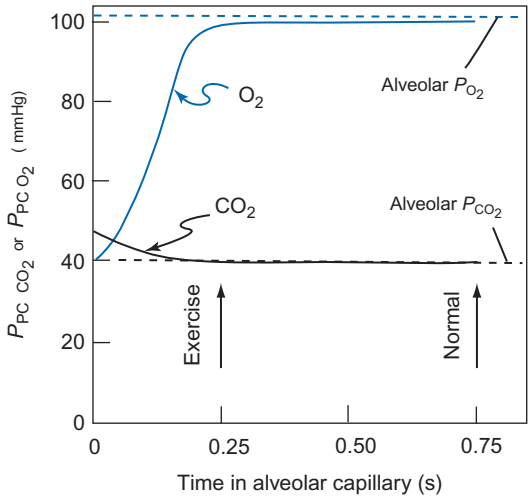

FIGURE 6.3.4 Time course of gas equilibration during the transit of blood through the pulmonary capillaries. Blood remains in the lung capillaries about 0.75 s. Venous blood has a $P_{V_{O_2}}$ of about 40 mmHg and a $P_{V_{CO_2}}$ of about 46 mmHg. The alveolar gas has a relatively constant composition of $P_{A_{O_2}}$ of about 102 mmHg and $P_{A_{CO_2}}$ of about 40 mmHg. Blood coursing through the capillaries equilibrates $P_{PC\ CO_2}$ and $P_{PC\ O_2}$ (pulmonary capillary plasma P_{CO_2} and P_{O_2}, respectively) with the alveolar gas within about 0.25 s, leaving considerable reserve time for gas exchange. During exercise, the cardiac output is increased and the dwell time of blood in the lung capillaries is reduced. Even in this case, the gases have enough time to equilibrate between alveolar gas and blood.

The result of decreased blood flow to Zone 1 because of gravity in an upright person is a mismatch between ventilation and perfusion. The ventilation/perfusion ratio, Q_A/Q_V, is about 3 in this region. At the base of the lung, the Q_A/Q_V ratio is about 0.6.

REGULATION OF THE PULMONARY CIRCULATION HELPS RESTORE THE VENTILATION/PERFUSION RATIO

Pulmonary circulation depends on the pulmonary vascular resistance (PVR), gravity, alveolar pressure, and the hydrostatic pressure gradient provided by the right heart. The PVR, in turn, is influenced mainly by two factors: the inflation of the lung and the reaction of the arterioles to the partial pressure of oxygen in the blood.

The PVR, which determines blood flow, is affected by three separate variables: the alveolar gas pressure that compresses the capillaries, the resistance of alveolar vessels (mainly the capillaries), and the resistance of extraalveolar vessels. Unlike the systemic circulation, the capillaries in the lungs accounts for about 40% of the PVR. At lung volumes greater than the FRC, capillaries are stretched and compressed, and the resistance of the alveolar vessels increases. At lower lung volumes, the extraalveolar vessels are not held open by their tethers to the alveolar tissues, and their resistance increases.

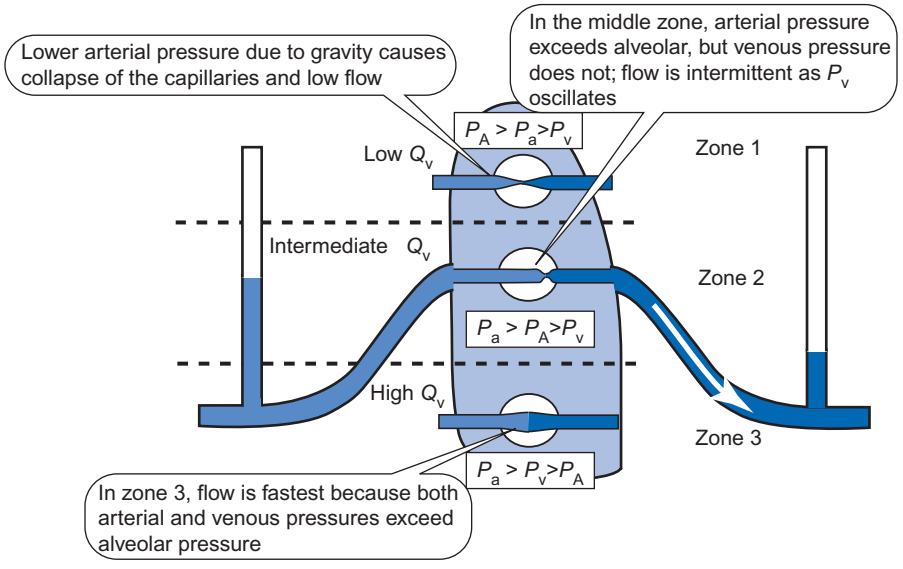

FIGURE 6.3.5 Effect of gravity on blood flow in the lung. The pulmonary arterial blood is shown in lighter blue; pulmonary venous blood, returning to the left heart, is shown in darker blue. The pressures are low, averaging around 15 mmHg in the pulmonary artery and about 8.5 mmHg in the venous circulation. Because gravity reduces the pressure, the perfusion pressures are lower at the top of the lung and therefore this area has less perfusion. In the middle zone, arterial pressure exceeds alveolar pressure, but venous pressure is sometimes insufficient, and the vessels close intermittently. In the lower zone, there is sufficient pressure to allow perfusion.

Clinical Applications: Asthma

Asthma is an obstructive disease marked by increased airway resistance. Although asthma is a chronic condition, it is differentiated from the other chronic obstructive pulmonary diseases by its reversibility. A sudden, acute exacerbation of the condition, with marked interference in gas flow, is called an asthma attack. The severity of the disease varies from rare and intermittent attacks to persistent symptoms requiring frequent use of short-term β_2 agonists.

The cause of asthma remains puzzling. At least two types of asthma exist: an allergic form, also called extrinsic or atopic, and a nonallergic, intrinsic nonatopic form. In the allergic form, asthma attacks are brought about by exposure to specific allergens. Exposure activates secretion of histamine from mast cells in the lung, inducing bronchospasm that is relieved by β_2 agonists such as salbutamol. About half of asthma cases, however, have no connection with allergic reactions.

Epidemiological studies show that the incidence of asthma is markedly increasing worldwide. Currently some 300 million people are affected, and in 2009 it caused an estimated 250,000 deaths globally. In 1989, David Strachan, a British epidemiologist, first proposed the **hygiene hypothesis** to explain the rapid rise in asthma. He observed that people living in unhygienic circumstances were less likely to have asthma than those living in cleaner environments. In this hypothesis, dirt, bacteria, and viruses in early life train the immune system to produce cells geared to defend against these problems, keeping the cells that respond to allergens in check. Unfortunately, more studies have shown that the hygiene hypothesis often does not explain all of the epidemiology. Children living on farms where cows and pigs are raised, drinking unpasteurized milk, almost never have asthma of any sort. There is something about Western, urban, or suburban culture that helps produce asthma, perhaps a change in the bacterial fauna both within the respiratory and GI tracts, but the full story is not yet known.

Other types of asthma also exist. Exercise-induced asthma, or exercise-induced bronchospasm (EIB), is thought to be related to dehydration of the airways when increased airflow requires increased humidification. This increases the osmolarity within cells in the airways, leading to release of histamine or leukotrienes, bringing on the bronchospasm.

These effects contribute to a biphasic relationship between PVR and lung volume. The PVR is minimum around the functional residual capacity, FRC.

When the ventilation/perfusion ratio is low, blood flow into the region carries away all of the excess oxygen until the alveolar gas becomes depleted of it. The resulting relative hypoxia causes a vasoconstriction called **hypoxic vasoconstriction**. This increases the local resistance to blood flow and shifts perfusion to underperfused regions of the lungs with high Q_A/Q_V ratios. Similar hypoxic vasoconstriction can occur in underventilated areas of the lungs. In these areas, neither ventilation nor perfusion is normal, but the hypoxic vasoconstriction helps to normalize their ratios.

SUMMARY

The total pressure within a gas mixture can be partitioned into the contributions made by each gas. These are their partial pressures, which are proportional to the mole fractions in the mixture. Water vapor also has a partial pressure, which is 47 mmHg at 37°C. The relation between partial pressure and mole fraction is written as

$$P_A = f_A(P_B - 47)$$

where P_A is the partial pressure in alveolar air (in mmHg), f_A is the mole fraction in dry air, P_B is the barometric pressure (mmHg), and 47 is the vapor pressure of water at body temperature.

The mole fraction of gases in liquid phases is also proportional to the partial pressure of gas in equilibrium with that phase (Henry's law). The solubility is the proportionality constant between mole fraction of gas in the aqueous phase and partial pressure; this solubility depends on the temperature. Gas content is always expressed as mL of gas per deciliter under STPD conditions: standard temperature and pressure, dry. Henry's law is written as

$$X_A = \beta_A P_A \quad [A] = \alpha_A P_A$$

Transport of gases into the blood traverses a thin sandwich of several layers including the alveolar lining, the alveolar cell, interstitial fluid, endothelial cell, plasma, and red cell membrane. Gas exchanges across these barriers occur passively. The rate of gas transport is linearly related to the partial pressure difference. The lung area is large (70 m^2) and thin (0.5 μm) so that gas exchange is rapid, reaching equilibrium within 0.25 s, whereas blood is in the exchanging pulmonary capillaries perhaps 0.8 s during quiet sitting. The overall transport of gas can be described by the diffusing capacity, the coefficient that relates flow of gas in mL min^{-1} to the partial pressure difference in mmHg. The diffusing capacity for CO_2 is about 20 times higher than that of O_2 because CO_2 has a higher solubility.

Each breath ventilates a dead space that does not exchange gas as well as regions of the lungs that do exchange. The anatomic dead space corresponds to areas of the tracheobronchial tree that do not exchange. The physiologic dead space includes areas that are underperfused. The alveolar P_{CO_2} can be calculated from the alveolar ventilation equation: it increases with increase in CO_2 production (as occurs in exercise) and decreases with increase in ventilation. The alveolar P_{CO_2} at rest is typically 40 mmHg. The alveolar P_{O_2} can be calculated from the alveolar gas equation and is typically about 100 mmHg. Because gas exchange is

complete, these are also the typical values of P_{CO_2} and P_{O_2} in the blood that leaves the lungs.

REVIEW QUESTIONS

1. Why is the expired gas generally not the same volume as inspired gas?
2. Why is it necessary to express Q_{O_2} and Q_{CO_2} in volumes at STPD?
3. What is pulmonary ventilation? If pulmonary ventilation increases, what happens to alveolar ventilation? How do the two differ? If alveolar ventilation increases, what happens to $P_{A_{CO_2}}$ if there is no change in Q_{CO_2}? What happens to $P_{A_{CO_2}}$ if pulmonary ventilation decreases?
4. What is the vapor pressure of water? Does it vary with P_B? Does it vary with temperature? What is its value at body temperature? Why do we have to be concerned about it?
5. Distinguish between anatomic dead space and physiological dead space. Which is larger and why?
6. Would alveolar air have a constant composition if V_T were much larger, as in exercise?
7. What is the relative solubility of CO_2 and O_2?
8. Why do we say that "pressure gradients drive gas flow" instead of concentration gradients?

APPENDIX 6.3.A1 DERIVATION OF THE STEADY-STATE GAS EXCHANGE EQUATIONS

ALVEOLAR VENTILATION CONSISTS OF THE PART OF PULMONARY VENTILATION THAT PARTICIPATES IN GAS EXCHANGE

As discussed in Chapter 6.2, the pulmonary ventilation is the total flow of air into and out of the lungs. This is given by the respiratory rate times the tidal volume:

$$[6.3.A1.1] \qquad Q_P = \nu_R \langle V_T \rangle$$

where Q_P is the pulmonary ventilation, ν_R is the respiratory frequency, and $\langle V_T \rangle$ is the average tidal volume. This equation is ambiguous because the inspired part of the tidal volume is not the same as the expired part. Three things happen to change the expired volume relative to the inspired volume: first, the air is equilibrated to body temperature. In temperate climates, this generally means that the expired air has a larger volume because the ambient temperature is usually below body temperature. Second, the inspired air is humidified. This also generally expands the volume of air. Both of these effects can be corrected by converting the volumes of gas to some standard such as STPD. Third, the volume of expired air is altered by the consumption of O_2 and the excretion of waste CO_2. If the consumption of O_2 is a mole-to-mole match for CO_2 excretion, then the expired volume will match the inspired volume. But in general the consumption of O_2 is not one-for-one with CO_2 production. We will return to these issues.

Part of each tidal volume renews air in regions of the lung where gas exchanges with the blood, whereas dead space ventilation does not exchange gas. The alveolar ventilation is given as

$$[6.3.A1.2] \qquad \begin{aligned} \nu_R V_A &= \nu_R V_E - \nu_R V_D \\ Q_A &= Q_E - Q_D \end{aligned}$$

where V_A is the part of the tidal volume that exchanges gas, V_E is the expired volume, V_D is the volume of gas in the dead space, Q_A is the alveolar ventilation, in $L\,min^{-1}$, Q_E is the flow of expired air, and Q_D is the dead space ventilation.

INSPIRED AIR DIFFERS IN VOLUME FROM EXPIRED AIR TO THE EXTENT OF DIFFERENCES BETWEEN Q_{O_2} AND Q_{CO_2}

Consider a single tidal volume. The volume of inspired air that exchanges gas is V_A^* and the volume of expired air that originates from the exchanging regions is V_A. The volume of this expired air is equal to the volume of inspired air less the volume of O_2 taken up plus the volume of CO_2 excreted:

$$[6.3.A1.3] \qquad V_A = V_A^* + V_{CO_2} - V_{O_2}$$

We can multiply both sides of this equation by ν_R to obtain the flow of gas instead of the volumes:

$$[6.3.A1.4] \qquad Q_A = Q_A^* + Q_{CO_2} - Q_{O_2}$$

This can be rewritten in terms of Q_A^*:

$$[6.3.A1.5] \qquad Q_A^* = Q_A - Q_{CO_2} + Q_{O_2}$$

Since there is no gas exchange in the dead-space volumes, we write the conservation of mass equations for CO_2 and O_2 as

$$[6.3.A1.6A] \qquad Q_{CO_2} = f_{A_{CO_2}} Q_A - f_{I_{CO_2}} Q_A^*$$

$$[6.3.A1.6B] \qquad Q_{O_2} = f_{I_{O_2}} Q_A^* - f_{A_{O_2}} Q_A$$

Substituting in for Q_A^* from Eqn [6.3.A1.5] into Eqn [6.3.A1.6A], we obtain

$$[6.3.A1.7] \qquad Q_{CO_2} = f_{A_{CO_2}} Q_A - f_{I_{CO_2}} (Q_A + Q_{O_2} - Q_{CO_2})$$

This can be solved for $f_{A_{CO_2}}$:

$$[6.3.A1.8] \qquad f_{A_{CO_2}} = \frac{Q_{CO_2}}{Q_A} + f_{I_{CO_2}} + f_{I_{CO_2}} \frac{(Q_{O_2} - Q_{CO_2})}{Q_A}$$

The mole fractions can be converted to partial pressures by multiplying by $(P_B - 47)$, because the mole fractions of gas are always expressed in terms of STPD. The results give

$$P_{A_{CO_2}} = \frac{Q_{CO_2}}{Q_A}(P_B - 47) + P_{I_{CO_2}} + P_{I_{CO_2}} \frac{(Q_{O_2} - Q_{CO_2})}{Q_A}$$

$$[6.3.A1.9]$$

When there is little CO_2 in the inspired air, the last two terms on the right-hand side of the equation can be ignored. The resulting approximation gives

$$[6.3.A1.10] \qquad P_{A_{CO_2}} = \frac{Q_{CO_2}}{Q_A}(P_B - 47)$$

This is the alveolar ventilation equation. It illustrates the inverse relationship between $P_{A_{CO_2}}$ and Q_A. When alveolar ventilation increases, for the same metabolic condition, $P_{A_{CO_2}}$ decreases and vice versa. Similarly, when CO_2 production increases, $P_{A_{CO_2}}$ increases in parallel unless Q_A also increases.

THE ALVEOLAR GAS EQUATION GIVES $P_{A_{O_2}}$

In the same manner as our derivation of $P_{A_{CO_2}}$, we can find an expression for $P_{A_{O_2}}$ by inserting the result for Q_A^* from Eqn [6.3.A1.5] into Eqn [6.3.A1.6B]. The result gives

$$[6.3.A1.11] \qquad Q_{O_2} = f_{I_{O_2}}(Q_A + Q_{O_2} - Q_{CO_2}) - f_{A_{O_2}} Q_A$$

Solving for $f_{A_{O_2}}$, we find

$$[6.3.A1.12] \qquad f_{A_{O_2}} = f_{I_{O_2}} + f_{I_{O_2}}\left(\frac{Q_{O_2}}{Q_A} - \frac{Q_{CO_2}}{Q_A}\right) - \frac{Q_{O_2}}{Q_A}$$

here we make use of another definition. We define the **respiratory quotient**, R, to be the ratio of Q_{CO_2}/Q_{O_2}. Then we substitute in for $Q_{O_2} = Q_{CO_2}/R$. Then Eqn [6.3.A1.12] becomes

$$f_{A_{O_2}} = f_{I_{O_2}} + f_{I_{O_2}}\left(\frac{1}{R}\frac{Q_{CO_2}}{Q_A} - \frac{Q_{CO_2}}{Q_A}\right) - \frac{1}{R}\frac{Q_{CO_2}}{Q_A}$$

[6.3.A1.13]

which is rearranged to

$$[6.3.A1.14] \qquad f_{A_{O_2}} = f_{I_{O_2}} + f_{I_{O_2}}\left(\frac{(1-R)}{R}\right)\frac{Q_{CO_2}}{Q_A} - \frac{1}{R}\frac{Q_{CO_2}}{Q_A}$$

Now we may substitute in for Q_{CO_2}/Q_A from Eqn [6.3.A1.10] to obtain

$$f_{A_{O_2}} = f_{I_{O_2}} + f_{I_{O_2}}\left(\frac{(1-R)}{R}\right)\frac{P_{A_{CO_2}}}{(P_B - 47)} - \frac{1}{R}\frac{P_{A_{CO_2}}}{(P_B - 47)}$$

[6.3.A1.15]

Multiplying both sides of Eqn [6.3.A1.15] by $P_B - 47$, we obtain the partial pressure of O_2 in the alveolar air:

$$[6.3.A1.15] \qquad P_{A_{O_2}} = P_{I_{O_2}} - \frac{1}{R}P_{A_{CO_2}} + f_{I_{O_2}}\left(\frac{(1-R)}{R}\right)P_{A_{CO_2}}$$

This last equation is the **alveolar gas equation**. To obtain this equation, we used Eqn [6.3.A1.10], which is an approximation for the case when the inspired CO_2 is negligible. Thus this equation is strictly true only when the inspired concentration of CO_2 is as low as it is in atmospheric air. In addition, this equation describes alveolar P_{O_2} during the steady state at constant R. At any time that the metabolic condition rapidly changes, such as the onset of or cool down in exercise, this equation will be only approximately true.

APPENDIX 6.3.A2 CONVERSION OF PARTIAL PRESSURES AND VOLUMES BETWEEN STPD AND BTPS

Respiratory physiologists report $P_{A_{O_2}}$ and $P_{A_{CO_2}}$, Q_{O_2} and Q_{CO_2} in terms of mmHg or mL min^{-1} STPD, or standard pressure and temperature, dry. The standard temperature is 0°C = 273.16 K, standard pressure is 1 atm = 760 mmHg = 1.01×10^5 Pa, and dry means $P_{H_2O} = 0$. Many measurements are not performed under these conditions, however. BTPS refers to body temperature (37°C = 310.16 K), pressure of 1 atm and saturated with water vapor so that $P_{H_2O} = 47$ mmHg. Sometimes measurements are also performed at ATPS. The relevant temperature then is the ambient temperature. If the measurements are performed at some elevation, the ambient pressure may be less than 1 atm, and P_{H_2O} depends on the temperature. Between 21°C and 37°C, the vapor pressure decreases by about 2 mmHg per °C change in temperature.

What we need is some way to convert partial pressures and volumes between conditions, particularly between STPD and BTPS. The equation we use is the Ideal Gas Equation

$$[6.3.A2.1] \qquad PV = nRT$$

If we consider two conditions, labeled by "1" and "2," this equation holds for both:

$$[6.3.A2.2] \qquad \begin{aligned} P_1V_1 &= nRT_1 \\ P_2V_2 &= nRT_2 \end{aligned}$$

If the number of moles of gas is constant ($n_1 = n_2$), then Eqn [6.3.A2.2] is easily rewritten as

$$[6.3.A2.3] \qquad V_2 = \frac{T_2}{T_1}\frac{P_1}{P_2}V_1$$

Suppose that state "1" represents STPD and state "2" represents BTPS. Now what complicates matters is that $n_1 \neq n_2$ because the total number of moles in the gas is increased by equilibration with water vapor at 37°C. However, the dry gases still obey this equation. What we need to use in Eqn [6.3.A2.3] is the pressure of the dry gases alone. This is $P_B - P_{H_2O}$, the difference between the barometric pressure and the vapor pressure of saturated air. For STPD conditions, $P_{H_2O} = 0$. For state 1, representing STPD, we have $T_1 = 273$ K, $P_1 = 760$ mmHg; for state 2, represented by BTPS, we have $T_2 = 310$ K (=273 K + 37 K) and $P_2 = 760 - 47$. Conversion of $V_2 = V_{BTPS}$ from $V_1 = V_{STPD}$ is thus given as

$$[6.3.A2.3] \qquad \begin{aligned} V_{BTPS} &= \frac{310}{273}\frac{760}{(760-47)}V_{STPD} \\ &= 1.2104\, V_{STPD} \end{aligned}$$