Problem Set 6.1
Airway Resistance and Alveolar Gas Exchange

1. The density of Hg is 13.6 g cm^3 and the acceleration due to gravity is 981 cm s^{-2}. Convert the pressure exerted by a column of mercury 1 mm high to pascals, N m^{-2}. If the atmospheric pressure on top of a 3000 m peak is 527 mmHg, what is it in pascals?
2. A. The typical diameter of an alveolus is about 0.3 mm. If there are 300×10^6 alveoli in the lungs, calculate the approximate area for gas exchange, assuming that the alveoli are spheres.
   B. Based on this number and size of alveoli, calculate the total volume of gas in the alveoli. How does this compare to typical values for FRC? Differences between these two ought to be due to errors or rounding in either the size or number of alveoli. If the number of alveoli is correct, what would the diameter have to be to make this volume equal to the FRC? If the diameter is correct, what would the number of alveoli have to be to make this volume equal to the FRC?
3. A. The typical diameter of an alveolus is 0.3 mm. The surface tension of water is about 70 dyne cm^{-1}. The dyne is the unit of force in the cgs system: 1 g cm s^{-2}. It is converted to N by multiplying by 10^{-3} kg g^{-1} $\times$ 10^{-2} m cm^{-1} = 10^{-5} kg g^{-1} m cm^{-1}. What is the pressure within this alveolus assuming no surfactant at mechanical equilibrium? Give the answer in pascals (N m^{-2}) and in mmHg.
   B. Surfactant lowers the surface tension to about 28 dyne cm^{-1}. What would the pressure be in an alveolus with a diameter of 0.3 mm?
   C. Expanding the lungs increases the size of the alveoli and increases the surface area. How much work is necessary to expand the alveoli in a normal tidal volume of 0.5 L when the FRC is 2.3 L and assuming there is surfactant. *Hint*: Calculate work either as $P\mathrm{d}V$ or $\gamma \mathrm{d}A$; use the change in volume given to calculate $\mathrm{d}V$ or $\mathrm{d}A$ given that the rest diameter of alveoli is 0.3 mm and that there are N of them (see Problem 2B).
   D. During normal tidal breathing, the airflow is about 0.5 L s^{-1} and the pressure driving airflow is about 1 mmHg. Calculate the overall airway resistance. Estimate the energy required to overcome this airway resistance for a tidal volume of 0.5 L.
   E. The value calculated in 3C is the work required per breath just to expand the alveoli. The work of breathing includes the work to overcome airway resistance and to expand the chest wall. Assume that the work of expanding the chest wall is about one-half of the work of expanding the lungs. If the resting metabolism is 5200 J min^{-1}, what fraction of resting metabolism is used in the work of breathing? If breathing is 8% efficient, about what fraction of resting metabolism is used for breathing?
4. The table below lists the airway diameter and aggregate area (the number of airways times the airway area per airway) as a function of generation number.

Generation No.	Diameter (cm)	Area (cm^2)	Reynold's No.
0	1.8	2.54	
1	1.22	2.33	
2	0.83	2.13	
3	0.56	2	
4	0.45	2.48	
5	0.35	3.11	
6	0.28	3.96	
7	0.23	5.1	
8	0.186	6.95	
9	0.154	9.56	
10	0.130	13.4	
11	0.109	19.6	
12	0.095	28.8	
13	0.082	44.5	
14	0.074	69.4	
15	0.066	113	
16	0.060	180	
17	0.054	300	
18	0.050	534	
19	0.047	944	
20	0.045	1600	
21	0.043	3220	
22	0.041	5880	
23	0.041	11,800	

   A. Calculate the Reynolds number for generation 0, 2, and 10 for airflow during normal resting respiration. The maximum flow rate during these conditions is about 0.5 L s^{-1}.

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What generalizations can you make about laminar and turbulent flows during rest?

B. Calculate the Reynolds number for generation 0, 2, and 10 for airflow during high ventilation that occurs during exercise, for example. Assume that the flow rate during these conditions is $5\ \mathrm{L\,s^{-1}}$. What conclusions can you make regarding laminar and turbulent flows during high flow conditions?

5. The surface area of the lungs is about $70\ \mathrm{m}^2$ and the thickness of the alveolar diffusion layer is about $0.5\ \mu\mathrm{m}$. The $P_{\mathrm{A_{O_2}}}$ is about 100 mmHg, whereas the $P_{\mathrm{v_{O_2}}}$, the partial pressure of O_2 in venous blood, is about 40 mmHg. The diffusion coefficient of O_2 in water is about $1.5 \times 10^{-5}\ \mathrm{cm^2\,s^{-1}}$. The solubility of O_2 is given by Henry's law with a coefficient α given in Table 6.3.3 (with gas volumes at STPD!). This gives

$$[O_2] = 0.024 P_{O_2}$$

where $[O_2]$ is expressed in mL O_2 per mL of water and P_{O_2} is in atmospheres. Only the dissolved O_2 diffuses. What is the initial rate of O_2 diffusion from the aggregate alveoli to the blood? Assume that the diffusion of O_2 across the lipid bilayers in the alveolar membrane is essentially instantaneous because the partition coefficient of O_2 is high and the diffusion distance, the thickness of the membrane, is very small (~ 10 nm). Thus to O_2 the alveolar membrane looks like a thin aqueous layer. The initial rate means clamp the venous P_{O_2} at 40 mmHg. Give the answer in mL O_2 (STPD) per min and in mol per min using the Ideal Gas Law. How does this compare to Q_{O_2} at rest? Explain the difference.

6. Resting O_2 consumption is about $250\ \mathrm{mL\,min^{-1}}$ while resting CO_2 production is about $200\ \mathrm{mL\,min^{-1}}$. Both these volumes refer to volumes at STPD. How much mass is lost per day through the lungs, exclusive of water loss, due to resting metabolism?

7. Assume that the inspired air is dry air. The vapor pressure of 47 mmHg at 37°C corresponds to a volume and mass of liquid water. If the tidal volume is 500 mL and respiration rate is $12\ \mathrm{min^{-1}}$, how much water is lost from the body per day? Would you expect this to increase or decrease with activity?

8. A. At rest V_T at BTPS is 510 mL. If the dead space is 150 mL, what is V_A?
   B. If the respiratory rate is $12\ \mathrm{min^{-1}}$, what is Q_A at BTPS?
   C. Convert Q_A from part B to STPD.
   D. Calculate $P_{\mathrm{A_{CO_2}}}$ if $Q_{\mathrm{A_{CO_2}}}$ is $200\ \mathrm{mL\,min^{-1}}$ and $P_B = 760$ mmHg.

9. A. The mole fraction of O_2 in dry atmospheric air is 0.21. If $P_B = 760$ mmHg, what is P_{O_2}?
   B. What is $P_{\mathrm{I_{O_2}}}$ in moist tracheal air?
   C. If $R = 0.8$ and $P_{\mathrm{A_{CO_2}}} = 40$ mmHg, calculate $P_{\mathrm{A_{O_2}}}$.

10. A number of empirical formulas have been developed to predict lung volumes for people as a function of body size and age. These are used to provide spirometric standards to determine what is normal and what is abnormal. Two such formulas are given below (C.J. Gore et al., Spirometric standards for healthy adult lifetime nonsmokers in Australia. *European Respiratory Journal* 8:773–782, 1995):

$$\mathrm{FVC\ (female)} = -3.598 - 0.0002525A^2 + 4.680H$$

$$\mathrm{FVC\ (male)} = 12.675 - 0.0002764A^2 + 10.736H^2 + 4.790H^3$$

Another empirical equation (J.F. Morris et al., Spirometric standards for healthy nonsmoking adults. *American Review of Respiratory Disease* **103**:57–67, 1971) gives

$$\mathrm{FVC\ (female)} = -2.852 - 0.024A + 4.528H$$
$$\mathrm{FVC\ (male)} = -4.241 - 0.025A + 5.827H$$

where A is the age in years and H is the height in m.

   A. For a male 5′10″ tall and 20 years of age, compare these two predictions for FVC.
   B. For a female 5′4″ tall and 20 years old, compare these two predictions for FVC.
   C. How could you judge which of the many available predictive equations was the best to use?

11. A common clinical unit in respiratory physiology is $\mathrm{cmH_2O}$ instead of mmHg. Convert the two units. Assume the density of water is $1.00\ \mathrm{g\,cm^{-3}}$.

12. The FRC can be measured using a whole body plethysmograph. This consists of a refrigerator-sized, air-tight chamber in which the patient can sit comfortably and which is usually transparent so that the patient can see the operator and vice versa. The patient breathes through a tube connected to a pneumotach, which can measure airflow and pressure at the mouthpiece and which is fitted with a shutter. After a normal expiration, the shutter closes and the patient pants against the closed shutter, while simultaneously holding the cheeks in with the hands. The pressure at the mouthpiece is recorded. The pressure goes up and down cyclically due to the panting effort. When the pressure goes up, the gas remaining in the lung is compressed—its volume decreases. This decrease in volume of the thoracic cavity is accompanied by an increase in volume outside the body, in the chamber. If the volume change measured by the body plethysmograph is 71 mL and the pressure change measured at the mouthpiece was 20 mmHg, estimate the FRC. (*Hint*: Let V be the FRC prior to expiratory effort, and $V - \Delta V$ be its volume after; the pressure prior to expiratory effort is $P_B = 760$ mmHg, and the pressure after is $P_B + \Delta P$. An important point to remember is that the vapor pressure of water does *not* change!)

13. Normal values of $Q_{CO_2} = 200$ mL min^{-1} (STPD), $V_T = 510$ mL (BTPS), $V_D = 150$ mL (BTPS), and $\nu_R = 12$ min^{-1}. During exercise, Q_{CO_2} increased to 1000 mL min^{-1} (STPD), and ν_R increased to 20 min^{-1}.
    A. What would V_T be if $P_{A_{CO_2}}$ did not change?
    B. If $R = 0.8$, calculate $P_{A_{O_2}}$ (assuming that $P_{A_{CO_2}} = 40$ mmHg).
14. The solubility of CO_2 in water at 37°C is given in Table 6.3.3 as 0.0747 mL dL^{-1} mmHg^{-1}. Blood is not water. Assume that CO_2 is soluble only in water, and that 93% of plasma is water by volume. Assume that the hematocrit is 0.45 and that in the erythrocytes only 72% of their volume is water. Calculate the amount of dissolved CO_2 in blood at 37°C at 40 mmHg in mL dL^{-1}.
15. The diffusion coefficient of O_2 is about 1.5×10^{-5} cm^2 s^{-1}. Its solubility given in Table 6.3.3 is 0.00317 mL dL^{-1} mmHg^{-1}.
    A. Given that plasma is 93% water by volume, the hematocrit is 0.45 and that in the erythrocytes only 72% of the volume is water, calculate the concentration of $[O_2]$ in the blood if $P_{O_2} = 40$ mmHg.
    B. Estimate the concentration of $[O_2]$ in the alveolar lining if its water volume is 0.80 of the tissue volume, and $P_{A_{O_2}} = 1000$ mmHg.
    C. The diffusing capacity of O_2 is 21 mL min^{-1} mmHg^{-1}. If $P_{A_{O_2}} = 100$ mmHg and $P_{v_{O_2}} = 40$ mmHg, what is the *initial* flow of O_2 from alveolar air to blood? (Initial means assume that the gradient of partial pressures remains constant.)
    D. What area of water surface would give the same diffusive flow from D_{O_2} and $[O_2]$ gradient if the diffusion distance is 0.5 μm?