Problem Set

Physical Foundations: Pressure and Electrical Forces and Flows 1.1

1. Identify whether the following variables are intensive or extensive and explain your reasoning.
   A. Temperature
   B. Heat content
   C. Volume
   D. Density
   E. Mass
   F. Concentration
   G. Moles
   H. Pressure
   I. Area
   J. Flow
   K. Flux
   L. Viscosity
2. Normal systolic blood pressure is about 120 mmHg
   A. Convert this to atmospheres.
   B. Convert this to pascals
3. Normal diastolic blood pressure is about 80 mmHg
   A. Convert this to atmospheres
   B. Convert this to pascals
4. The density of whole blood is typically 1.055 g cm^{-3}. The density of Hg is 13.6 g cm^{-3}.
   A. Derive an equation to express the hydrostatic pressure of a column of blood as the height of Hg that would produce the same hydrostatic pressure as that column of blood
   B. Use the equation to determine the pressure of a 20-cm column of blood, expressed in mmHg
   C. If systolic blood pressure is 120 mmHg, what is the height of blood that this could support (if it was constant)?
5. For dialysis membranes, the L_p was determined to be 6.34×10^{-7} cm min^{-1} $mmHg^{-1}$. A cylindrical hole 1 cm in diameter was cut into two Lexan pieces that were then bolted together with the membrane between. Fluid entered one side through a pump and a pressure transducer was affixed. The flow was adjusted until the pressure (above atmospheric) was 20,000 pascals. The pressure on the opposite side was atmospheric (zero). Assume steady-state flow. What was the flow through the membrane?
6. The viscosity of water at 25 °C is about 0.00089 Pa s. The inner diameter of a PE160 polyethylene tubing is 1.14 mm. Assume steady-state laminar flow.
   A. What pressure is necessary to get a flow of 5 mL min^{-1} through a 20-cm length of this tubing?
   B. What is the velocity of the flow?
   C. What pressure is necessary to get a flow of 5 mL min^{-1} through the same PE160 tubing if plasma is used, with viscosity of 0.002 Pa s?
   D. What pressure would be needed for the same flow of water through a 20-cm length of PE60 tubing with i.d. = 0.76 mm?
7. After the end of a normal inspiration, the volume of air in the lungs is about 2.8 L. Normally quiet inspiration is driven by a pressure difference of about 2 mmHg. The air in the lungs is at 37 °C and after normal expiration it is at atmospheric pressure. Quiet inspiration is driven by the expansion of the chest cavity by contraction of the diaphragm, which expands the air in the lungs. How much is the air expanded to produce an decrease of 2 mmHg in pressure? Use the ideal gas equation, $PV = nRT$, where P is the pressure, V is the volume, n is the number of moles, $R = 0.082$ L atm mol^{-1} $°K^{-1}$ is the gas constant, and T is the absolute temperature.
8. A burette is a vertical, right circular cylinder that is open at the top and has a stopcock valve at the bottom to let fluid out. Assume that you have a burette that has an inner diameter of 1 cm and a height of 100 cm.
   A. Assume that you fill the burette with water to some height, h. What is the relation between h and the pressure at the base of the burette?
   B. What is the relation between volume of water in the burette and the pressure?
   C. How does this relation map onto the relation between charge and voltage on a capacitor?
   D. What is the hydraulic analogue of voltage?
   E. What is the hydraulic analogue of charge?

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F. Suppose you open the stopcock fully. The fluid will drain out. Assume a constant diameter of the opening of the stopcock that provides a constant resistance, R. Derive an equation that describes the time course of draining the burette.
G. Identify the time constant for the burette emptying.

9. You are given a long vertical tube filled with fluid of viscosity η at sea level where the acceleration due to gravity is $g = 9.81\ \mathrm{m\ s^{-2}}$. You have a steel ball or radius r and density ρ_{steel} that you carefully drop into the fluid. Assume that the drag force on the ball obeys Stokes' Law: the drag coefficient is $6\ \pi r \eta$, where $\mathbf{F}_{\mathrm{drag}} = -6\ \pi r \eta\ \mathbf{v}$. Remember that the steel ball is subjected to a buoyant force equal to the volume of the ball times the density of the fluid, ρ_{fluid}, times the acceleration of gravity.
   A. Derive an expression for the time of approach of the steel ball to terminal velocity.
   B. Derive an expression for the viscosity of the fluid as a function of the terminal velocity. (This is how you can determine the viscosity of a fluid.)
10. The Poiseuille equation that relates flow in narrow tubes to the pressure difference is analogous to Ohm's Law for current flow.
   A. What is the resistance to flow in terms of the parameters of the tube?
   B. What happens to resistance if the radius of the tube is halved?
   C. What happens to resistance if the radius of the tube is doubled?
11. Suppose that you begin to exercise and as a consequence you begin to produce more heat.
   A. What do you suppose will happen to the body temperature if, when environmental temperature does not change, the rate of heat production increases.
   B. How is this change in body temperature described by the continuity equation applied to heat?
   C. How do you suppose the body will shed this excess heat production?
12. You are out camping and it is very cold outside. You are losing heat faster than you can produce it.
   A. What happens to body temperature?
   B. How is this change in body temperature described by the continuity equation applied to heat?
   C. How do you suppose you can prevent hypothermia?
13. Consider a capacitor with a capacitance of $10\ \mu\mathrm{F}$. You connect it to a variable DC voltage source with a switch in the circuit and a resistor of 1000Ω in series with the capacitor.
   A. What is the relation between the steady-state voltage across the capacitor and the charge?
   B. Before you close the switch, there is no charge on the capacitor. When you close the switch, current begins to flow from the voltage source at E volts. What is the relation between current and the voltage drop across the resistor in terms of the current and the resistance?
   C. What is the voltage drop across the capacitor in terms of charge and capacitance?
   D. Kirchoff's voltage law says that the voltage drop around any loop must be zero. Write the equation for the voltage drop across the resistor, capacitor, and voltage source.
   E. Solve the equation in part D to derive the time course of charging of the capacitor.
   F. Solve the equation in part C to derive the time course of the current.
   G. Identify the time constant for the charging of the capacitor.
14. An unmyelinated axon can be considered to be a long right circular cylinder. Consider that an axon is 10 cm long with a diameter of $1.0\ \mu$ ($1\ \mu = 10^{-6}$ m).
   A. If the specific capacitance of the membrane is $1\ \mu\mathrm{F\ cm^{-2}}$, what is the capacitance of the axon membrane?
   B. How much charge is separated by this membrane to give a potential of 70 mV?
15. A muscle cell approximates a right circular cylinder 10 cm long and $70\ \mu$ in diameter. The specific capacitance of the membrane (the capacitance per unit area) is $1\ \mu\mathrm{F\ cm^{-2}}$.
   A. What is the capacitance of the muscle membrane?
   B. How much charge is separated by this membrane to give a potential of -85 mV?
16. A bubble is held at a radius of $250\ \mu\mathrm{m}$. The transmural pressure difference is 2 mm Hg. What is the tension in the wall?
17. The thickness of a single membrane is about 7 nm with a specific capacitance $C_{\mathrm{m}} = 1\ \mu\mathrm{F\ cm^{-2}}$. A myelin sheath consists of multiple membranes produced by coils of Schwann cell or oligodendroglia cells. Suppose a myelin sheath results from 100 membranes stacked on top of each other.
   A. What is the specific capacitance of the myelin?
   B. If the myelin sheath is a right circular cylinder 1 mm long with a radius of 3 m, what is its total capacitance?
   C. How much charge does it take to produce a voltage of -70 mV across this capacitor?
   D. If the myelin were just 1 membrane—i.e., not myelin—what would be its capacitance? How much charge would it take to produce a voltage of -70 mV across the single membrane?
18. The transmural pressure difference across a small vein is 20 mmHg. The radius is 1 mm. What is the wall tension?

19. The heat capacity of the human body is about 3500 joules $kg^{-1}\,{}^{\circ}C^{-1}$. Suppose that a person weighs 75 kg. How much energy does it take to raise the body temperature from 98°C to 102°C?
20. The charge on the electron is 1.602×10^{-19} coulombs. How much energy, in joules, is gained when an electron is accelerated across a potential difference of 1 v in a vacuum? This amount of energy is called the electron-volt.
21. Suppose that the average diameter of the aorta is 1.1 cm. The flow through the aorta is nearly the entirety of the cardiac output.
    A. If the cardiac output is 5 L/min, what is the average flow through the aorta?
    B. Suppose further that the cardiovascular system is nearly closed to fluid transfer. That is, that on a short-term basis the volume of the blood does not change. This means that ALL of the blood that leaves the heart goes through the arteries, and then capillaries, and then returns to the heart through the veins. Using the continuity equation, what is the flow through the aggregate capillaries?
    C. If the average diameter of the capillary is 4×10^{-4} cm, and flow through the capillary is 0.1 cm s^{-1}, how many capillaries are there?