Q4 Expand $e^x \log(1+y)$ in powers of x and y upto 3^{rd} degree

Sol^n:- $f(x,y) = e^x \log(1+y)$ Here, powers of x & y $\rightarrow$ $a = 0$, $b = 0$

$(x-0)$, $(y-0)$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[(x-0) f_x(0,0) + (y-0) f_y(0,0)\right] + \frac{1}{2!}\left[(x-0)^2 f_{xx}(0,0) + 2(x-0)(y-0) f_{xy}(0,0) + (y-0)^2 f_{yy}(0,0)\right]$$

$$+ \frac{1}{3!}\left[(x-0)^3 f_{xxx}(0,0) + 3(x-0)^2(y-0) f_{xxy}(0,0) + 3(x-0)(y-0)^2 f_{xyy}(0,0) + (y-0)^3 f_{yyy}(0,0)\right]$$

Here:- $f(0,0) = e^0 \log(1+0) = 0$

1° $f_x = \frac{d}{dx}\left(e^x \log(1+y)\right) = e^x \log(1+y) \longrightarrow f_x(0,0) = e^0 \log(1+0) = 0$

$f_y = \frac{d}{dy}\left(e^x \log(1+y)\right) = \frac{e^x}{1+y} \longrightarrow f_y(0,0) = \frac{e^0}{1+0} = 1$

2° $f_{xx} = \frac{d}{dx}(0) = 0$ $\therefore f_{xx}(0,0) = 0$ $\quad f_{yy} = \frac{d}{dy}\left(\frac{e^x}{1+y}\right) = e^x\left(\frac{-1}{(1+y)^2}\right)$ $\therefore f_{yy}(0,0) = \frac{-e^0}{1+0} = -1$

$f_{xy} = \frac{d}{dx}\left(\frac{e^x}{1+y}\right) = \frac{e^x}{1+y}$ $\therefore f_{xy}(0,0) = \frac{e^0}{1+0} = 1$

3° $f_{xxx} = \frac{d}{dx}(0) = 0$ $\therefore f_{xxx}(0,0) = 0$ $\quad f_{xxy} = \frac{d}{dx}\left(\frac{e^x}{1+y}\right) = \frac{e^x}{1+y}$ $\therefore f_{xxy}(0,0) = \frac{e^0}{1+0} = 1$

$f_{yyy} = \frac{d}{dy}\left(\frac{-e^x}{(1+y)^2}\right) = \frac{2e^x}{(1+y)^3}$ $\therefore f_{yyy}(0,0) = \frac{2e^0}{(1+0)^3} = 2$ $\quad f_{xyy} = \frac{d}{dx}\left(\frac{-e^x}{(1+y)^2}\right) = \frac{-e^x}{(1+y)^2}$ $\therefore f_{xyy}(0,0) = \frac{-e^0}{(1+0)^2} = -1$

$$\therefore e^x \log(1+y) = 0 + \frac{1}{1!}\left[x(0) + y(1)\right] + \frac{1}{2!}\left[x^2(0) + 2xy(1) + y^2(-1)\right]$$

$$+ \frac{1}{3!}\left[x^3(0) + 3x^2y(1) + 3xy^2(-1) + y^3(2)\right]$$

$$\Rightarrow e^x \log(1+y) = y + xy - \frac{y^2}{2} + \frac{x^2y}{2} - \frac{xy^2}{2} + \frac{y^3}{3}$$

Qs Expand $\sin xy$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ upto 2^{nd} degree

Sol^n:- $f(x,y) = \sin xy$ Here, $\therefore (x-1) \longrightarrow a = 1$

$\left(y-\frac{\pi}{2}\right) \longrightarrow b = \frac{\pi}{2}$

$$f(x,y) = f\left(1,\frac{\pi}{2}\right) + \frac{1}{1!}\left[(x-1) f_x\left(1,\frac{\pi}{2}\right) + \left(y-\frac{\pi}{2}\right) f_y\left(1,\frac{\pi}{2}\right)\right]$$

$$+ \frac{1}{2!}\left[(x-1)^2 f_{xx}\left(1,\frac{\pi}{2}\right) + 2(x-1)\left(y-\frac{\pi}{2}\right) f_{xy}\left(1,\frac{\pi}{2}\right) + \left(y-\frac{\pi}{2}\right)^2 f_{yy}\left(1,\frac{\pi}{2}\right)\right]$$

$f\left(1,\frac{\pi}{2}\right) = \sin\left(1\times\frac{\pi}{2}\right) = 1$

1° $f_x = \frac{d}{dx}(\sin xy) = y\cos xy \quad \therefore f_x\left(1,\frac{\pi}{2}\right) = \frac{\pi}{2}\cos\left(1\times\frac{\pi}{2}\right) = 0$

$f_y = \frac{d}{dy}(\sin xy) = x\cos xy \quad \therefore f_y\left(1,\frac{\pi}{2}\right) = 1\times\cos\left(\frac{\pi}{2}\right) = 0$

2° $f_{xx} = \frac{d}{dx}(y\cos xy) = y(-\sin xy)\cdot y = -y^2\sin xy \quad \therefore f_{xx}\left(1,\frac{\pi}{2}\right) = -\left(\frac{\pi}{2}\right)^2 \sin\left(1\times\frac{\pi}{2}\right) = -\frac{\pi^2}{4}$

$f_{xy} = \frac{d}{dx}(x\cos xy) = x(-\sin xy)y + \cos xy$
$= -xy\sin xy + \cos xy \quad \therefore f_{xy}\left(1,\frac{\pi}{2}\right) = -(1\times\pi/2)\sin(\pi/2) + \ldots = -\frac{\pi}{2}$

$f_{yy} = \frac{d}{dy}(x\cos xy) = x(-\sin xy)\cdot x = -x^2\sin xy \quad \therefore f_{yy}\left(1,\frac{\pi}{2}\right) = -(1)^2\sin\ldots = -1$

$$\therefore \sin xy = 1 + \frac{1}{1!}\left[(x-1)(0) + \left(y-\frac{\pi}{2}\right)(0)\right] + \frac{1}{2!}\left[(x-1)^2\left(-\frac{\pi^2}{4}\right) + 2(x-1)\left(y-\frac{\pi}{2}\right)\left(-\frac{\pi}{2}\right) + \left(y-\frac{\pi}{2}\right)^2(-1)\right]$$

$$\Rightarrow \sin xy = 1 - \frac{\pi^2}{8}(x-1)^2 - \frac{\pi}{2}(x-1)\left(y-\frac{\pi}{2}\right) - \frac{1}{2}\left(y-\frac{\pi}{2}\right)^2$$

Q6. Expand $e^x \sin y$ about $\left(1, \frac{\pi}{2}\right)$ upto 3rd degree

Sol:- $f(x,y) = e^x \sin y$ Here $\left(1, \frac{\pi}{2}\right) \longrightarrow a = 1, \; b = \pi/2$

$(x-1) \qquad (y-\pi/2)$

$$f(x,y) = f\left(1,\frac{\pi}{2}\right) + \frac{1}{1!}\left[(x-1)f_x + \left(y-\frac{\pi}{2}\right)f_y\right] + \frac{1}{2!}\left[(x-1)^2 f_{xx} + 2(x-1)\left(y-\frac{\pi}{2}\right)f_{xy} + \left(y-\frac{\pi}{2}\right)^2 f_{yy}\right]$$
$$+ \frac{1}{3!}\left[(x-1)^3 f_{xxx} + 3(x-1)^2\left(y-\frac{\pi}{2}\right)f_{xxy} + 3(x-1)\left(y-\frac{\pi}{2}\right)f_{xyy} + \left(y-\frac{\pi}{2}\right)^3 f_{yyy}\right]$$

Here:- $f\left(1,\frac{\pi}{2}\right) = e^1 \sin(\pi/2) = e$

1° $f_x = \frac{d}{dx}(e^x \sin y) = e^x \sin y \;\therefore f_x(1,\pi/2) = e^1 \sin(\pi/2) = e$

$f_y = \frac{d}{dy}(e^x \sin y) = e^x \cos y \;\therefore f_y(1,\pi/2) = e^1 \cos(\pi/2) = 0$

2° $f_{xx} = \frac{d}{dx}(e^x \sin y) = e^x \sin y \;\therefore f_{xx} = e^1 \sin(\pi/2) = e$ $\quad f_{yy} = \frac{d}{dx}(e^x \cos y) = -e^x \sin y \;\therefore f_{yy} = -e^1 \sin(\pi/2) = -e$

$f_{xy} = \frac{d}{dx}(e^x \cos y) = e^x \cos y \;\therefore f_{xy} = e^1 \cos(\pi/2) = 0$

3° $f_{xxx} = \frac{d}{dx}(e^x \sin y) = e^x \sin y \;\therefore f_{xxx} = e^1 \sin(\pi/2) = e$ $\quad f_{xxy} = \frac{d}{dx}(e^x \cos y) = e^x \cos y \;\therefore f_{xxy} = e^1 \cos(\pi/2) = 0$

$f_{yyy} = \frac{d}{dy}(-e^x \sin y) = -e^x \cos y \;\therefore f_{yyy} = -e^1 \cos(\pi/2) = 0$ $\quad f_{xyy} = \frac{d}{dx}(-e^x \sin y) = -e^x \sin y \;\therefore f_{xyy} = -e^1 \sin(\pi/2) = -e$

$$\therefore e^x \sin y = e + \frac{1}{1!}\left[(x-1)e + \left(y-\frac{\pi}{2}\right)(0)\right] + \frac{1}{2!}\left[(x-1)^2 e + 2(x-1)\left(y-\frac{\pi}{2}\right)(0) + \left(y-\frac{\pi}{2}\right)^2(-e)\right]$$
$$+ \frac{1}{3!}\left[(x-1)^3 e + 3(x-1)^2\left(y-\frac{\pi}{2}\right)(0) + 3(x-1)\left(y-\frac{\pi}{2}\right)(-e) + \left(y-\frac{\pi}{2}\right)^2(0)\right]$$

$3 \times 2 = 6$

$$\Rightarrow e^x \sin y = e + (x-1)e + \frac{(x-1)^2 e}{2} + \frac{(y-\pi/2)^2 e}{2} + \frac{(x-1)^3 e}{6} \cancel{+ \frac{(y-\pi/2)^3 e}{6}} - \frac{e(x-1)(y-\pi/2)}{2}$$