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Q1. $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x^2+\frac{1}{x^2}$

Sol:- $(x^2D^2+4xD+2)y=x^2+\frac{1}{x^2}$ $\quad D=\frac{d}{dx}$, $D'=\frac{d}{dz}$

$e^z=x$

$xD=D'$

$x^2D^2=D'(D'-1)=D'^2-D$ $\quad \Rightarrow \log x=z$

$\therefore [D'^2-D+4D'+2]y=e^{2z}+e^{-2z}$

$\Rightarrow (D'^2+3D'+2)y=e^{2z}+e^{-2z}$

$m^2+3m+2=0$ $\quad \therefore m=-1,-2$

$CF=C_1e^{-z}+C_2e^{-2z}$

$=C_1(e^z)^{-1}+C_2(e^z)^{-2}$

$=\frac{C_1}{x}+\frac{C_2}{x^2}$

$PI_1=\frac{e^{2z}}{D'^2+3D'+2}$

$=\frac{e^{2z}}{2^2+3(2)+2}$

$=\frac{e^{2z}}{12}=\frac{(e^z)^2}{12}$

$\Rightarrow PI_1=\frac{x^2}{12}$

$PI_2=\frac{e^{-2z}}{D'^2+3D+2}$

$=\frac{e^{-2z}}{(-2)^2+3(2)+2} \rightarrow 0$

$=\frac{e^{-2z}}{2D'+3}\times z=\frac{e^{-2z}\times z}{(-1)}$

$\Rightarrow PI_2=\frac{-1}{x^2}\cdot\log x$

$\therefore y=\frac{C_1}{x}+\frac{C_2}{x^2}+\frac{x^2}{12}-\frac{\log x}{x^2}$

$a_0(ax+b)^n\frac{d^ny}{dx^n}+a_1(ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+\dots\dots+a_ny$

Let $(ax+b)=e^z$ $\Rightarrow x=\frac{e^z-b}{a}$

$\Rightarrow z=\log(ax+b)$

Q2. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4\sin(\log x)$

Sol:- $(x^2D^2 + xD + 1)y = 4\sin(\log x)$

$e^z = x \Rightarrow \log x = z$ $\quad xD = D'$

$x^2D^2 = D'^2 - D$

$\therefore [D'^2 - \cancel{D} + \cancel{D'} + 1]y = 4\sin z$

$\Rightarrow (D'^2 + 1)y = 4\sin z$

$m^2 + 1 = 0 \Rightarrow$ $\therefore m = \pm i$ $[\alpha = 0, \beta = 1]$

$CF = e^{0x}(C_1\cos z + C_2\sin z)$

$= C_1\cos(\log x) + C_2\sin(\log x)$

$PI = \frac{4\sin z}{D'^2+1}$ $\quad$ Type 2 $\quad D^2 = -a^2 = -1$

$= \frac{\cancel{4}^{2}\sin z}{\cancel{2}D'} \cdot z$

$= 2\int \sin z\, dz$

$= -2\cos z \cdot z$

$= -2\cos(\log x)\cdot(\log x)$

For Circle :- $x = r\cos\theta$, $y = r\sin\theta$

$\rightarrow x^2 + y^2 = r^2$

For Parabola :- $y^2 = 4ax$

$\rightarrow x = at^2$, $y = 2at$

For Hyperbola :- $x = a\sec\theta$, $y = b\tan\theta$

$\rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For Ellipse :- $x = a\cos\theta$, $y = b\sin\theta$

$\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q1. $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$

Sol^n:- Let $(1+x) = e^z$

$\Rightarrow \log(1+x) = z$

$\therefore (1+x)D = D'$

$(1+x)^2D^2 = D'(D'-1)$

$\therefore [(1+x)^2D^2 + (1+x)D + 1]y = 2\sin[\log(1+x)]$

$\Rightarrow [D'^2 - D' + D' + 1]y = 2\sin z$

$\Rightarrow (D'^2+1)y = 2\sin z$

$\therefore m^2+1 = 0 \Rightarrow m = \pm i$

CF:- $CF = e^{0z}(C_1\cos z + C_2\sin z)$

$= C_1\cos(\log(1+x)) + C_2\sin(\log(1+x))$

PI:- $PI = \frac{2\sin z}{D'^2+1}$ Type 1:- $D'^2 = -a^2 = -1$

$= \frac{2\sin z}{2D'} \times z$ $D'^2+1 = -1+1 = 0$

$= \frac{\cancel{2}z}{\cancel{2}}\int \sin z\, dz$

$= -z\cos z$

$= -\log(1+x)\cdot\cos(\log(1+x))$

Q2. $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

Sol:- Let $(3x+2) = e^z$ $\rightarrow x = \frac{e^z - 2}{3}$

$\Rightarrow \log(3x+2) = z$

We know, $(ax+b)D = aD'$

$(ax+b)^2 D^2 = a^2 D'(D'-1)$

Here, $a = 3$

$\therefore (3x+2)D = 3D'$

$(3x+2)^2 D^2 = 9D'(D'-1) = 9D'^2 - 9D'$

$\therefore [(3x+2)D^2 + 3\cdot 3D - 36]y = 3x^2 + 4x + 1$

$\Rightarrow [9D'^2 - \cancel{9D'} + \cancel{9D'} - 36]y = 3\frac{(e^z-2)^2}{3^2} + 4\left(\frac{e^z-2}{3}\right) + 1$

$\Rightarrow 9(D'^2 - 4)y = \frac{(e^{2z} + 4 - \cancel{4e^z}) + \cancel{4e^z} - 8 + 3}{3}$

$\Rightarrow (D'^2 - 4)y = \frac{e^{2z} - 1}{27}$

$\therefore m^2 - 4 = 0 \Rightarrow m = \pm 2$ $\quad \cancel{CF = C_1 e^{2z} + C_2 e^{-2z} = C_1}$

$CF = C_1 e^{2z} + C_2 e^{-2z}$

$\cancel{= C_1 e}$

$= C_1 e^{2\log(3x+2)} + C_2 e^{-2\log(3x+2)}$ $\quad e^{\log a} = a$

$= C_1 e^{\log(3x+2)^2} + C_2 e^{\log(3x+2)^{-2}}$

$= C_1(3x+2)^2 + \frac{C_2}{(3x+2)^2}$

$PI = \left(\frac{e^{2z}}{27} - \frac{e^{0z}}{27}\right) \cdot \frac{1}{D'^2 - 4}$

$= \frac{e^{2z} \times z}{27 \times 2D'} - \frac{1}{27(-4)}$

$= \frac{z e^{2z}}{108} + \frac{1}{108} = \frac{\log(3x+2)\cdot(3x+2) + 1}{108}$