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Lagrange's Method (2 variable)

Steps 1) Let $f(x,y) =$
$\phi(x,y) =$

2) $F(x,y) = f(x,y) + \lambda\, \phi(x,y)$

3) $dF = \left[\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy\right] + \lambda \left[\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy\right]$

4) $dF = 0$ (dF w.r.t x & y → partial differentiated)

5) $\boxed{x = }$ & $\boxed{y = }$ (Find x & y)

↓ put in ϕ

$\boxed{\lambda = }$

Q1. Find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ using method of Lagrange's multiplier.

Soln:- Let $f(x,y) = 3x + 4y$ & $\phi(x,y) = x^2 + y^2 - 1$ → 1st step

$\therefore F(x,y) = f(x,y) + \lambda\, \phi(x,y)$
$= 3x + 4y + \lambda(x^2 + y^2 - 1)$ → 2nd step

3rd step:- Now,

$$dF = \left[\left(\frac{\partial f}{\partial x}\right) \cdot dx + \left(\frac{\partial f}{\partial y}\right) \cdot dy\right] + \lambda \left[\left(\frac{\partial \phi}{\partial x}\right) dx + \left(\frac{\partial \phi}{\partial y}\right) dy\right]$$

Here, $\cdot\ \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(3x + 4y) = 3$

$\cdot\ \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(3x + 4y) = 4$

$\cdot\ \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2 - 1) = 2x$

$\cdot\ \frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2 - 1) = 2y$

$\therefore dF = \left[(3)dx + (4)dy\right] + \lambda\left[(2x)dx + (2y)dy\right]$

$\Rightarrow dF = dx(3 + 2x\lambda) + dy(4 + 2y\lambda)$

4th step:- $dF = 0$

$\Rightarrow (3 + 2\lambda x)dx + (4 + 2\lambda y)dy = 0$

(*) $A\,dx + B\,dy = 0 \quad \left[\begin{array}{l} A = 3 + 2\lambda x \\ B = 4 + 2\lambda y \end{array}\right]$

Here, dx & dy $\rightarrow$ independent variables

5th step:- For above eqⁿ to hold true, each coefficient must be independently $= 0$

$\therefore A = 0$ & $B = 0$

$\Rightarrow 3 + 2\lambda x = 0$ $\Rightarrow 4 + 2\lambda y = 0$

$\Rightarrow x = \frac{-3}{2\lambda}$ $\Rightarrow y = \frac{-4}{2\lambda}$

Find λ:- Put x & y in ϕ:-

$\therefore \phi(x,y) = \left(\frac{-3}{2\lambda}\right)^2 + \left(\frac{-4}{2\lambda}\right)^2 - 1 = 0$

$\Rightarrow \frac{9 + 16}{4\lambda^2} = 1$

$\Rightarrow \frac{25}{4} = \lambda^2$ $\Rightarrow \lambda = \frac{5}{2}, \frac{-5}{2}$

Find x & y:- When $\lambda = \frac{5}{2}$, $x = \frac{-3}{2\left(\frac{5}{2}\right)} = \frac{-3}{5}$ & $y = \frac{-4}{2\left(\frac{5}{2}\right)} = \frac{-4}{5}$

When $\lambda = \frac{-5}{2}$, $x = \frac{-3}{2\left(\frac{-5}{2}\right)} = \frac{3}{5}$ & $y = \frac{-4}{2\left(\frac{-5}{2}\right)} = \frac{4}{5}$

Maxima & Minima:- For $\left(\frac{-3}{5}, \frac{-4}{5}\right)$:- $f\left(\frac{-3}{5}, \frac{-4}{5}\right) = 3\left(\frac{-3}{5}\right) + 4\left(\frac{-4}{5}\right) = \frac{-9}{5} - \frac{16}{5} = \frac{-25}{5} = -5$ (Minima)

For $\left(\frac{3}{5}, \frac{4}{5}\right)$:- $f\left(\frac{3}{5}, \frac{4}{5}\right) = 3\left(\frac{3}{5}\right) + 4\left(\frac{4}{5}\right) = \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$ (Maxima)

Q2. Find the value of x^2+y^2 subjected to the condition $x+y=2$ by Lagrange Multiplier

Sol:- Let $f(x,y) = x^2+y^2$ & $\phi(x,y) = x+y-2$

$\therefore F(x,y) = f(x,y) + \lambda\,\phi(x,y)$

$= x^2+y^2+\lambda(x+y-2)$

Here, • $\frac{\partial f}{\partial x} = 2x$ • $\frac{\partial f}{\partial y} = 2y$ • $\frac{\partial \phi}{\partial x} = 1$ • $\frac{\partial \phi}{\partial y} = 1$

$\therefore dF = \left[2x\,dx + 2y\,dy\right] + \left[\lambda\,dx + \lambda\,dy\right]$

$\Rightarrow dF = dx(2x+\lambda) + dy(2y+\lambda)$

$dF = 0$

$\Rightarrow (2x+\lambda)\,dx + (2y+\lambda)\,dy = 0$

$\therefore 2x+\lambda = 0$ & $2y+\lambda = 0$

$\Rightarrow x = \frac{-\lambda}{2}$ $\Rightarrow y = \frac{-\lambda}{2}$

Find λ:- Put x & y in $\phi(x,y)$:-

$\therefore \phi(x,y) = \left(\frac{-\lambda}{2}\right) + \left(\frac{-\lambda}{2}\right) - 2 = 0$

$\Rightarrow \frac{-\cancel{2}\lambda}{\cancel{2}} = 2 \Rightarrow \lambda = -2$

Find x & y:- $\therefore x = \frac{-(-2)}{2} = 1$ & $y = \frac{-(-2)}{2} = 1$

Maxima & Minima:- For $(1,1)$:- $f(1,1) = (1)^2+(1)^2 = 2$ (maxima)

No minima Max value $= 2$