Q. Test for Convergence for following:-

(i) $a = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots$

Sol:- $a_n = \frac{n}{n+1}$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}(1+\frac{1}{n})}$$

$$= \frac{1}{(1+\frac{1}{\infty})}$$

$\Rightarrow l = 1$ (finite)

$\therefore$ a_n sequence $\rightarrow$ convergent

(ii) $a = 1, 4, 9, 16, \ldots$

Sol:- $a_n = n^2$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2$$

$\Rightarrow l = \infty^2$ (infinite)

$\therefore$ a_n sequence $\rightarrow$ divergent

In sequence, every element is related to previous or next element and follow a general formula/form

For series :- (S)

$a = a_1, a_2, a_3, a_4 \ldots a_n$

$\therefore S = a_1 + a_2 + a_3 + \ldots + a_n$

$\Rightarrow S = \Sigma a$

$$\Rightarrow \boxed{S = \sum_{n=1}^{\infty} a_n}$$

- In Sequence, elements are arranged in proper order
- In Series, elements are added

$\therefore$ Series $\rightarrow$ Sum of elements in sequence

Eg (i) $a = 1, 4, 9, 16, \ldots n^2$

$\therefore S =$ Sumation of sequence in a

$= 1 + 4 + 9 + 16 + \ldots n^2$

$$\boxed{S = \sum_{n=1}^{\infty} n^2}$$

Convergency check :-

- Find n^{th} term (S_n)
- $\lim_{n \to \infty} S_n$

defined $\rightarrow$ convergent

undefined $\rightarrow$ divergent

Geometric Series :- $\left[S_n = \sum_{n=1}^{\infty} a r^{n-1} \right]$

$\lim_{n \to \infty} \frac{a}{1-r}$:-
- $r < 1 \rightarrow$ converging
- $r \geq 1 \rightarrow$ diverging
- $r = -1 \rightarrow$ oscillate

Q. Test Convergence of series :-

(i) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

Sol^n: $\sum_{n=1}^{\infty} \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1}$ $\left[\text{Write in form of } \sum_{n=1}^{\infty} a r^{n-1} \right]$

$a = 1/3$ $\quad r = 1/3$ $\quad (r < 1)$

Now, we find $\lim_{n \to \infty} \frac{a}{1-r}$

$\lim_{n \to \infty} \frac{a}{1-r} = \lim_{n \to \infty} \frac{1/3}{1-\frac{1}{3}} = \frac{1}{2}$

Therefore,
$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \rightarrow$ convergent

(ii) $\sum_{n=1}^{\infty} 3^n$

Sol^n:- $\sum_{n=1}^{\infty} 3 \cdot (3)^{n-1}$ $\left[\text{Write in form of } \sum_{n=1}^{\infty} a r^{n-1} \right]$

$a = 3$ $\quad r = 3$ $\quad (r \geq 1)$

Now, we find $\lim \frac{a}{1-r}$

$\frac{a}{1-r} = \frac{3}{1-3} = \frac{-3}{2}$

Therefore,
$\sum_{n=1}^{\infty} 3^n \rightarrow$ divergent