Radius of curvature :-

① $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

Q1. $y = 4\sin x - \sin 2x$
Find radius of curvature at $x = \frac{\pi}{2}$

Sol:- $y = 4\sin x - \sin 2x$

For y_1 :- $y_1 = \frac{d}{dx}(4\sin x - \sin 2x)$

$\Rightarrow y_1 = 4\cos x - 2\cos 2x$

At $x = \pi/2$,

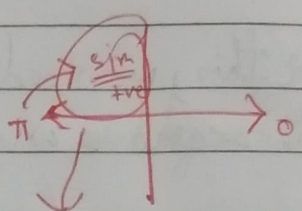
$y_1 = 4\cos\left(\frac{\pi}{2}\right) - 2\cos\cancel{2}\times\frac{\pi}{\cancel{2}}$

$= 4(0) - 2\cos(\pi - 0)$

$= 0 - 2(-\cos 0)$

$= 2(1)$

$\Rightarrow y_1 = 2$



cos → −ve

↳ $-\cos\theta = \cos(\pi - \theta)$

For y_2 :- $y_2 = \frac{d}{dx}(4\cos x - 2\cos 2x)$

$= 4(-\sin x) - 2(-2\sin 2x)$

$\Rightarrow y_2 = -4\sin x + 4\sin 2x$

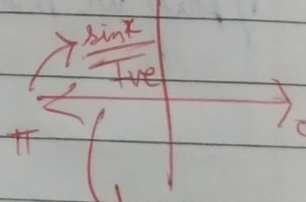
At $x = \pi/2$,

$y_2 = -4\sin(\pi/2) + 4\sin\left(\cancel{2}\times\frac{\pi}{\cancel{2}}\right)$

$= -4(1) + 4\sin(\pi - 0)$

$= -4 + 4(0)$

$\Rightarrow y_2 = -4$



sin → +ve

↳ $\sin\theta = \sin(\pi - \theta)$

Now,

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1+2^2)^{3/2}}{(-4)}$$

$$= \frac{(1+\cancel{4})^{3/2}}{-4}$$

Radius can't be -ve

$$\therefore \rho = \frac{5^{3/2}}{4}$$

Q2. $y^2 = 4ax$ (Parabola)
Find radius of curvature at any point

Sol:- $y^2 = 4ax$ ——— (1)

For y_1 :- $y_1 = \frac{d}{dx}$ (Differentiate both sides of (1) w.r.t x

$$2y \cdot \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y}$$

$$\Rightarrow y_1 = \frac{2a}{y}$$

For y_2 :- $$y_2 = \frac{d}{dx}\left(\frac{2a}{y}\right)$$

$$= 2a\left(-\frac{1}{y^2}\right) \cdot \frac{dy}{dx}$$

$y_1 = \frac{2a}{y}$

$$= \frac{-2a}{y^2} \cdot \frac{2a}{y}$$

$$\Rightarrow y_2 = \frac{-4a^2}{y^3}$$

Now,
$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$
$$= \frac{\left[1+\left(\frac{2a}{y}\right)^2\right]^{3/2}}{\left(\frac{-4a^2}{y^3}\right)}$$
$$= \frac{(y^2+4a^2)^{3/2}}{(y^2)^{3/2}} \times \frac{y^3}{-4a^2}$$
Radius cant be -ve

$$= \frac{(y^2+4a^2)^{3/2}}{\cancel{y^3}} \times \frac{\cancel{y^3}}{4a^2}$$
$[y^2 = 4ax$
$$= \frac{(4ax+4a^2)^{3/2}}{4a^2}$$
$$= \frac{[4a(x+a)]^{3/2}}{4a^2}$$
$$= \frac{(4a)^{3/2} \cdot (x+a)^{3/2}}{4a^2}$$
$$= 4^{3/2-1} \cdot a^{3/2-2} (x+a)^{3/2}$$
$$= 4^{1/2} \cdot a^{-1/2} (x+a)^{3/2}$$

$$\therefore \rho = \frac{2}{\sqrt{a}} (x+a)^{3/2}$$

Q3. Find the radius of curvature at any point of curve :-
$y = c \log \sec\left(\frac{x}{c}\right)$

Sol :- $y = c \log \sec\left(\frac{x}{c}\right)$

For y_1 :- $y_1 = \frac{d}{dx}\left(c \log \sec\left(\frac{x}{c}\right)\right)$
$$= c \cdot \frac{1}{\sec\left(\frac{x}{c}\right)} \cdot \frac{d}{dx}\left(\sec\left(\frac{x}{c}\right)\right) \cdot \frac{d}{dx}\left(\frac{x}{c}\right)$$
$$= \cancel{c} \cdot \frac{1}{\sec\left(\frac{x}{c}\right)} \cdot \sec\left(\frac{x}{c}\right) \tan\left(\frac{x}{c}\right) \cdot \frac{1}{\cancel{c}}$$

$$\Rightarrow y_1 = \tan\left(\frac{x}{c}\right)$$

For y_2 :- $y_2 = \frac{d}{dx}\left(\tan\left(\frac{x}{c}\right)\right)$

$$= \sec^2\left(\frac{x}{c}\right) \cdot \frac{d}{dx}\left(\frac{x}{c}\right)$$

$$\Rightarrow y_2 = \frac{1}{c}\sec^2\left(\frac{x}{c}\right)$$

Now,

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{\left(1+\tan^2\left(\frac{x}{c}\right)\right)^{3/2}}{\frac{1}{c}\sec^2\left(\frac{x}{c}\right)}$$

We know, $1+\tan^2\theta = \sec^2\theta$

$$\therefore \rho = \frac{\left[\sec^{\cancel{2}}\left(\frac{x}{c}\right)\right]^{3/\cancel{2}}}{\frac{1}{c}\sec^2\left(\frac{x}{c}\right)}$$

$$= \frac{\sec^3(x/c) \times c}{\sec^2(x/c)}$$

$$\Rightarrow \rho = c\sec\left(\frac{x}{c}\right)$$