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Q1. Find the radius of curvature for $\sqrt{\frac{x}{a}} - \sqrt{\frac{y}{b}} = 1$ at the point where it touches the coordinate axises.

Sol:- Given curve :- $\sqrt{\frac{x}{a}} - \sqrt{\frac{y}{b}} = 1$ —— (1)

(1) When it touches x-axis, $y = 0$:-

$$(1) \Rightarrow \sqrt{\frac{x}{a}} - \sqrt{\frac{0}{b}} = 1 \Rightarrow \sqrt{\frac{x}{a}} = 1$$

(Square both sides)

$\therefore x = a$

$\therefore$ Point $(a, 0)$ [Case 1]

(2) When it touches y-axis, $x = 0$:-

$$(1) \Rightarrow \sqrt{\frac{0}{a}} - \sqrt{\frac{y}{b}} = 1 \Rightarrow -\sqrt{\frac{y}{b}} = 1$$

(Square both sides)

$\therefore y = b$

$\therefore$ Point $(0, b)$ [Case 2]

For y_1 :- Differentiate both sides w.r.t x :-

$$\frac{1}{\sqrt{a}} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{b}} \cdot \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{a} \cdot \cancel{2}\sqrt{x}} \cdot \sqrt{b} \cdot \cancel{2}\sqrt{y}$$

$$\Rightarrow y_1 = \frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{\sqrt{y}}{\sqrt{x}} \rightarrow \frac{\sqrt{b}}{\sqrt{a}} \frac{\sqrt{y}}{\sqrt{x}}$$

For y_2 :- $y_2 = \frac{d}{dx}\left(\frac{\sqrt{b}}{\sqrt{a}} \frac{\sqrt{y}}{\sqrt{x}}\right)$

$$= \frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{d}{dx}\left(\frac{\sqrt{y}}{\sqrt{x}}\right)$$

$$= \frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{\sqrt{x} \frac{1}{2\sqrt{y}}\frac{dy}{dx} - \sqrt{y} \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{\sqrt{x} \cdot \frac{1}{2\sqrt{y}}\left(\frac{\sqrt{b}}{\sqrt{a}} \frac{\sqrt{y}}{\sqrt{x}}\right) - \sqrt{y} \cdot \frac{1}{2\sqrt{x}}}{x}$$

$$\Rightarrow y_2 = \frac{\sqrt{b}}{x\sqrt{a}} \cdot \frac{1}{2}\left[\frac{\sqrt{b}}{\sqrt{a}} - \frac{\sqrt{y}}{\sqrt{x}}\right]$$

A/a,

$$\frac{\sqrt{x}}{\sqrt{a}} - \frac{\sqrt{y}}{\sqrt{b}} = 1$$

$$\Rightarrow \sqrt{x}\sqrt{b} - \sqrt{y}\sqrt{a} = \sqrt{a}\sqrt{b} \quad \text{——— (2)}$$

$$\therefore y_2 = \frac{\sqrt{b}}{x\sqrt{a}} \cdot \frac{1}{2}\left[\frac{\sqrt{x}\sqrt{b} - \sqrt{y}\sqrt{a}}{\sqrt{a}\sqrt{x}}\right]$$

(2) in y_2 :-

$$\rightarrow y_2 = \frac{\sqrt{b}}{2x\sqrt{a}}\left[\frac{\cancel{\sqrt{a}}\sqrt{b}}{\cancel{\sqrt{a}}\sqrt{x}}\right]$$

$$= \frac{b}{2x^{3/2}\sqrt{a}}$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{\left[1+\left(\frac{\sqrt{b}}{\sqrt{a}}\frac{\sqrt{y}}{\sqrt{x}}\right)^2\right]^{3/2}}{\left(\frac{b}{2x^{3/2}a^{1/2}}\right)}$$

$$= \left(\frac{ax+by}{ax}\right)^{3/2} \cdot \frac{2x^{3/2}\cdot a^{1/2}}{b}$$

$$= \frac{(ax+by)^{3/2}}{a^{3/2}\cdot \cancel{x^{3/2}}} \cdot \frac{2\cancel{x^{3/2}}\cdot a^{1/2}}{b}$$

$$= \frac{2(ax+by)^{3/2}}{b\cdot a^{3/2-1/2}}$$

$$\rightarrow \Rightarrow \rho = \frac{2(ax+by)^{3/2}}{ab}$$

- At $(a,0)$:- $\rho = \frac{2[a(a)+b(0)]^{3/2}}{ab} = \frac{2(a^2)^{3/2}}{ab} = \frac{2a^2}{b}$

  (x-axis)

- At $(0,b)$:- $\rho = \frac{2[a(0)+b(b)]^{3/2}}{ab} = \frac{2(b^2)^{3/2}}{ab} = \frac{2b^2}{a}$

Q2. Find the radius of curvature at point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3+y^3=3axy$

Sol:- Given curve :- $x^3+y^3=3axy$ → uv form

For y_1:- Differentiate both sides w.r.t x

$$\Rightarrow 3x^2+3y^2\cdot\frac{dy}{dx}=3a\left[x\frac{dy}{dx}+y\right]$$

Incorrect $\left\{ \cancel{\Rightarrow y_1=\frac{3ay-3x^2}{3y^2}} \quad \cancel{\Rightarrow y_1=\frac{ay-x^2}{y^2}} \right.$

$$\Rightarrow x^2-ay=\frac{dy}{dx}\left[ax-y^2\right]$$

$$\Rightarrow y_1=\frac{x^2-ay}{ax-y^2}$$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right)$:- $y_1=\dfrac{\left(\frac{3a}{2}\right)^2-a\left(\frac{3a}{2}\right)}{a\left(\frac{3a}{2}\right)-\left(\frac{3a}{2}\right)^2}$

$$=\frac{\frac{9a^2}{4}-\frac{3a^2\times 2}{2\times 2}}{\frac{3a^2\times 2}{2\times 2}-\frac{9a^2}{4}}$$

$$\Rightarrow y_1=\frac{9a^2-6a^2}{4}\times\frac{4}{6a^2-9a^2}$$

$$=\frac{\cancel{3a^2}}{\cancel{4}}\times\frac{\cancel{4}}{-\cancel{3a^2}}$$

$$\Rightarrow y_1=-1$$

$$y_1=-1$$

For y_2 :- $y_2 = \frac{d}{dx}\left(\frac{x^2-ay}{ax-y^2}\right)$

$$= \frac{(ax-y^2)\,\frac{d}{dx}(x^2-ay) - (x^2-ay)\cdot\frac{d}{dx}(ax-y^2)}{(ax-y^2)^2}$$

$$= \frac{(ax-y^2)\left[2x - a\left(\frac{dy}{dx}\right)\right] - (x^2-ay)\left[a - 2y\left(\frac{dy}{dx}\right)\right]}{(ax-y^2)^2}$$

Here $\frac{dy}{dx} = y_1 = -1$

$$\cancel{= (ax-y^2)\left[2x - a\left(\frac{x^2-ay}{ax-y^2}\right)\right]}$$

$$= \frac{(ax-y^2)\left[2x - a(-1)\right] - (x^2-ay)\left[a - 2y(-1)\right]}{(ax-y^2)^2}$$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right)$:- $y_2 = \frac{\left[a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2\right]\left[2\left(\frac{3a}{2}\right) + a\right] - \left[\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)\right]\left[a + 2\left(\frac{3a}{2}\right)\right]}{\left[a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2\right]^2}$

$$= \frac{\left[\frac{3a^2 \times 2}{2 \times 2} - \frac{9a^2}{4}\right](4a) - \left[\frac{9a^2}{4} - \frac{3a^2 \times 2}{2 \times 2}\right](4a)}{\left[\frac{3a^2 \times 2}{2 \times 2} - \frac{9a^2}{4}\right]^2}$$

$$= \frac{\left(\frac{-3a^2}{\cancel{4}}\right)(\cancel{4}a) - \left(\frac{3a^2}{\cancel{4}}\right)(\cancel{4}a)}{\left(\frac{3a^2}{4}\right)^2}$$

$$\Rightarrow y_2 = \frac{-3a^3 - 3a^3}{6a^4} \times 16$$

$$= -\frac{\cancel{6}^{\,2}\,a^3}{\cancel{9}_{\,3}\,a^4} \times 16$$

$$\Rightarrow y_2 = -\frac{32}{3a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1+(-1)^2)^{3/2}}{\left(\frac{-32}{3a}\right)} \rightarrow \text{can't be -ve}$$

$$= \frac{2^{3/2}}{2^5} \times 3a$$

$$= 2^{3/2-5} \times 3a$$

$$\Rightarrow \rho = 2^{-7/2} \times 3a \quad \text{(or)} \quad \frac{2\sqrt{2}}{32} \times 3a$$

$2 \times 16 = 32$

Q.3: Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = 1$

Sol^n:- Given curve :- $\sqrt{x} + \sqrt{y} = 1$

For y_1 :- Differentiate both sides w.r.t x

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\cancel{2}\sqrt{x}} \times \cancel{2}\sqrt{y}$$

$$\Rightarrow y_1 = -\frac{\sqrt{y}}{\sqrt{x}}$$

For y_2 :- $y_2 = \frac{d}{dx}\left(\frac{-\sqrt{y}}{\sqrt{x}}\right)$

$$= -\left[\frac{\sqrt{x} \cdot \frac{d}{dx}(\sqrt{y}) - \sqrt{y}\,\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}\right]$$

$$= -\left[\frac{\sqrt{x} \cdot \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} - \sqrt{y}\,\frac{1}{2\sqrt{x}}}{x}\right] \quad \left[\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}\right]$$

$$= -\left[\frac{\cancel{\sqrt{x}} \cdot \frac{1}{2\cancel{\sqrt{y}}}\left(\frac{-\cancel{\sqrt{y}}}{\cancel{\sqrt{x}}}\right) - \sqrt{y} \cdot \frac{1}{2\sqrt{x}}}{x}\right]$$

$$\therefore y_2 = \frac{\left(-\frac{1}{2} - \frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}\right)}{x}$$

$$\Rightarrow y_2 = \left(\frac{1}{2} + \frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}\right) \cdot \frac{1}{x}$$

At $\left(\frac{1}{4}, \frac{1}{4}\right)$:-

- $y_1 = -\sqrt{\cancel{\frac{1}{4}}} \times \cancel{\sqrt{4}} = \underline{\underline{-1}}$
- $y_2 = \left(\frac{1}{2} + \frac{1}{2}\cancel{\frac{\sqrt{1/4}}{\sqrt{1/4}}}\right) \cdot \frac{1}{(1/4)}$

$$= (1)(4)$$

$$\Rightarrow y_2 = 4$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1+(-1)^2)^{3/2}}{4}$$

$$= \frac{2^{3/2}}{2^2}$$

$$= 2^{3/2-2}$$

$$\Rightarrow \rho = 2^{-1/2}$$

$$= \frac{1}{\sqrt{2}}$$