

21MAB101T MATRICES - UNIT-I

Matrix from Quadratic form



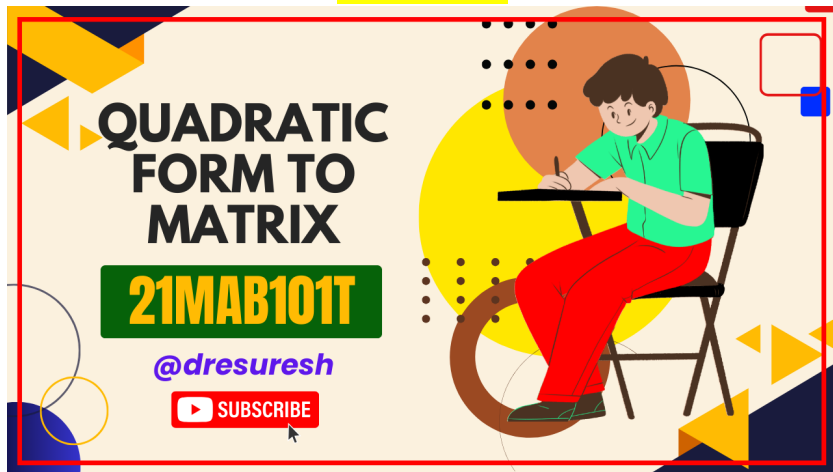
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Quadratic form to Real Symmetric Matrix

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Quadratic Form

The given quadratic form is $Q = X^T A X$, i.e.

$$a x_1^2 + b x_2^2 + c x_3^2 + d x_1 x_2 + e x_2 x_3 + f x_1 x_3$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$X^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

Hence the matrix of the quadratic form is

$$A = \begin{pmatrix} \text{coefficient of } x_1^2 & \frac{1}{2} \text{coefficient of } x_1 x_2 & \frac{1}{2} \text{coefficient of } x_1 x_3 \\ \frac{1}{2} \text{coefficient of } x_1 x_2 & \text{coefficient of } x_2^2 & \frac{1}{2} \text{coefficient of } x_2 x_3 \\ \frac{1}{2} \text{coefficient of } x_1 x_3 & \frac{1}{2} \text{coefficient of } x_2 x_3 & \text{coefficient of } x_3^2 \end{pmatrix}$$

Example 1.

Write down the matrix of the quadratic form

$$3x_1^2 + 5x_2^2 + 7x_3^2 + 4x_1x_2 + 6x_2x_3 + 8x_1x_3.$$

Solution. The given quadratic form is $Q = X^TAX$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad X^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

Hence the matrix of the quadratic form is

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 5 & 3 \\ 4 & 3 & 7 \end{bmatrix}$$

Example 2.

Write down the matrix of the quadratic form

$$x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_3x_2 + 6x_3x_1 .$$

Solution. The given quadratic form is $Q.F. = X^T A X$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad X^T = [x_1 \quad x_2 \quad x_3]$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Example 3.

Write down the matrix of the quadratic form $2x^2 + 6y^2 + 2z^2 + 2xz$.

Solution. The given quadratic form is $Q.F. = X^T A X$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad X^T = [x_1 \quad x_2 \quad x_3]$$

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 6 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Example 4.

Write down the matrix of the quadratic form

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3.$$

Solution. The given quadratic form is $Q.F. = X^TAX$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad X^T = [x_1 \quad x_2 \quad x_3]$$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Example 5.

Write down the matrix of the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_1x_3$.

Solution. The given quadratic form is $Q.F. = X^TAX$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad X^T = [x_1 \quad x_2 \quad x_3]$$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Example 6.

Write down the matrix of the quadratic form $7x_1^2 - 10x_1x_2 + 6x_2^2$.

Solution. The given quadratic form is $Q.F. = X^T A X$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad X^T = [x_1 \quad x_2]$$

$$A = \begin{bmatrix} 7 & -5 \\ -5 & 6 \end{bmatrix}$$