Gibbs-Helmholtz Equation

By Gibb's Energy :- $G = H - TS$

enthalpy → entropy

Enthalpy → $\Delta H = \Delta E + p\Delta V$ (at const pressure)

Entropy → $T\Delta S = \Delta q$ (at const temp)

$$\therefore G = E + pV - TS$$

$$\Rightarrow \Delta G = \Delta E + (p\Delta V + V\Delta p) - (T\Delta S + S\Delta T)$$

By 1st law :- $dE = dq - pdV$

By 2nd law :- $dq = TdS$

$$\therefore dG = dE + pdV + Vdp - TdS - SdT$$

$$= \cancel{dq} - \cancel{pdV} + \cancel{pdV} + Vdp - \cancel{dq} - SdT$$

$$\therefore dG = Vdp - SdT$$

But At constant pressure ($dp = 0$)

$$\therefore dG = -SdT$$

$$\Rightarrow \frac{dG}{dT} = -S$$

for any 2 states of system :- $\left\{ \begin{array}{l} dG_1 = -S_1 dT \\ dG_2 = -S_2 dT \end{array} \right\}$

$$\Rightarrow dG_2 - dG_1 = -\{S_2 dT - (-S_1 dT)\}$$

$$\Rightarrow d\Delta G = -(S_2 - S_1) dT$$

$$= -\Delta S dT$$

$$\Rightarrow \boxed{\frac{d\Delta G}{dT} = -\Delta S}$$

By Gibbs energy :- $\Delta G = \Delta H - T\Delta S$ (const temp)

$$\Rightarrow -\Delta S = \frac{\Delta G - \Delta H}{T}$$

But ~~$-\Delta S$~~ $\boxed{-\Delta S = \frac{d\Delta G}{dT}}$ → at const pressure

$$\therefore \frac{d\Delta G}{dT} = \frac{\Delta G - \Delta H}{T}$$

$$\Rightarrow T\left(\frac{d\Delta G}{dT}\right)_P = \Delta G - \Delta H$$

$$\Rightarrow \Delta G = \Delta H + T\left(\frac{d\Delta G}{dT}\right)_P$$

This is Gibb-Helmholtz eq^n interms of
- Gibbs' free energy
- Enthalpy