Monotonic Sequence :-

A sequence is said to be monotonic if it is either monotonically increasing/decreasing

- Monotonically increasing :- If $S_{n+1} \geq S_n \ \forall n$

$$S_1 \leq S_2 \leq S_3 \leq \dots S_n \leq S_{n+1} \leq \dots$$

Eg:- $S_n = \{2, 4, 6, 8, 10 \dots\}$ (unbounded)

Here,

$$2 \leq 4 \leq 6 \leq 8 \leq 10 \leq \dots S_n \leq S_{n+1}$$

- Monotonically decreasing :- If $S_{n+1} \leq S_n \ \forall n$

$$S_1 \geqslant S_2 \geqslant S_3 \geqslant \dots S_n \geqslant S_{n+1} \geqslant \dots$$

Eg:- $S_n = \{10, 8, 6, 4, 2, \dots\}$ (unbounded)

Here,

$$10 \geqslant 8 \geqslant 6 \geqslant 4 \geqslant 2 \geqslant \dots S_n \geqslant S_{n+1}$$

Limit of a Sequence :-

If $\lim_{n\to\infty} S_n = l$, then l → limit of sequence

Eg:- $\langle S_n \rangle = \frac{n}{n+1}$ [To find limit of this sequence]

Sol:- $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n}{n+1}$

Take n common :-

$$= \lim_{n\to\infty} \frac{n}{n\left(1+\frac{1}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{\infty}\right)} \quad \left[\text{We know, } \frac{1}{\infty} = 0\right]$$

$$\therefore \lim_{n\to\infty} = \frac{1}{1+0} = 1$$

$\therefore l = 1$

1 → limit of the sequence $\langle S_n \rangle$

Convergent Sequence :-

If $\lim_{n\to\infty} S_n = l$, then we can say that the sequence $\langle S_n \rangle$ converges to 'l'

(or)

$\langle S_n \rangle$ is convergent to limit 'l'

Eg:- $\langle S_n \rangle = \frac{1}{n}$ $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n}\right\}$

Sol:- $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{n}$

$= \frac{1}{\infty} = 0$

→ This means all the points are converging to the point 0 (value)

Divergent Sequence :-

A sequence which is not convergent is called a divergent sequence where $\lim_{n\to\infty} S_n = \pm\infty$

Eg:- $\langle S_n \rangle = n$

Sol:- $\lim_{n\to\infty} S_n = \lim_{n\to\infty} n$

$= \infty$

→ This means all points/values are diverting/undetermined

∴ In Convergent, Sequence approaches a finite real number

In Divergent, Sequence tend to $+\infty$ or $-\infty$

Types of Divergent Sequences:-

(a) Divergent to $+\infty$:- $\lim_{n\to\infty} S_n = \lim_{n\to\infty} n = +\infty$ $\quad [\langle S_n \rangle = n]$

(b) Divergent to $-\infty$:- $\lim_{n\to\infty} S_n = \lim_{n\to\infty} -n = -\infty$ $\quad [\langle S_n \rangle = -n]$

(c) Oscillating finitely :- $\langle S_n \rangle = (-1)^n \to -1, 1, -1, \ldots\ldots$

(d) Oscillating infinitely :- $\langle S_n \rangle = (-1)^n n \to -1, 2, -3, 4, \ldots$