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Particular Integral (Type 1 → $X = e^x$)

Q. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = e^{3x}$

Solⁿ:- Auxillary Function :- $D^3y - 3D^2y + 4y = e^{3x}$

$\Rightarrow (D^3 - 3D^2 + 4)y = e^{3x}$

∵ In auxillary, we replace RHS with 0

$\therefore D^3 - 3D^2 + 4 = 0$ ∴ $D = -1, \underline{2, 2}$ (repeated)

$C.F = c_1 e^{m_1 x} + (c_2 + c_3 x) e^{m_2 x}$

$= c_1 e^{-1} + (c_2 + c_3 x) e^{2x}$

* Here, RHS of given differential eqⁿ is not zero

∴ $\boxed{y = C.F + P.I}$

For P.I :- ($X = e^{ax}$ type)

$$P.I = \frac{1}{f(D)} e^{ax}$$

Here, $f(D)$ = LHS of axillary eqⁿ

$\Rightarrow f(D) = D^3 - 3D^2 + 4$

$\therefore P.I = \frac{1}{D^3 - 3D^2 + 4} \cdot e^{3x}$ [Here, $a = 3$]

Here, ∵ $a = 3$, we put this value in place of D

$\therefore P.I = \frac{1}{(3)^3 - 3(3)^2 + 4} \cdot e^{3x}$

$= \frac{1}{4} e^{3x}$

∴ General solⁿ is given by :- $y = C.F + P.I$

$\Rightarrow y = c_1 e^{-x} + (c_2 + c_3 x) e^{2x} + \frac{e^{3x}}{4}$

Q2. $(D^3-2D^2-5D+6)y = e^{3x}+8$

Sol^n:- $D^3-2D^2-5D+6=0$ [Auxillary eq^n → RHS becomes 0]

$\therefore D = -2, 3, 1$

For CF :- $CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

$= c_1 e^{-2x} + c_2 e^{3x} + c_3 e^{x}$

For P.I :- $P.I = \dfrac{1}{f(D)} \cdot (e^{3x}+8)$

Here, $f(D) = D^3-2D^2-5D+6$

We can sepperate $\dfrac{e^{3x}+8}{f(D)}$ as following :-

$$P.I = \frac{1}{D^3-2D^2-5D+6}(e^{3x}+8)$$

$8 \to e^{0x}$

$$= \frac{e^{3x}}{D^3-2D^2-5D+6} + \frac{8e^{0x}}{D^3-2D^2-5D+6}$$

(In the form of e^{ax})

$(a=3)$ $(a=0)$

Let's take $P1 = \dfrac{e^{3x}}{D^3-2D^2-5D+6}$ & $P2 = \dfrac{8e^{0x}}{D^3-2D^2-5D+6}$

For P1:- $(a=3)$

$$P1 = \frac{e^{3x}}{(3)^3-2(3)^2-5(3)+6} = \frac{e^{3x}}{33-33}$$

$$= \frac{e^{3x}}{0}$$

~~Denominator is zero,~~

$\because$ Denominator is 0, we will differentiate $f(D)$ w.r.t D

$\therefore$ We get :-

$$P1 = \frac{e^{3x}}{3D^2-4D-5}(x)$$

After differentiating denominator, we multiply numerator by 'x'

Now,

$$P1 = \frac{e^{3x} \cdot x}{3(3)^2-4(3)-5}$$

$$= \frac{xe^{3x}}{10}$$

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* For P2 :- $(a=0)$ $P2 = \dfrac{8e^{0x}}{(0)^3-2(0)^2-5(0)+6}$

$= \dfrac{8\cancel{e^{0x}}(1)}{6}$

$\Rightarrow P2 = \dfrac{4}{3}$

$\therefore P.I = P1 + P2$

$= \dfrac{xe^{3x}}{10} + \dfrac{4}{3}$

General sol^n given by :- $y = C.F + P.I$

$\Rightarrow y = c_1e^{-2x} + c_2e^{3x} + c_3e^{x} + \dfrac{xe^{3x}}{10} + \dfrac{4}{3}$

Q3. $\dfrac{d^3y}{dx^3} - 4\dfrac{dy}{dx} = 2\cosh^2 2x$

$\hookrightarrow \cosh(x) \rightarrow$ hyperbolic function $= \dfrac{e^{x}+e^{-x}}{2}$

Sol^n :- Auxillary eq^n :- $D^3 - 4D = 0$

$\therefore D = 2, 0, -2$

Given eq^n becomes :- $D^3 - 4D = 2\left(\dfrac{e^{2x}+e^{-2x}}{2}\right)^2 = \cancel{\dfrac{1}{2}\left(e^{2x}+e^{-2x}\right)}$

$= \dfrac{1}{2}\left[(e^{2x})^2 + (e^{-2x})^2 + 2\cdot e^{2x}\cdot e^{-2x}\right]$

$\Rightarrow D^3 - 4D = \dfrac{e^{4x}}{2} + \dfrac{e^{-4x}}{2} + 1$

For C.F :- $C.F = c_1e^{2x} + c_2e^{0x} + c_3e^{-2x}$

$= c_1e^{2x} + c_2 + c_3e^{-2x}$

For P.I :- $P.I = \dfrac{1}{f(D)}\cdot\left[\dfrac{e^{4x}}{2} + \dfrac{e^{-4x}}{2} + 1\right]$

$= \dfrac{e^{4x}}{2f(D)} + \dfrac{e^{-4x}}{2f(D)} + \dfrac{e^{0x}}{f(D)}$

$(a=4)$ $(a=-4)$ $(a=0)$

$f(D) = D^3 - 4D$

$\therefore$ For $a = 4$, $f(4) = 4^3 - 4(4) = 48$ $\quad f(4) \neq 0$

For $a = -4$, $f(-4) = (4)^3 - 4(-4) = -48$ $\quad f(-4) \neq 0$

For $a = 0$, $f(0) = 0^3 - 4(0) = 0$ $\quad f(0) = 0$

$f(0) = 0$, we differentiate Denominator & multiply by x

$\therefore f'(D) = 3D^2 - 4$ $\quad f'(0) = 3(0)^2 - 4 = -4$

$$\therefore P.I = \frac{e^{4x}}{2(48)} + \frac{e^{-4x}}{2(-48)} + \frac{e^{0x}}{f'(0)} \cdot x$$

$$= \frac{e^{4x}}{96} + \frac{e^{-4x}}{-96} + \frac{x}{-4}$$

$$= \frac{-x}{4} + \frac{1}{48}\left[\frac{e^{4x} - e^{-4x}}{2}\right]$$

$$\Rightarrow P.I = \frac{-x}{4} + \frac{1}{48}\sinh 4x \quad \left[\because \sinh(x) = \frac{e^{x} - e^{-x}}{2}\right]$$

General solⁿ is given by :-

$$y = C.F + P.I$$

$$= c_1 e^{2x} + c_2 e^{0x} + c_3 e^{-2x} - \frac{x}{4} + \frac{1}{48}\sinh 4x$$