MATRICES - UNIT-I

Eigen Vectors of a Symmetric matrix

(Non-repeated Eigen Values)

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Example 3.

Find the Eigen Values and Eigen vectors for the following matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

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Eigen Vectors

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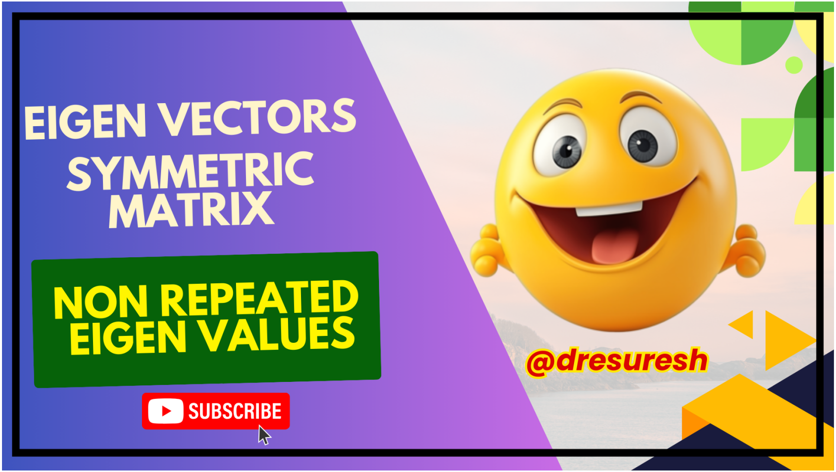

The characteristic equation is $|A - \lambda I| = 0$

$$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\begin{aligned}
S_1 &= 8 + 7 + 3 = 18 \\
S_2 &= \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} \\
&= (21 - 6) + (24 - 4) + (56 - 36) \\
&= 45 \\
S_3 &= |A| = 8(21 - 6) + 6(-18 + 8) + 2(24 - 14) \\
&= 40 - 60 + 20 = 0
\end{aligned}$$

$$\boxed{S_1 = 18, S_2 = 45, S_3 = 0}$$

$$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

The characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$
$\Rightarrow \lambda(\lambda - 3)(\lambda - 15) = 0$

$$\lambda = 0, 3, 15.$$

The eigenvalues are $\lambda = 0, 3, 15$.

The eigenvectors are given by $[A - \lambda I]\ X = 0$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} &(8-\lambda)x_1 - 6x_2 + 2x_3 = 0 \\ &-6x_1 + (7-\lambda)x_2 - 4x_3 = 0 \\ &2x_1 - 4x_2 + (3-\lambda)x_3 = 0 \end{aligned}\right\} \qquad (1)$$

Case (i): For $\lambda = 0$ in (1), we get

$$8x_1 - 6x_2 + 2x_3 = 0$$

$$-6x_1 + 7x_2 - 4x_3 = 0$$

$$2x_1 - 4x_2 + 3x_3 = 0$$

Solving second and third equations we have

2312

$$\begin{array}{cccc} 7 & -4 & -6 & 7 \\ -4 & 3 & 2 & -4 \end{array}$$

$$\frac{x_1}{21-16} = \frac{x_2}{-8+18} = \frac{x_3}{24-14}$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

The eigenvector

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

Case (ii): For $\lambda = 3$ in (1), we get

$$5x_1 - 6x_2 + 2x_3 = 0$$

$$-6x_1 + 4x_2 - 4x_3 = 0$$

$$2x_1 - 4x_2 + 0x_3 = 0$$

Solving second and third equations we have

2312

$$\begin{array}{cccc} 4 & -4 & -6 & 4 \\ -4 & 0 & 2 & -4 \end{array}$$

$$\frac{x_1}{0-16} = \frac{x_2}{-8+0} = \frac{x_3}{24-8}$$

$$\frac{x_1}{-16} = \frac{x_2}{-8} = \frac{x_3}{16}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

The eigenvector

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}.$$

Case (iii): For $\lambda = 15$ in (1), we get

$$-7x_1 - 6x_2 + 2x_3 = 0$$

$$+6x_1 + 8x_2 + 4x_3 = 0$$

$$2x_1 - 4x_2 - 12x_3 = 0$$

Solving first two equations we have

2312

$$\begin{matrix} -6 & 2 & -7 & -6 \\ 8 & 4 & 6 & 8 \end{matrix}$$

$$\frac{x_1}{-24-16} = \frac{x_2}{12+28} = \frac{x_3}{-56+36}$$

$$\frac{x_1}{-40} = \frac{x_2}{40} = \frac{x_3}{-20}$$

$$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

The eigenvector

$$X_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}.$$

Modal Matrix

$$M = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Eigen Values	Eigen Vectors
$\lambda = 0$	$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$
$\lambda = 3$	$X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$
$\lambda = 15$	$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$