Q1. Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ upto 3^{rd} degree

Sol^n:- $f(x,y) = x^2y+3y-2$

$\because (x-1) \longrightarrow$ In terms of $(x-a)$ $\therefore a=1$

$\because (y+2) \longrightarrow$ In terms of $(y-b) \rightarrow (y-(-2))$ $\therefore b=-2$

By Taylor's Expansion:-

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] \cancel{+\frac{1}{2!}}$$

$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$

$$+ \frac{1}{3!}\left[(x-a)^3 f_{xxx}(a,b) + 3(x-a)^2(y-b) f_{xxy}(a,b) + 3(x-a)(y-b)^2 f_{xyy}(a,b) + (y-b)^3 f_{yyy}(a,b)\right]$$

Here:- $f(x,y) = x^2y+3y-2$ ~~$f(a,b) = f(1,2) =$~~

$\therefore f(a,b) = f(1,-2) = 1^2(-2)+3(-2)-2 = -2-6-2 = -10$

1°
$\therefore f_x = \frac{d}{dx}(x^2y+3y-2) = 2xy \longrightarrow f_x(1,-2) = 2(1)(-2) = -4$

$f_y = \frac{d}{dx}(\;"\;\;"\;\;"\;) = x^2+3 \longrightarrow f_y(1,-2) = 1+3 = 4$

2°
$f_{xx} = \frac{d}{dx}(2xy) = 2y \longrightarrow f_{xx}(1,-2) = 2(-2) = -4$

$f_{xy} = \frac{d}{dx}\left(\frac{df}{dy}\right) = \frac{d}{dx}(x^2) = 2x \longrightarrow f_{xy}(1,-2) = 2(1) = 2$

$f_{yy} = \frac{d}{dy}(x^2+3) = 0 \longrightarrow f_{yy}(1,-2) = 2(1) = 0$

3°
$f_{xxx} = \frac{d}{dx}(2y) = 0 \therefore f_{xxx}(1,-2) = 0$ | $f_{xxy} = \frac{d}{dx}(2x) = 2 \therefore f_{xxy}(1,-2) = 2$

$f_{yyy} = \frac{d}{dy}(2x) = 0 \therefore f_{yyy}(1,-2) = 0$ | $f_{xyy} = \frac{d}{dx}(0) = 0 \therefore f_{xyy}(1,-2) = 0$

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$$f(x,y) = -10 + \frac{1}{1!}\left[(x-1)(-4) + (y+2)(4)\right] + \frac{1}{2!}\left[(x-1)^2(-4) + 2(x-1)(y+2)(2) + (y+2)^2(0)\right]$$
$$+ \frac{1}{3!}\left[(x-1)^3(0) + 3(x-1)^2(y+2)(2) + 3(x-1)(y+2)^2(0) + (y+2)^3(0)\right]$$

$$\therefore x^2y + 3y - 2 = -10 - 4(x-1) + 4(y+2) - 2(x-1)^2 + 2(x-1)(y+2) + (x-1)^2(y+2)$$

Q2. Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in the powers of $(x+2)$ and $(y-1)$ upto 3^{rd} degree

Solⁿ: $f(x,y) = x^2y^2 + 2x^2y + 3xy^2$

$\because (x+2) \longrightarrow$ In terms of $(x-a) \Rightarrow (x-(-2)) \therefore a = -2$

$(y-1) \longrightarrow$ In terms of $(y-b) \therefore b = 1$

Here $f(-2,1) = (-2)^2(1)^2 + 2(-2)^2(1) + 3(-2)(1)^2 = 4+8-6 = 6$

1° $f_x = \frac{d}{dx}(x^2y^2 + 2x^2y + 3xy^2) = 2xy^2 + 4xy + 3y^2 \therefore f_x(-2,1) = 2(-2)(1)^2 + 4(-2)(1) + 3(1)^2 = -9$

$f_y = \frac{d}{dy}(x^2y^2 + 2x^2y + 3xy^2) = 2x^2y + 2x^2 + 6xy \therefore f_y(-2,1) = 2(-2)^2(1) + 2(-2)^2 + 6(-2)(1) = 4$

2° $f_{xx} = \frac{d}{dx}(2xy^2 + 4xy + 3y^2) = 2y^2 + 4y \therefore f_{xx}(-2,1) = 2(1)^2 + 4(1) = 6$

$f_{xy} = \frac{d}{dx}(2x^2y + 2x^2 + 6xy) = 4xy + 4x + 6y \therefore f_{xy}(-2,1) = 4(-2)(1) + 4(-2) + 6(1) = -10$

$f_{yy} = \frac{d}{dy}(2x^2y + 2x^2 + 6xy) = 2x^2 + 6x \therefore f_{yy}(-2,1) = 2(-2)^2 + 6(-2) = -4$

3° $f_{xxx} = \frac{d}{dx}(2y^2 + 4y) = 0 \therefore f_{xxx}(-2,1) = 0$ $\quad f_{xxy} = \frac{d}{dx}(4xy + 4x + 6y) = 4y + 4 \therefore f_{xxy}(-2,1) = 4(1) + 4 = 8$

$f_{yyy} = \frac{d}{dy}(2x^2 + 6x) = 0 \therefore f_{yyy}(-2,1) = 0$ $\quad f_{xyy} = \frac{d}{dx}(2x^2 + 6x) = 4x + 6 \therefore f_{xyy}(-2,1) = 4(-2) + 6 = -2$

$$\therefore x^2y^2 + 2x^2y + 3xy^2 = 6 + \frac{1}{1!}\left[(x+2)(-9) + (y-1)(4)\right] + \frac{1}{2!}\left[(x+2)^2(6) + 2(x+2)(y-1)(-10) + (y-1)^2(-4)\right]$$
$$+ \frac{1}{3!}\left[(x+2)^3(0) + 3(x+2)^2(y-1)(8) + 3(x+2)(y-1)^2(-2) + (y-1)^3(0)\right]$$

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Q3. Expand $f(x,y)=e^{xy}$ in Taylor series at $(1,1)$ upto 2^{nd} degree

Sol:- $f(x,y)=e^{xy}$ Here, $(1,1) \longrightarrow a=1,\ b=1$
$(x-1)\quad (y-1)$

$$f(x,y)=f(1,1)+\frac{1}{1!}\left[(x-1)f_x(1,1)+(y-1)f_y(1,1)\right]$$
$$+\frac{1}{2!}\left[(x-1)^2 f_{xx}(1,1)+2(x-1)(y-1)f_{xy}(1,1)+(y-1)^2 f_{yy}(1,1)\right]$$

Here :-

$f(1,1)=e^{1\times 1}=e$

1°
$$f_x=\frac{d}{dx}\left(e^{xy}\right)=e^{xy}\frac{d}{dx}(xy)=y e^{xy} \longrightarrow f_x(1,1)=(1)e^{1\times 1}=e$$
$$f_y=\frac{d}{dy}\left(e^{xy}\right)=x e^{xy} \longrightarrow f_y(1,1)=(1)e^{1\times 1}=e$$

2°
$$f_{xx}=\cancel{\frac{d}{dx}(ye)}=\frac{d}{dx}\left(y e^{xy}\right)=y\left(y e^{xy}\right)=y^2 e^{xy} \longrightarrow f_{xx}(1,1)=(1)^2 e^{1\times 1}=e$$
$$f_{xy}=\frac{d}{dx}\left(\underset{u}{x}\cdot \underset{v}{e^{xy}}\right)=e^{xy}\frac{d}{dx}(x)+x\cdot\frac{d}{dx}\left(e^{xy}\right)=e^{xy}+xye^{xy} \longrightarrow f_{xy}(1,1)=e+e=2e$$
$$f_{yy}=\frac{d}{dy}\left(x e^{xy}\right)=x\left(x e^{xy}\right)=x^2 e^{xy} \longrightarrow f_{yy}(1,1)=(1)^2 e^{1\times 1}=e$$

$$\therefore e^{xy}=e+\frac{1}{1!}\left[(x-1)e+(y-1)e\right]+\frac{1}{2!}\left[(x-1)^2 e+2(x-1)(y-1)(2e)+(y-1)^2 e\right]$$

$$\Rightarrow e^{xy}=e\left[1+(x-1)+(y-1)+\frac{1}{2}(x-1)^2+2(x-1)(y-1)+\frac{1}{2}(y-1)^2\right]$$