Particular Integral (Type 2 → X = Trigonometric function)

Q1. Find PI of $(D^2-4D+4)y = e^x + \cos 2x$

Ans. $f(D) = D^2-4D+4$

For P.I :- $P.I = \frac{1}{f(D)} \cdot (e^x + \cos 2x)$

$$= \frac{e^x}{D^2-4D+4} + \frac{\cos 2x}{D^2-4D+4}$$

Here,
In case of trigonometric functions, we take $\boxed{D^2 = -a^2}$
Do not change/put value in 'D' here

In this case, a = coefficient of x in trigonometric function
$\therefore a = 2$

Here, $D^2 = -(2)^2 = -4$

$$\therefore P.I = \frac{e^x}{(1)^2-4(1)+4} + \frac{\cos 2x}{\cancel{-4}-4D+\cancel{4}}$$

$$= \frac{e^x}{1} + \frac{\cos 2x}{-4D}$$

$$= e^x - \frac{1}{4D} \cos 2x$$

* Here, D → derivative
$\therefore \frac{1}{D}$ → integration ⇒ $\frac{1}{D} = \int$ [$\because \int$ is opposite of $\frac{d}{dx}$]

$$\therefore P.I = e^x - \frac{1}{4}\int \cos 2x$$

$$= e^x - \frac{1}{4} \cdot \frac{\sin 2x}{\frac{d}{dx}(2x)}$$

$$= e^x - \frac{1}{4} \cdot \frac{\sin 2x}{2}$$

$$\Rightarrow P.I = e^x - \frac{\sin 2x}{8}$$

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Q2. ~~Solve~~ Solve $(D^2+4)y = \cos 2x$

Ans. For A.E :- $D^2+4=0$ $\rightarrow 0 \pm 2i$

$\Rightarrow D = \sqrt{-4} = \pm 2i$ (Imaginary & distinct)

Here, $\alpha = 0$ & $\beta = 2$

For C.F :- $C.F = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$= e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

For PI :- $P.I = \dfrac{\cos 2x}{f(D)}$ [Here, $f(D) = D^2+4$, $a = 2$]

$= \dfrac{\cos 2x}{D^2+4}$

Here, $D^2 = -a^2 = -(2)^2$

$\Rightarrow D^2 = -4$

$\cancel{\therefore P.I = \dfrac{\cos 2x}{(-4+4)}}$

$f(-4) = -4+4 = 0$ $\because$ Denominator is zero

We differentiate $f(D) \longrightarrow f'(D) = 2D$

$\therefore P.I = \dfrac{\cos 2x}{f'(-4)} \cdot x$

$= \dfrac{\cos 2x}{2 \cdot D} \cdot x$

$= \dfrac{x}{2} \cdot \dfrac{\cos 2x}{D}$

Here, $1/D \rightarrow \int$

$\therefore P.I = \dfrac{x}{2} \cdot \int \cos 2x$

$= \dfrac{x}{2} \cdot \dfrac{\sin 2x}{\frac{d}{dx}(2x)} = \dfrac{x}{2} \dfrac{\sin 2x}{2}$

$\Rightarrow P.I = \dfrac{x \sin 2x}{4}$

$\therefore$ General sol^n :- $y = C.F + P.I$

$= c_1 \cos 2x + c_2 \sin 2x + \dfrac{x \sin 2x}{4}$

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Q3. Solve $(D^2 - 5D + 6)y = \sin 3x$

Ans. For A.E :- $D^2 - 5D + 6 = 0$

$\Rightarrow (D-2)(D-3) = 0 \quad \therefore D = 2, 3$ [Real & distinct]

For C.F :- $C.F = c_1 e^{2x} + c_2 e^{3x}$

For P.I :- $P.I = \dfrac{\sin 3x}{f(D)}$

Here, $a = 3 \quad D^2 = -(a)^2 = -9$

$$\therefore P.I = \frac{\sin 3x}{f(-9)}$$

$$= \frac{\sin 3x}{-9 - 5D + 6}$$

$$= \frac{\sin 3x}{-3 - 5D}$$

In these type, we take ~~conj~~ conjugate

$$\therefore P.I = \frac{\sin 3x}{-3-5D} \cdot \frac{-3+5D}{-3+5D}$$

$$= \frac{\sin 3x\,(-3+5D)}{(-3)^2 - (5D)^2}$$

$$= \frac{\sin 3x\,(-3+5D)}{9 - 25D^2}$$

$$= \frac{-3\sin 3x + 5(D\sin 3x)}{9 - 25(-9)} \quad \left[\text{Here, } D^2 = -9\right]$$

$$= \frac{-3\sin 3x + 5\frac{d}{dx}(\sin 3x)}{9 + 225} \quad \left[D \to \frac{d}{dx} \therefore \frac{d}{dx}(\sin 3x)\right]$$

$$= \frac{-3\sin 3x + 5\cos 3x \cdot (3)}{234}$$

$$= \frac{\cancel{3}(-\sin 3x + 5\cos 3x)}{\cancel{234}\ 78} \quad \therefore P.I = \frac{5\cos 3x - \sin 3x}{8}$$

$\therefore$ General solⁿ :- $y = c_1 e^{2x} + c_2 e^{3x} + \dfrac{5\cos 3x - \sin 3x}{8}$