Q. Solve $(D^2+a^2)y = \tan ax$

Soln:- A.E:- $m^2+a^2=0$

$\Rightarrow m^2 = -a^2$

$\Rightarrow m = \pm\sqrt{-a^2} = \pm ai$ $\quad \therefore m = 0+ai,\ 0-ai$

$\alpha = 0,\ \beta = a$

CF:- $CF = e^{0x}(C_1 \cos ax + C_2 \sin ax)$

$= C_1 \cos ax + C_2 \sin ax$

Here, $y_1 = \cos ax$ $\quad y_2 = \sin ax$

$y_1' = a(-\sin ax)$ $\quad y_2' = a\cos ax$

Here, $X = \tan ax$

$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ a(-\sin ax) & a\cos ax \end{vmatrix}$

$= a\cos^2 ax + a\sin^2 ax$

$= a(\cos^2 ax + \sin^2 ax) \Rightarrow W = a$

• $P = -\int \frac{y_2 X\,dx}{W}$ $\qquad$ $\cancel{\cdot Q = \int y_1 X\,dx}$

$= -\int \frac{(\sin ax)(\tan ax)\,dx}{a}$

$= -\frac{1}{a}\int \sin ax \left(\frac{\sin ax}{\cos ax}\right) dx$ $\qquad$ $\frac{1}{a}\int \frac{\sin^2 ax}{\cos ax}\,dx$

$= -\frac{1}{a}\int \frac{(1-\cos^2 ax)\,dx}{\cos ax}$ $\quad = -\frac{1}{a}\int(\sec ax - \cos ax)\,dx$

$\cancel{= -\frac{1}{a}\int(\sec ax - 1)\,dx}$

$= -\frac{1}{a}\left[\log(\sec ax+\tan ax)\cdot\frac{1}{\frac{d}{dx}(ax)} - \frac{\sin ax}{\frac{d}{dx}(ax)}\right]$

$\ldots + \tan ax) - \sin ax\}$

$\therefore P = \frac{1}{a^2}\left[\sin ax - \log(\sec ax + \tan ax)\right]$

$$Q = \int \frac{y_1 X \, dx}{W}$$

$$= \int \frac{(\cos ax)(\tan ax)}{a} dx$$

$$= \frac{1}{a} \int \cancel{(\cos ax)} \cdot \frac{\sin ax}{\cancel{(\cos ax)}} dx$$

$$= \frac{1}{a} \left(\frac{-\cos ax}{\frac{d}{dx}(ax)} \right) = \frac{1}{a} \left(\frac{-\cos ax}{a} \right)$$

$$\therefore Q = \frac{-\cos ax}{a^2}$$

P.I :- $PI = Py_1 + Qy_2$

$$= \frac{1}{a^2} \left[\sin ax - \log(\sec ax + \tan ax) \right] \cos ax$$

$$+ \left(\frac{-\cos ax}{a^2} \right) (\sin ax)$$

$$= \cancel{\frac{\sin ax \cos ax}{a^2}} - \frac{\log(\sec ax + \tan ax) \cdot \cos ax}{a^2} - \cancel{\frac{\sin ax \cos ax}{a^2}}$$

$$\Rightarrow PI = \frac{-\log(\sec ax + \tan ax)}{a^2}$$

$$\therefore y = CF + PI$$

$$y = C_1 \cos ax + C_2 \sin ax - \frac{\log(\sec ax + \tan ax)}{a^2}$$

Q2. Solve $(D^2+a^2)y = \sec ax$

Sol:- A.E:- $m^2+a^2=0$

$\Rightarrow m^2 = -a^2$

$\Rightarrow m = \pm\sqrt{-a^2} = \pm ai$

$\therefore m = 0+ai, 0-ai$

$\alpha = 0, \beta = a$

C.F:- $CF = e^{0}(C_1 \cos ax + C_2 \sin ax)$

$= C_1 \cos ax + C_2 \sin ax$

Here, $y_1 = \cos ax$ $\qquad$ $y_2 = \sin ax$

$y_1' = -a \sin ax$ $\qquad$ $y_2' = a \cos ax$

$X = \sec ax$

$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$

$= a(\cos^2 ax + \sin^2 ax)$

$\Rightarrow W = a$

• $Q = -\int \frac{y_1 X \, dx}{W} = -\int \frac{(\cos ax)(\sec ax)\,dx}{a}$

$= \frac{1}{a}\int \frac{(\cos ax)}{(\cos ax)}$

$= \frac{1}{a} x$

$\therefore Q = \frac{x}{a}$

$$P = -\int \frac{y_2 X}{W}\, dx$$

$$= -\int \frac{(\sin ax)(\sec ax)}{a}\, dx$$

$$= -\int \frac{\sin ax}{\cos ax} \cdot \frac{1}{a} \cdot dx$$

$$= \cancel{\frac{-1}{a} \int \tan ax\, dx}$$

Let $t = \cos ax$

$$\frac{dt}{dx} = -a \sin ax$$

$$\Rightarrow dx = \frac{-dt}{a \sin ax}$$

$$\int \frac{\quad}{\cos ax} \cdot \frac{1}{a} \cdot \left(\frac{\cancel{-} dt}{a \cancel{\sin} ax} \right)$$

$$= \frac{1}{a^2} \int \frac{1}{t} \cdot dt$$

$$= \frac{1}{a^2} \cdot \ln(t) \qquad \therefore P = \frac{1}{a^2} \cdot \log(\cos ax)$$

PI:- $PI = P y_1 + Q y_2$

$$\Rightarrow PI = \frac{1}{a^2} \log(\cos ax) \cdot (\cos ax) + \frac{x}{a} \cdot (\sin ax)$$

$$\therefore y = CF + PI$$

$$= C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2} \cos ax \log(\cos ax) + \frac{x}{a} (\sin ax)$$

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Q3. Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

Sol:- ~~A.E~~ $(D^2+1)y = \operatorname{cosec} x$

A.E:- $m^2+1=0$

$\Rightarrow m^2 = -1$

$\Rightarrow m = \pm\sqrt{-1} = i$ $\therefore m = 0+i, 0-i$

$\alpha = 0, \beta = 1$

CF:- $CF = e^0(C_1\cos x + C_2 \sin x)$

$= C_1 \cos x + C_2 \sin x$

Here, $y_1 = \cos x$ $y_2 = \sin x$

$y_1' = -\sin x$ $y_2' = \cos x$

$X = \operatorname{cosec} x$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = (\cos^2 x) + (\sin^2 x) \Rightarrow W = 1$

- $P = -\int \frac{y_2 X\,dx}{W}$

$= -\int \frac{(\sin x)(\operatorname{cosec} x)}{1}\,dx$

$= -\int (\cancel{\sin x}) \frac{1}{(\cancel{\sin x})}\,dx \Rightarrow P = -x$

- $Q = \int \frac{y_1 X\,dx}{W}$

$= \int \frac{(\cos x)(\operatorname{cosec} x)}{1}\,dx$

$= \int \frac{\cos x}{\sin x}\,dx$

$\boxed{\begin{aligned} &\text{Let } t = \sin x \\ &\frac{dt}{dx} = \cos x \\ &\Rightarrow dx = \frac{dt}{\cos x} \end{aligned}}$

$= \int \frac{\cancel{\cos x}}{\sin x} \cdot \frac{dt}{\cancel{\cos x}} = \int \frac{1}{t} \cdot dt$

$= \ln(t)$ $\Rightarrow Q = \ln(\sin x)$

$\underline{PI}$:- $PI = Py_1 + Qy_2$

$= (-x)\cos x + \ln(\sin x) \cdot (\sin x)$

$\Rightarrow PI = \sin x \ln(\sin x) - x\cos x$

$\therefore y = CF + PI$

$= C_1 \cos ax + C_2 \sin ax + \sin x \ln(\sin x) - x\cos x$