Q7 Expand $e^x \cos y$ about $\left(0, \frac{\pi}{2}\right)$ upto 3rd degree

Solⁿ:- $f(x,y) = e^x \cos y$ Here, $\left(0, \frac{\pi}{2}\right) \longrightarrow a = 0, \; b = \pi/2$

$(x-0) \quad (y-\pi/2)$

$$f(x,y) = f(0,\pi/2) + \frac{1}{1!}\left[(x-0)f_x + (y-\tfrac{\pi}{2})f_y\right] + \frac{1}{2!}\left[(x-0)^2 f_{xx} + 2(x-0)(y-\pi/2)f_{xy} + (y-\pi/2)^2 f_{yy}\right]$$
$$+ \frac{1}{3!}\left[(x-0)^3 f_{xxx} + 3(x-0)^2(y-\tfrac{\pi}{2})f_{xxy} + 3(x-0)(y-\tfrac{\pi}{2})^2 f_{xyy} + (y-\tfrac{\pi}{2})^3 f_{yyy}\right]$$

Here:- $f\left(0, \frac{\pi}{2}\right) = e^0 \cos(\pi/2) = 0$

1° $f_x = \frac{d}{dx}(e^x \cos y) = e^x \cos y = e^0 \cos(\pi/2) = 0$ | $f_y = \frac{d}{dy}(e^x \cos y) = -e^x \sin y = -e^0 \sin(\pi/2) = -1$

2° $f_{xx} = \frac{d}{dx}(e^x \cos y) = e^x \cos y = e^0 \cos(\pi/2) = 0$, $f_{yy} = \frac{d}{dy}(-e^x \sin y) = -e^x \cos y = -e^0 \cos(\pi/2) = 0$

$f_{xy} = \frac{d}{dx}(-e^x \sin y) = -e^x \sin y = -e^0 \sin(\pi/2) = -1$

3° $f_{xxx} = \frac{d}{dx}(e^x \cos y) = e^x \cos y = e^0 \cos(\pi/2) = 0$ | $f_{xxy} = \frac{d}{dx}(-e^x \sin y) = -e^x \sin y = -e^0 \sin(\pi/2) = -1$

$f_{yyy} = \frac{d}{dy}(-e^x \cos y) = e^x \sin y = e^0 \sin(\pi/2) = 1$ | $f_{xyy} = \frac{d}{dx}(-e^x \cos y) = -e^x \cos y = -e^0 \cos(\pi/2) = 0$

$$\therefore e^x \cos y = 0 + \frac{1}{1!}\left[x(0) + (y-\tfrac{\pi}{2})(-1)\right] + \frac{1}{2!}\left[x^2(0) + 2x(y-\tfrac{\pi}{2})(-1) + (y-\tfrac{\pi}{2})^2(0)\right]$$
$$+ \frac{1}{3!}\left[x^3(0) + 3x^2(y-\tfrac{\pi}{2})(-1) + 3x(y-\tfrac{\pi}{2})^2(0) + (y-\tfrac{\pi}{2})^3(1)\right]$$

$$\Rightarrow e^x \cos y = -\left(y-\frac{\pi}{2}\right) - x\left(y-\frac{\pi}{2}\right) - \frac{x^2(y-\pi/2)}{2} + \frac{(y-\pi/2)^3}{6}$$

Q8. Expand $e^x \sin y$ at $\left(-1, \frac{\pi}{4}\right)$ upto 3rd degree

Soln:- $f(x,y) = e^x \sin y$ Here, $\left(-1, \frac{\pi}{4}\right) \longrightarrow a = -1$, $b = \pi/4$

$(x+1)$, $(y - \pi/4)$

$$f(x,y) = f\left(-1, \frac{\pi}{4}\right) + \frac{1}{1!}\left[(x+1)f_x + \left(y - \frac{\pi}{4}\right)f_y\right] + \frac{1}{2!}\left[(x+1)^2 f_{xx} + 2(x+1)\left(y-\frac{\pi}{4}\right)f_{xy} + (y-\pi/4)^2 f_{yy}\right]$$

$$+ \frac{1}{3!}\left[(x+1)^3 f_{xxx} + 3(x+1)^2\left(y-\frac{\pi}{4}\right)f_{xxy} + 3(x+1)\left(y-\frac{\pi}{4}\right)^2 f_{xyy} + \left(y-\frac{\pi}{4}\right)^3 f_{yyy}\right]$$

Here:- $f\left(-1, \frac{\pi}{4}\right) = e^{-1}\sin(\pi/4) = \frac{1}{e\sqrt{2}}$

1° $f_x = \frac{d}{dx}(e^x \sin y) = e^x \sin y = e^{-1}\sin(\pi/4) = \frac{1}{e\sqrt{2}}$ | $f_y = \frac{d}{dy}(e^x \sin y) = e^x \cos y = e^{-1}\cos(\pi/4) = \frac{1}{e\sqrt{2}}$

2° $f_{xx} = \frac{d}{dx}(e^x \sin y) = e^x \sin y = e^{-1}\sin\left(\frac{\pi}{4}\right) = \frac{1}{e\sqrt{2}}$ $f_{yy} = \frac{d}{dy}(e^x \cos y) = -e^x \sin y = -e^{-1}\sin\left(\frac{\pi}{4}\right) = \frac{-1}{e\sqrt{2}}$

$f_{xy} = \frac{d}{dx}(e^x \cos y) = e^x \cos y = e^{-1}\cos(\pi/4) = \frac{1}{e\sqrt{2}}$

3° $f_{xxx} = \frac{d}{dx}(e^x \sin y) = e^x \sin y = e^{-1}\sin\left(\frac{\pi}{4}\right) = \frac{1}{e\sqrt{2}}$ | $f_{xxy} = \frac{d}{dx}(e^x \cos y) = e^x \cos y = e^{-1}\cos(\pi/4) = \frac{1}{e\sqrt{2}}$

$f_{yyy} = \frac{d}{dy}(-e^x \sin y) = -e^x \cos y = -e^{-1}\cos\left(\frac{\pi}{4}\right) = \frac{-1}{e\sqrt{2}}$ | $f_{xyy} = \frac{d}{dx}(-e^x \sin y) = -e^x \sin y = -e^{-1}\sin\left(\frac{\pi}{4}\right) = \frac{-1}{e\sqrt{2}}$

$$\therefore e^x \sin y = \frac{1}{e\sqrt{2}} + \frac{1}{1!}\left[(x+1)\frac{1}{e\sqrt{2}} + \left(y - \frac{\pi}{4}\right)\frac{1}{e\sqrt{2}}\right] + \frac{1}{2!}\left[(x+1)^2 \frac{1}{e\sqrt{2}} + 2(x+1)\left(y-\frac{\pi}{4}\right)\frac{1}{e\sqrt{2}} + \left(y-\frac{\pi}{4}\right)\left(\frac{-1}{e\sqrt{2}}\right)\right]$$

$$+ \frac{1}{3!}\left[(x+1)^3 \frac{1}{e\sqrt{2}} + 3(x+1)^2\left(y-\frac{\pi}{4}\right)\frac{1}{e\sqrt{2}} + 3(x+1)\left(y-\frac{\pi}{4}\right)^2\left(\frac{-1}{e\sqrt{2}}\right) + \left(y-\frac{\pi}{4}\right)^3\left(\frac{-1}{e\sqrt{2}}\right)\right]$$

Q9. Expand x^y at $(1,1)$ upto 1^{st} degree

Sol^n:- $f(x,y) = x^y$ Here, $(1,1) \longrightarrow a = 1 , b = 1$

$(x-1) \quad (y-1)$

Here:- $f(x,y) = f(1,1) + \frac{1}{1!}\left[(x-1)f_x + (y-1)f_y\right]$

Here:- $f(1,1) = 1^1 = 1$

$f_x = \frac{d}{dx}(x^y) = yx^{y-1} = 1 \cdot x^{1-1} = 1$

$f_y = \frac{d}{dy}(x^y) = x^y \log x = 1^1 \log(1) = 0$

$\left[\because \frac{d}{dx}(a^x) = a^x \log a\right]$

$\therefore x^y = 1 + \frac{1}{1!}\left[(x-1)(1) + (y-1)(0)\right]$

$\Rightarrow x^y = 1 + x - 1$

$\Rightarrow x^y = x$