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Q1. Find the radius of curvature on $y = e^x$ at the point where the curve cuts y-axis

Sol:- Curve cuts at y-axis : $(0, y)$

Given curve:- $y = e^x$

At $(0,y)$:- For y_1:- $y_1 = e^x = e^0 \Rightarrow y_1 = 1$ & For y_2:- $y_2 = e^x = e^0 \Rightarrow y_2 = 1$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+1^2)^{3/2}}{1} \Rightarrow \rho = 2^{3/2} = 2\sqrt{2}$$

Q2. Find radius of curvature for $y = \frac{\log x}{x}$ at $x = 1$

Sol:- Given curve:- $y = \frac{\log x}{x}$

For y_1:- $y_1 = \frac{d}{dx}\left(\frac{\log x}{x}\right)$

$$= \frac{x \cdot \frac{d}{dx}(\log x) - \log x \cdot \frac{d}{dx}(x)}{x^2}$$

$$= \frac{x \cdot \frac{1}{x} - \log x}{x^2} \Rightarrow y_1 = \frac{1 - \log x}{x^2}$$

At $x=1$:- $y_1 = \frac{1 - \log(1)}{1^2} \Rightarrow y_1 = 1 - 0 = 1$

For y_2:- $y_2 = \frac{d}{dx}\left(\frac{1-\log x}{x^2}\right)$

$$= \frac{x^2 \cdot \frac{d}{dx}(1-\log x) - (1-\log x) \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 \cdot -\frac{1}{x} - 2x + 2x \log x}{x^4} \Rightarrow y_2 = \frac{-3x + 2x \log x}{x^4}$$

At $x=1$:- $y_2 = \frac{-3(1)+2(1)\cancel{\log(1)}}{(1)^4} \Rightarrow y_2 = -3$

$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$= \frac{(1+1^2)^{3/2}}{(-3)}$ → cant be -ve $\Rightarrow \rho = \frac{2^{3/2}}{3} = \frac{2\sqrt{2}}{3}$

Q3. Show that the radius of curvature $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ at any point on it is $\frac{2(ax+by)^{3/2}}{ab}$

Sol:- Given curve:- $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$.

For y_1:- Differentiate both sides w.r.t x

$$\frac{d}{dx}\left(\frac{\sqrt{x}}{\sqrt{a}}\right) + \frac{d}{dx}\left(\frac{\sqrt{y}}{\sqrt{b}}\right) = \frac{d}{dx}(1)$$

$$\Rightarrow \frac{1}{\sqrt{a}} \cdot \frac{1}{2\cdot\sqrt{x}} + \frac{1}{\sqrt{b}} \cdot \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{a}\cdot\cancel{2}\sqrt{x}} \times \sqrt{b}\cdot\cancel{2}\sqrt{y} \Rightarrow y_1 = -\frac{\sqrt{by}}{\sqrt{ax}}$$

For y_2:- Differentiate ~~both~~ y_1

$$y_2 = \frac{d}{dx}\left(\frac{-\sqrt{b}\sqrt{y}}{\sqrt{a}\sqrt{x}}\right)$$

$$= -\frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{d}{dx}\left(\frac{\sqrt{y}}{\sqrt{x}}\right) \quad \left[\frac{u}{v} \text{ rule}\right]$$

$$= -\frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{\sqrt{x}\cdot\frac{1}{2\sqrt{y}}\frac{dy}{dx} - \sqrt{y}\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= -\frac{\sqrt{b}}{\sqrt{a}} \cdot \frac{\sqrt{x}\cdot\frac{1}{2\sqrt{y}}\cdot\left(-\frac{\sqrt{b}\sqrt{y}}{\sqrt{a}\sqrt{x}}\right) - \sqrt{y}\cdot\frac{1}{2\sqrt{x}}}{x}$$

$$= -\frac{\sqrt{b}}{x\sqrt{a}} \cdot \frac{-1}{2}\left[\frac{\sqrt{b}}{\sqrt{a}} + \frac{\sqrt{y}}{\sqrt{x}}\right]$$

$$\therefore y_2 = \frac{\sqrt{b}}{2x\sqrt{a}}\left[\frac{\sqrt{b}\sqrt{x}+\sqrt{a}\sqrt{y}}{\sqrt{a}\sqrt{x}}\right]$$

A/Q,

$$\frac{\sqrt{x}}{\sqrt{a}} + \frac{\sqrt{y}}{\sqrt{b}} = 1$$

$$\Rightarrow \frac{\sqrt{x}\sqrt{b}+\sqrt{y}\sqrt{a}}{\sqrt{a}\sqrt{b}} = 1$$

$$\Rightarrow \sqrt{b}\sqrt{x}+\sqrt{a}\sqrt{y} = \sqrt{a}\sqrt{b} \quad \text{———} \quad \#$$

Use # on y_2:-

$$y_2 = \frac{\sqrt{b}}{2x\sqrt{a}}\left[\frac{\cancel{\sqrt{a}}\sqrt{b}}{\cancel{\sqrt{a}}\sqrt{x}}\right]$$

$$\Rightarrow y_2 = \frac{b}{2\sqrt{a}\,x\sqrt{x}}$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{\left[1+\left(\frac{-\sqrt{by}}{\sqrt{ax}}\right)^2\right]^{3/2}}{\left(\frac{b}{2\sqrt{a}\sqrt{x}\,x}\right)} = \left(\frac{ax+by}{ax}\right)^{3/2} \times \frac{2\sqrt{a}\left(\sqrt{x}\cdot x\right)}{b}$$

$x^{3/2}$

$$\Rightarrow \rho = \frac{(ax+by)^{3/2}}{a^{3/2}\cancel{x^{3/2}}}\cdot\frac{2a^{1/2}\cdot\cancel{x^{3/2}}}{b}$$

$$= \frac{2(ax+by)^{3/2}}{ba}$$