MATRICES - UNIT-I

Eigen Vectors of a Symmetric matrix

(Repeated Eigen Values)

Dr. E. Suresh,
Assistant Professor, Department of Mathematics,
SRM Institute of Science and Technology,
Kattankulathur - 603203.

September 4, 2024

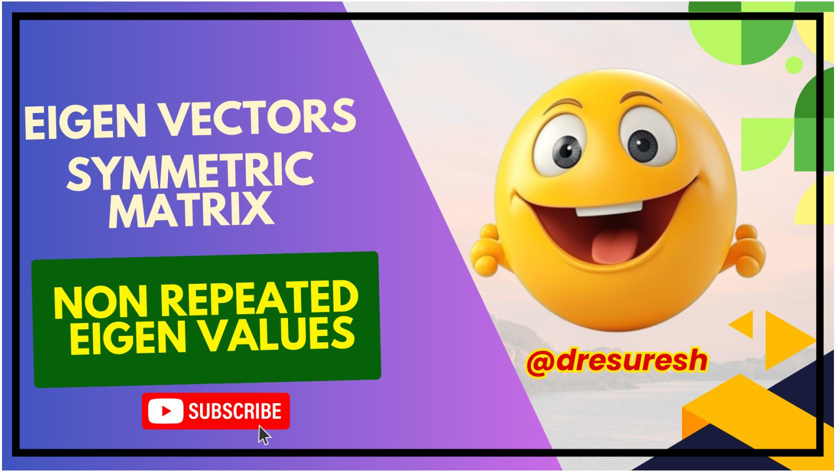
Eigen Values

For Video lectures use the following youtube link

Click here

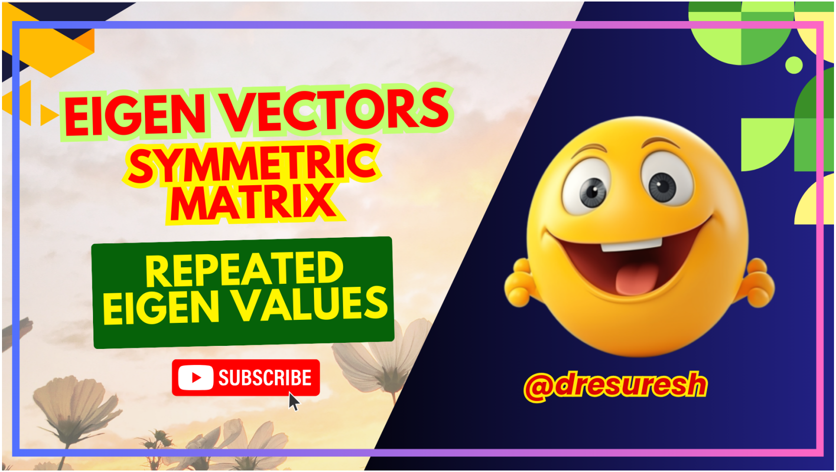
Eigen Vectors

For Video lectures use the following youtube link
Click here



For Video lectures use the following youtube link

Click here



Example 4.

Find the Eigen Values and Eigen vectors for the following matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

The characteristic equation is $|A - \lambda I| = 0$

$$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\begin{aligned}
S_1 &= 2 + 2 + 2 = 6 \\
S_2 &= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \\
&= (4-1) + (4-1) + (4-1) \\
&= 9 \\
S_3 = |A| &= \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} \\
&= (2)(4-1) - (-1)(-2+1) + (1)(1-2) \\
&= 6 - 1 - 1 = 4
\end{aligned}$$

$$\boxed{S_1 = 6, S_2 = 9, S_3 = 4}$$

$$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

The characteristic equation is $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$$\Rightarrow (\lambda - 4)(\lambda - 1)(\lambda - 1) = 0$$

$$\lambda = 4, 1, 1.$$

The eigenvalues are $\lambda = 4, 1, 1.$

The eigenvectors are given by $[A - \lambda I]\ X = 0$

$$\begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} (2-\lambda)x_1 - x_2 + x_3 = 0 \\ -x_1 + (2-\lambda)x_2 - x_3 = 0 \\ x_1 - x_2 + (2-\lambda)x_3 = 0 \end{aligned}\right\} \tag{1}$$

Case (i): For $\lambda = 4$ in (1), we get

$$-2x_1 - x_2 + x_3 = 0$$

$$-x_1 - 2x_2 - x_3 = 0$$

$$x_1 - x_2 - 2x_3 = 0$$

Solving first and third equations we have

2312

$$\begin{array}{cccc} -1 & 1 & -2 & -1 \\ -1 & -2 & 1 & -1 \end{array}$$

$$\frac{x_1}{2+1} = \frac{x_2}{1-4} = \frac{x_3}{2+1}$$

$$\frac{x_1}{3} = \frac{x_2}{-3} = \frac{x_3}{3}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{1}$$

The eigenvector

$$X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

Case (ii): For $\lambda = 1$ in (1), we get

$$x_1 - x_2 + x_3 = 0$$

$$-x_1 + x_2 - x_3 = 0$$

$$x_1 - x_2 + x_3 = 0$$

All equations are same, so put $x_1 = 0$.

$$x_2 - x_3 = 0$$

$$x_2 = x_3$$

The eigenvector

$$X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Case (iii): For $\lambda = 1$

Since the Matrix A is symmetric, all three eigen vectors are mutually orthogonal.

Let $X_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be orthogonal to X_1 and X_2, then

$$X_1^T X_3 = 0 \quad \& \quad X_2^T X_3 = 0$$

$$X_1^T X_3 = 0$$

$$\begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$X_2^T X_3 = 0$$

$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

Then we get

$$a - b + c = 0$$
$$0a + b + c = 0$$

Solving first and second equations we have

2312

$$\begin{array}{cccc} -1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array}$$

$$\frac{a}{-1-1} = \frac{b}{0-1} = \frac{c}{1-0}$$

$$\frac{a}{-2} = \frac{b}{-1} = \frac{c}{1}$$

The Eigen Vector $X_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

Modal Matrix

$$M = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Eigen Values	Eigen Vectors
$\lambda = 4$	$X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
$\lambda = 1$	$X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
$\lambda = 1$	$X_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$