COLLEGE OF ENGINEERING AND TECHNOLOGY
CYCLE TEST PAPER

SRM INSTITUTE OF SCIENCE & TECHNOLOGY (Deemed to be University u/s 3 of UGC Act, 1956)

Set - B.

REG. No. | R | A | 2 | 4 | 1 | 1 | 0 | 0 | 3 | 0 | 1 | 1 | 2 | 4 | 2 | | |

DATE ~~5/12~~ 5/12/24

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DEGREE B. Tech.

SPECIALISATION CSE (Core)

COURSE Calculus and Linear Algebra

SEMESTER 1st.

Part A.

1) C

2) B

3) B

4) A

40/50

37/50

$\left(\frac{n}{2n+1}\right)^n$.

$\frac{n}{2n+1}$

$\frac{\cancel{n}}{2\cancel{n}\left(1+\frac{1}{n}\right)}$ $\frac{1}{2}$.

$\frac{a}{1-r}$.

$\frac{1}{1-\frac{1}{2}} = 2$.

Part B

5) $r = a(1+\cos\theta)$

$\Rightarrow \frac{dr}{d\theta} = a(-\sin\theta) = r_1$

$\frac{d^2r}{d\theta^2} = a(-\cos\theta) = r_2$.

No Additional Sheets will be issued

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - r r_2}$$

$$\Rightarrow \frac{(a^2(1+\cos\theta)^2 + a^2\sin^2\theta)^{3/2}}{a^2(1+\cos\theta)^2 + 2a^2\sin^2\theta + a^2(1+\cos\theta)(\cos\theta)}$$

$$\Rightarrow \frac{[a^2(1+\cos^2\theta + 2\cos\theta + \sin^2\theta)]^{3/2}}{a^2(1+\cos^2\theta + 2\cos\theta + 2\sin^2\theta + \cos\theta + \cos^2\theta)}$$

$$\Rightarrow \frac{[a^2(2+2\cos\theta)]^{3/2}}{a^2(3+3\cos\theta)}$$

$$\Rightarrow \frac{[2a^2(1+\cos\theta)]^{3/2}}{3a^2(1+\cos\theta)}$$

$$\Rightarrow \frac{[2ar]^{3/2}}{3ar} \Rightarrow \frac{2^{3/2} \cdot a^{1/2} \cdot r^{1/2}}{3 \cdot a \cdot r}$$

$$\Rightarrow \frac{2\sqrt{2}\sqrt{ar}}{3} = \rho$$

Now, $$\frac{\rho^2}{r} = \frac{\left(\frac{2\sqrt{2}\sqrt{ar}}{3}\right)^2}{r} = \frac{8 \cdot ar}{9} \times \frac{1}{r}$$

$$\Rightarrow \frac{8a}{9}$$

constant proved.

Ratio test = $l \nless 1$
converges.

7) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} \cdots\cdots$

$u_n = \frac{1}{\sqrt{n}+\sqrt{n+1}}$

Now, $\cancel{n^2\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{1}{n}}\right)}$

Now, $\frac{1}{\sqrt{n}\left(1+\sqrt{1+\frac{1}{\sqrt{n}}}\right)}$.

$\Rightarrow \frac{1}{n^{1/2}\cdot\left(1+\sqrt{1+\frac{1}{\sqrt{n}}}\right)}$.

let $v_n = \frac{1}{n^{1/2}}$.

$\lim_{n\to\infty} \Rightarrow \frac{u_n}{v_n} = \frac{1}{1+\sqrt{1+\frac{1}{\sqrt{n}}}}$.

$\Rightarrow \frac{1}{1+1} = \frac{1}{2} \; (\neq 0)$.

$v_n = \frac{1}{n^{1/2}}$ Also, $v_n = \frac{1}{n^p}$

$\therefore p = 1/2$. as $p < 1$ = divergent.

$\therefore V_n$ is divergent.

$\therefore$ using the limit comparison test, u_n is also divergent.

Part C.

8) $\frac{x^2}{8} + \frac{y^2}{18} = 1.$

$\Rightarrow \frac{2x}{8} + \frac{2y}{18} \cdot \frac{dy}{dx} = 0.$

$\Rightarrow \frac{dy}{dx} \cdot \frac{y}{9} = \frac{-x}{4}.$

$\Rightarrow \frac{dy}{dx} = \frac{-9x}{4y} = y_1$

$$\frac{d^2y}{dx^2} = \frac{4y \cdot -9 - (-9x) 4 \frac{dy}{dx}}{16y^2}.$$

$$\Rightarrow \frac{-36y + 36x \cdot \frac{dy}{dx}}{16y^2} = y_2.$$

y_1 at $(2,3)$: $\frac{-9 \cdot 2}{4 \cdot 3} = \frac{-3}{2}.$

$$y_2 \text{ at } (2,3) = \frac{-36.3 + 36.2 \times -3/2}{16.9}$$

$$\Rightarrow \frac{-108 - 72 \times 3/2}{144} = \frac{-108 - 108}{144}$$

$$\Rightarrow \cancel{\frac{-108 - 72}{144}} = \frac{-216}{144}$$

$$\Rightarrow \cancel{\frac{-180}{144}} \quad \frac{-216}{144}$$

Now, $f = \dfrac{(1+y_1^2)^{3/2}}{|y_2|}$

$$= \cancel{\frac{(1+9/4)^{3/2}}{\frac{180}{144}}} = \frac{(1+9/4)^{3/2}}{-\frac{216}{144}}$$

$$\Rightarrow \frac{\left(\frac{13}{4}\right)^{3/2}}{\frac{\cancel{180}\,216}{144}} = \frac{144}{216} \times \sqrt{\frac{2197}{64}} = f$$

$$\Rightarrow \cancel{\frac{144}{180} \times \sqrt{\frac{2197}{64}} = f}$$

$$\bar{x} = x - y_1\left(\frac{1+y_1^2}{y_2}\right)$$

$$\Rightarrow 2 - \left(-3/2\right)\left(\frac{1 + 9/4}{\frac{-\cancel{180}\,216}{144}}\right)$$

$2 - \frac{3}{2} \times \frac{13}{4} \times \frac{144}{\cancel{180}\ 216}$

$2 - \frac{5616}{\cancel{1440}\ 1728}$

$\Rightarrow \frac{\cancel{2880 - 5616}}{\cancel{1440}\ 1728}$ $\qquad \frac{3456 - 5616}{1728}$

$\Rightarrow \frac{-2736}{1440}$ $\qquad \Rightarrow \frac{-2160}{1728}$

$\bar{y} = y + \left(\frac{1+y_1^2}{y_2}\right)$

$\Rightarrow 3 - \frac{13}{4} \times \frac{144}{\cancel{180}\ 216}$

$\Rightarrow 3 - \frac{1872}{\cancel{720}\ 864}$

$\Rightarrow \frac{\cancel{2160 - 1872}}{\cancel{720}\ 864} = \frac{2592 - 1872}{864}$

$\Rightarrow \frac{\cancel{288}}{\cancel{2}720} = \frac{720}{864}$

therefore, $\bar{x} = \frac{\cancel{-2736}}{\cancel{1440}}\ \frac{-2160}{1728}$

$\bar{y} = \frac{\cancel{288}}{\cancel{720}}\ \frac{720}{864}$ $\qquad \rho = \frac{144}{\cancel{180}\ 216} \times \sqrt{\frac{2197}{64}}$

therefore the equation :.

$$(x-\bar{x})^2+(y-\bar{y})^2=r^2.$$

$$\Rightarrow \cancel{\left(x+\frac{2736}{1440}\right)^2+\left(y-\frac{288}{720}\right)^2=20736.}$$

$$\Rightarrow \left(x+\frac{2160}{1728}\right)^2+\left(y-\frac{720}{864}\right)^2=\frac{20736}{46656}\times\frac{2197}{64}.$$

$$\Rightarrow (x+1.25)^2+(y-0.8333)^2=15.2569$$

The equation of the circle.

10). $\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\ldots\ldots$

~~Finding U_n~~ :

~~numerator~~

let $\sum U_n$ be $\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1}{2}\cdot\frac{3}{4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}\ldots$

Finding U_n.

numerator. $1.3.5\ldots\ldots$

$\Rightarrow a+d(n-1)$

$\Rightarrow 1+2(n-1)$

$\Rightarrow 2n-2+1=2n-1.$

$\Rightarrow (2n-1)(2n+1)(2n+3)\ldots\ldots$

$\frac{x^3}{3}, \frac{x^5}{5}, \frac{x^7}{7}$.

$\Rightarrow 3, 5, 7$.

$\Rightarrow 3 + 2(n-1)$.

$\Rightarrow 3 + 2n - 2 = 2n+1$.

$\Rightarrow \frac{x^{2n+1}}{2n+1}$.

denominator.

$2.4.6.8 \ldots$

$\Rightarrow \cancel{2a}\, a + d(n-1)$

$\Rightarrow 2 + 2(n-1)$.

$\Rightarrow \cancel{2} + 2n - \cancel{2}$.

$\Rightarrow 2n . (2n+2) . (2n+4)$.

$\Rightarrow u_n = \frac{(2n-1)(2n+1)(2n+3) \ldots}{2n(2n+2)(2n+4) \ldots} \frac{x^{2n+1}}{2n+1}$.

$u_n = \frac{(2n-1)(2n+1)(2n+3) \ldots}{2n(2n+2)(2n+4) \ldots} \times \frac{x^{2n+1}}{2n+1}$.

Now, $u_{n+1} = \frac{(2n+1)(2n+3)(2n+5) \ldots}{(2n+2)(2n+4)(2n+6) \ldots} \times \frac{x^{2n+3}}{2n+3}$.

$\frac{u_{n+1}}{u_n} = \dfrac{\frac{(2n+1)\cancel{(2n+3)}\cancel{(2n+5)} \ldots}{(2n+2)\cancel{(2n+4)}\cancel{(2n+6)} \ldots} \times \frac{x^{2n} . x^{3}}{2n+3}}{\frac{(2n-1)\cancel{(2n+1)}\cancel{(2n+3)} \ldots}{(2n)(2n+2)(2n+4) \ldots} \times \frac{x^{2n} . x}{2n+1}}$

$\frac{1\cdot 3 \cdots 2n)}{2\cdot 4 \cdots 6 \cdot 2n} \cdot \frac{1}{2n+3}$

$$\frac{2n}{(2n-1)} \cdot \frac{x^2 (2n+1)}{2n+3}.$$

$$\Rightarrow x^2 \left[\frac{2n(2n+1)}{(2n-1)(2n+3)} \right] = \frac{u_{n+1}}{u_n}.$$

$$\Rightarrow \lim_{n\to\infty} \frac{u_{n+1}}{u_n}.$$

$$\lim_{n\to\infty} \Rightarrow x^2 \left[\frac{\cancel{2n} \cdot 2n\left(1+\frac{1}{2n}\right)}{\cancel{2n} \cdot 2n\left(1-\frac{1}{2n}\right)\left(1+\frac{3}{2n}\right)} \right]$$

$$\Rightarrow \frac{x^2 \cdot 1+0}{1 \cdot 1} = x^2 = l.$$

as l depends on x.

Now, given $x > 0$.

value of x.	convergence or Divergence.	Test used.
$x < 1$	convergent.	Using D'Alembert's Ratio test.
$x > 1$	divergent.	Using D'Alembert's Ratio test.
$x = 1$.	—	Test case fails.

Now, we use Raabe's Formula to check convergence at $x=1$.

$$\frac{u_{n+1}}{u_n}\ (\text{at } x=1) = \frac{2n(2n+1)}{(2n-1)(2n+3)}$$

$$\frac{u_n}{u_{n+1}} = \frac{(2n-1)2n}{(2n-1)(2n+3)}$$

$$\lim_{n\to\infty}\left(\frac{u_n}{u_{n+1}} - 1\right)$$

$$\lim_{n\to\infty} \Rightarrow \frac{2n(2n-1)}{(2n-1)(2n+3)} - 1$$

$$\lim_{n\to\infty} \frac{4n^2-2n-(4n^2+6n-2n-3)}{4n^2+6n-2n-3}$$

$$\lim_{n\to\infty} \frac{\cancel{4n^2}-2n-\cancel{4n^2}-4n+3}{4n^2+4n-3}$$

$$\lim_{n\to\infty} \frac{-6n+3}{4n^2+4n-3}$$

$$\lim_{n\to\infty} \Rightarrow \frac{\cancel{n}\left(-6+\frac{3}{n}\right)}{n^{\cancel{2}}\left(4+\frac{4}{n}-\frac{3}{n^2}\right)}$$

$\Rightarrow 0.$

here, $l = 0$.

as $l<1$, it is divergent

~~using~~:

Therefore.

value of x.	convergence.	Test used.
$x < 1$	convergent	using D'Alembert's Ratio Test.
$x > 1$	divergent	using D'Alembert's Ratio Test.
$x = 1$	divergent	using Raabe's Test

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