classmate

Date
Page

Q. $6x^2+3y^2+3z^2-4xy-2yz+4xz$

Soln:- $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ same same same

(sum of diagonal elements)

$S_1 = \text{Trace of } A = 6+3+3 = 12$

$S_2 = 36$

$S_3 = 32$

$\therefore \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0 \Rightarrow \lambda = 2, 2, 8$ (eigen values)

For eigen vector:- $(A-\lambda I)\vec{X} = 0$ $\Rightarrow \left\{ \begin{array}{l} (6-\lambda)x_1 - 2x_2 + 2x_3 = 0 \\ -2x_1 + (3-\lambda)x_2 - x_3 = 0 \\ 2x_1 - x_2 + (3-\lambda)x_3 = 0 \end{array} \right\}$ ⊛

For $\lambda = 8$:- ⊛ $\Rightarrow \left\{ \begin{array}{l} -2x_1 - 2x_2 + 2x_3 = 0 \\ -2x_1 - 5x_2 - x_3 = 0 \\ 2x_1 - x_2 - 5x_3 = 0 \end{array} \right\}$ $\rightarrow \dfrac{x_1}{\begin{vmatrix} -2 & 2 \\ -5 & -1 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} -2 & 2 \\ -2 & -1 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} -2 & -2 \\ -2 & -5 \end{vmatrix}}$

$\Rightarrow \dfrac{x}{2} = \dfrac{x_2}{-1} = \dfrac{x_3}{1}$

$\therefore X_1 = [2 \ -1 \ 1]$

For $\lambda = 2$:- ⊛ $\Rightarrow 4x_1 - 2x_2 + 2x_3 = 0$

Let $x_1 = 0$ (or) $x_2 = 0$ (or) $x_3 = 0$

$\therefore -2x_2 = -2x_3$ $\qquad \therefore 4x_1 = -2x_3$ $\qquad \therefore 4x_1 = 2x_2$

$\Rightarrow \dfrac{x_2}{1} = \dfrac{x_3}{1}$ $\qquad \Rightarrow \dfrac{x_1}{-1} = \dfrac{x_3}{2}$ $\qquad \Rightarrow \dfrac{x_1}{1} = \dfrac{x_2}{2}$

$\therefore X_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\qquad \therefore X_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ $\qquad \therefore X_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

From these 3 eigen vectors (X_2) we may take 'any' one of them for solving for X_3

- $X_1^T \cdot X_3 = 0 \Rightarrow (2 \ -1 \ 1)\begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0 \Rightarrow 2l - m + n = 0$ — ①
- $X_2^T \cdot X_3 = 0 \Rightarrow (0 \ 1 \ 1)\begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0 \Rightarrow 0l + m + n = 0$ — ②

(For $x_1 = 0$)

$\therefore \dfrac{l}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}} = \dfrac{-m}{\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}} = \dfrac{n}{\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}} \Rightarrow \dfrac{l}{-2} = \dfrac{m}{-2} = \dfrac{n}{2}$ $\quad \therefore X_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\therefore X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

For normalised vector:-

$$N = \begin{bmatrix} X_1/\sqrt{X_1 \text{ values}} & X_2/\sqrt{X_2 \text{ values}} & X_3/\sqrt{X_3 \text{ values}} \end{bmatrix}$$

where $\sqrt{X_1 \text{ values}} = \sqrt{2^2+(-1)^2+(1)^2} = \sqrt{6}$

(same for X_2 & X_3)

$$\therefore N = \begin{bmatrix} 2/\sqrt{6} & 0 & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{bmatrix}$$

For diagonalised vector:-

$$D = N^T A N = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

For cannonical form:-

$$C.F = Y^T \cdot D \cdot Y$$

$$= \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= 8y_1^2 + 2y_2^2 + 2y_3^2$$