Particular Integral (Type 4 → $e^{ax} \cdot V$)

Q1. $(D^2 - 3D + 2)y = x^2 \cdot e^{2x}$

Sol^n:- For A.E :- $D^2 - 3D + 2 = 0$

$\Rightarrow (D-2)(D-1) = 0 \quad \therefore D = 1, 2$ (Real & distinct)

For C.F :- $CF = c_1 e^{1x} + c_2 e^{2x}$

For P.I :- $P.I = \frac{1}{f(D)} \cdot x^2 \cdot e^{2x}$

Whenever we have e^{2x} write it as $e^{2x} \frac{1}{f(D)} \cdot x^2$

Here, $f(D) = D^2 - 3D + 2$

Substitute D by $(D+a)$ $\left[\because e^{ax}, \boxed{D+a}\right]$

$$\therefore P.I = e^{2x} \frac{1}{(D+2)^2 - 3(D+2) + 2} \cdot x^2$$

$$= e^{2x} \cdot \frac{1}{D^2 + 4 + 4D - 3D - 6 + 2} \cdot x^2$$

$$= e^{2x} \cdot \frac{1}{D^2 + D} \cdot x^2$$

$$= e^{2x} \cdot \frac{1}{D(1+D)} x^2$$

$(1+D)$ is in the form of $(1+x)^{-1}$

$$\therefore P.I = e^{2x} \cdot \frac{(1+D)^{-1}}{D} \cdot x^2$$

$$= e^{2x} \cdot \frac{1}{D} \cdot (1 - D + D^2 - D^3 + \dots) \cdot x^2$$

Here, $\cdot Dx^2 = \frac{d}{dx}(x^2) = 2x$

$\cdot D^2x^2 = \frac{d}{dx}(2x) = 2$

$\cdot D^3x^2 = \frac{d}{dx}(2) = 0$

$\therefore D^3x^2$ onwards it's 0

$$\therefore P.I = e^{2x} \cdot \frac{1}{D} \cdot [x^2 - 2x + 2 - 0]$$

We know, $D \to \frac{d}{dx}$ $\quad \frac{1}{D} \Rightarrow \int dx$

$$\therefore P.I = e^{2x} \int (x^2 - 2x + 2)\, dx$$

$$= e^{2x} \left[\frac{x^3}{3} - \frac{2x^2}{2} + 2x\right] \quad \left[\because \int x^n = \frac{x^{n+1}}{n+1}\right]$$

General sol^n :- $y = c_1 e^x + c_2 e^{2x} + e^{2x}\left[\frac{x^3}{3} - \frac{2x^2}{2} + 2x\right]$

Q2. Find PI of $(D^2+2)y = x^2 e^{3x} + e^x - \cos 2x$

Sol^n:- $P.I = \frac{1}{f(D)} \cdot [x^2 e^{3x} + e^x - \cos 2x]$ Here, $f(D) = D^2 + 2$

$$= \frac{x^2 e^{3x}}{f(D)} + \frac{e^x}{f(D)} + \frac{\cos 2x}{f(D)}$$

[Type 4] [Type 1] [Type 2]

* For Type 1, $a = 1 \to f(1) = 1^2 + 2 = 3$

For Type 2, $a = 2 \to D^2 = -a^2 = -4$

For Type 4, $a = 3 \to D$ ~~$D = a$~~ substituted by $D - a = D + 3$

$$\therefore P.I = e^{3x} \cdot \frac{1}{(D+3)^2 + 2} \cdot x^2 + e^x \frac{1}{(1)^2 + 2} - \frac{\cos 2x}{D(-4) + 2}$$

$$= e^{3x} \cdot \frac{1}{D^2 + 6D + 9 + 2} x^2 + \frac{e^x}{3} + \frac{\cos 2x}{2}$$

$$= e^{3x} \cdot \frac{1}{D^2 + 6D + 11} x^2 + \frac{e^x}{3} + \frac{\cos 2x}{2}$$

Now, write in form of $(1+x)^{-1}$

$$= e^{3x} \cdot \frac{1}{11\left(\frac{D^2}{11} + \frac{6D}{11} + 1\right)} \cdot x^2 + \dots$$

$$\therefore P.I = \frac{e^{3x}}{11}\left[1+\left(\frac{D^2+6D}{11}\right)\right]^{-1} x^2 + \dots$$

$$= \frac{e^{3x}}{11}\left[1 - \left(\frac{D^2+6D}{11}\right) + \left(\frac{D^2+6D}{11}\right)^2 + \dots\right] x^2 + \dots$$

Here, $= \frac{e^{3x}}{11}\Big[$ $Dx^2 = \frac{d}{dx}(x^2) = 2x$

$D^2x^2 = \frac{d}{dx}(2x) = 2$

$D^3x^2 = 0$ $\therefore \left(\frac{D^2+6D}{11}\right)^2 x^2$ onwards its 0

$$\therefore P.I = \frac{e^{3x}}{11}\left[x^2 - \left(\frac{2+6(2x)}{11}\right) + \left(\frac{36(2)}{121}\right)\right] + \dots$$

$$= \frac{e^{3x}}{11}\left[x^2 - \frac{2}{11} + \frac{12x}{11} + \frac{72}{121}\right] + \dots$$

$$= \frac{e^{3x}}{11}\left[x^2 + \frac{12x}{11} + \frac{72-22}{121}\right] + \dots$$

$$\Rightarrow P.I = \frac{e^{3x}}{11}\left[x^2 + \frac{12x}{11} + \frac{50}{121}\right] + \frac{e^x}{3} + \frac{\cos 3x}{2}$$

(*) We can also divide this whole sol^n for PI into :-

1) $(P.I)_1 = \frac{x^2 e^{3x}}{f(D)}$

2) $(P.I)_2 = \frac{e^x}{f(D)}$

3) $(P.I)_3 = \frac{\cos 3x}{f(D)}$