Curve Tracing

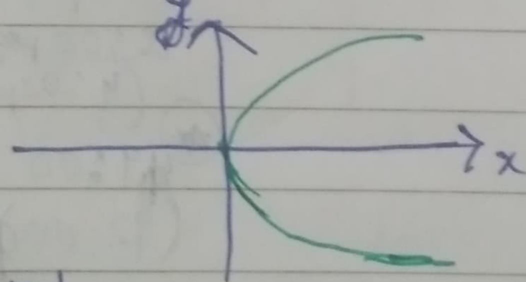
1) Symmetry :- • If y = even → x-axis is the symmetry line
• If x = even → y-axis is the symmetry line

Eg :- Parabola :-

1) $y^2 = 4ax$

Here, $y^{2} \to$ even [y's power → even (2)
$x^{1} \to$ odd x's power → odd (1)]

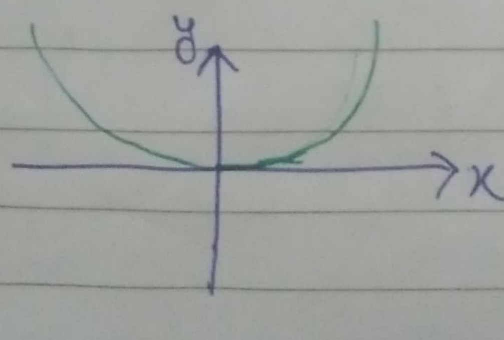
$\therefore$ x-axis is the symmetry line



2) $x^2 = 4by$

Here, $x^{2} \to$ even
$y^{1} \to$ odd

$\therefore$ y-axis is the symmetry line



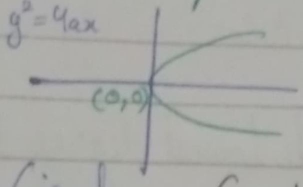
3) Origin :- If $F(x,y) = f(0,0)$

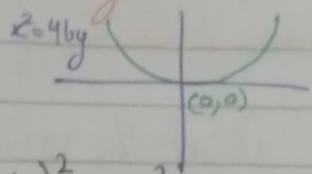
i) **Parabola** :- $f(0,0) = y^2 = 4ax$

$\Rightarrow 0^2 = 4a(0)$

$\Rightarrow f(0,0) = 0$

$\therefore$ Curve passes through origin





2) **Circle** :- $(x-h)^2+(y-k)^2 = r^2$

where (h,k) is centre of circle

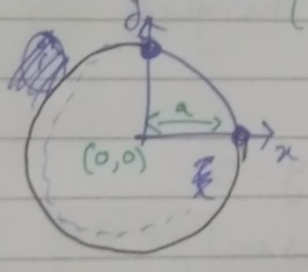
r is the radius

If circle's centre is at origin, $(h,k) = (0,0)$

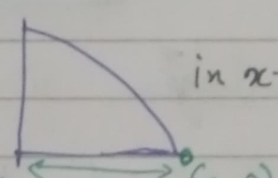
& $r = a$

$\therefore x^2+y^2=a^2$ ——— (1)

Now, for any part of curve, it must pass through :-
(in the circle)

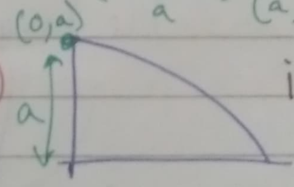


$\Rightarrow$ (i)



in x-axis

(ii)



in y-axis

For x-axis $(a,0)$:-

(1) $\Rightarrow x^2+y^2=a^2$

$\Rightarrow a^2+y^2=a^2$

$\Rightarrow \boxed{y = 0}$

$\therefore x = \pm a$

$(a,0)$ & $(-a,0)$

For y-axis $(0,a)$:-

(1) $\Rightarrow x^2+y^2=a^2$

$\Rightarrow x^2+a^2=a^2$

$\Rightarrow \boxed{x=0}$

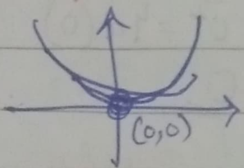
Hence, the curve passes through these 4 points

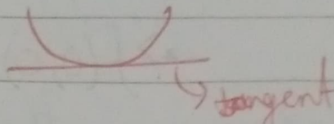
$y = \pm a$

$(0,a)$ & $(0,-a)$

3) Tangent at origin :-

(i) If curve passed through origin, then tangent exists



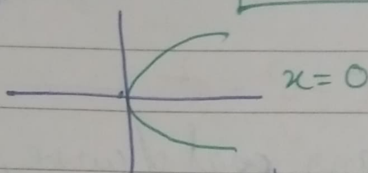


(ii) For finding eqⁿ of tangent :-

1) Set the variable with lowest degree in curve eqⁿ to zero

Eg :- $y^2 = 4ax$ → lowest degree $\therefore x = 0$

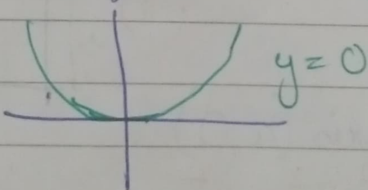
2) The tangent lies in $\boxed{x = 0}$



$\therefore$ y-axis is the tangent

Similarly :- $x^2 = 4by$ → $y = 0$

$\therefore$ The tangent lies in $\boxed{y = 0}$



$\therefore$ x-axis is the tangent

- For $y^2 = 4ax$, tangent lies in $(0, y)$
- For $x^2 = 4by$, tangent lies in $(x, 0)$

4) Intersection on coordinate axis :-

(i) To find intersection of curve on x-axis :-
Put $y = 0$

(ii) To find intersection of curve on y-axis :-
Put $x = 0$

Eg :- $x^2 + y^2 = a^2$ (Eqⁿ of circle at (0,0) & centre origin point)

- To find intersection of curve at x-axis :- Put $y = 0$

$\therefore x^2 + 0^2 = a^2$

$\Rightarrow x = \sqrt{a^2} = \pm a$

$\therefore$ Curve of above eqⁿ intersects x-axis at points $(a, 0)$ & $(-a, 0)$

- To find intersection of curve at y-axis :- Put $x = 0$

$\therefore 0^2 + y^2 = a^2$

$\Rightarrow y = \sqrt{a^2} = \pm a$

$\therefore$ Curve of above eqⁿ intersects y-axis at points $(0, a)$ & $(0, -a)$

5) Asymptotes :- Tangents at infinity

It is a straight line that a curve approaches but never actually touches or intersects, even when curve extends to ∞

① Even as the curve continues, the curve gets closer to asymptote
② These type of curve stretches out to ∞
③ They are straight line and it represents the direction that the curve is heading towards but won't intersect

∞

Eg :- Hyperbola :- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$