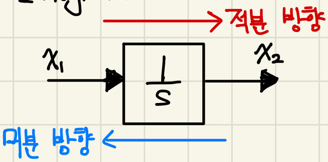
82~86강

1. 적분기 (Integrator)

Integrator



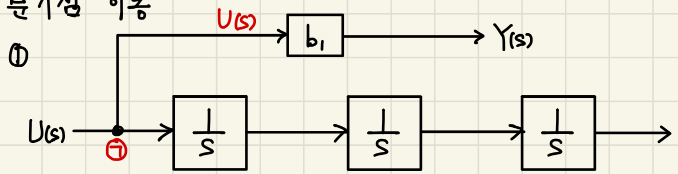
- $x_2 = \int x_1 dt$
- $x_1 = \dot{x}_2$

$\rightarrow$

- $\mathcal{L}\{x_2\} = X_2(s) = \frac{1}{s}X_1(s)$
- $\mathcal{L}\{x_1\} = X_1(s) = sX_2(s)$

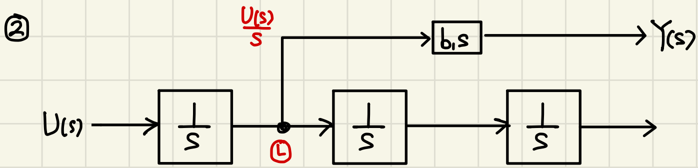
분기점 이동

①



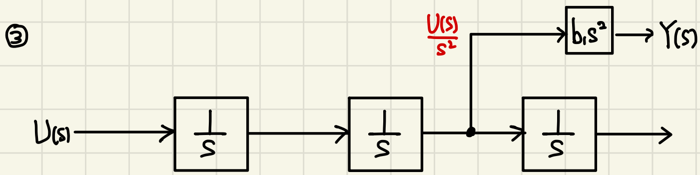
$\therefore Y(s) = b_1 U(s)$

②

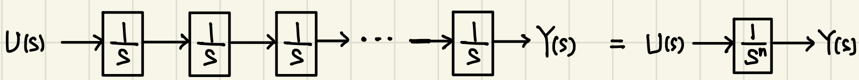


- 적분 흐름 방향으로 분기점 이동 시, 적분기를 거친 횟수만큼 s를 곱한다.

③

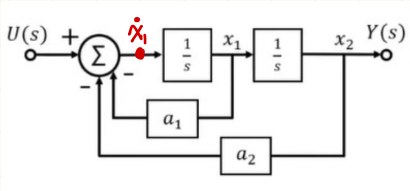


번외



2. 유리수 전달함수

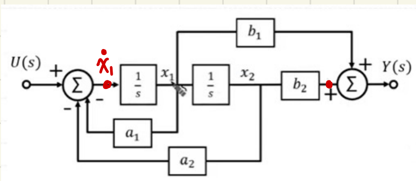
①



- $X_2(s) = \frac{1}{s}X_1(s)$ → • $X_1(s) = sX_2(s) = sY(s)$
- $Y(s) = X_2(s)$
- $\mathcal{L}\{\dot{x}_1\} = s\underbrace{X_1(s)}_{sY(s)} = U(s) - a_1\underbrace{X_1(s)}_{sY(s)} - a_2\underbrace{X_2(s)}_{Y(s)}$

$\therefore U(s) = (s^2 + a_1 s + a_2)Y(s)$; $\frac{Y(s)}{U(s)} = \frac{1}{s^2 + a_1 s + a_2}$

②



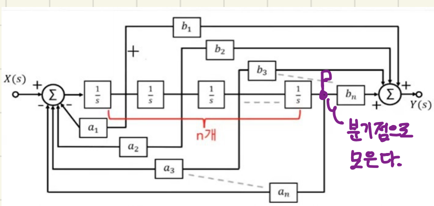
- $X_2(s) = \frac{1}{s}X_1(s)$ → • $X_1(s) = sX_2(s)$
- $\mathcal{L}\{\dot{x}_1\} = sX_1(s) = U(s) - a_1X_1(s) - a_2X_2(s)$ ⋯ (i)
- $Y(s) = b_1X_1(s) + b_2X_2(s)$ ⋯ (ii)

$\therefore \overset{(i)}{U(s)} - a_1\{sX_2(s)\} - a_2X_2(s) = s\{sX_2(s)\}$; $U(s) = (s^2 + a_1 s + a_2)X_2(s)$

(ii) $b_1\{sX_2(s)\} + b_2X_2(s) = Y(s)$; $Y(s) = (b_1 s + b_2)X_2(s)$ → $\frac{Y(s)}{U(s)} = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$

일반화

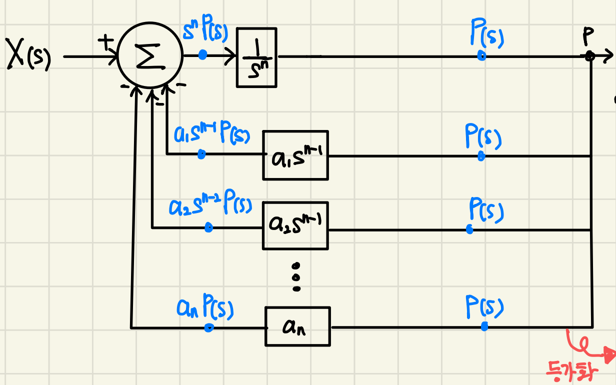
③



- a_1 : (n−1) 개의 적분기 거침. → $a_1 s^{n-1}$
- a_2 : (n−2) 개의 적분기 거침. → $a_2 s^{n-2}$
- a_3 : (n−3) 개의 적분기 거침. → $a_3 s^{n-3}$

⋮

- a_n : 0 개의 적분기 거침. → a_0



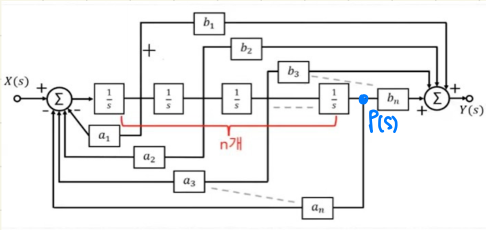
$\therefore X(s) - P(s)(a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n) = s^nP(s)$

; $X(s) = (s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n)P(s)$

; $\frac{P(s)}{X(s)} = \frac{1}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}$

$$\boxed{X(s) \to \boxed{\frac{1}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}} \to P(s)}$$

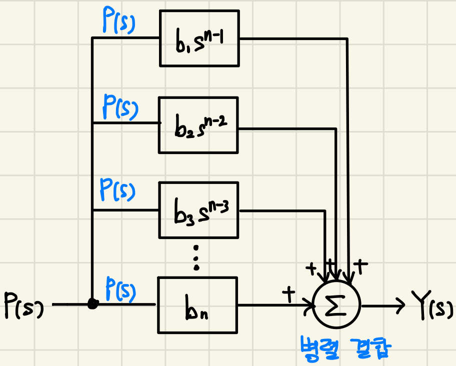
(③ 이어서)



- b_1 : (n-1)개의 적분기 거침. → $b_1 s^{n-1}$
- b_2 : (n-2)개의 적분기 거침. → $b_2 s^{n-2}$
- b_3 : (n-3)개의 적분기 거침. → $b_3 s^{n-3}$

⋮

- b_n : 0 개의 적분기 거침. → b_n



$\therefore Y(s) = (b_1 s^{n-1} + b_2 s^{n-2} + b_3 s^{n-3} + \cdots + b_n) P(s)$

; $\dfrac{Y(s)}{P(s)} = b_1 s^{n-1} + b_2 s^{n-2} + b_3 s^{n-3} + \cdots + b_n$

$P(s) \rightarrow \boxed{b_1 s^{n-1} + b_2 s^{n-2} + b_3 s^{n-3} + \cdots + b_n} \rightarrow Y(s)$

$$\boxed{\therefore \frac{Y(s)}{X(s)} = \frac{P(s)}{X(s)} \cdot \frac{Y(s)}{P(s)} = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}}$$

1. 상태공간 설계 (Space State Design)

■ Space - State Design

- 미분방정식 → 행렬 상태 → Laplace 변환 → 역행렬 곱함 → 라플라스 역변환 곱함.
- 컴퓨터 기술 사용에 적합, 엔지니어들이 많이 사용.

■ 수식

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

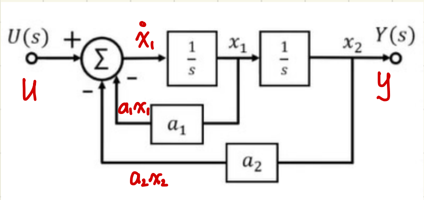
- x : system 상태
- A : $n\times n$ 시스템 행렬
- B : $n\times 1$ 입력 행렬
- C : $1\times n$ 출력 행렬
- D : Scalar 직접전달항.

■ 응용

- 유리수 전달함수 → 상태공간 설계
- 유리수 전달함수의 피드백 블록선도 → 유리수 전달함수 → 상태공간 설계
- 유리수 전달함수의 피드백 블록선도 → 상태공간 설계
- 상태공간 설계 → 유리수 전달함수

2. 피드백 블록선도 → 상태공간 설계

1)



$\therefore x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ -- (i)

① $\dot{x}_1 = u - a_1x_1 - a_2x_2$

② $x_2 = y$

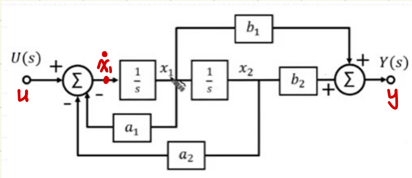
③ $\dot{x}_2 = x_1$

①,③ : $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

② : $y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$

(이어서)

2)



① $\dot{x}_2 = x_1$

② $\dot{x}_1 = u - a_1 x_1 - a_2 x_2$

③ $y = b_1 x_1 + b_2 x_2$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u \end{cases}$$

3. 상태공간 → 전달함수

$$\begin{cases} \dot{x} = Ax + Bu & \cdots ① \\ y = Cx + Du & \cdots ② \end{cases}$$

①: $\mathcal{L}\{\dot{x}\} = A\mathcal{L}\{x\} + B\mathcal{L}\{u\}$

; $sX(s) - x(0) = AX(s) + BU(s)$

; $(sI - A)X(s) = x(0) + BU(s)$

; $X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$

②: $Y(s) = CX(s) + DU(s)$

; $Y(s) = \underbrace{C(sI-A)^{-1}x(0)}_{\text{제차}} + \underbrace{C(sI-A)^{-1}BU(s)}_{\text{비제차}} + DU(s)$

전달함수 $x(0)=0$ → $Y(s) = C(sI-A)^{-1}BU(s) + DU(s)$; $\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$ (전달함수)

각 요소를 $\mathcal{L}^{-1}$ 변환!

- $(sI-A)^{-1} = \Delta(s)$: 각 요소가 s에 대한 함수인 행렬.
- $\mathcal{L}^{-1}\{(sI-A)^{-1}\} = \Delta(t)$: 각 요소가 t에 대한 함수인 행렬.

⟹

- $X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$

$\Downarrow \mathcal{L}^{-1}$

- $x(t) = x(0)\Delta t + \mathcal{L}^{-1}\{(sI-A)^{-1}BU(s)\}$